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Multi-attribute group decision making method under 2-dimension uncertain linguistic variables

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Abstract: A method is proposed to deal with the uncertain multiple attribute group decision making problems, where 2-dimension uncertain linguistic variables (2DULVs) are used as the reliable way for the experts to express their fuzzy subjective evaluation information. Firstly, in order to measure the 2DULVs more accurately, a new method is proposed to compare two 2DULVs, called a score function, while a new function is defined to measure the distance between two 2DULVs. Secondly, two optimization models are established to determine the weight of experts and attributes based on the new distance formula and a weighted average operator is used to determine the comprehensive evaluation value of each alternative. Then, a score function is used to determine the ranking of the alternatives. Finally, the effectiveness of the proposed method is proved by an illustrated example.

Keywords: 2-dimension uncertain linguistic variables (2DULVs), multi-attribute group decision making problem, score function, distance formula.

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1. Introduction

Multi-attribute group decision making (MAGDM) problem is a process in which decision makers choose the most satisfactory alternative from limited alternatives according to the evaluation information of experts [1−8]. To obtain the most satisfactory alternative, experts are invited to give their preference information, which may be expressed in the form of numerical value, such as clear number, interval number, fuzzy number, interval fuzzy number and so on [9−18]. However, some attribute values can only be evaluated qualitatively rather than quantitatively in real scenes, such as automobile comfort. In fuzzy linguistic methods, qualitative attribute values are expressed with fuzzy values through linguistic variables [19−24].

However, sometimes experts are unable to give definite linguistic variables in the course of evaluation due to the lack of expertise and uncertain cognition. For this reason, the concept of uncertain linguistic variables was proposed [25], however, it failed to describe the reliability of the evaluation information. In response to this problem, the concept of 2-dimension linguistic variables was proposed [26]. The 2-dimension linguistic variables haveattractedtheattentionofscholarsbecausetheycanaccurately describe evaluation information [27−30]. However, experts may hesitate between consecutive linguistic terms. Thus, the concept of 2-dimension uncertain linguistic variables (2DULVs) was proposed [31]. The 2DULVs can clearly and intuitively reflect the expert's subjective information, which is helpful to improve the accuracy of decision results. They have been widely used for the risk assessment of public private partnership (PPP) waste-to-energy incineration projects, optimal site selection of straw biomass power plant, sustainable supplier selection, energy policy and so on [32−35].

Although in the literature, there is a method comparing two 2DULVs [36], it compares two 2DULVs according to the product of the median of two dimensional linguistic intervals. Obviously, there is a situation where two 2DULVs are not equal and their medians are equal. Thus, this paper proposes a new method comparing two 2DULVs according to two indicators. Moreover, this paper defines a new distance formula to solve the problem that the existed distance formula is imprecise [37−39].

The remainder of this paper is organized as follows. Section 2 briefly reviews some preliminary concepts related to our research. We propose the new score function and the distance formula of 2DULVs in Section 3. Section 4 gives the application method in the MAGDM problem. Section 5 gives two examples to prove the effectiveness and advantage of the proposed method. The final section summarizes the main work of this paper with a discussion of implications for the future research.

2. Preliminaries

Definition 1 [39] Let $s=((S_{g_1},S_{g_2}),(S^*_{h_1},S^*_{h_2}))$, where

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(S_{g_1}, S_{g_2}) is I class uncertain linguistic information, which represents the decision maker's judgment to of an evaluated object, and S_{g_1}, S_{g_2} are the elements from the predefined linguistic assessment set $S_l = \{S_0, S_1, \cdots, S_{l-1}\}$, while $(S^*_{h_1}, S^*_{h_2})$ is II class uncertain linguistic information, which represents the subjective evaluation on the reliability of their given results, and $S^*_{h_1}, S^*_{h_2}$ are the elements from the predefined linguistic assessment set $S^*_{\mathrm{II}} = \{S^*_0, S^*_1, \cdots, S^*_{t-1}\}$, then s is called 2DULV.

For any two 2DULVs, $s_1 = ((S_{g_1}, S_{g_2}), (S^*_{h_1}, S^*_{h_2}))$ and $s_2 = ((S_{p_1}, S_{p_2}), (S^*_{q_1}, S^*_{q_2}))$, the operational rules are shown as follows [39]:

(i) $s_1 \oplus s_2 = ((S_{g_1}, S_{g_2}), (S^*_{h_1}, S^*_{h_2})) \oplus ((S_{p_1}, S_{p_2}), (S^*_{q_1}, S^*_{q_2})) = ((S_{g_1+p_1}, S_{g_2+p_2}), (S^*_{h_1+q_1-h_1q_1/(t-1)}, S^*_{h_2}+q_2-h_2q_2/(t-1);$

(ii) $s_1 \otimes s_2 = ((S_{g_1}, S_{g_2}), (S^*_{h_1}, S^*_{h_2})) \otimes ((S_{p_1}, S_{p_2}), (S^*_{q_1}, S^*_{q_2})) = ((S_{g_1p_1}, S_{g_2p_2}), (S^*_{h_1q_1/(t-1)}, S^*_{h_2q_2/(t-1)}));$

