Condition-based maintenance optimization for continuously monitored degrading systems under imperfect maintenance actions

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Abstract: Condition-based maintenance (CBM) is receiving increasing attention in various engineering systems because of its effectiveness. This paper formulates a new CBM optimization problem for continuously monitored degrading systems considering imperfect maintenance actions. In terms of maintenance actions, in practice, they scarcely restore the system to an as-good-as new state due to residual damage. According to up-to-data researches, imperfect maintenance actions are likely to speed up the degradation process. Regarding the developed CBM optimization strategy, it can balance the maintenance cost and the availability by the searching the optimal preventive maintenance threshold. The maximum number of maintenance is also considered, which is regarded as an availability constraint in the CBM optimization problem. A numerical example is introduced, and experimental results can demonstrate the novelty, feasibility and flexibility of the proposed CBM optimization strategy.

Keywords: condition-based maintenance (CBM), imperfect maintenance, maintenance cost, availability constraint, optimization.

1. Introduction

A large number of modern engineering systems are incredibly large-scale and complex [1 – 3]. Reliability and safety are two main concerns in their whole life time. However, system performance will inevitably deteriorate with the increase of serving time due to comprehensive influences of various internal causes such as aging of component, mechanical wearing and material fatigue, and external causes such as vibration and shock [4 – 6]. When the performance degradation accumulates to a certain extent,

it may lead to malfunctions or failures, even accidents. Obviously, unexpected shutdown will cause high safety risks, serious economic loss and decrease of system availability. Early and timely maintenance is a core desire in all engineering systems [7].

In general, maintenance can be divided into corrective maintenance (CM), time-based maintenance (TBM) and condition-based maintenance (CBM) [8]. CM refers to the maintenance that restores the system to a specified functional state after repairing or replacing the failed components. This maintenance strategy, however, has high downtime loss and poor safety. TBM is based on the serving time and the probability distribution of the trouble-free operation time span of the system. Due to its conservation, TBM results in excessive maintenance. Unlike CM and TBM, CBM relies on the condition of the monitored system over time. It is performed when there is evidence that maintenance is required [9,10]. Nowadays, CBM has become the most popular maintenance strategy in engineering applications [11 – 16].

To develop a feasible and effective CBM strategy, the following two factors should be carefully considered: repair degree and optimization objective. According to the repair degree, maintenance actions are divided into three categories: minor maintenance, perfect maintenance and imperfect maintenance [17,18]. Perfect maintenance can completely restore the system to an as-good-as new state, which is suitable for systems with a simple structure. Minor maintenance means that the degradation condition of the system cannot be improved by maintenance actions, which is possible for some complex systems. However, in practice, most of maintenance actions are imperfect ones. The system is restored to be an intermediate state between the as-good-as new and the as-bad-as old states. Imperfect

maintenance is more general and has attracted a lot of attentions [19 – 21].

In the earlier stage, the influence of imperfect maintenance was generally described by a virtual age or a risk rate [22 – 24]. With the development of advanced sensors such as laser scanners and infrared sensors, maintenance decision based on the degradation model becomes more accurate. In [25], the degradation degree after maintenance was used at the first time to describe the influence of imperfect maintenance actions. Systems after imperfect maintenance will be restored to an intermediate state between an as-good-as new and as-bad-as old state. Subsequently, residual damage was used to characterize the degradation degree [26]. In [27], the authors held the view that maintenance actions may accelerate the degradation process, and under imperfect maintenance, study on the optimal maintenance for gyroscope was carried out. In general, after maintenance, the degree and speed of system degradation will be changed simultaneously. For example, welding can reduce the crack length but it may destroy some physical features of the material. As a consequence, after maintenance, the degradation degree of the system may be reduced, while the degradation speed may be increased.

The CBM strategy is essentially an optimization problem. For a typical optimization problem, the objective criterion, the decision variables, the constraint condition and the optimization algorithm are the four key elements. Existing studies on the CBM optimization objectives mainly focus on single-objective optimization. In [28], a CBM strategy that minimized the total maintenance cost over an infinite horizon was proposed to obtain the optimal inspection times and replacement threshold. In [29], the authors derived an expression to limit the average availability for the system with non-self-announcing failures and suggested the opportunities for effective inspection strategies based on the availability model. To obtain an optimal preventive maintenance threshold, a CBM strategy was proposed and solved by maximizing the long-run availability [30].

In practice, CBM strategies with a single-optimization objective are, however, difficult to guarantee high requirements on multiple goals, such as minimizing the average maintenance cost and maximizing the availability at the same time. Existing studies on the CBM constraint conditions mainly consider the preventive maintenance threshold, the long-run availability and the total uptime separately. In [31], constraints to the preventive maintenance threshold were considered in optimizing the CBM strategy. In [32], the long-run availability threshold was considered as a constraint. In [33], the total uptime constraint was introduced to ensure that the system would not be removed from service nor be replaced before a period of time. However, these works ignored the short-run availability which is also an important constraint, because the short-run availability can determine the maximum number of maintenance to ensure high availability.

This paper aims to develop a CBM strategy for continuously monitored degrading systems under imperfect maintenance actions. Main contributions of this work are summarized as below.

