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Renormalization: single-photon processes of two-level system in free space

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Abstract: The investigation on quantum radar requires accurate computation of the state vectors of the single-photon processes of the two-level system in free space. However, the traditional Weisskopf-Wigner (W-W) theory fails to deal with those processes other than spontaneous emission. To solve this problem, we provide a new method based on the renormalization theory. We evaluate the renormalized time-ordered Green functions associated with the single-photon processes, and relate them to the corresponding state vectors. It is found that the ultraviolet divergences generated by the Lamb shift and higher-order interactions can be systematically subtracted in the state vectors. The discussions on spontaneous emission and single-photon absorption are then presented to illustrate the proposed method. For spontaneous emission, we obtain the same results of the W-W theory. For single-photon absorption where W-W theory fails, we find that the two-level electric dipole first gets excited rapidly and then decays exponentially, and that the efficiency of the single-photon absorption declines as the bandwidth of the incident photon becomes narrow. The proposed method can improve the investigation on quantum radar.

Keywords: single-photon process, two-level system (2LS), renormalization time-ordered Green function (RTGF), quantum radar.

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1. Introduction

Single-photon processes exist in numerous phenomena, such as Rabi flopping [1 – 3], resonance fluorescence [4 – 6], spontaneous emission [3], etc. In describing these processes, the two-level system (2LS) that a two-level electric dipole couples to the electromagnetic field via rotating-wave interaction is widely used. Even recently, the single-photon processes of the 2LS in free space are introduced in the study of quantum radar [7,8], where the target is regarded as a two-level electric dipoles. The dynamics of the single-photon processes of the 2LS are determined by the corresponding state vectors. Generally, if the 2LS is confined in a cavity with a finite size, the state vector can be obtained by solving the equation of motion (EOM) and is found oscillating periodically, as we see in Rabi flopping. However, things get subtle when the 2LS is in free space. The single-photon processes taking place in free space are usually irreversible, and therefore the state vector should contain terms that are damping rather than oscillating. However, this cannot be easily achieved, because the EOM in free space cannot be solved analytically, and the perturbative or numerical solutions of the EOM can only give oscillating results. To handle this problem, Weisskopf and Wigner introduced Markov approximation in solving the EOM for spontaneous emission and obtained the exponential decay of the excited electric dipole [3], which has been confirmed by experiments. All the ultraviolet divergences in the Weisskopf-Wigner (W-W) theory are owed to Lamb shift and are dropped explicitly. Nevertheless, the direct use of the W-W theory on single-photon absorption will lead to an unphysical result that the electric dipole keeps unexcited throughout the process, in contrast with which the electric dipole in reality will first get excited by absorbing the incident photon and will then decay by emitting another photon. Though there are other ways to investigate the single-photon processes of the 2LS in the one-dimensional system with infinite linear extension [9 – 20], the dynamics of the single-photon processes in free space except for spontaneous emission remain uncovered.

In this article, we shall not solve the EOM, but use renormalization theory [21 – 23] to study the single-photon processes of the 2LS in free space. In Section 2, we derive the renormalized Lagrangian and the corresponding Feynman rules in the mass-shell scheme by analyzing the bare time-ordered Green functions (BTGFs). In Section 3, the renormalized time-ordered Green functions (RTGFs) are computed with the renormalized Lagrangian and the Feynman rules, and the state vector is obtained as the superposition of the RTGFs. We find that the ultraviolet diver-

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gences originate not only from the Lamb shift but also from higher-order interactions, and that these divergences are systematically subtracted in the state vector. In Section 4, spontaneous emission and single-photon absorption are discussed to illustrate the proposed method. For spontaneous emission, our results are the same as the W-W theory. While for single-photon absorption, it is found that the surviving amplitude of the excited electric dipole first increases rapidly to its maximum and then gradually decays to zero, and that the efficiency of single-photon absorption declines as the bandwidth of the incident photon becomes narrow, which has not been noticed by the previous studies. As for quantum radar, we obtain a more accurate normalized energy density of the scattered quantum electromagnetic field (SQEF) than [7].

2. Renormalized Lagrangian and Feynman rules

Suppose a two-level electric dipole possesses a ground state $|0\rangle_d$ with the energy 0 and a bare excited state $|e\rangle$ with the energy $\hbar\omega_A$, and denote $|0\rangle_p$ and $|\boldsymbol{k},s\rangle$ as the vacuum state of the electromagnetic field and the single-photon state with the momentum $\boldsymbol{k}$ and the helicity index s, respectively. Then the dynamics of the 2LS are well described by the following Hamiltonian in the Schrödinger picture [3]:

$$H=\hbar\omega_A\widehat{a}^+\widehat{a}+\sum_{\boldsymbol{k},s}\hbar\omega_{\boldsymbol{k}}\widehat{b}^+_{\boldsymbol{k},s}\widehat{b}_{\boldsymbol{k},s}+$$
$$\mathrm{i}\hbar\sum_{\boldsymbol{k},s}\boldsymbol{\lambda}\cdot\boldsymbol{u}^*_{\boldsymbol{k},s}(0)\widehat{b}^+_{\boldsymbol{k},s}\widehat{a}-\mathrm{i}\hbar\sum_{\boldsymbol{k},s}\boldsymbol{\lambda}\cdot\boldsymbol{u}_{\boldsymbol{k},s}(0)\widehat{b}_{\boldsymbol{k},s}\widehat{a}^+ \quad (1)$$

where $\omega_{\boldsymbol{k}}=c|\boldsymbol{k}|$, $\boldsymbol{\lambda}$ is the bare moment of the electric dipole, and

$$\widehat{a}=|0\rangle_d\langle e|,\quad \widehat{b}_{\boldsymbol{k},s}=|0\rangle_p\langle\boldsymbol{k},s| \quad (2)$$

are the annihilation operators of $|e\rangle$ and $|\boldsymbol{k},s\rangle$, respectively. Confine the 2LS in a cubic with the volume V and impose periodic boundary conditions, then $\boldsymbol{u}_{\boldsymbol{k},s}$ reads