(iii) $s_1 / s_2 = ((S_{g_1}, S_{g_2}), (S^*_{h_1}, S^*_{h_2})) / ((S_{p_1}, S_{p_2}), (S^*_{q_1}, S^*_{q_2})) = ((S_{g_1/p_2}, S_{p_1/g_2}), (S^*_{\min(h_1,q_1)}, S^*_{\min(h_2,q_2)}))$, where $p_2, g_2 \neq 0$;

(iv) $\lambda s_1 = ((S_{\lambda\times g_1}, S_{\lambda\times g_2}), (S^*_{(t-1)[1-(1-h_1/(t-1))^\lambda]}, S^*_{(t-1)[1-(1-h_2/(t-1))^\lambda]})), \lambda > 0;$

(v) $(s_1)^\lambda = ((S_{(g_1)^\lambda}, S_{(g_2)^\lambda}), (S^*_{h_1^\lambda/(t-1)^{\lambda-1}}, S^*_{h_2^\lambda/(t-1)^{\lambda-1}})), \lambda > 0$.

Let $s_1 = ((S_{g_1}, S_{g_2}), (S^*_{h_1}, S^*_{h_2}))$, $s_2 = ((S_{p_1}, S_{p_2}), (S^*_{q_1}, S^*_{q_2}))$ and $s_3 = ((S_{k_1}, S_{k_2}), (S^*_{l_1}, S^*_{l_2}))$ be any three 2DULVs, and $\lambda, \lambda_1, \lambda_2 \geqslant 0$. The 2DULVs satisfy the following properties [39]:

(i) $s_1 \oplus s_2 = s_2 \oplus s_1$;

(ii) $s_1 \otimes s_2 = s_2 \otimes s_1$;

(iii) $s_1 \oplus s_2 \oplus s_3 = s_1 \oplus (s_2 \oplus s_3)$;

(iv) $s_1 \otimes s_2 \otimes s_3 = s_1 \otimes (s_2 \otimes s_3)$;

(v) $s_1 \otimes (s_2 \oplus s_3) = (s_1 \otimes s_2) \oplus (s_1 \otimes s_3)$;

(vi) $\lambda(s_1 \oplus s_2) = (\lambda s_1) \oplus (\lambda s_2)$;

(vii) $(\lambda_1 + \lambda_2)s_1 = (\lambda_1 s_1) \oplus (\lambda_2 s_1)$.

Definition 2 [36] Let $s_1 = ((S_{g_1}, S_{g_2}), (S^*_{h_1}, S^*_{h_2}))$ be a 2DULV, then the expectation $E(s_1)$ of s_1 is defined as

$$E(s_1) = \frac{g_1 + g_2}{2\times(l-1)} \times \frac{h_1 + h_2}{2\times(t-1)}. \tag{1}$$

Definition 3 [36] Let $s_1 = ((S_{g_1}, S_{g_2}), (S^*_{h_1}, S^*_{h_2}))$ and $s_2 = ((S_{p_1}, S_{p_2}), (S^*_{q_1}, S^*_{q_2}))$ be any two 2DULVs, the Hamming distance of s_1 and s_2 is defined as follows:

$$d(s_1, s_2) = \frac{1}{4(l-1)}\left(\left|g_1 \times \frac{h_1}{t-1} - p_1 \times \frac{q_1}{t-1}\right| + \left|g_1 \times \frac{h_2}{t-1} - p_1 \times \frac{q_2}{t-1}\right| + \left|g_2 \times \frac{h_1}{t-1} - p_2 \times \frac{q_1}{t-1}\right| + \left|g_2 \times \frac{h_2}{t-1} - p_2 \times \frac{q_2}{t-1}\right|\right). \tag{2}$$

Definition 4 [38] Let $s_j = ((S_{g_{1j}}, S_{g_{2j}}), (S^*_{h_{1j}}, S^*_{h_{2j}}))$ $(j = 1, 2, \cdots, n)$ be a collection of the 2DULVs, and the 2-dimension uncertain linguistic weighted averaging operator (2DULWA): $\boldsymbol{\Omega}^n \to \boldsymbol{\Omega}$, if

$$\mathrm{2DULWA}(s_1, s_2, \cdots, s_n) = \bigoplus_{j=1}^n (\omega_j s_j) = ((S_{\sum_{j=1}^n(\omega_j g_{1j})}, S_{\sum_{j=1}^n(\omega_j g_{2j})}), (S^*_{(t-1)-(t-1)\prod_{j=1}^n(1-\frac{h_{1j}}{t-1})^{\omega_j}}, S^*_{(t-1)-(t-1)\prod_{j=1}^n(1-\frac{h_{2j}}{t-1})^{\omega_j}})) \tag{3}$$

where $\boldsymbol{\Omega}$ is the set of all 2DULVs; $\boldsymbol{\omega}=(\omega_1, \omega_2, \cdots, \omega_n)^{\mathrm{T}}$ is the weight vector of $(s_1, s_2, \cdots, s_n)$, which meets $0 \leqslant \omega_j \leqslant 1$ and $\sum_{j=1}^n \omega_j = 1$.

