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A super-network equilibrium optimization method for operation architecture with fuzzy demands

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Abstract: From the view of information flow, a super-network equilibrium optimization model is proposed to compute the solution of the operation architecture which is made up of a perceptive level, a command level and a firepower level. Firstly, the optimized conditions of the perceptive level, command level and firepower level are analyzed respectively based on the demand of information relation, and then the information supply-and-demand equilibrium model of the operation architecture super-network is established. Secondly, a variational inequality transformation (VIT) model for equilibrium optimization of the operation architecture is given. Thirdly, the contraction projection algorithm for solving the operation architecture super-network equilibrium optimization model with fuzzy demands is designed. Finally, numerical examples are given to prove the validity and rationality of the proposed method, and the influence of fuzzy demands on the super-network equilibrium solution of operation architecture is discussed.

Keywords: super-network equilibrium, operation architecture, fuzzy demand, information flow, variational inequality transformation.

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1. Introduction

Ballistic missiles play an significant role in the future military struggle due to their own advantages. All countries in the world regard it as the punch weapon in future wars and put it in the first place in the development of military forces. The research of anti-missile warfare has become a hot topic at present [1,2]. As a complex and large system, the operation system consists of three parts: early warning detection system, command and control system, and intercept and strike system [3]. Each part is made up of many entities, which interact, complement and restrict each other according to a certain hierarchy. It stimulates a characteristic of emergence [4] that a system as a whole has but that the constituent entities alone do not, such as the ability emergence. The emergence effect of system capability often depends on the complex interaction between entities. This interaction is essentially determined by the information flows between entity nodes. Therefore, information flow [5] is very important in the operation architecture. The more quantity and quality of the battlefield information are obtained, the more comprehensive the battlefield situation is available. However, a mass of information means huge time cost. Excessive information also causes the information process and distribution system to become congested or ineffective. Therefore, the value of information flow of the operation architecture is to provide the suitable quantity and quality of information at right time to meet operational task demands. The supply and demand relationship of the information value among different levels of the operation architecture network should be adjusted reasonably to achieve the balance, in which each node of the architecture can achieve maximum operational profits without full load running, and it is helpful to advance the networked warfare efficiency. In this paper, the network equilibrium steady state problem of the operation architecture based on the information flow is studied, which has an important supporting role for the research on emergence and adaptive evolution of the system.

Nowadays, most research on the network equilibrium optimization is about route planning [6,7], supply chain network [8 – 10] and transportation network [11 – 13]. Unfortunately, there are few studies on super-network equilibrium optimization for the operation architecture. Many experts and scholars have paid great attention on super-network modeling and network performance analysis.

Jeffrey et al. [14,15] of the United States explored the principle of distributed networked warfare and then conducted relevant research. They first used the complex network theory to build a model of warfare in the information age, which divided the force nodes in the battlefield into

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four categories, decision node, sensor node, response node and target node, according to the different roles played by different forces in the battlefield. Then the network was established according to the relationship between the nodes, and a method of measuring network efficiency by the adjacency matrix was proposed. Although Jeffrey only describes the combat network at a macro level and lacks the description of micro behaviors, it still has a high reference value. Sean et al. [16] of Old Dominion University in 2009 further discussed and improved the information age engagement model established by Jeffrey, and analyzed an example.

Also, multiple quality assessments of the system of systems architecture were discussed as reported in [17], where an extended influence diagram method was developed to assess the operation architecture functionality. Huang et al. [18] proposed the construction and analysis method of the combat system based on the extension space. Zou et al. [19,20] established the super-network model of the air defense system and its evolution method was studied. Then, he proposed an extended granularity calculation method to solve the quantitative mapping process and the system quantification problem of functional networks at different granularity layers. Zhang's team [21,22] established a super-network model of the networked command, control, communication, computer, intelligence, surveillance, reconnaissance (C4ISR) system structure, and described the characteristics and applicability of the super-network. Hu [23], a professor of National Defense University, built a three-layer view of combat system from the physical domain, information domain and cognitive domain. Through the analysis of information interaction and characteristics in the command process, the influence of information interaction on the command efficiency was studied and the observe-orient-decide-act (OODA) combat ring extraction method of the system super-network model was discussed. Yang et al. [24] put forward the concept of information flow super-network, established a hierarchical network structure, and built an information flow equilibrium model considering optimization objectives of every layer and the overall equilibrium. Wang et al. [25] optimized the modeling of the combat network structure of surface warship formation in the anti-ship combat. Zhao et al. [26] proposed a weapon system of systems based on a granular analysis of super-network. Xing et al. [27] proposed a missile defense system-of-systems architecture super-network equilibrium optimization model based on information stream. Shao et al. [28] established the network structure of the shore-to-ship missile combat system under the platform central warfare mode based on the complex network theory. The network structure of the system was optimized through the network coordination of firepower layer, command layer, and pre-alarming detection layer.

The operation system is generally composed of early warning detection equipment, command and control equipment, interception and strike equipment. The optimization of the operation architecture has always been a difficult problem in the field of military operations.

This paper tries to improve the equilibrium optimization model of Xing [27]. In the revised objective function model, the information conversion rate and the information value coefficient are used to replace the information value conversion rate. It advances the scope of application of the model where information value conversion rate must be less than 1. In the revised constraints model, the constraint between the amount of information transmitted from the command level to the firepower level and the amount transmitted from the perception level to the command level is present, and the contraction projection algorithm is used to replace the projection algorithm for solving the variational inequality model iteratively.

2. Problem description and modeling

2.1 Problem analysis

Operation architecture super-network is composed of perception node (PN), command node (CN) and firepower node (FN). Engagement chain is a complete information flow link, which completes a process of perceiving target information, making interception decisions and destroying targets. Each flow can be seen as a combat alliance, which can be represented by a hyper-edge in the logic layer. As shown in Fig. 1, the dotted line in the diagram is the hyper-edge indication of the engagement chains of the operation architecture. If the numbers of PNs, CNs and FNs of the operation architecture are I, J and K, then the operation architecture has $I \times J \times K$ engagement chains.

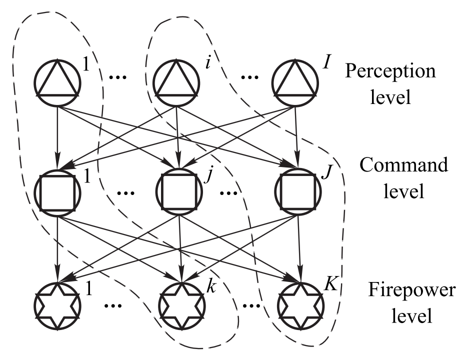


Fig. 1 Network structure of engagement chains

Super-network equilibrium state (SNES) refers to the state in which the information flow among the nodes of perception level, command level and firepower level is in a stable state and the information value generated by the information flow among the nodes of all levels and the degree to which the information value demand is satisfied are the largest. The information flows of operational tasks in the operation architecture super-network correspond to the structure of engagement chains. When the information flow is transmitted step by step on the link, not only the information value at each node of the flow is guaranteed to be maximum, but also the information value between levels is guaranteed to match the supply and demand relationship. Therefore, the operation architecture super-network achieves the overall equilibrium state.

In the operation architecture super-network, the PN realizes battlefield situational awareness by obtaining target state information. On the one hand, the PNs share information with each other to further enhance the value of their existing information. On the other hand, the PNs also need to transfer the information they acquire to command levels. Information value increment and transformation of PNs is achieved only by realizing this kind of information flow. However, information value loss is caused by wastage or error information in the process of information transmission.

The command level is a connecting link between the preceding and the following in the engagement chain. On the one hand, it needs to receive the target situation information from the perception level for decision-making. On the other hand, it needs to send operational command information to the firepower level, so that the firepower level equipment could respond quickly to complete the interception task in time. Only when the CNs obtain enough valuable information can they make correct decisions and select appropriate firepower equipment for interception. This requires that the information value provided by the perception level must match the information value demand of the command level. Only in this way can the information flow of the engagement chain be guaranteed to continue. Thus, for CNs, the supply-demand matching relationship between the perception level and the command level is mainly manifested in two aspects. Firstly, when the information quantity transmitted by PNs to CNs is more than that required by CNs, it will impose additional burden on the information processing of the CN. What is worse, if the information value of the excessive information cannot be transformed in time, the generation efficiency of the information value would be affected and information value loss would occur. Secondly, when the information quantity transmitted by PNs to CNs is less than that required by CNs, the information quantity received does not meet the requirement of CNs. The information quantity of the missing part may affect the information fusion effect and the information value generation, which will result in the loss of information value.