$$\boldsymbol{u}_{\boldsymbol{k},s}(\boldsymbol{r})=\sqrt{\frac{\hbar\omega_{\boldsymbol{k}}}{2V\varepsilon_0}}\boldsymbol{e}(\boldsymbol{k},s)\mathrm{e}^{\mathrm{i}\boldsymbol{k},\boldsymbol{r}},\quad s=\pm1 \quad (3)$$

with $\boldsymbol{e}(\boldsymbol{k},s)$ as the unit-polarization vector:

$$\begin{cases}\boldsymbol{e}(\boldsymbol{k},1)\cdot\boldsymbol{e}(\boldsymbol{k},-1)=\boldsymbol{e}(\boldsymbol{k},-1)\cdot\boldsymbol{k}=0\\ \boldsymbol{e}(\boldsymbol{k},1)\times\boldsymbol{e}(\boldsymbol{k},-1)=\boldsymbol{k}/|\boldsymbol{k}|\\ \displaystyle\sum_s e^\alpha(\boldsymbol{k},s)e^\beta(\boldsymbol{k},s)=\delta^{\alpha\beta}-\frac{k^\alpha k^\beta}{\boldsymbol{k}^2}\end{cases}. \quad (4)$$

where $\delta^{\alpha\beta}$ $(\alpha,\beta=1,2,3)$ is the Kronecker delta.

We will take $V\to\infty$ in the following to obtain a continuous spectrum of the electromagnetic field. By employing Hamilton's principle, one explicitly writes down the bare Lagrangian corresponding to (1) as

$$L=\hbar\varPhi^+_A(t)\left[\mathrm{i}\frac{\partial}{\partial t}-\omega_A\right]\varPhi_A(t)+$$
$$\hbar\sum_{\boldsymbol{k},s}\varPhi^+_{\boldsymbol{k},s}(t)\left[\mathrm{i}\frac{\partial}{\partial t}-\omega_{\boldsymbol{k}}\right]\varPhi_{\boldsymbol{k},s}(t)-$$
$$\mathrm{i}\hbar\sum_{\boldsymbol{k},s}\boldsymbol{\lambda}\cdot\boldsymbol{u}^*_{\boldsymbol{k},s}(0)\varPhi^+_{\boldsymbol{k},s}(t)\varPhi_A(t)+$$
$$\mathrm{i}\hbar\sum_{\boldsymbol{k},s}\boldsymbol{\lambda}\cdot\boldsymbol{u}_{\boldsymbol{k},s}(0)\varPhi_{\boldsymbol{k},s}(t)\varPhi^+_A(t) \quad (5)$$

where

$$\varPhi_A(t)=\mathrm{e}^{\mathrm{i}Ht/\hbar}\widehat{a}\mathrm{e}^{-\mathrm{i}Ht/\hbar}, \quad (6)$$

$$\varPhi_{\boldsymbol{k},s}(t)=\mathrm{e}^{\mathrm{i}Ht/\hbar}\widehat{b}_{\boldsymbol{k},s}\mathrm{e}^{-\mathrm{i}Ht/\hbar}. \quad (7)$$

Via functional derivation [21, 22], we achieve the corresponding Feynman rules as shown in Fig. 1. The straight line and the fold line on the first row come from the first and the second terms of (5), respectively. The solid vertices in the second row represent the interaction between the electric dipole and electromagnetic field. All the unfixed $\boldsymbol{k},s$ should be summed up.

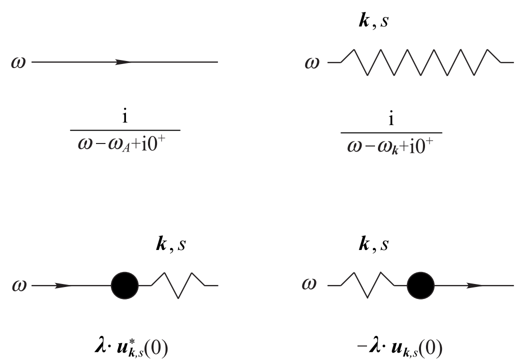


Fig. 1 Feynman rules for (5)

The BTGFs can be evaluated with the above Feynman rules. Consider $\int\mathrm{e}^{\mathrm{i}\omega t}\langle\mathrm{VAC}|T\{\varPhi_A(t)\varPhi^+_A(0)\}|\mathrm{VAC}\rangle\mathrm{d}t$ where $|\mathrm{VAC}\rangle=|0\rangle_d\otimes|0\rangle_p$. The non-vanishing Feynman diagrams are illustrated in Fig. 2.

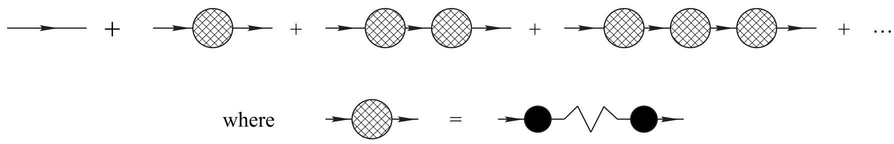


Fig. 2 Feynman diagram of $\int \mathrm{e}^{\mathrm{i}\omega t}\left\langle\mathrm{VAC}\right| T\{\varPhi_A(t)\varPhi_A^+(0)\}\left|\mathrm{VAC}\right\rangle \mathrm{d}t$

It is straightforward to obtain

$$\int \mathrm{e}^{\mathrm{i}\omega t}\left\langle\mathrm{VAC}\right| T\{\varPhi_A(t)\varPhi_A^+(0)\}\left|\mathrm{VAC}\right\rangle \mathrm{d}t =$$

$$\frac{\mathrm{i}}{\omega-\omega_A+\mathrm{i}0^+}\cdot$$

$$\sum_{N=0}^{\infty}\left(\sum_{\boldsymbol{k},s}\frac{-\mathrm{i}\left|\boldsymbol{\lambda}\cdot\boldsymbol{u}_{\boldsymbol{k},s}(0)\right|^2}{\omega-\omega_{\boldsymbol{k}}+\mathrm{i}0^+}\frac{\mathrm{i}}{\omega-\omega_A+\mathrm{i}0^+}\right)^N =$$

$$\frac{\mathrm{i}}{\omega-\omega_A-\sum\limits_{\boldsymbol{k},s}\dfrac{\left|\boldsymbol{\lambda}\cdot\boldsymbol{u}_{\boldsymbol{k},s}(0)\right|^2}{\omega-\omega_{\boldsymbol{k}}+\mathrm{i}0^+}}. \tag{8}$$