3. Score function and distance formula of 2DULVs

We call the function used to measure the size of 2DULVs score function. In the previous studies, the product of the median of two dimensional language intervals is regarded as a scoring function of 2DULVs. However, there may be a situation where two 2DULVs are not equal and their medians are equal. Thus, the existing score function of 2DULVs is not precise. To overcome this shortcoming, this paper adds an index to measure the size of 2DULVs according to the concept of variance of a random variable.

3.1 Scoring function of 2DULVs

Definition 5 Let $s = ((S_{g_1}, S_{g_2}), (S^*_{h_1}, S^*_{h_2}))$ be a 2DULV, $S_{g_1}, S_{g_2} \in S_1 = \{S_\alpha | \alpha \in [0, q]\}$, $S^*_{h_1}, S^*_{h_2} \in S^*_{\mathrm{II}} = \{S^*_\beta | \beta \in [0, q']\}$. If $X = g_1 + a_1(a_1 \in [0, g_2 - g_1])$ and $Y = h_1 + b_1(b_1 \in [0, h_2 - h_1])$, then X and Y are two random variables subject to normal distribution $N(\mu_x, \sigma^2_x)$ and $N(\mu_y, \sigma^2_y)$. According to the 3σ principle of the normal distribution, mean μ and variance σ^2 can be obtained as follows:

$$\mu_x = \frac{1}{2}(g_1 + g_2), \tag{4}$$

$$\sigma^2_x = \frac{1}{36}(g_2 - g_1)^2, \tag{5}$$

$$\mu_y = \frac{1}{2}(h_1 + h_2), \tag{6}$$

$$\sigma^2_y = \frac{1}{36}(h_2 - h_1)^2. \tag{7}$$

According to Definition 5, the scoring function of 2DULVs can be obtained as follows.

Definition 6 Let $s = ((S_{g_1}, S_{g_2}), (S^*_{h_1}, S^*_{h_2}))$ be a 2DULV. If $X = g_1 + a_1(a_1 \in [0, g_2 - g_1])$ and $Y = h_1 + b_1$ $(b_1 \in [0, h_2 - h_1))$, then the scoring function of 2DULVs can be expressed as follows:

$$\beta(s) = \{\eta, \gamma^2\} \tag{8}$$

where

$$\eta = \mu_x\mu_y = \frac{1}{4}(g_1 + g_2)(h_1 + h_2), \tag{9}$$

$$\gamma^2=\sigma_{xy}^2=\mu_{x^2y^2}-\mu_{xy}^2=\mu_{x^2}\mu_{y^2}-\mu_x^2\mu_y^2=\frac{1}{1\,296}[(g_2-g_1)^2(h_2-h_1)^2+9(g_2-g_1)^2\times(h_1+h_2)^2+9(h_2-h_1)^2(g_1+g_2)^2]. \tag{10}$$

Theorem 1 Let $s_1=((S_{g_1},S_{g_2}),(S^*_{h_1},S^*_{h_2}))$ and $s_2=((S_{p_1},S_{p_2}),(S^*_{q_1},S^*_{q_2}))$ be two 2DULVs. η_1 is the mathematical expectation of s_1. η_2 is the mathematical expectation of s_2. γ_1^2 is the variance of s_1. γ_2^2 is the variance of s_2. If $\eta_1>\eta_2$, then $s_1\succ s_2$. If $\eta_1<\eta_2$, then $s_1\prec s_2$. When $\eta_1=\eta_2$, if $\gamma_1^2>\gamma_2^2$, then $s_1\prec s_2$; if $\gamma_1^2<\gamma_2^2$, then $s_1\succ s_2$.

Theorem 2 Let $s_1=((S_{g_1},S_{g_2}),(S^*_{h_1},S^*_{h_2}))$ and $s_2=((S_{p_1},S_{p_2}),(S^*_{q_1},S^*_{q_2}))$ be two 2DULVs. When $q_1=h_1$ and $q_2=h_2$, if $g_2-g_1>p_2-p_1$ and $0.5(g_1+g_2)=0.5(p_1+p_2)$, then $s_1<s_2$.

Proof When the information in the second dimension is exactly the same, if $0.5(g_1+g_2)=0.5(p_1+p_2)$, then $\eta_1=\eta_2$. In this time, the size of 2DULVs depends on the size of γ^2 according to Theorem 1. Because $g_2-g_1>p_2-p_1$, $q_1=h_1$ and $q_2=h_2$, $(g_2-g_1)^2(h_2-h_1)^2>(p_2-p_1)^2(q_2-q_1)^2$ and $(g_2-g_1)^2(h_2+h_1)^2>(p_2-p_1)^2\cdot(q_2+q_1)^2$. Because $0.5(g_1+g_2)=0.5(p_1+p_2)$, $q_1=h_1$ and $q_2=h_2$, $(g_2+g_1)^2(h_2-h_1)^2=(p_2+p_1)^2(q_2-q_1)^2$. Thus $\gamma_1^2>\gamma_2^2$. According to Theorem 1, $s_1<s_2$. □

In Definition 6, we propose a new score function of 2DULVs to overcome the existing score function of 2DULVs. Then we compare the two scoring functions.