The FN completes the interception task by receiving command instructions from the command level. Correct and timely command instructions are particularly important for response, tracking and target interception. In the information transmission process of the whole engagement chain, the firepower level is the final demander. It does not produce information value, but expects to obtain information that can meet the needs of firepower interception. The matching relationship between the command level and the firepower level is mainly manifested in two aspects. When the information quantity transmitted by CNs to FNs is more than that required by FNs, it will impose additional burden on the target allocation of the FNs. Too much task instruction information may exceed the interception capability of the FN, so the information value of the excessive information quantity cannot be transformed in time and the efficiency of information value generating would be affected. Thus the loss of information value would occur. However, when the amount of information transmitted by the command level to the firepower level is less than that required by FNs, the information quantity received does not reach the amount required by FNs. The missing part of information may affect the information value needed by FNs, e.g., guidance and tracking accuracy, and affect the generation of information value. This will result in the loss of information value.

2.2 Problem formulation

The operation architecture super-network information flow realizes the information transmission and increment of information value through the engagement chain. For the operation mission requirements, the perception level is mainly to generate the target situation information for the command level. The greater the effective information value is generated, the better the combat status of the perception level is. The information generated by the perception level and required by the command level is not as much as possible, but rather the information can meet the desired information value needs of command and decision. Therefore, the perception level should consider the command level's demand for information value while pursuing the maximization of information value. The relationship between supply and demand of the information value must reach a balanced state. Similarly, between the command level and the firepower level, since the command level mainly produces decision instruction information and the target situation information which is processed by the command level and used by the firepower level, the greater the effective information value generates, the better the operational status of the command level is. However, the firepower level does not require the information value of the command level to be as large as possible. Instead, it can obtain information that can be used for fire interception to meet the desired information value requirement. In the pursuit of maximizing the value of information, the command level must also take into account the firepower level of the information demand. The information supply and demand relationship between the two levels must also reach a balanced state. In this way, for the entire operation architecture super-network, it is necessary to comprehensively balance the optimization objectives at all levels in order to achieve the optimal equilibrium state of the architecture.

Accordingly, the equilibrium state of the super-network based on the information value demand mainly depends on the decision-making state at all levels. When the effective information value and information value demand of all levels in the super-network reach their optimal state at the same time, the state parameters of the super-network are the characteristic parameters of the operation architecture super-network, of which the information flow achieves equilibrium. That is to say, we must find a set of solutions to maximize the effective information value generated by the perception level and the command level, and to maximize the information value expected by the command level and the firepower level. For the purpose, Xing et al. [27] set up a hypernetwork equilibrium optimization model. However, in the subsequent research, it is found that there are the following two expressions in the model:

$$\left\{\begin{array}{l}\max(v_j^2\mathrm{E}[\min\{q_j^1,\widetilde{d}_j(v_j^2)\}]-\alpha_j^1\mathrm{E}[\max\{0,q_j^1-\widetilde{d}_j(v_j^2)\}]-\alpha_j^2\mathrm{E}[\max\{0,\widetilde{d}_j(v_j^2)-q_j^1\}])\\ \max(v_k^3\mathrm{E}[\min\{q_k^2,\widetilde{d}_k(v_k^3)\}]-\beta_k^1\mathrm{E}[\max\{0,q_k^2-\widetilde{d}_k(v_k^3)\}]-\beta_k^2\mathrm{E}[\max\{0,\widetilde{d}_k(v_k^3)-q_k^2\}])\end{array}\right., \tag{1}$$

where q_j^1 is the information quantity of perception node j, $q_j^1=\sum_{i=1}^{I}q_{ij}^1$, I is the number of nodes at the perception level, q_{ij}^1 is the information transmitted by perception node i to command node j; v_j^2 is an information conversion rate of perception node (ICRPN); α_j^1/α_j^2 is the information value loss coefficient (IVLC) of unit information redundancy/deficiency between command node j and perception nodes; $\widetilde{d}_j(v_j^2)$ is an information fuzzy demand function of command node j; $\mathrm{E}[\min\{q_j^1,\widetilde{d}_j(v_j^2)\}]$ is the expected information quantity value transmitted by perception nodes to command node j; $\mathrm{E}[\max\{0,q_j^1-\widetilde{d}_j(v_j^2)\}]$ is the expected information redundancy value transmitted by perception nodes to command node j; $\mathrm{E}[\max\{0,\widetilde{d}_j(v_j^2)-q_j^1\}]$ is the expected information deficiency value transmitted by perception nodes to command node j; q_k^2 is the information quantity got by firepower node k from nodes at the command level, $q_k^2=\sum_{j=1}^{J}q_{jk}^2$, J is the number of nodes at the command level, q_{jk}^2 is the information quantity transmitted by command node j to firepower node k; v_k^3 is an information conversion rate of command node (ICRCN); β_k^1/β_k^2 is the IVLC of unit information redundancy/deficiency between firepower node k and the command level; $\widetilde{d}_k(v_k^3)$ is the information fuzzy demand function of firepower node k; $\mathrm{E}[\min\{q_k^2,\widetilde{d}_k(v_k^3)\}]$ is the expected information quantity value transmitted by command nodes to firepower node k; $\mathrm{E}[\max\{0,q_k^2-\widetilde{d}_k(v_k^3)\}]$ is the expected information redundancy value transmitted by the command nodes to firepower node k; $\mathrm{E}[\max\{0,\widetilde{d}_k(v_k^3)-q_k^2\}]$ is the expected information deficiency transmitted by command nodes to firepower node k.

In the model, $v_j^2\mathrm{E}[\min\{q_j^1,\widetilde{d}_j(v_j^2)\}$ is not on the same level as $\alpha_j^2\mathrm{E}[\max\{0,\widetilde{d}_j(v_j^2)-q_j^1\}]$ and $\alpha_j^1\mathrm{E}[\max\{0,q_j^1-\widetilde{d}_j(v_j^2)\}]$, $v_k^3\mathrm{E}[\min\{q_k^2,\widetilde{d}_k(v_k^3)\}]$ is not on the same level as $\beta_k^1\mathrm{E}[\max\{0,q_k^2-\widetilde{d}_k(v_k^3)\}]$ and $\beta_k^2\mathrm{E}[\max\{0,\widetilde{d}_k(v_k^3)-q_k^2\}]$. $v_j^2\mathrm{E}[\min\{q_j^1,\widetilde{d}_j(v_j^2)\}$ reflects the amount of information, while the latter two reflect the value of information. They are not on the same level. The condition for direct subtraction is that the conversion rate of the information value is 1.

In addition, we find that the model lacks constraints on the flow of information between different levels, so the constraints model of the information flow between different levels is added. The constraints formula is $\sum_{k=1}^{K}q_{jk}^2<\sum_{i=1}^{I}v_{ij}^1q_{ij}^1$. Thus, we set up the following objective function and constraints formula groups:

$$\left\{\begin{array}{l}\max\sum_{i=1}^{I}\left(\sum_{j=1}^{J}v_{ij}^1q_{ij}^1+f_i^1(\boldsymbol{Q}^1)-\sum_{j=1}^{J}c_{ij}^1(q_{ij}^1)\right)\\ \max(\varepsilon_j^2v_j^2\mathrm{E}[\min\{q_j^1,\widetilde{d}_j(v_j^2)\}]-\\ \quad\alpha_j^1\mathrm{E}[\max\{0,q_j^1-\widetilde{d}_j(v_j^2)\}]-\\ \quad\alpha_j^2\mathrm{E}[\max\{0,\widetilde{d}_j(v_j^2)-q_j^1\}])\\ \max\sum_{j=1}^{J}\left(\sum_{k=1}^{K}v_{jk}^2q_{jk}^2+f_j^2(\boldsymbol{Q}^2)-\sum_{k=1}^{K}c_{jk}^2(q_{jk}^2)\right)\\ \max(\varepsilon_k^3v_k^3\mathrm{E}[\min\{q_k^2,\widetilde{d}_k(v_k^3)\}]-\\ \quad\beta_k^1\mathrm{E}[\max\{0,q_k^2-\widetilde{d}_k(v_k^3)\}]-\\ \quad\beta_k^2\mathrm{E}[\max\{0,\widetilde{d}_k(v_k^3)-q_k^2\}])\end{array}\right.$$