Replace $\dfrac{1}{V}\sum\limits_{\boldsymbol{k}}$ with $\displaystyle\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}$, then the denominator of the right hand side of (8) reads

$$D(\omega)=\omega-\omega_A-\sum_s\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\frac{\left|\boldsymbol{\lambda}\cdot\boldsymbol{e}(\boldsymbol{k},s)\right|^2}{\omega-\omega_{\boldsymbol{k}}+\mathrm{i}0^+}\frac{\hbar\omega_{\boldsymbol{k}}}{2\varepsilon_0}. \tag{9}$$

The zero point of $\operatorname{Re}D(\omega)$ is the physical frequency ω_{AR} of the excited state:

$$\omega_{AR}-\omega_A-\sum_s P\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{\left|\boldsymbol{\lambda}\cdot\boldsymbol{e}(\boldsymbol{k},s)\right|^2}{\omega_{AR}-\omega_{\boldsymbol{k}}}\frac{\hbar\omega_{\boldsymbol{k}}}{2\varepsilon_0}=0 \tag{10}$$

with $P\displaystyle\int\dots$ as the principle-valued integral. Expand $\operatorname{Re}D(\omega)$ around $\omega=\omega_{AR}$:

$$\operatorname{Re}D(\omega)=\omega-\omega_A-C_0(\lambda^2)-C_1(\lambda^2)(\omega-\omega_{AR})-$$

$$C_2(\lambda^2)(\omega-\omega_{AR})^2-C_3(\lambda^2)(\omega-\omega_{AR})^3+F(\lambda^2,\omega) \tag{11}$$

where

$$\begin{cases} C_0(\lambda^2)=\sum\limits_s P\displaystyle\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\frac{\left|\boldsymbol{\lambda}\cdot\boldsymbol{e}(\boldsymbol{k},s)\right|^2}{\omega_{AR}-\omega_{\boldsymbol{k}}}\frac{\hbar\omega_{\boldsymbol{k}}}{2\varepsilon_0}\\ C_1(\lambda^2)=-\sum\limits_s P\displaystyle\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\frac{\left|\boldsymbol{\lambda}\cdot\boldsymbol{e}(\boldsymbol{k},s)\right|^2}{(\omega_{AR}-\omega_{\boldsymbol{k}})^2}\frac{\hbar\omega_{\boldsymbol{k}}}{2\varepsilon_0}\\ C_2(\lambda^2)=\sum\limits_s P\displaystyle\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\frac{\left|\boldsymbol{\lambda}\cdot\boldsymbol{e}(\boldsymbol{k},s)\right|^2}{(\omega_{AR}-\omega_{\boldsymbol{k}})^3}\frac{\hbar\omega_{\boldsymbol{k}}}{2\varepsilon_0}\\ C_3(\lambda^2)=-\sum\limits_s P\displaystyle\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\frac{\left|\boldsymbol{\lambda}\cdot\boldsymbol{e}(\boldsymbol{k},s)\right|^2}{(\omega_{AR}-\omega_{\boldsymbol{k}})^4}\frac{\hbar\omega_{\boldsymbol{k}}}{2\varepsilon_0} \end{cases}, \tag{12}$$

$$F(\lambda^2,\omega)=\frac{\hbar\lambda^2}{\pi^2c^3\varepsilon_0}\left(\frac{11}{48}\omega^3-\frac{3}{8}\omega^2\omega_{AR}+\frac{3}{16}\omega\omega_{AR}^2-\right.$$

$$\left.\frac{1}{24}\omega_{AR}^3-\frac{1}{8}\omega^3\ln\frac{\omega}{\omega_{AR}}\right). \tag{13}$$

It is obvious that not only C_0, which is exactly the ultraviolet divergence owed to the Lamb shift, but also C_1, C_2 and C_3 diverge when $\boldsymbol{k}$ is integrated over $|\boldsymbol{k}|\gg\omega_{AR}$. In fact, the latter three terms come from higher-order interactions between the electric dipole and the electromagnetic field, and are ignored in the W-W theory. Though these divergences may interfere with the validity of our calculation, they can be subtracted by introducing the renormalized Lagrangian, so that the RTGFs and thus the state vector are well-defined. Specifically, the renormalized Lagrangian, as suggested by the renormalization theory, should contain four species of counter terms to eliminate C_n for $n=0,1,2,3$, respectively.

$$L_R=\hbar\varPhi_{AR}^+(t)\left[\mathrm{i}\frac{\partial}{\partial t}-\omega_{AR}\right]\varPhi_{AR}(t)+$$

$$\hbar\sum_{\boldsymbol{k},s}\varPhi_{\boldsymbol{k},s}^+(t)\left[\mathrm{i}\frac{\partial}{\partial t}-\omega_{\boldsymbol{k}}\right]\varPhi_{\boldsymbol{k},s}(t)-$$

$$\mathrm{i}\hbar\sum_{\boldsymbol{k},s}\boldsymbol{\lambda}_R\cdot\boldsymbol{u}_{\boldsymbol{k},s}^*(0)\varPhi_{\boldsymbol{k},s}^+(t)\varPhi_{AR}(t)+$$

$$\mathrm{i}\hbar\sum_{\boldsymbol{k},s}\boldsymbol{\lambda}_R\cdot\boldsymbol{u}_{\boldsymbol{k},s}(0)\varPhi_{\boldsymbol{k},s}(t)\varPhi_{AR}^+(t)+$$

$$\hbar\sum_{n=0}^{3}D_n\varPhi_{AR}^+(t)\left[\mathrm{i}\frac{\partial}{\partial t}-\omega_{AR}\right]^n\varPhi_{AR}(t) \tag{14}$$