Example 1 Let $S=\{S_0,S_1,S_2,S_3,S_4,S_5,S_6\}$ be the linguistic term set of the first dimension linguistic variable and $S^*=\{S_0^*,S_1^*,S_2^*,S_3^*,S_4^*\}$ be the linguistic term set of the second dimension linguistic variable. Let $s_1=((S_1,S_4),(S_2^*,S_3^*))$ and $s_2=((S_2,S_3),(S_2^*,S_3^*))$ be two 2DULVs.

According to Definition 2, the score functions of two 2DULVs are as follows:

$$E(s_1)=\frac{\frac{1+4}{2}\times\frac{2+3}{2}}{7\times5}=0.179,$$

$$E(s_2)=\frac{\frac{2+3}{2}\times\frac{2+3}{2}}{7\times5}=0.179.$$

Thus, $s_1=s_2$.

However, according to Definition 6, the score functions of two 2DULVs are as follows:

$$\beta(s_1)=\{6.25,1.743\},$$

$$\beta(s_2)=\{6.25,0.348\}.$$

According to Theorem 1, $s_1\prec s_2$.

Obviously, the calculation for Definition 6 is more accurate than that for Definition 2.

3.2 Distance formula for 2DULVs

Definition 7 Let $s_1=((S_{g_1},S_{g_2}),(S^*_{h_1},S^*_{h_2}))$ and $s_2=((S_{p_1},S_{p_2}),(S^*_{q_1},S^*_{q_2}))$ be two 2DULVs. Then the distance between two uncertain linguistic variables in the first dimension can be expressed as follows:

$$d_x=\sqrt[\lambda]{\left|\frac{1}{2}(g_1+g_2-p_1-p_2)\right|^\lambda+\frac{1}{3}\left|\frac{1}{2}(g_2-g_1-p_2+p_1)\right|^\lambda} \tag{11}$$

where $\lambda\geqslant1$.

When $\lambda=1$, (11) is the Hamming distance, i.e.,

$$d_x=\left|\frac{1}{2}(g_1+g_2-p_1-p_2)\right|+\frac{1}{3}\left|\frac{1}{2}(g_2-g_1-p_2+p_1)\right|. \tag{12}$$

When $\lambda=2$, (11) is the Euclidean distance, i.e.,

$$d_x=\sqrt{\left|\frac{1}{2}(g_1+g_2-p_1-p_2)\right|^2+\frac{1}{3}\left|\frac{1}{2}(g_2-g_1-p_2+p_1)\right|^2}. \tag{13}$$

Definition 8 Let $s_1=((S_{g_1},S_{g_2}),(S^*_{h_1},S^*_{h_2}))$ and $s_2=((S_{p_1},S_{p_2}),(S^*_{q_1},S^*_{q_2}))$ be two 2DULVs. Then the distance between two uncertain linguistic variables in the second dimension can be expressed as follows:

$$d_y=\sqrt[\lambda]{\left|\frac{1}{2}(h_1+h_2-q_1-q_2)\right|^\lambda+\frac{1}{3}\left|\frac{1}{2}(h_2-h_1-q_2+q_1)\right|^\lambda} \tag{14}$$

where $\lambda\geqslant1$.

When $\lambda=1$, (14) is the Hamming distance, i.e.,

$$d_y=\left|\frac{1}{2}(h_1+h_2-q_1-q_2)\right|+\frac{1}{3}\left|\frac{1}{2}(h_2-h_1-q_2+q_1)\right|. \tag{15}$$

When $\lambda=2$, (14) is the Euclidean distance, i.e.,

$$d_y=\sqrt[2]{\left|\frac{1}{2}(h_1+h_2-q_1-q_2)\right|^2+\frac{1}{3}\left|\frac{1}{2}(h_2-h_1-q_2+q_1)\right|^2}. \tag{16}$$

Definition 9 Let $s_1=((S_{g_1},S_{g_2}),(S^*_{h_1},S^*_{h_2}))$ and $s_2=((S_{p_1},S_{p_2}),(S^*_{q_1},S^*_{q_2}))$ be two 2DULVs. Then the distance between two 2DULVs can be expressed as follows:

$$d(s_1,s_2)=\sqrt[\lambda]{d_x^\lambda+d_y^\lambda}. \tag{17}$$

Property 1 $d(s_1,s_1)=0$.