$$\text{s.t.}\begin{cases}0 \leqslant q_{ij}^1 \leqslant q_{ij}^{1\max} \\ 0 \leqslant v_{ij}^1 \leqslant v_{ij}^{1\max} \\ 0 \leqslant q_{jk}^2 \leqslant q_{jk}^{2\max} \\ 0 \leqslant v_{jk}^2 \leqslant v_{jk}^{2\max} \\ \sum_{k=1}^{K} q_{jk}^2 < \sum_{i=1}^{I} v_{ij}^1 q_{ij}^1\end{cases}, \quad i = \{1, 2, \ldots, I\};$$
$$j = \{1, 2, \ldots, J\}; k = \{1, 2, \ldots, K\} \tag{2}$$

where $\boldsymbol{Q}^1$ is the IJ dimensional vector matrix being composed of information transmitted by nodes at the perception level to nodes at the command level, $\boldsymbol{Q}^1 \in \mathbf{R}_+^{IJ}$; v_{ij}^1 is the information value conversion coefficient (IVCC) from PNi to CNj; $f_i^1(\boldsymbol{Q}^1)$ is the information value gain function of PNi, which reflects the influence of the synergistic relationship of the peer PNs on the information value of the PNi; $c_{ij}^1(q_{ij}^1)$ is the information value wastage function from PNi to CNj; ε_j^2 is IVCCPN; K is the number of FNs; $\boldsymbol{Q}^2$ is the JK dimensional vector matrix composed of information transmitted by all nodes at the command level to all nodes at the firepower level, $\boldsymbol{Q}^2 \in \mathbf{R}_+^{JK}$; $f_j^2(\boldsymbol{Q}^2)$ is the information value gain function of CNj, which reflects the influence of the synergistic relationship of the peer CNs on the information value of the CNj; v_{jk}^2 is the information value conversion coefficient from CNj to FNk; $c_{jk}^2(q_{jk}^2)$ is the information value wastage function from CNj to FNk; and ε_k^3 is an IVCCCN.

Due to the influence of equipment performance, the information quantity generated by PNs is limited. Assuming that the maximum amount of information transmitted by PNi to CNj is $q_{ij}^{1\max}$, then $q_{ij}^1 \in [0, q_{ij}^{1\max}]$. Similarly, the IVCC between adjacent nodes from the perception to the command level is also limited, when the equipment performance characteristics of PNs and CNs are considered. Assuming that the upper limit of the IVCC from PNi to CNj is $v_{ij}^{1\max}$, then $v_{ij}^1 \in [0, v_{ij}^{1\max}]$. In this way, the optimization goal of information value at the perception level is to maximize the effective information value generated by all PNs.

Similar to the optimization condition of information value at the perception level, the information quantity generated by each CN is limited due to the influence of equipment performance. Assuming that the maximum amount of information transmitted by CNj to FNk is $q_{jk}^{2\max}$, then $q_{jk}^2 \in [0, q_{jk}^{2\max}]$. Similarly, the information value conversion coefficient between adjacent nodes from the command level to the firepower level is also limited, when the equipment performance characteristics of nodes at the command level and the firepower level are considered. Assuming that the upper limit of the IVCC transmitted by CNj to the FNk is $v_{jk}^{2\max}$, then $v_{jk}^2 \in [0, v_{jk}^{2\max}]$. When there is no other information source provided to the command level, the amount of information transmitted from the command level to the firepower level and the amount transmitted from the perception level to the command level satisfy the constraint $\sum_{k=1}^{K} q_{jk}^2 < \sum_{i=1}^{I} v_{ij}^1 q_{ij}^1$. In this way, the optimization goal of the information value of the command level is to maximize the expected information value obtained and the effective information value generated by all CNs as much as possible.

If there is a set of solutions $(\boldsymbol{Q}^{1*}, \boldsymbol{Q}^{2*})$, the operation architecture super-network achieves a balanced stability in the information value demand at all levels. The value of the super-network information relationship is the greatest, and the structural benefit is the best. In this case, the operation architecture is the optimal information relationship structure that can carry out an operation task, and its optimized information flow can effectively enhance the execution of operation tasks.

2.3 Variational inequality transformation (VIT) for optimization model with fuzzy demands

For (2), it is difficult to directly prove the existence and uniqueness of its solution, and it is also difficult to solve the model directly. Considering the relationship between the optimization problem and the variational inequality problem, the equilibrium model can be transformed into a variational inequality form with more convenient analysis and solution.

Variational inequality originates from mathematical problems, physical problems and non-linear programming problems. Its modeling framework is rigorous and smooth and very suitable for theoretical analysis. At present, it has been widely used in supply chain network equilibrium optimization and traffic flow network equilibrium optimization [29,30]. Generally speaking, an optimization problem can be transformed into a variational inequality problem when certain conditions are satisfied.

Consider the constrained optimization problems [31]:

$$\min \sum_{i}^{m} F_i(\boldsymbol{X}_i)$$

$$\begin{cases}\boldsymbol{a}_j^{\mathrm{T}} \boldsymbol{X} \leqslant \boldsymbol{b}_j, \quad j = 1, \ldots, r \\ \boldsymbol{X}_i \in \boldsymbol{U}_i, \quad i = 1, \ldots, m\end{cases} \tag{3}$$

where $\boldsymbol{U}_i$ is a closed convex set, $F_i : \mathbf{R}^{n_i} \mapsto \mathbf{R}$ is a differentiable convex function, $\boldsymbol{a}_j^{\mathrm{T}}$ is the vector matrix composed of correlation coefficient of constraint condition j, $\boldsymbol{X} = (\boldsymbol{X}_1, \ldots, \boldsymbol{X}_m)$. Such an optimization problem (3) can be transformed into a variational inequality problem

$\boldsymbol{X}_i^* \in \boldsymbol{U}_i$, $\mu_j^* \geqslant 0$, which is shown as

$$\sum_{i=1}^{m}\left\langle\left(\nabla F_i(\boldsymbol{X}_i^*)+\sum_{j=1}^{r}\mu_j^* a_{ji}\right)^{\mathrm{T}},(\boldsymbol{X}_i-\boldsymbol{X}_i^*)\right\rangle+$$
$$\sum_{j=1}^{r}(\boldsymbol{b}_j-\boldsymbol{a}_j^{\mathrm{T}}\boldsymbol{X}^*)(\mu_j-\mu_j^*)\geqslant 0,$$
$$\forall \boldsymbol{X}_i\in\boldsymbol{U}_i;\mu_j\geqslant 0;\forall j. \tag{4}$$

This is a basic principle of solving variational optimization problems by variational inequalities. In the optimization model given in (2), the command level contains fuzzy variables [32,33] and their expected expressions. In order to realize the transformation from the optimization problem to the variational inequality problem, the properties of fuzzy variables and their expected expressions need to be studied. Assuming that $\widetilde{d}_j(v_j^2)$ is an information fuzzy demand of CNj, when its demand for the information value conversion rate of the perception level is v_j^2, its support is regarded as $[d_j^-(v_j^2), d_j^+(v_j^2)]$, and its confidence distribution function [34,35] is regarded as

$$\varPsi_j(x,v_j^2)=Cr\{\widetilde{d}_j(v_j^2)\leqslant x\}. \tag{5}$$

Theorem 1 For the CNj, the information quantity expectation, the information quantity redundancy expectation and the information quantity deficiency expectation transmitted by the perception level are as follows:

$$\begin{cases}\mathrm{E}[\min\{q_j^1,\widetilde{d}_j(v_j^2)\}]=q_j^1-\displaystyle\int_{d_j^-(v_j^2)}^{q_j^1}(q_j^1-x)\mathrm{d}[\varPsi_j(x,v_j^2)]\\ \mathrm{E}[\max\{0,q_j^1-\widetilde{d}_j(v_j^2)\}]=\displaystyle\int_{d_j^-(v_j^2)}^{q_j^1}(q_j^1-x)\mathrm{d}[\varPsi_j(x,v_j^2)]\\ \mathrm{E}[\max\{0,\widetilde{d}_j(v_j^2)-q_j^1\}]=\displaystyle\int_{q_j^1}^{d_j^+(v_j^2)}(x-q_j^1)\mathrm{d}[\varPsi_j(x,v_j^2)]\end{cases}. \tag{6}$$

Corollary 1 For CNj, let

$$Y(\boldsymbol{Q}^1)=\varepsilon_j^2 v_j^2\mathrm{E}[\min\{q_j^1,\widetilde{d}_j(v_j^2)\}]-\alpha_j^1\mathrm{E}[\max\{0,q_j^1-\widetilde{d}_j(v_j^2)\}]-\alpha_j^2\mathrm{E}[\max\{0,\widetilde{d}_j(v_j^2)-q_j^1\}], \tag{7}$$

then $Y(\boldsymbol{Q}^1)$ is a continuous concave function for q_{ij}^1.