where the last line are the counter terms with D_n to be determined. Then the denominator of $\int \mathrm{e}^{\mathrm{i}\omega t}\left\langle\mathrm{VAC}\right| T\{\varPhi_{AR}(t)\varPhi_{AR}^+(0)\}\left|\mathrm{VAC}\right\rangle \mathrm{d}t$, analogous to (10), reads

$$D_R(\omega)\equiv\frac{\mathrm{i}}{\int \mathrm{e}^{\mathrm{i}\omega t}\left\langle\mathrm{VAC}\right| T\{\varPhi_{AR}(t)\varPhi_{AR}^+(0)\}\left|\mathrm{VAC}\right\rangle \mathrm{d}t}=$$

$$\omega-\omega_{AR}-\sum_s\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\frac{\left|\boldsymbol{\lambda}_R\cdot\boldsymbol{e}(\boldsymbol{k},s)\right|^2}{\omega-\omega_{\boldsymbol{k}}+\mathrm{i}0^+}\frac{\hbar\omega_{\boldsymbol{k}}}{2\varepsilon_0}+$$

$$\sum_{n=0}^{3}D_n(\omega-\omega_{AR})^n=\omega-\omega_{AR}+F(\lambda_R^2,\omega)+$$

$$\mathrm{Im}\, D_R(\omega) + \sum_{n=0}^{3}[D_n - C_n(\lambda_R^2)](\omega - \omega_{AR})^n. \quad (15)$$

Next, we shall take the mass shell scheme that $\boldsymbol{\lambda}_R$ and $\Phi_{AR}(t)$ are defined as the physical moment and the physical operator of the electric dipole, respectively. This demands $\mathrm{Re}\, D_R(\omega) = \omega - \omega_{AR} + F(\lambda_R^2, \omega)$, and thus $D_n = C_n(\lambda_R^2)$. The Feynman rules for the renormalized Lagrangian are therefore given in Fig. 3. The shaded vertex in the third row originates from the counter terms. Remark that, since $\mathrm{Im}\, D_R(\omega)$ is finite with the notion $\mathrm{Im}\,(\omega - \omega_{\boldsymbol{k}} + \mathrm{i}0^+)^{-1} = -\mathrm{i}\pi\delta(\omega - \omega_{\boldsymbol{k}})$, $\int \mathrm{e}^{\mathrm{i}\omega t} \langle \mathrm{VAC}| T\{\Phi_{AR}(t)\Phi_{AR}^+(0)\} |\mathrm{VAC}\rangle\, \mathrm{d}t$ is indeed free from ultraviolet divergences.

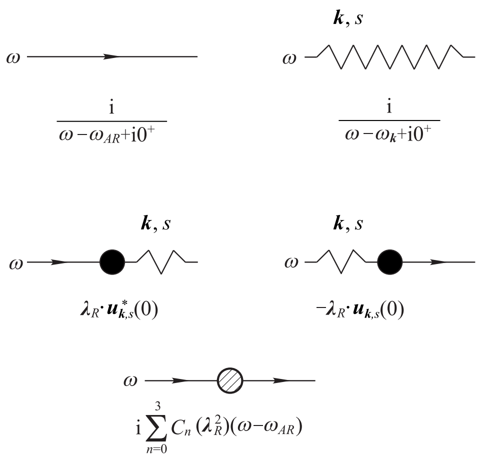


Fig. 3 Feynman rules for (14)

3. RTGFs and state vector

The state vector $|\boldsymbol{\psi}(t)\rangle$ describing the arbitrary single-photon process of the 2LS can be decomposed as the superposition of $|e\rangle$ and $|\boldsymbol{k}, s\rangle$.

$$|\boldsymbol{\psi}(t)\rangle = C_e(t)\,|e\rangle + \sum_{\boldsymbol{k},s} C_{\boldsymbol{k},s}(t)\,|\boldsymbol{k}, s\rangle \quad (16)$$

where $C_e(t)$ and $C_{\boldsymbol{k},s}(t)$ are the surviving amplitudes of $|e\rangle$ and $|\boldsymbol{k}, s\rangle$, respectively. Noting that, in the mass-shell scheme,

$$\Phi_{AR}^+(0)\,|\mathrm{VAC}\rangle = |e\rangle \quad (17)$$

$$\Phi_{\boldsymbol{k},s}^+(0)\,|\mathrm{VAC}\rangle = |\boldsymbol{k}, s\rangle \quad (18)$$

where $|\mathrm{VAC}\rangle = |0\rangle_d\,|0\rangle_p$ is the vacuum state of the system, and one has

$$C_e(t) = C_e(0)\,\langle \mathrm{VAC}| T\{\Phi_{AR}(t)\Phi_{AR}^+(0)\} |\mathrm{VAC}\rangle + \sum_{\boldsymbol{k}',s'} C_{\boldsymbol{k}',s'}(0)\,\langle \mathrm{VAC}| T\{\Phi_{AR}(t)\Phi_{\boldsymbol{k}',s'}^+(0)\} |\mathrm{VAC}\rangle\,, \quad (19)$$

$$C_{\boldsymbol{k},s}(t) = C_e(0)\,\langle \mathrm{VAC}| T\{\Phi_{\boldsymbol{k},s}(t)\Phi_{AR}^+(0)\} |\mathrm{VAC}\rangle + \sum_{\boldsymbol{k}',s'} C_{\boldsymbol{k}',s'}(0)\,\langle \mathrm{VAC}| T\{\Phi_{\boldsymbol{k},s}(t)\Phi_{\boldsymbol{k}',s'}^+(0)\} |\mathrm{VAC}\rangle\,. \quad (20)$$