Proof

$$d_x=\sqrt[\lambda]{\left|\frac{1}{2}(g_1+g_2-g_1-g_2)\right|^\lambda+\frac{1}{3}\left|\frac{1}{2}(g_2-g_1-g_2+g_1)\right|^\lambda}=0,$$

$$d_y=\sqrt[\lambda]{\left|\frac{1}{2}(h_1+h_2-h_1-h_2)\right|^\lambda+\frac{1}{3}\left|\frac{1}{2}(h_2-h_1-h_2+h_1)\right|^\lambda}=0.$$

Thus, $d(s_1,s_1)=0$. □

Property 2 $d(s_1,s_2)=d(s_2,s_1)$.

Proof Let $s_1=((S_{g_1},S_{g_2}),(S^*_{h_1},S^*_{h_2}))$ and $s_2=((S_{p_1},S_{p_2}),(S^*_{q_1},S^*_{q_2}))$ be two 2DULVs. Then

$$d(s_1,s_2)=\sqrt[\lambda]{d_x^\lambda+d_y^\lambda}=$$

$$\sqrt[\lambda]{\left|\frac{1}{2}(g_1+g_2-p_1-p_2)\right|^\lambda+\frac{1}{3}\left|\frac{1}{2}(g_2-g_1-p_2+p_1)\right|^\lambda+\left|\frac{1}{2}(h_1+h_2-q_1-q_2)\right|^\lambda+\frac{1}{3}\left|\frac{1}{2}(h_2-h_1-q_2+q_1)\right|^\lambda},$$

$$d(s_2,s_1)=\sqrt[\lambda]{d_x^\lambda+d_y^\lambda}=$$

$$\sqrt[\lambda]{\left|\frac{1}{2}(p_1+p_2-g_1-g_2)\right|^\lambda+\frac{1}{3}\left|\frac{1}{2}(p_2-p_1-g_2+g_1)\right|^\lambda+\left|\frac{1}{2}(q_1+q_2-h_1-h_2)\right|^\lambda+\frac{1}{3}\left|\frac{1}{2}(q_2-q_1-h_2+h_1)\right|^\lambda}.$$

Because $|(g_1+g_2)-(p_1+p_2)|=|(p_1+p_2)-(g_1+g_2)|$, $|(g_2-g_1)-(p_2-p_1)|=|(p_2-p_1)-(g_2-g_1)|$, $|(h_1+h_2)-(q_1+q_2)|=|(q_1+q_2)-(h_1+h_2)|$ and $|(h_2-h_1)-(q_2-q_1)|=|(q_2-q_1)-(h_2-h_1)|$, $d(s_1,s_2)=d(s_2,s_1)$. □

4. AGDM method for 2DULVs with completely unknown weight information

In the existing MAGDM methods with 2DULVs, experts and attribute weights are mostly known [37−39]. Therefore, this section proposes an MAGDM method with completely unknown weight information under 2DULVs. Firstly, two optimization models are established based on the measure formula to determine the weight of experts and attributes. Then the comprehensive evaluation value of each alternative is determined based on the weighted average operator. Finally, the ranking of alternatives is determined based on the score function of 2DULVs.

4.1 Problem descriptions

Consider the MAGDM problems based on 2-dimension uncertain linguistic information. Let $AS=\{AS_1,AS_2,\cdots,AS_m\}$ be the set of alternatives of MAGDM problems, and $C=C_1,C_2,\cdots,C_n\}$ be the set of attributes of the MAGDM problem. ω_j is the weight of the attribute $C_j(j=1,2,\cdots,n)$, which is completely unknown. Suppose that $E=\{E_1,E_2,\cdots,E_s\}$ is the set of experts, and $\lambda_l(l=1,2,\cdots,s)$ is a weight of expert E_l, which is completely unknown. Suppose that $\boldsymbol{S}=[s^l_{ij}]_{mn}$ is the decision matrix, where $s^l_{ij}=((S^L,S^U),(S^{L*},S^{U*}))=((S_{a^l_{ij}},S_{b^l_{ij}}),(S^*_{c^l_{ij}},S^*_{d^l_{ij}}))$ takes the form of the 2DULVs and $S_{a^l_{ij}},S_{a^l_{ij}}\in S_I=\{S_\alpha|\alpha\in[0,q]\}$,$S^*_{c^l_{ij}},S^*_{d^l_{ij}}\in S^*_{II}=\{S^*_\beta|\beta\in[0,q']\}$, which represents that the expert E_l evaluates the attribute C_j with respect to the alternative AS_i.

4.2 Weight determination model

In this paper, the weighted average operator is used as the aggregation operator to solve the MAGDM problem. Therefore, expert and attribute weights should be determined before expert and attribute information are fused. For the expert weight, the greater the consistency between individual preference and group preference, the greater the weight of the individual should be. Based on above principles, it is suggested to calculate the expert weight model as follows:

$$\min J=\sum_{i=1}^m\sum_{j=1}^n\sum_{l=1}^s\lambda_l\sum_{k=1}^s d(s^l_{ij},s^k_{ij}),$$

$$\text{s.t.}\begin{cases}\sum\limits_{l=1}^s\lambda_l^2=1\\ 0\leqslant\lambda_l\leqslant 1\end{cases}. \tag{18}$$

To solve this model, we construct the Lagrange function:

$$L(\lambda,\pi)=\sum_{i=1}^m\sum_{j=1}^n\sum_{l=1}^s\lambda_l\sum_{k=1}^s d(s^l_{ij},s^k_{ij})+\pi\left(\sum_{l=1}^s\lambda_l^2-1\right) \tag{19}$$

where π is the Lagrange multiplier.