For the firepower level, there are a fuzzy demand variable $\widetilde{d}_k(v_k^3)$ and its expectation expression in the equilibrium model of the firepower level, whose support is denoted as $[d_k^-(v_k^3), d_k^+(v_k^3)]$. Its confidence distribution function is denoted as

$$\varPsi_k(x,v_k^3)=Cr\{\widetilde{d}_k(v_k^3)\leqslant x\}. \tag{8}$$

Theorem 2 For FNs, the information quantity expectation, the information quantity redundancy expectation and the information quantity deficiency expectation transmitted by the command level are as follows:

$$\begin{cases}\mathrm{E}[\min\{q_k^2,\widetilde{d}_k(v_k^3)\}]=q_k^2-\displaystyle\int_{d_k^-(v_k^3)}^{q_k^2}(q_k^2-x)\mathrm{d}[\varPsi_k(x,v_k^3)]\\ \mathrm{E}[\max\{0,q_k^2-\widetilde{d}_k(v_k^3)\}]=\displaystyle\int_{d_k^-(v_k^3)}^{q_k^2}(q_k^2-x)\mathrm{d}[\varPsi_k(x,v_k^3)]\\ \mathrm{E}[\max\{0,\widetilde{d}_k(v_k^3)-q_k^2\}]=\displaystyle\int_{q_k^2}^{d_k^+(v_k^3)}(x-q_k^2)\mathrm{d}[\varPsi_k(x,v_k^3)]\end{cases}. \tag{9}$$

Corollary 2 For FNk, let

$$Y(\boldsymbol{Q}^2)=\varepsilon_k^3 v_k^3\mathrm{E}[\min\{q_k^2,\widetilde{d}_k(v_k^3)\}]-\beta_k^1\mathrm{E}[\max\{0,q_k^2-\widetilde{d}_k(v_k^3)\}]-\beta_k^2\mathrm{E}[\max\{0,\widetilde{d}_k(v_k^3)-q_k^2\}], \tag{10}$$

then $Y(\boldsymbol{Q}^2)$ is a continuous concave function for q_{jk}^2.

The proof of theorem and corollary is in [27]. On this basis, an equilibrium state optimal model of the operation architecture super-network based on the information demand value can be obtained.

In the SNES, the information quantity $(\boldsymbol{Q}^{1*},\boldsymbol{Q}^{2*})\in\boldsymbol{U}=\mathbf{R}_+^{IJ+JK}$ transmitted by the perception level to the command level, and the command level to the firepower level, satisfy the following variational inequalities as

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\left(\frac{\partial c_{ij}(q_{ij}^{1*})}{\partial q_{ij}^1}-\frac{\partial f_i(\boldsymbol{Q}^{1*})}{\partial q_{ij}^1}-v_{ij}^1\right)(q_{ij}^1-q_{ij}^{1*})+$$
$$\sum_{i=1}^{I}\sum_{j=1}^{J}\left[\alpha_j^1\varPsi_j(q_j^{1*},v_j^2)-(\alpha_j^2+\varepsilon_j^2v_j^2)\cdot\right.$$
$$\left.[1-\varPsi_j(q_j^{1*},v_j^2)]\right][q_{ij}^1-q_{ij}^{1*}]+$$
$$\sum_{j=1}^{J}\sum_{k=1}^{K}\left(\frac{\partial c_{jk}(q_{jk}^{2*})}{\partial q_{jk}^2}-\frac{\partial f_j(\boldsymbol{Q}^{2*})}{\partial q_{jk}^2}-v_{jk}^2\right)(q_{jk}^2-q_{jk}^{2*})+$$
$$\sum_{j=1}^{J}\sum_{k=1}^{K}\left[[\beta_k^1\varPsi_k(q_k^{2*},v_k^3)]-(\beta_k^2+\varepsilon_k^3v_k^3)\cdot\right.$$
$$\left.[1-\varPsi_k(q_k^{2*},v_k^3)]\right][q_{jk}^2-q_{jk}^{2*}]\geqslant 0,$$
$$\forall(\boldsymbol{Q}^1,\boldsymbol{Q}^2)\in\boldsymbol{U}=\mathbf{R}_+^{IJ+JK} \tag{11}$$

where $\boldsymbol{U}=\{(\boldsymbol{Q}^1,\boldsymbol{Q}^2)|q_{ij}^{1\max}\geqslant q_{ij}^1\geqslant 0, q_{jk}^{2\max}\geqslant q_{jk}^2\geqslant 0,\forall i,j,k\}$, $\varPsi_j(x,v_j^2)$ and $\varPsi_k(x,v_k^3)$ are confidence distribution of the fuzzy demands $\widetilde{d}_j(v_j^2)$ and $\widetilde{d}_k(v_k^3)$.

Variational inequality (11) is the condition for the operation architecture super-network to reach an equilibrium state based on the information value demand, but whether this equilibrium state can be achieved depends on whether there is a solution in (11). Considering that its feasible region $\boldsymbol{U}$ is a bounded compact convex set, according to the existence condition of solutions in the theory of variational inequalities, it is easy to know that the solutions of the above-mentioned variational inequality (11) exist.

3. Solution algorithm

Generally, projection algorithms [36,37], improved projection algorithms [38] and external gradient methods [39–41] are used to solve the problem of variational inequalities. The projection method is widely used as one of the most important and effective methods in many algorithms. The basic idea of the projection method is to find out some connection between the variational inequality problem and the fixed point problem according to projection, and then some equations that have been proved are used to solve problems [42,43]. The projection method is considered to be one of the most effective algorithms for solving variational inequality problems because of its low computational complexity and easy operation in each iteration. It is very suitable for solving some large-scale optimization problems, the author [27] presented revised projection algorithms for solving the variational inequality model. In the algorithm, parameter β is constant, and cannot be adjusted with the error vector. In this paper, the contraction projection algorithm is used to solve problems iteratively.

For the sake of expression, the solution of the network equilibrium problem is expressed in the vector form. $\Omega = \{(q_{ij}^1)_{IJ}, (q_{jk}^2)_{JK} | q_{ij}^{1\max} \geqslant q_{ij}^1 \geqslant 0, q_{jk}^{2\max} \geqslant q_{jk}^2 \geqslant 0\}$, $\boldsymbol{u} = (\boldsymbol{Q}^1, \boldsymbol{Q}^2)$, and $P_\Omega(\boldsymbol{u})$ represents the projection of $\boldsymbol{u}$ on Ω. Let

$$\boldsymbol{F} = \{(F_{ij}^1)_{IJ}, (F_{ij}^2)_{IJ}, (G_{jk}^1)_{JK}, (G_{jk}^2)_{JK}\} \tag{12}$$

where

$$\begin{cases} F_{ij}^1 = \dfrac{\partial c_{ij}(q_{ij}^{1*})}{\partial q_{ij}^1} - \dfrac{\partial f_i(\boldsymbol{Q}^{1*})}{\partial q_{ij}^1} - v_{ij}^1 \\ F_{ij}^2 = \alpha_j^1 \Psi_j(q_j^{1*}, v_j^2) - (\alpha_j^2 + \varepsilon_j^2 v_j^2)[1 - \Psi_j(q_j^{1*}, v_j^2)] \\ G_{jk}^1 = \dfrac{\partial c_{jk}(q_{jk}^{2*})}{\partial q_{jk}^2} - \dfrac{\partial f_j(\boldsymbol{Q}^{2*})}{\partial q_{jk}^2} - v_{jk}^2 \\ G_{jk}^2 = \beta_k^1 \Psi_k(q_k^{2*}, v_k^3) - (\beta_k^2 + \varepsilon_k^3 v_k^3)[1 - \Psi_k(q_k^{2*}, v_k^3)] \end{cases}. \tag{13}$$

Then the original variational inequality (4) is transformed to solve $\boldsymbol{u}^* = (\boldsymbol{Q}^{1*}, \boldsymbol{Q}^{2*})$, that satisfies

$$\sum_{i=1}^{I}\sum_{j=1}^{J}(F_{ij}^1(q_{ij}^{1*}) + F_{ij}^2(q_{ij}^{1*}))(q_{ij}^1 - q_{ij}^{1*}) + \sum_{j=1}^{J}\sum_{k=1}^{K}(G_{jk}^1(q_{jk}^{2*}) + G_{jk}^2(q_{jk}^{2*}))(q_{jk}^2 - q_{jk}^{2*}) \geqslant 0. \tag{14}$$

The contraction projection algorithm for solving the operation architecture super-network equilibrium optimization model with fuzzy demands is as follows.