Therefore, the four RTGFs in the right hand side of (19) and (20) are the key to evaluating $|\boldsymbol{\psi}(t)\rangle$. Fig. 4 shows the corresponding Feynman diagrams of these RTGFs, then we get

$$\int \mathrm{e}^{\mathrm{i}\omega t}\,\langle \mathrm{VAC}| T\{\Phi_{AR}(t)\Phi_{AR}^+(0)\} |\mathrm{VAC}\rangle\,\mathrm{d}t = \frac{\mathrm{i}}{\omega - \omega_{AR} + F(\lambda_R^2, \omega) + \mathrm{i}\Gamma(\omega)/2}, \quad (21)$$

$$\int \mathrm{e}^{\mathrm{i}\omega t}\,\langle \mathrm{VAC}| T\{\Phi_{\boldsymbol{k},s}(t)\Phi_{AR}^+(0)\} |\mathrm{VAC}\rangle\,\mathrm{d}t = \frac{\mathrm{i}}{\omega - \omega_{AR} + F(\lambda_R^2, \omega) + \mathrm{i}\Gamma(\omega)/2}\boldsymbol{\lambda}_R\cdot \boldsymbol{u}_{\boldsymbol{k},s}^*(0)\frac{\mathrm{i}}{\omega - \omega_{\boldsymbol{k}} + \mathrm{i}0^+}, \quad (22)$$

$$\int \mathrm{e}^{\mathrm{i}\omega t}\,\langle \mathrm{VAC}| T\{\Phi_{AR}(t)\Phi_{\boldsymbol{k}',s'}^+(0)\} |\mathrm{VAC}\rangle\,\mathrm{d}t = -\frac{\mathrm{i}}{\omega - \omega_{\boldsymbol{k}'} + \mathrm{i}0^+}\boldsymbol{\lambda}_R\cdot \boldsymbol{u}_{\boldsymbol{k}',s'}(0)\frac{\mathrm{i}}{\omega - \omega_{AR} + F(\lambda_R^2, \omega) + \mathrm{i}\Gamma(\omega)/2}, \quad (23)$$

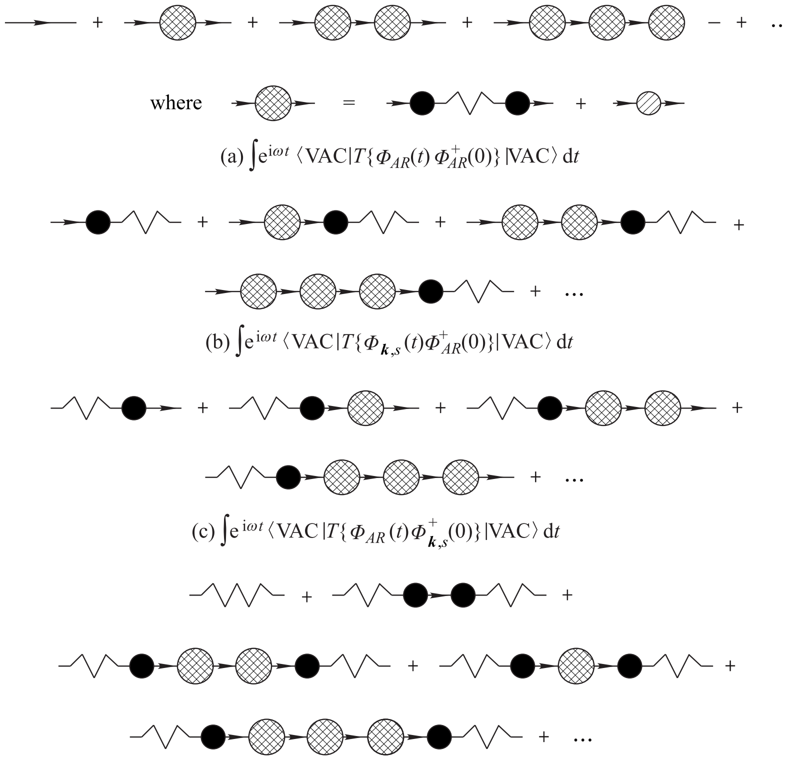
$$\int \mathrm{e}^{\mathrm{i}\omega t}\,\langle \mathrm{VAC}| T\{\Phi_{\boldsymbol{k},s}(t)\Phi_{\boldsymbol{k}',s'}^+(0)\} |\mathrm{VAC}\rangle\,\mathrm{d}t = -\frac{\mathrm{i}}{\omega - \omega_{\boldsymbol{k}'} + \mathrm{i}0^+}\boldsymbol{\lambda}_R\cdot \boldsymbol{u}_{\boldsymbol{k}',s'}(0)\frac{\mathrm{i}}{\omega - \omega_{AR} + F(\lambda_R^2, \omega) + \mathrm{i}\Gamma(\omega)/2}\cdot \boldsymbol{\lambda}_R\cdot \boldsymbol{u}_{\boldsymbol{k},s}^*(0)\frac{\mathrm{i}}{\omega - \omega_{\boldsymbol{k}} + \mathrm{i}0^+} + \frac{\mathrm{i}}{\omega - \omega_{\boldsymbol{k}} + \mathrm{i}0^+}\delta_{\boldsymbol{k}\boldsymbol{k}'}\delta_{ss'} \quad (24)$$

where

$$\Gamma(\omega) = 2\pi \sum_s \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\,|\boldsymbol{\lambda}\cdot \boldsymbol{e}(\boldsymbol{k}, s)|^2\,\frac{\hbar\omega_{\boldsymbol{k}}}{2\varepsilon_0}\delta(\omega - \omega_{\boldsymbol{k}}) = \frac{\hbar\lambda_R^2\omega^3}{3\pi\varepsilon_0 c^3}. \quad (25)$$

In the above computation, all the ultraviolet divergences are subtracted by the counter terms in (14), and therefore the RTGFs and $|\boldsymbol{\psi}(t)\rangle$ are well-defined.

Once $C_e(0)$ and $C_{\boldsymbol{k},s}(0)$ are given, $C_e(t)$, $C_{\boldsymbol{k},s}(t)$, and hence $|\boldsymbol{\psi}(t)\rangle$ can be obtained explicitly by substituting (21)–(25) into (19) and (20).