Then we compute the partial derivatives of L as follows:

$$\begin{cases} \dfrac{\partial L}{\partial \lambda_l} = \displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{s} d(s_{ij}^l, s_{ij}^k) + 2\pi\lambda_l = 0 \\ \dfrac{\partial L}{\partial \pi} = \displaystyle\sum_{l=1}^{s} \lambda_l^2 - 1 = 0 \end{cases}.$$

From (19), we get a simple and exact formula for determining the experts weight as follows:

$$\lambda_l^* = \frac{\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{s} d(s_{ij}^l, s_{ij}^k)}{\sqrt{\displaystyle\sum_{l=1}^{s}\left[\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{s} d(s_{ij}^l, s_{ij}^k))\right]^2}}. \tag{20}$$

By normalizing $\lambda_l^*(l = 1, 2, \cdots, s)$ as a unit, we have

$$\lambda_l = \frac{\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{s} d(s_{ij}^l, s_{ij}^k)}{\displaystyle\sum_{l=1}^{s}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{s} d(s_{ij}^l, s_{ij}^k)}. \tag{21}$$

As the expert weight determined by (21) and the weighted average operator given by (3), we can aggregate individual preferences to form group preferences:

$$s_{ij} = \text{2DULWA}(s_{ij}^1, s_{ij}^2, \cdots, s_{ij}^s). \tag{22}$$

Next, we establish the attribute weight model with the attribute value as the 2DULV:

$$\max\ J = \sum_{j=1}^{n}\omega_j \sum_{i=1}^{m}\sum_{k=1}^{m} d(s_{ij}, s_{kj}),$$

$$\text{s.t.} \begin{cases} \displaystyle\sum_{j=1}^{n}\omega_j^2 = 1 \\ 0 \leqslant \omega_j \leqslant 1 \end{cases}. \tag{23}$$

To solve this model, we construct the Lagrange function:

$$L(\omega, \lambda) = \sum_{j=1}^{n}\omega_j \sum_{i=1}^{m}\sum_{k=1}^{m} d(s_{ij}, s_{kj}) + \lambda\Big(\sum_{j=1}^{n}\omega_j^2 - 1\Big) \tag{24}$$

where λ is the Lagrange multiplier.

Then we compute the partial derivatives of L as follows:

$$\begin{cases} \dfrac{\partial L}{\partial \omega_j} = \displaystyle\sum_{i=1}^{m}\sum_{k=1}^{m} d(s_{ij}, s_{kj}) + 2\lambda\omega_j = 0 \\ \dfrac{\partial L}{\partial \lambda} = \displaystyle\sum_{j=1}^{n}\omega_j^2 - 1 = 0 \end{cases}.$$

From (24), we get a simple and exact formula for determining the attributets weights as follows:

$$\omega_j^* = \frac{\displaystyle\sum_{i=1}^{m}\sum_{k=1}^{m} d(s_{ij}, s_{kj})}{\sqrt{\displaystyle\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{k=1}^{m} d(s_{ij}, s_{kj})\right]^2}}. \tag{25}$$

By normalizing $\omega_j^*(j = 1, 2, \cdots, n)$ as a unit, we have

$$\omega_j = \frac{\displaystyle\sum_{i=1}^{m}\sum_{k=1}^{m} d(s_{ij}, s_{kj})}{\displaystyle\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} d(s_{ij}, s_{kj})}. \tag{26}$$

4.3 MAGDM method for 2DULVs

The steps for solving the MAGDM of 2DULVs are as follows:

Step 1 Establish the distance matrix between experts;

Step 2 Calculate the weight of experts;

Step 3 Aggregate evaluation information given by experts;

Step 4 Establish the distance matrix between alternatives;

Step 5 Calculate the weight of attributes;

Step 6 Aggregate attribute information;

Step 7 Rank each alternative.

5. An illustrated example

Example 2 This example is adopted from [39]. A practical use of the proposed approach involves the technological innovation ability evaluation of four enterprises $\{AS_1, AS_2, AS_3, AS_4\}$, the attributes are shown as follows: the ability of innovative resources input (C_1), the ability of innovation management (C_2), the ability of innovation tendency (C_3) and the ability of research and development (C_4). Based on the four attributes, three experts $\{E_1, E_2, E_3\}$ evaluate the technological innovation ability of the four enterprises. $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3)^{\mathrm{T}}$ is the weight vector of the three experts, which is completely unknown. $\boldsymbol{\omega} = (\omega_1, \omega_2, \omega_3, \omega_4)^{\mathrm{T}}$ is the weight vector of the four attributes, which is completely unknown. The attribute values given by the experts take the form of 2DULVs, which are shown in Tables 1–3. The experts utilize I class linguistic set $S_{\mathrm{I}} = \{S_0, S_1, S_2, S_3, S_4, S_5, S_6\}$ and the II class linguistic set $S_{\mathrm{II}} = \{S_0^*, S_1^*, S_2^*, S_3^*, S_4^*\}$. Rank the four enterprises based on their technological innovation ability.