Step 1 Initialize parameter β_0, accuracy ε, and initial iteration point $\boldsymbol{u}_0$, and set $k = 0$.

Step 2 Calculate $\boldsymbol{u}_{k+1} = P_\Omega(\boldsymbol{u}_k - \beta_k F(\boldsymbol{u}_k))$.

Step 3 Calculate error $e(\boldsymbol{u}_{k+1}, \beta_k) = \boldsymbol{u}_{k+1} - \boldsymbol{u}_k$ for variational inequalities on Ω.

Step 4 If $\|e(\boldsymbol{u}_{k+1}, \beta_k)\| \leqslant \varepsilon$, then return the optimal value $\boldsymbol{u}_{k+1}$ and the algorithm stops; otherwise, go to Step 5.

Step 5 Adjust parameters $\beta_{k+1} = \dfrac{\|e(\boldsymbol{u}_{k+1}, \beta_k)\|}{\|F(\boldsymbol{u}_{k+1})\|}$, $k = k + 1$, and go to Step 2.

The information value of each engagement chain is

$$f_l = \left(F_{ij}^1(q_{ij}^{1*}) + F_{ij}^2(q_{ij}^{1*})\right) q_{ij}^{1*} + \left(G_{jk}^1(q_{jk}^{2*}) + G_{jk}^2(q_{jk}^{2*})\right) q_{jk}^{2*},$$
$$l = J \cdot K \cdot (i-1) + K \cdot (j-1) + k \tag{15}$$

where $i \in \{1, 2, \ldots, I\}$, $j \in \{1, 2, \ldots, J\}$, $k \in \{1, 2, \ldots, K\}$.

The total information value of the network (TIVN) can be calculated as

$$f_{\text{alls}} = \sum_{i=1}^{I}\sum_{j=1}^{J}(F_{ij}^1(q_{ij}^{1*}) + F_{ij}^2(q_{ij}^{1*})) \cdot q_{ij}^{1*} + \sum_{j=1}^{J}\sum_{k=1}^{K}(G_{jk}^1(q_{jk}^{2*}) + G_{jk}^2(q_{jk}^{2*})) \cdot q_{jk}^{2*}. \tag{16}$$

4. Numerical example and discussion

We present a numerical example to demonstrate how the proposed model can be used in practice. The information flow of the operation architecture is transmitted among the perception level, the command level and the firepower level. No loss of generality, it is assumed that there exist two PNs noted as PN1 and PN2, two CNs noted as CN1 and CN2, two FNs noted as FN1 and FN2, in the operation architecture super-network. Suppose that the information flow is a unit value and its interval is $[0, 1]$.

The information value gain functions for PN are

$$\begin{cases} f_1^1(\boldsymbol{Q}^1) = 0.5(q_1^1)^2 + q_1^1 q_2^1 + 3.5 q_1^1 = 0.5(q_{11}^1 + q_{12}^1)^2 + \\ \quad (q_{11}^1 + q_{12}^1)(q_{21}^1 + q_{22}^1) + 3.5(q_{11}^1 + q_{12}^1) \\ f_2^1(\boldsymbol{Q}^1) = (q_1^1)^2 + q_1^1 q_2^1 + 2.5 q_2^1 = (q_{11}^1 + q_{12}^1)^2 + \\ \quad (q_{11}^1 + q_{12}^1)(q_{21}^1 + q_{22}^1) + 2.5(q_{21}^1 + q_{22}^1) \end{cases}. \tag{17}$$

The information value gain functions for CN are

$$\begin{cases} f_1^2(\boldsymbol{Q}^2) = 2.5(q_1^2)^2 + q_1^2 q_2^2 + q_1^2 = 2.5(q_{11}^2 + q_{12}^2)^2 + \\ \quad (q_{11}^2 + q_{12}^2)(q_{21}^2 + q_{22}^2) + (q_{11}^2 + q_{12}^2) \\ f_2^2(\boldsymbol{Q}^2) = (q_1^2)^2 + q_1^2 q_2^2 + 3q_2^2 = (q_{11}^2 + q_{12}^2)^2 + \\ \quad (q_{11}^2 + q_{12}^2)(q_{21}^2 + q_{22}^2) + 3(q_{21}^2 + q_{22}^2) \end{cases}. \tag{18}$$

The information value loss functions between the perception level and the command level are

$$\begin{cases} c_{11}^1(q_{11}^1) = 0.5(q_{11}^1)^2 + q_{11}^1 \\ c_{12}^1(q_{12}^1) = 0.4(q_{12}^1)^2 + q_{12}^1 \\ c_{21}^1(q_{21}^1) = 0.9(q_{21}^1)^2 + 2q_{21}^1 \\ c_{22}^1(q_{22}^1) = 0.8(q_{22}^1)^2 + 2q_{22}^1 \end{cases}. \tag{19}$$

The information value loss functions between the command level and the firepower level are

$$\begin{cases} c_{11}^2(q_{11}^2) = 2(q_{11}^2)^2 + q_{11}^2 \\ c_{12}^2(q_{12}^2) = 3(q_{12}^2)^2 + q_{12}^2 \\ c_{21}^2(q_{21}^2) = 0.5(q_{21}^2)^2 + 3q_{21}^2 \\ c_{22}^2(q_{22}^2) = 0.5(q_{22}^2)^2 + 0.2q_{22}^2 \end{cases}. \tag{20}$$

Let $\alpha_1^1 = \alpha_2^1 = 0.45$, $\alpha_1^2 = \alpha_2^2 = 0.52$, $\beta_1^1 = \beta_2^1 = 0.35$, $\beta_1^2 = \beta_2^2 = 0.41$, $v_{11}^1 = 0.65$, $v_{12}^1 = 0.75$, $v_{21}^1 = 0.85$, $v_{22}^1 = 0.85$, $v_{11}^2 = 0.75$, $v_{12}^2 = 0.82$, $v_{21}^2 = 0.75$, $v_{22}^2 = 0.80$.

Suppose that fuzzy demands $\widetilde{d}_j(v_j^2)$ and $\widetilde{d}_k(v_k^3)$ are triangular fuzzy variables. The triangular fuzzy number is $\left[\dfrac{b_j}{r_j} - \underline{\Delta}, \dfrac{b_j}{r_j}, \dfrac{b_j}{r_j} + \overline{\Delta}\right]$, b_j is the fuzzy demand mean (FDM), and r_j is the information conversion rate (ICR). The confidence distribution of the variable is defined as

$$\Psi(x, r_j) = \begin{cases} 0, \quad x < \dfrac{b_j}{r_j} - \underline{\Delta} \\ \dfrac{1}{2\underline{\Delta}} \cdot \left(x + \underline{\Delta} - \dfrac{b_j}{r_j}\right), \quad \dfrac{b_j}{r_j} - \underline{\Delta} \leqslant x < \dfrac{b_j}{r_j} \\ \dfrac{1}{2\overline{\Delta}} \cdot \left(x + \overline{\Delta} - \dfrac{b_j}{r_j}\right), \quad \dfrac{b_j}{r_j} \leqslant x < \dfrac{b_j}{r_j} + \overline{\Delta} \\ 1, \quad x \geqslant \dfrac{b_j}{r_j} + \overline{\Delta} \end{cases}. \tag{21}$$

For CN1, CN2, FN1 and FN2, the mean value of the fuzzy demands are 0.6, 0.7, 0.7 and 0.8, respectively. $q_{ij}^{1\max}$, $v_{ij}^{1\max}$, $q_{jk}^{2\max}$, $v_{jk}^{2\max}$, ICRPN v_j^2, IVCCPN ε_j^2, ICRCN v_k^3, and IVCCCN ε_k^3 $(i = 1, 2; j = 1, 2; k = 1, 2)$ are set to 1 respectively. The initial value of β_0 is set to 0.001 and the accuracy which is less than or equal to 10^{-6} is the convergence criterion. When $(\underline{\Delta}, \overline{\Delta})$ is set to different values, according to the variational inequality formula (14), the super-network equilibrium of the operation architecture with fuzzy demands is solved.The corresponding objective function values are shown in Table 1 and Table 2.