(a) $\int \mathrm{e}^{\mathrm{i}\omega t} \langle \mathrm{VAC}| T\{\Phi_{AR}(t)\,\Phi^{+}_{AR}(0)\} |\mathrm{VAC}\rangle\, \mathrm{d}t$

(b) $\int \mathrm{e}^{\mathrm{i}\omega t} \langle \mathrm{VAC}| T\{\Phi_{\boldsymbol{k},s}(t)\Phi^{+}_{AR}(0)\} |\mathrm{VAC}\rangle\, \mathrm{d}t$

(c) $\int \mathrm{e}^{\mathrm{i}\omega t} \langle \mathrm{VAC}| T\{\Phi_{AR}(t)\Phi^{+}_{\boldsymbol{k},s}(0)\} |\mathrm{VAC}\rangle\, \mathrm{d}t$

(d) $\int \mathrm{e}^{\mathrm{i}\omega t} \langle \mathrm{VAC}| T\{\Phi_{\boldsymbol{k},s}(t)\Phi^{+}_{\boldsymbol{k}',s'}(0)\} |\mathrm{VAC}\rangle\, \mathrm{d}t$

Fig. 4 Feynman diagrams of the considered RTGFs

4. Spontaneous emission and single-photon absorption

It is straightforward but quite lengthy to evaluate $C_e(t)$ and $C_{\boldsymbol{k},s}(t)$ numerically. However, in free space, $\boldsymbol{\lambda}_R$ is small enough for one to apply the well-known Breit-Wigner approximation [21, 22].

$$\Gamma(\omega) \approx \Gamma := \frac{\hbar\lambda_R^2\omega_{AR}^3}{3\pi\varepsilon_0 c^3}, \quad F(\lambda_R^2, \omega) \approx 0 \tag{26}$$

where Γ denotes the decay width of the excited state. Substituting (26) into (21)–(24) and performing the Fourier transformations, we have

$$\langle \mathrm{VAC}| T\{\Phi_{AR}(t)\Phi^{+}_{AR}(0)\} |\mathrm{VAC}\rangle = \mathrm{e}^{-\mathrm{i}\omega_{AR}t-\Gamma t/2}, \tag{27}$$

$$\langle \mathrm{VAC}| T\{\Phi_{\boldsymbol{k},s}(t)\Phi^{+}_{AR}(0)\} |\mathrm{VAC}\rangle = \mathrm{i}\boldsymbol{\lambda}_R \cdot \boldsymbol{u}^*_{\boldsymbol{k},s}(0)\frac{\mathrm{e}^{-\mathrm{i}\omega_{AR}t-\Gamma t/2} - \mathrm{e}^{-\mathrm{i}\omega_{\boldsymbol{k}}t}}{\omega_{AR} - \omega_{\boldsymbol{k}} - \mathrm{i}\Gamma/2}, \tag{28}$$

$$\langle \mathrm{VAC}| T\{\Phi_{AR}(t)\Phi^{+}_{\boldsymbol{k}',s'}(0)\} |\mathrm{VAC}\rangle = -\mathrm{i}\boldsymbol{\lambda}_R \cdot \boldsymbol{u}_{\boldsymbol{k}',s'}(0)\frac{\mathrm{e}^{-\mathrm{i}\omega_{AR}t-\Gamma t/2} - \mathrm{e}^{-\mathrm{i}\omega'_{\boldsymbol{k}}t}}{\omega_{AR} - \omega'_{\boldsymbol{k}} - \mathrm{i}\Gamma/2}, \tag{29}$$

$$\langle \mathrm{VAC}| T\{\Phi_{\boldsymbol{k},s}(t)\Phi^{+}_{\boldsymbol{k}',s'}(0)\} |\mathrm{VAC}\rangle = \mathrm{e}^{-\mathrm{i}\omega_{\boldsymbol{k}}t}\delta_{ss'}\delta_{\boldsymbol{k}\boldsymbol{k}'} + \boldsymbol{\lambda}_R \cdot \boldsymbol{u}_{\boldsymbol{k}',s'}(0)\boldsymbol{\lambda}_R \cdot \boldsymbol{u}^*_{\boldsymbol{k},s}(0)\left[\frac{1}{\omega_{\boldsymbol{k}} - \omega'_{\boldsymbol{k}}}\frac{1}{\omega_{AR} - \omega'_{\boldsymbol{k}} - \mathrm{i}\Gamma/2}\mathrm{e}^{-\mathrm{i}\omega'_{\boldsymbol{k}}t} - \frac{1}{\omega_{\boldsymbol{k}} - \omega'_{\boldsymbol{k}}}\frac{1}{\omega_{AR} - \omega_{\boldsymbol{k}} - \mathrm{i}\Gamma/2}\mathrm{e}^{-\mathrm{i}\omega_{\boldsymbol{k}}t} + \frac{1}{\omega_{AR} - \omega_{\boldsymbol{k}} - \mathrm{i}\Gamma/2}\frac{1}{\omega_{AR} - \omega'_{\boldsymbol{k}} - \mathrm{i}\Gamma/2}\mathrm{e}^{-\mathrm{i}\omega_{AR}t-\Gamma t/2}\right]. \tag{30}$$

Note that since there may exsit a virtual state rather than a resonance [24–28] for some $|\boldsymbol{\lambda}_R|$, which would violate Breit-Wigner approximation, and we assume that $|\boldsymbol{\lambda}_R|$ is sufficiently small to satisfy (26).

In the following, we shall discuss spontaneous emission and single-photon absorption with the aid of (27)–(30).

(i) Spontaneous emission. For spontaneous emission, the incident condition reads

$$C_e(0) = 1, \quad C_{\boldsymbol{k},s}(0) = 0. \tag{31}$$

Combining (19), (20), (27), (28) and (31), one has

$$C_e(t) = \mathrm{e}^{-\mathrm{i}\omega_{AR}t-\Gamma t/2}, \tag{32}$$

$$C_{\boldsymbol{k},s}(t) = \mathrm{i}\boldsymbol{\lambda}_R \cdot \boldsymbol{u}^*_{\boldsymbol{k},s}(0)\frac{\mathrm{e}^{-\mathrm{i}\omega_{AR}t-\Gamma t/2} - \mathrm{e}^{-i\omega_{\boldsymbol{k}}t}}{\omega_{AR} - \omega_{\boldsymbol{k}} - \mathrm{i}\Gamma/2}. \tag{33}$$

Indeed, (32) and (33) agree with the results achieved by the W-W theory [3] exactly.