Step 1 Establish the distance matrix between experts.

We establish the distance matrix $\boldsymbol{D}_{ij} = [d_{ij}^{kl}]_{3\times 3}$ between experts according to (17), where d_{ij}^{kl} represents the distance between the evaluation information of E_k and the evaluation information of E_l with respect to s_{ij} (see Tables 1−3).

Table 1 Attribute values with respect to four enterprises given by expert E_1

Enterprise	Attribute(C_1)	Attribute(C_2)	Attribute(C_3)	Attribute(C_4)
AS_1	$((S_5,S_5),(S_2^*,S_3^*))$	$((S_2,S_3),(S_2^*,S_3^*))$	$((S_4,S_5),(S_4^*,S_4^*))$	$((S_3,S_4),(S_1^*,S_2^*))$
AS_2	$((S_3,S_4),(S_2^*,S_3^*))$	$((S_5,S_5),(S_3^*,S_3^*))$	$((S_3,S_3),(S_4^*,S_4^*))$	$((S_4,S_4),(S_1^*,S_2^*))$
AS_3	$((S_2,S_3),(S_2^*,S_3^*))$	$((S_3,S_4),(S_3^*,S_3^*))$	$((S_3,S_4),(S_4^*,S_4^*))$	$((S_4,S_5),(S_1^*,S_2^*))$
AS_4	$((S_5,S_6),(S_2^*,S_3^*))$	$((S_1,S_2),(S_3^*,S_3^*))$	$((S_2,S_3),(S_4^*,S_4^*))$	$((S_3,S_4),(S_1^*,S_2^*))$

Table 2 Attribute values with respect to four enterprises given by expert E_2

Enterprise	Attribute(C_1)	Attribute(C_2)	Attribute(C_3)	Attribute(C_4)
AS_1	$((S_4,S_4),(S_3^*,S_4^*))$	$((S_3,S_4),(S_2^*,S_3^*))$	$((S_3,S_4),(S_3^*,S_3^*))$	$((S_5,S_6),(S_3^*,S_4^*))$
AS_2	$((S_4,S_5),(S_3^*,S_4^*))$	$((S_2,S_3),(S_2^*,S_3^*))$	$((S_4,S_5),(S_3^*,S_3^*))$	$((S_2,S_3),(S_3^*,S_4^*))$
AS_3	$((S_3,S_4),(S_3^*,S_4^*))$	$((S_4,S_4),(S_2^*,S_3^*))$	$((S_2,S_3),(S_3^*,S_3^*))$	$((S_3,S_4),(S_3^*,S_4^*))$
AS_4	$((S_5,S_5),(S_3^*,S_4^*))$	$((S_4,S_5),(S_2^*,S_3^*))$	$((S_1,S_2),(S_3^*,S_3^*))$	$((S_4,S_4),(S_3^*,S_4^*))$

Table 3 Attribute values with respect to four enterprises given by expert E_3

Enterprise	Attribute(C_1)	Attribute(C_2)	Attribute(C_3)	Attribute(C_4)
AS_1	$((S_5,S_5),(S_2^*,S_3^*))$	$((S_2,S_3),(S_3^*,S_3^*))$	$((S_4,S_4),(S_3^*,S_4^*))$	$((S_4,S_5),(S_1^*,S_1^*))$
AS_2	$((S_4,S_4),(S_2^*,S_3^*))$	$((S_4,S_5),(S_2^*,S_2^*))$	$((S_1,S_2),(S_3^*,S_4^*))$	$((S_3,S_3),(S_1^*,S_1^*))$
AS_3	$((S_3,S_4),(S_2^*,S_3^*))$	$((S_5,S_5),(S_2^*,S_2^*))$	$((S_1,S_1),(S_3^*,S_4^*))$	$((S_4,S_4),(S_1^*,S_1^*))$
AS_4	$((S_2,S_3),(S_2^*,S_3^*))$	$((S_2,S_3),(S_2^*,S_2^*))$	$((S_4,S_5),(S_3^*,S_4^*))$	$((S_4,S_5),(S_1^*,S_1^*))$

Step 2 Calculate the weight of experts.

We establish the model (18) according to $\boldsymbol{D}_{ij}$ to calculate the weight of experts. Solving this model, we obtain the weight vector of experts $\boldsymbol{\lambda} = (0.310\,2, 0.370\,4, 0.319\,4)^{\mathrm{T}}$.

Step 3 Aggregate evaluation information given by experts.