Table 1 Information quantity and its value of SNES with invariant lower limit and variant upper limit of fuzzy demands

Solution	$(\underline{\Delta}, \overline{\Delta})$		
	(0.1,0.1)	(0.1,0.2)	(0.1,0.3)
$(q_{ij}^{1*})_{2\times 2}$	$\begin{bmatrix} 0.738\,751 & 0.740\,729 \\ 0.540\,398 & 0.746\,047 \end{bmatrix}$	$\begin{bmatrix} 0.745\,517 & 0.741\,164 \\ 0.540\,444 & 0.748\,228 \end{bmatrix}$	$\begin{bmatrix} 0.748\,187 & 0.741\,311 \\ 0.540\,460 & 0.748\,979 \end{bmatrix}$
$\left(q_{jk}^{2*}\right)_{2\times 2}$	$\begin{bmatrix} 0.332\,965 & 0.461\,255 \\ 0.354\,305 & 0.537\,711 \end{bmatrix}$	$\begin{bmatrix} 0.332\,965 & 0.461\,255 \\ 0.354\,305 & 0.537\,711 \end{bmatrix}$	$\begin{bmatrix} 0.332\,965 & 0.461\,255 \\ 0.354\,305 & 0.537\,711 \end{bmatrix}$
f_1	5.383 737	5.618 464	5.804 293
f_2	5.484 444	5.719 171	5.905 000
f_3	5.037 161	5.192 352	5.245 138
f_4	6.946 881	7.102 072	7.154 858
f_5	3.502 892	3.506 619	3.508 080
f_6	3.603 598	3.607 325	3.608 787
f_7	2.202 057	2.369 927	2.429 921
f_8	4.111 776	4.279 647	4.339 641
f_{alls}	36.272 547	37.395 578	37.995 718

As shown in Table 1 and Table 2, $f_1 - f_8$ are the expected information values of the engagement chains and f_{alls} is the total information value of the operation architecture super-network. As revealed in Table 1, with the increase of the upper limit of fuzzy demands, the information value of each engagement chain and the sum information value increase. In Table 2, with the increase of the lower limit of the fuzzy demands, the information value of each engagement chain and the sum of the information value decrease.

Table 2 Information quantity and its value of SNES with invariant upper limit and variant lower limit of fuzzy demands

Solution	$(\underline{\Delta},\overline{\Delta})$		
	(0.05,0.15)	(0.15,0.15)	(0.25,0.15)
$\left(q_{ij}^{1*}\right)_{2\times2}$	$\begin{bmatrix} 0.746\,082 & 0.746\,623 \\ 0.542\,023 & 0.753\,959 \end{bmatrix}$	$\begin{bmatrix} 0.739\,391 & 0.736\,535 \\ 0.535\,276 & 0.742\,692 \end{bmatrix}$	$\begin{bmatrix} 0.732\,455 & 0.728\,982 \\ 0.522\,958 & 0.737\,964 \end{bmatrix}$
$\left(q_{jk}^{2*}\right)_{2\times2}$	$\begin{bmatrix} 0.332\,965 & 0.461\,255 \\ 0.354\,305 & 0.537\,711 \end{bmatrix}$	$\begin{bmatrix} 0.332\,965 & 0.461\,255 \\ 0.354\,305 & 0.537\,711 \end{bmatrix}$	$\begin{bmatrix} 0.332\,965 & 0.461\,255 \\ 0.354\,305 & 0.537\,711 \end{bmatrix}$
f_1	5.450 146	5.429 956	5.412 106
f_2	5.550 852	5.530 662	5.512 813
f_3	5.153 658	5.125 557	5.099 989
f_4	7.063 378	7.035 277	7.009 708
f_5	3.728 150	3.403 675	3.309 222
f_6	3.828 857	3.504 381	3.409 929
f_7	2.290 700	2.326 105	2.334 961
f_8	4.200 420	4.235 825	4.244 681
f_{alls}	37.266 160	36.591 438	36.333 409

If $(\underline{\Delta},\overline{\Delta}) = (0.1, 0.2)$, three FDMs of CN1, CN2, FN1 and FN2 are fixed and one is variant. The change of the objective function value with fuzzy demands is shown in Fig. 2. When the FDM is certain, the relationship of the objective function value and the ICR is shown in Fig. 3.

If $(\underline{\Delta},\overline{\Delta}) = (0.1, 0.2)$, when the FDMs of FN1 and FN2 are fixed and the FDMs of CN1 and CN2 are variant, the change of the objective function value is shown in Fig. 4. Similarly, when the FDMs of CN1 and CN2 are fixed and the FDMs of FN1 and FN2 change, the change of the objective function value is shown in Fig. 5.

When the FDM and the ICR of FN1 and FN2 are fixed, and the ICRs of CN1 and CN2 change, the change of the objective function value is shown in Fig. 6. Similarly, when the ICRs of CN1 and CN2 are fixed and the ICRs of FN1 and FN2 change, the change of the objective function value is shown in Fig. 7.