(ii) Single-photon absorption. For single-photon absorption where the W-W theory fails, the electric dipole is initially on the ground state, and the incident photon is prepared in a wave packet.

$$C_e(0) = 0,\ \ C_{\boldsymbol{k},s}(0) = \frac{1}{\sqrt{V}} f(\boldsymbol{k}, s) \tag{34}$$

where $f(\boldsymbol{k}, s)$ is a smooth function with respect to $\boldsymbol{k}$, satisfying

$$\sum_s \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} \left|f(\boldsymbol{k}, s)\right|^2 = 1. \tag{35}$$

Analogous to spontaneous emission, we can obtain

$$C_e(t; f) = \frac{1}{\sqrt{V}} \sum_{\boldsymbol{k}', s'} f(\boldsymbol{k}', s') \cdot$$

$$\langle \mathrm{VAC}|\, T\{\varPhi_{AR}(t)\varPhi^{+}_{\boldsymbol{k}',s'}(0)\}\, |\mathrm{VAC}\rangle\,, \tag{36}$$

$$C_{\boldsymbol{k},s}(t; f) = \frac{1}{\sqrt{V}} \sum_{\boldsymbol{k}', s'} f(\boldsymbol{k}', s') \cdot$$

$$\langle \mathrm{VAC}|\, T\{\varPhi_{\boldsymbol{k},s}(t)\varPhi^{+}_{\boldsymbol{k}',s'}(0)\}\, |\mathrm{VAC}\rangle\,. \tag{37}$$

Remark that, no matter what form $f(\boldsymbol{k}, s)$ takes, it is governed by the Riemann-Lebesgue lemma that

$$\lim_{t\to\infty} C_e(t; f) = 0, \quad \forall f. \tag{38}$$

Equation (38) indicates that the electric dipole first gets excited by absorbing the incident photon and eventually decays with the emission of another photon, which is consistent with the physics of single-photon absorption.

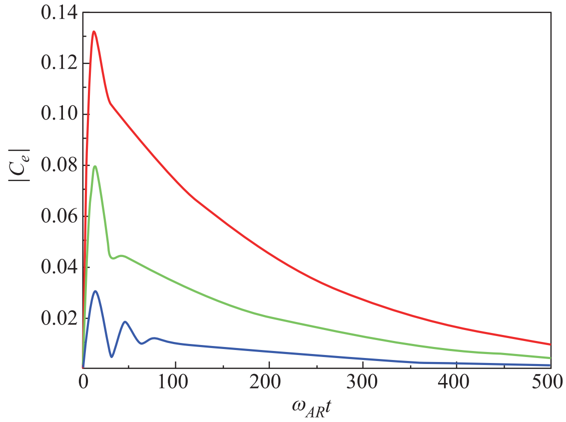
In particular, consider that the incident photon is prepared in a Gaussian packet with $s = +1$ helicity,

$$f(\boldsymbol{k}, s) = \left(\frac{4\pi}{\alpha}\right)^{\frac{3}{4}} \exp\{-\frac{(\boldsymbol{k}-\boldsymbol{q}_0)^2}{2\alpha}\}\delta_{s+1}. \tag{39}$$

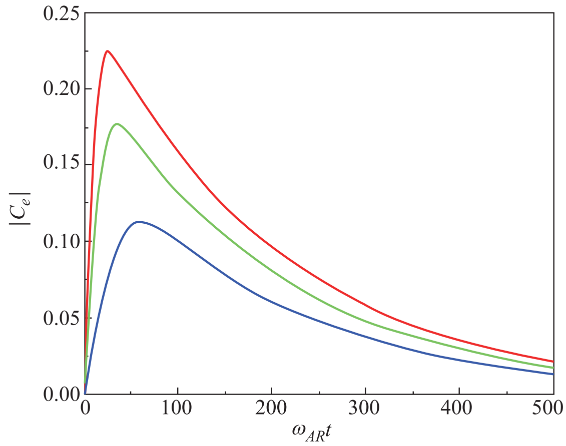
The coordinate is chosen such that $\boldsymbol{q}_0$ is along the z-axis and $\boldsymbol{\lambda}_R$ is written as

$$\boldsymbol{\lambda} = \lambda(\sin\delta, 0, \cos\delta) \tag{40}$$

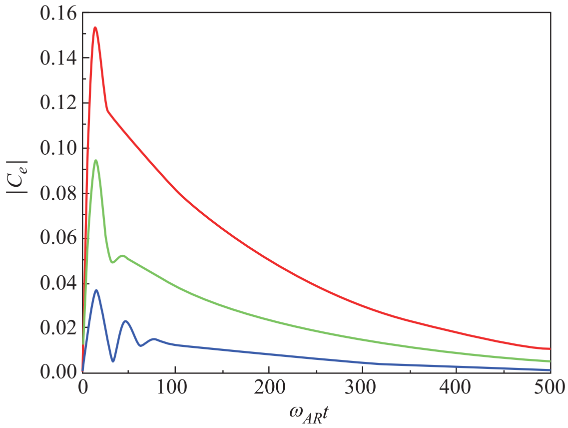
where δ is the intersecting angle between $\boldsymbol{\lambda}_R$ and $\boldsymbol{q}_0$. By combining (29), (36), (39) and (40), we get $|C_e(t; f)|$ for different α and $|\boldsymbol{q}_0|$ and plot them in Fig. 5.



(a) $|\boldsymbol{q}_0| = 0.8\omega_{AR}c^{-1}$, $\Gamma = 0.01\omega_{AR}$, $\delta = \pi/2$



(b) $|\boldsymbol{q}_0| = \omega_{AR}c^{-1}$, $\Gamma = 0.01\omega_{AR}$, $\delta = \pi/2$



(c) $|\boldsymbol{q}_0| = 1.2\omega_{AR}c^{-1}$, $\Gamma = 0.01\omega_{AR}$, $\delta = \pi/2$

: $\alpha = 0.009\omega_{AR}^2 C^{-2}$; : $\alpha = 0.004\omega_{AR}^2 C^{-2}$; : $\alpha = 0.001\omega_{AR}^2 C^{-2}$.