According to the weight vector of experts, multiple attributes decision matrix $\boldsymbol{S} = [s_{ij}]_{4\times 4}$ is obtained as follows:

$$\boldsymbol{S} = \begin{bmatrix} ((S_{4.629\,6},S_{4.629\,6}),(S^*_{2.452\,8},S^*_4)) & ((S_{2.370\,4},S_{3.370\,4}),(S^*_{2.397\,2},S^*_3)) & ((S_{3.629\,6},S_{4.310\,2}),(S^*_4,S^*_4)) & ((S_{4.060\,2},S_{5.060\,2}),(S^*_{2.002\,9},S^*_4)) \\ ((S_{3.689\,8},S_{4.689\,8}),(S^*_{2.452\,8},S^*_4)) & ((S_{3.569\,4},S_{4.259\,3}),(S^*_{2.386\,9},S^*_{2.752\,2})) & ((S_{2.731\,5},S_{3.421\,3}),(S^*_4,S^*_4)) & ((S_{2.939\,8},S_{3.310\,2}),(S^*_{2.002\,9},S^*_4)) \\ ((S_{2.689\,8},S_{3.689\,8}),(S^*_{2.422\,8},S^*_4)) & ((S_{4.009\,3},S_{4.319\,4}),(S^*_{2.386\,9},S^*_{2.752\,2})) & ((S_{1.990\,7},S_{2.671\,3}),(S^*_4,S^*_4)) & ((S_{3.629\,6},S_{4.310\,2}),(S^*_{2.002\,9},S^*_4)) \\ ((S_{4.041\,7},S_{4.671\,3}),(S^*_{2.452\,8},S^*_4)) & ((S_{2.430\,6},S_{3.430\,6}),(S^*_{2.386\,9},S^*_{2.752\,2})) & ((S_{2.268\,5},S_{3.268\,5}),(S^*_4,S^*_4)) & ((S_{3.689\,8},S_{4.319\,4}),(S^*_{2.002\,9},S^*_4)) \end{bmatrix}.$$

Step 4 Establish the distance matrix between alternatives.

We establish the distance matrix $\boldsymbol{D}_j = [d^j_{ik}]_{4\times 4}$ between alternatives according to (17), where d^j_{ik} represents the distance between AS_i and AS_k under C_j.

Step 5 Calculate the weight of attributes.

We establish the model (23) according to D_j to calculate the weight of attribute. Solving this model, we obtain the weight vector of attributes $\boldsymbol{\omega} = (0.244\ 1, 0.275\ 7, 0.257\ 4, 0.222\ 9)^{\mathrm{T}}$.

Step 6 Aggregate attribute information:

$$z_1 = ((S_{3.622\,9}, S_{4.296\,7}), (S_4, S_4)),$$

$$z_2 = ((S_{3.243\,1}, S_{3.937\,6}), (S_4, S_4)),$$

$$z_3 = ((S_{3.083\,4}, S_{3.739\,9}), (S_4, S_4)),$$

$$z_4 = ((S_{3.063\,1}, S_{3.890\,2}), (S_4, S_4)).$$

Step7 Rank each alternative.

To rank each alternative, the comprehensive evaluation value of each alternative is compared.

$$\eta_1 = 15.839\,1,$$

$$\eta_2 = 14.361\,4,$$

$$\eta_3 = 13.646\,5,$$

$$\eta_4 = 13.906\,5.$$

Therefore, the obtained alternatives ordering is as follows:

$$AS_1 \succ AS_2 \succ AS_4 \succ AS_3.$$

Example 3 In the Example 2, the weight vector of experts depends on $\boldsymbol{D}_{ij}$. Thus, the accuracy of $\boldsymbol{D}_{ij}$ directly affects the accuracy of the weight vector of experts.

However, the $\boldsymbol{D}_{ij}$ calculated according to (2) is imprecise in some cases. For example, the attribute values given by expert E_1 is $s_{ij}^1 = ((S_1, S_1), (S_1^*, S_1^*))$ and the attribute values given by expert E_2 is $s_{ij}^2 = ((S_{0.5}, S_{0.5}), (S_2^*, S_2^*))$. In this case, $d(s_{ij}^1,\ s_{ij}^2) = 0$ according to (2), but $d(s_{ij}^1, s_{ij}^2) = 1.5$ according to (17). Obviously, two linguistic variables are not equal. Although the attribute values given by expert E_1 is a 2-dimension certain linguistic variable, some attribute values given by experts may be certain.

6. Conclusions

The score function and distance formula of 2DULVs are two important criteria in the MAGDM problem. However, the existing score function of 2DULVs cannot compare two 2DULVs with the same product of the median of two dimensional linguistic intervals. On the other hand, the existing distance formula of 2DULVs is imprecise in some cases. To overcome these disadvantages, this paper proposes a new scoring function and distance formula of 2DULVs. Comparing with the existing score function and the distance formula of 2DULVs, it is more accurate. On this basis, we propose a method with completely unknown weight information under 2DULVs based on the score function and the distance formula. In further research, it is necessary and meaningful to propose the score function of 2DULVs based on non-normal distribution.

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