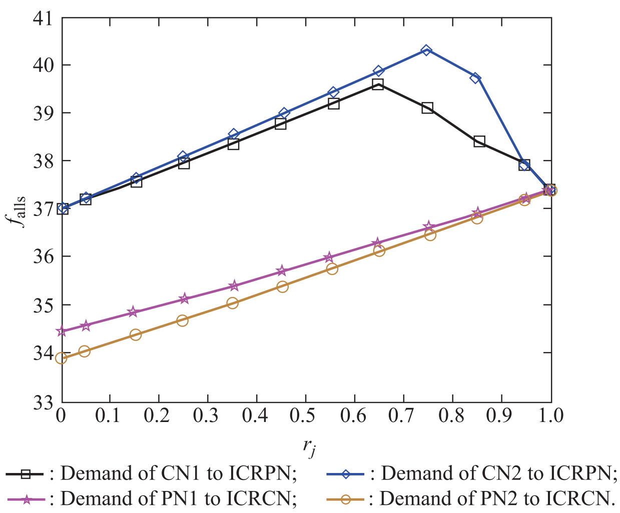


Fig. 3 Relationship between TIVN and ICR

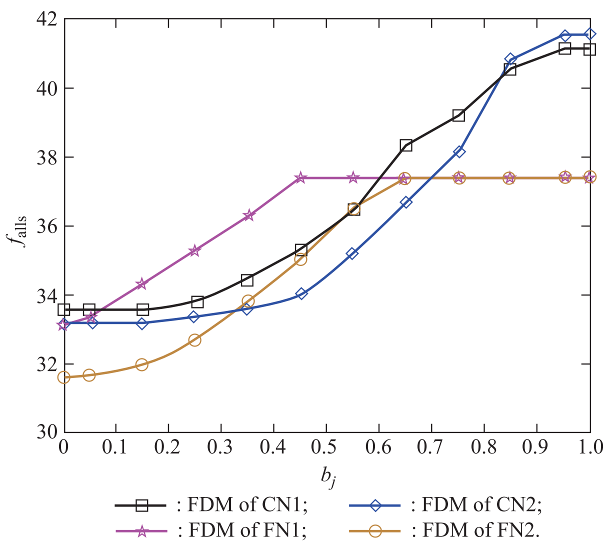


Fig. 2 Relationship between TIVN and FDM

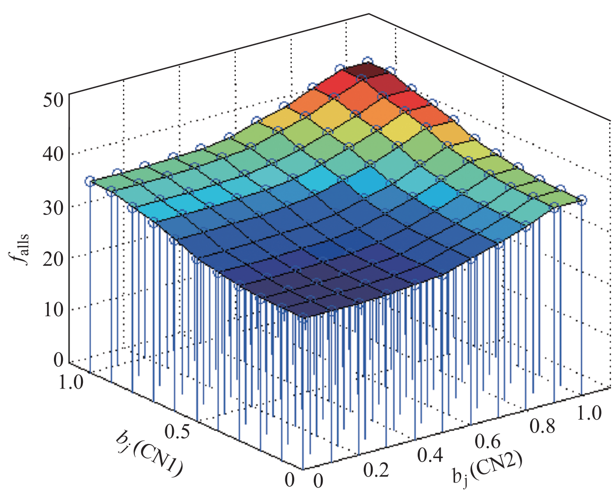


Fig. 4 Relationship between TIVN and FDM of CNs

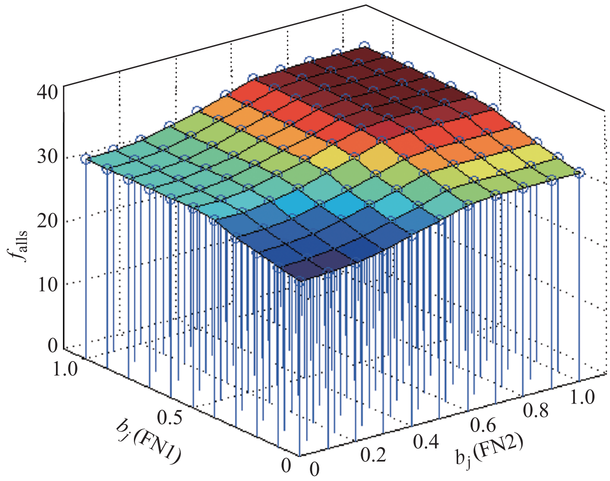


Fig. 5 Relationship between TIVN and FDM of FNs

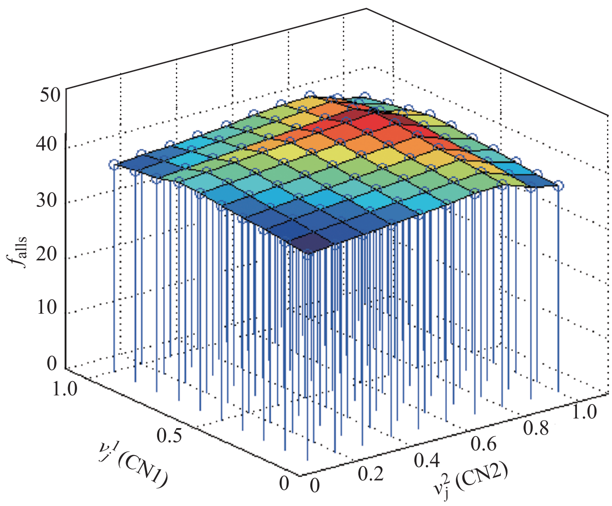


Fig. 6 Relationship between TIVN and ICRPN

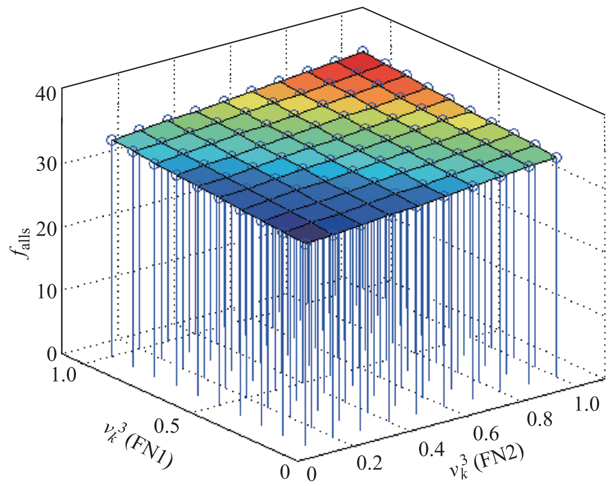


Fig. 7 Relationship between TIVN and ICRCN

The results of Fig. 2, Fig. 4 and Fig. 5 show that with the increase of the mean value of fuzzy demands, the information value increases. When the demand increases to a certain extent, the information value does not increase or grows slowly due to the limitation of the capacity of the information provider. As revealed in Fig. 3, Fig. 6 and Fig. 7, when the ICRPN increases to a certain extent, the value of the objective function is no longer improved. This is because the mean value of the fuzzy information demand is related to the ICR. If the ICR increases, the information quantity provided by PNs will increase. However, the increase of information will inevitably lead to the increase of information loss and redundancy, which will lead to the decrease of the objective function value of the whole network. For FNs, the value of the objective function will be improved with the increase of ICR. That is because the increase of the information value gain function is larger than the decrease of the objective function caused by the increase of information loss and redundancy.

When the fuzzy information demand function of the CN $\widetilde{d}_j(v_j^2)$ $(j=1,2)$, the fuzzy information demand function of the FN $\widetilde{d}_k(v_k^3)$ $(k=1,2)$, and the IVCC between the PN and CN, CN and FN v_{ij}^1, v_{jk}^2 $(i,j,k=1,2)$ are fixed, the relationship of the objective function value and the IVLC of the CN/FN are shown in Figs. 8 – 10.

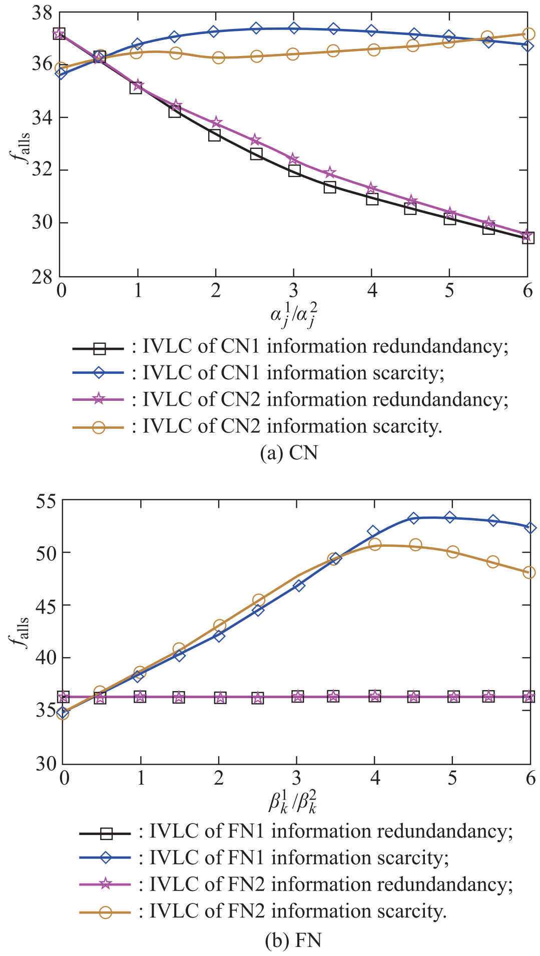
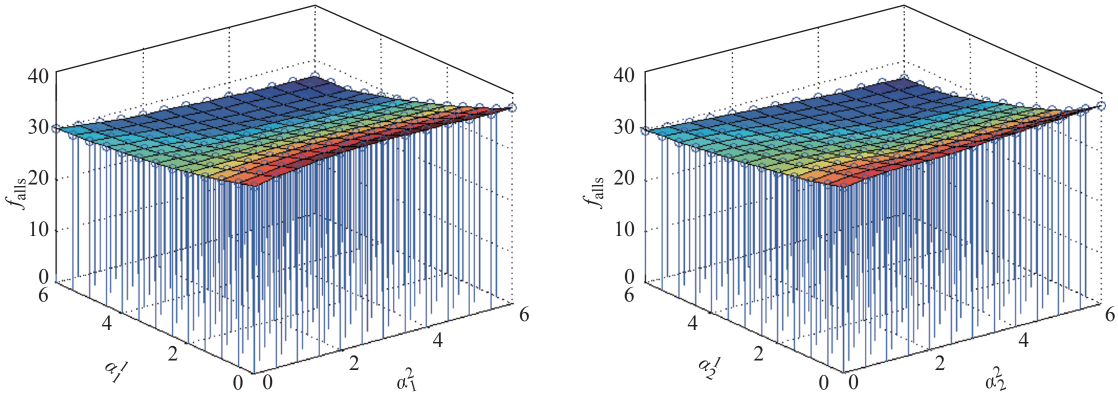
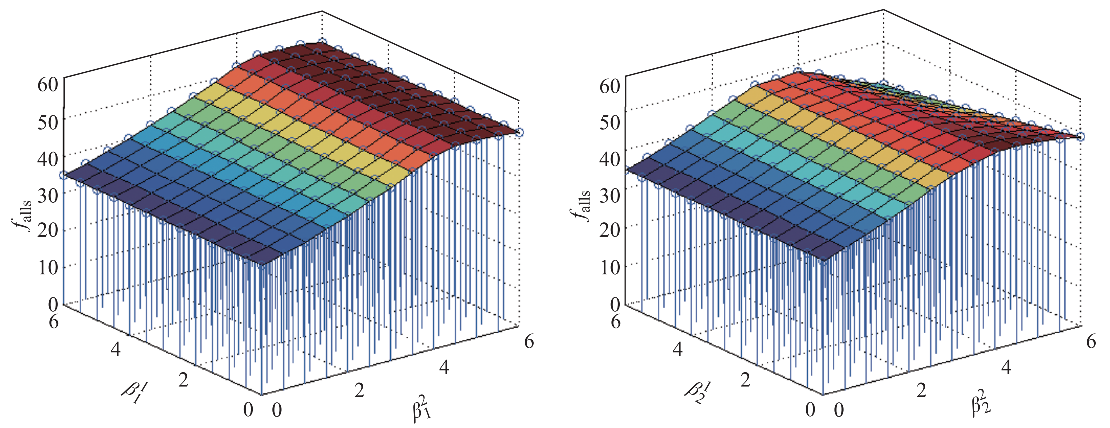


Fig. 8 Relationship between TIVN and IVLC



(a) CN1 (b) CN2

Fig. 9 Relationship between TIVN and IVLC of CN information redundancy/deficiency