Fig. 5 $|C_e(t; f)|$ for different Gaussian packets

It is easy to see that

(i) $|C_e(t; f)|$ arises rapidly to its maximum (< 1) and then decays slowly.

(ii) The non-vanishing detuning $\varDelta = \omega_{AR} - |\boldsymbol{q}_0|\,c$, shown in Fig. 5(a) and Fig. 5(c), causes a dissipative oscillation of $|C_e(t; f)|$. This phenomenon, analogous to Rabi flopping, comes from reemission and reabsorption of a single photon.

(iii) The efficiency of absorption of the incident photon declines as $\alpha \to 0$, i.e., as the bandwidth of the incident photon becomes narrow. This phenomenon has not been noticed by previous investigations, and is easy to be understood: the density of the photon around the electric dipole is reciprocal to the bandwidth of the photon, and is proportional to the strength of the interaction between the photon and the electric dipole, and therefore a smaller bandwidth of the incident photon yields stronger absorption by the electric dipole.

The investigation on quantum radar concerns the normalized energy density of the SQEF. It is obvious that the scattering of the incident photon by the target is just single-photon absorption of the 2LS by regarding the target as a two-level electric dipole, and therefore the normalized energy density can be obtained from (37). Specifically, note that the first and the second term and of the right hand side of (29) come from the incident photon and the SQEF, respectively, and thus the electromagnetic component of the state vector can be divided into two parts:

$$|\psi_{SC}(t;f)\rangle = \sum_{\boldsymbol{k},s}\left[C_{\boldsymbol{k},s}(t;f) - \frac{1}{\sqrt{V}} f(\boldsymbol{k},s)\mathrm{e}^{-\mathrm{i}\omega_{\boldsymbol{k}}t}\right]|\boldsymbol{k},s\rangle, \tag{41}$$

$$|\psi_{IN}(t;f)\rangle = \frac{1}{\sqrt{V}}\sum_{\boldsymbol{k},s} f(\boldsymbol{k},s)\mathrm{e}^{-\mathrm{i}\omega_{\boldsymbol{k}}t}|\boldsymbol{k},s\rangle. \tag{42}$$

Then the normalized energy density reads

$$\rho(\boldsymbol{r},t;f) = 4\pi r^2\frac{\langle\psi_{SC}(t;f)|\,\boldsymbol{E}^{+}(\boldsymbol{r})\cdot\boldsymbol{E}(\boldsymbol{r})\,|\psi_{SC}(t;f)\rangle}{\langle\psi_{IN}(0;f)|\,\boldsymbol{E}^{+}(0)\cdot\boldsymbol{E}(0)\,|\psi_{IN}(0;f)\rangle} \tag{43}$$

where

$$E^{\alpha}(\boldsymbol{r}) = \mathrm{i}\sum_{\boldsymbol{k},s}\sqrt{\frac{\hbar\omega_{\boldsymbol{k}}}{2\varepsilon_0 V}}\mathrm{e}^{\alpha}(\boldsymbol{k},s)\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}\varPhi_{\boldsymbol{k},s}(0) \tag{44}$$

is the electric field intensity operator. In contrast, the previous studies, e.g. [7], neglect the arising period of $C_e(t;f)$ and approximate the decaying period as spontaneous emission. As a consequence, the normalized energy density in them is actually obtained from (28), and thus is free from the incident photon. The $\rho - \delta$ relation achieved by the proposed approach and by [7] are shown in Fig. 6 where $\boldsymbol{r} = (0,0,-1\,000\omega_{AR}^{-1}c)$, $t = 1\,035\omega_{AR}^{-1}$, $|\boldsymbol{q}_0| = \omega_{AR}c^{-1}$, $\varGamma = 0.01\omega_{AR}$, $\alpha = 0.009\omega_{AR}^2 c^{-2}$. We see that, by considering the entire evolution of $C_e(t;f)$, the normalized energy density indeed depends on the wave packet of the incident photon. Therefore, one has to adopt the proposed method, instead of [7], to achieve an accurate normalized energy density of the SQEF.

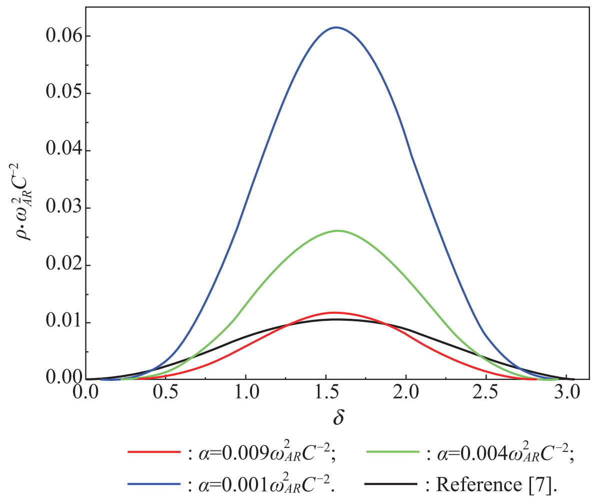


Fig. 6 $\rho - \delta$ relations obtained by the proposed approach and [7]

5. Conclusions

We present a new approach based on the renormalization theory to the single-photon processes of the 2LS in free space. The state vector of the 2LS is obtained as the superposition of the RTGFs involved in the single-photon processes, where all the ultraviolet divergences originating from the Lamb shift and higher-order interactions are systematically subtracted. We include the discussions on spontaneous emission and single-photon absorption with the proposed method. For spontaneous emission, we obtain the same result with the W-W theory. For single-photon absorption, we find that the electric dipole first gets excited rapidly and then decays slowly, and that the efficiency of single-photon absorption declines as the bandwidth of the incident photon becomes narrow, which has not been noticed before. The proposed method improves the computation of the normalized energy density of the SQEF.

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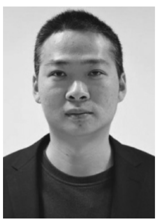

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