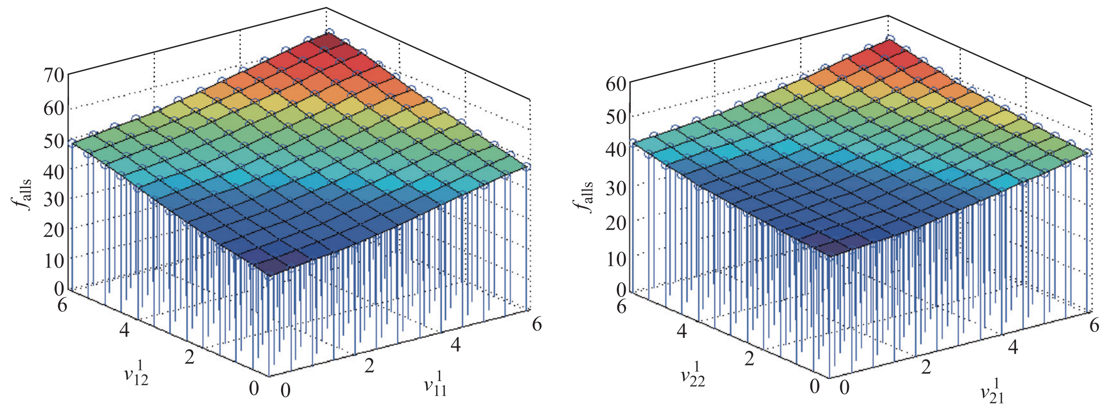
(a) FN1 (b) FN2

Fig. 10 Relationship between TIVN and IVLC of FN information redundancy/deficiency

The results of Fig. 8 and Fig. 9 show that, when the IVLC of information redundancy/deficiency between PN and CN, CN and FN changes slightly, the network information flow changes a little, and the change of the network information value is relatively stable. Fig. 10 shows that when IVLC of the information redundancy/deficiency between PN and CN, CN and FN changes greatly, the value of the network information changes greatly. Moreover, when the IVLC between nodes becomes larger, the network information value also becomes larger. This indicates that when the i IVLC between nodes becomes larger, the network information quantity optimization should avoid information deficiency and increase the information quantity as much as possible. Therefore, the value of network information is increased. This is consistent with the actual situation.

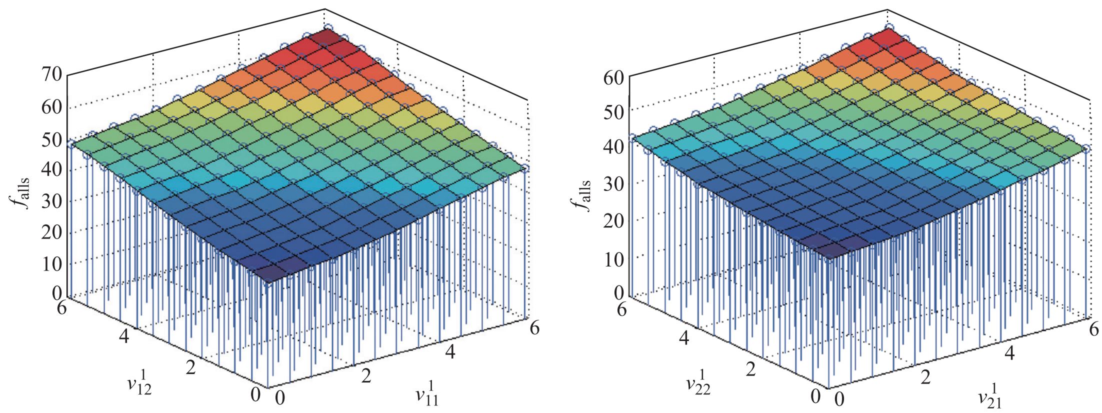
When the fuzzy demand function of the CN $\widetilde{d}_j(v_j^2)$ $(j = 1, 2)$ and the FN $\widetilde{d}_k(v_k^3)$ $(k = 1, 2)$, the IVLC of the information redundancy/deficiency $\alpha_j^1, \beta_k^1(\alpha_j^2, \beta_k^2)$ $(j, k = 1, 2)$ are certain, $v_{ij}^{1\max}$, $v_{jk}^{2\max}$ $(i = 1, 2; j = 1, 2; k = 1, 2)$ are set to 6 respectively, and the IVCC between PN and CN, CN and FN are variable. The overall network information value changes as shown in Figs. 11 – 13.

The result of Fig. 11(a) shows that the changes of IVCC between PN1 and CN1 has the greatest influence on the overall network information value, and the second influence is the IVCC between PN1 and CN2. There is a relatively stable impact on the network information value when the IVCC between PN2 and CN is less than 2. Similarly, the result of Fig. 11(b) shows that the change of the IVCC between CN1 and FN1 has the greatest impact on the value of the overall network information, and the second influence is CN1 to FN2 and CN2 to FN2. Moreover, the impact on the network information value of the IVCC between CN2 and FN1 is relatively stable. The results of Fig. 12 and Fig. 13 show that the value of the overall network information increases with the increment of the IVCC, which is consistent with the actual situation.



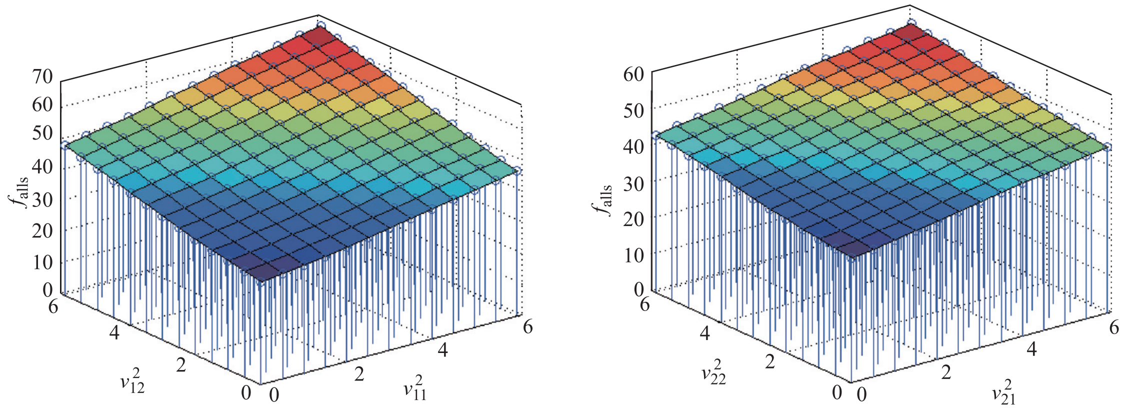
(a) PN to CN (b) CN to FN

Fig. 11 Relationship between TIVN and IVCC



(a) PN1 to CN (b) PN2 to CN

Fig. 12 Relationship between TIVN and IVCC of PN to CN



(a) CN1 to FN (b) CN2 to FN

Fig. 13 Relationship between TIVN and IVCC of CN to FN

5. Conclusions

In this paper, the optimization method of the super-network equilibrium for the operation architecture with fuzzy demands is studied. Firstly, the hypothesis of the information relationship modeling of the operation architecture engagement chain is given, the decision variables of the information relationship are analyzed, and the value function of the information relationship is constructed. Secondly, the optimization conditions of the perception level, the command level and the firepower level of the operation architecture engagement chain are analyzed respectively based on information fuzzy demands. On this basis, the information equilibrium model of the operation architecture super-network is established. Thirdly, the equilibrium model is analyzed and transformed using the theory of variational inequality, and the improved projection algorithm is used to solve the variational inequality optimization model. Finally, a case study is carried out to verify the proposed super-network equilibrium optimization method. The results show that there exists a state of information flow, which can make the generation and usage of information value for the operation architecture reach a balanced state, in which the operation architecture super-network gets the maximum benefit. Therefore, the balance of information of the operation architecture can be achieved by adjusting the information quantity transmitted among the perception level, the command level and the firepower level without changing the physical relationship of the operation architecture. At this time, the information utilization rate of the engagement chain is the highest. In addition, through the analysis and verification from multiple aspects, we can find the key links to improve the value of the overall network information, which provides support for super-network optimization. The research result of this paper is very useful for constructing the equilibrium and the steady state of the operation architecture engagement, and it has an important supporting role for the study of emergence properties and evolution properties of the operation architecture super-network.

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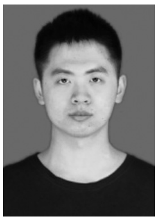

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