(i) The residual damage and accelerated degradation are investigated comprehensively. The proof that the expectation of the degradation degree after maintenance is an increasing function of the number of imperfect maintenance actions is given.

(ii) A new CBM optimization problem formulation is proposed for continuously monitored degrading systems, which can well balance the maintenance cost and the long-run and short-run availabilities by searching the optimal preventive maintenance threshold.

The rest of this paper is organized as follows. Section 2 presents a general system description and assumptions. Section 3 investigates the influences of imperfect maintenance actions on the degradation degree and the degradation speed. Section 4 proposes the CBM optimization formulation, including the details on the optimization objective and the optimization algorithm. A numerical example is given in Section 5, to demonstrate the novelty, feasibility and flexibility of the proposed CBM optimization strategy. Conclusion and further work are given in Section 6.

2. System description and main assumptions

2.1 General assumptions

In reality, system performance gradually deteriorates to failure due to aging and accumulated wear. In condition monitoring, changes in condition values, such as vibration, pressure and temperature, are mainly reflected. Let $X(t)$ denote the system condition at time t, D_{PM} denote the preventive maintenance threshold, and D_F denote the failure threshold. When $X(t) < D_{PM}$, the system continues to operate without maintenance; when $D_{PM} \leqslant X(t) < D_F$, preventive maintenance is carried out; when $X(t) \geqslant D_F$, the system is forced to shut down for CM. According to the actual operation of the general system, the following assumptions are made on the research object.

(i) The system is a single-unit system with a single degradation process, and the case of multiple system associations and multiple degradation modes is not considered.

(ii) The system is continuously monitored, and the condition monitoring is perfect, which can truly reflect the actual operation condition of the system [30].

(iii) The initial state $X(0)$ starts from 0.

(iv) Maintenance actions are imperfect, so the condition

$X(t)$ after maintenance will not restore to 0.

(v) The degradation process shows an accelerating tendency.

(vi) The system will be recovered to the as-good-as new state after the replacement of the failed unit.

2.2 System description

In this paper, the Gamma process is used to describe the degradation process of the system. The reasons are twofold: (i) the Gamma process is a stochastic process with independent non-negative increments and (ii) its paths can be regarded as the accumulation of an infinite number of small shocks [31,34,35]. Then, it is assumed that the system degradation between the $(i-1)$th and the ith maintenance actions evolves like a Gamma stochastic process.

For $0 < s < t$, $X(t) - X(s)$ follows a Gamma probability density function as

$$f_{\alpha_i(t-s),\beta}(x) = \frac{\beta^{\alpha_i(t-s)} x^{\alpha_i(t-s)-1} \exp(-\beta x)}{\Gamma(\alpha_i(t-s))}, \quad x \geqslant 0 \tag{1}$$

where $\alpha_i(t-s)$ is the shape parameter; β is the scale parameter; $\exp(\cdot)$ and $\Gamma(\cdot)$ are exponential and Gamma functions, respectively. Thus, the degradation speed between the $(i-1)$th and the ith maintenance actions is obtained as $\upsilon_i = \alpha_i/\beta$ [36].

A typical continuously monitored degrading system with imperfect maintenance actions is shown in Fig. 1. Following the work in [30], the life cycle of the system is defined as the time period between two consecutive replacements. Let R_i denote the time of the ith maintenance action, and R_i^+ denote the time of restart after the ith maintenance. Let T_i denote the ith operating time period, M_i denote the ith maintenance duration and Q denote the replacement duration.
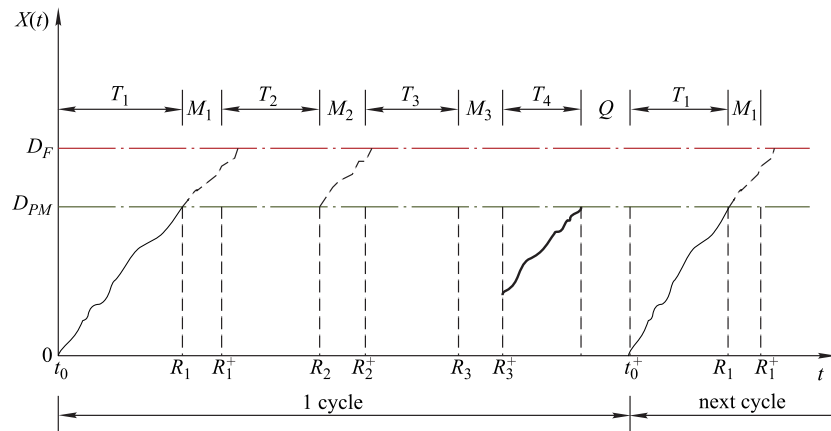

Fig. 1 Typical maintenance process of the system with imperfect maintenance actions [30]

When the degradation condition indicator $X(t)$ violates the preventive maintenance threshold D_{PM}, preventive maintenance should be carried out, e.g., at the time R_1, R_2 and R_3. Due to imperfect maintenance actions, the system is restored to an intermediate state, i.e., $X(R_i^+) \in (0, D_{PM})$. Moreover, the system after the maintenance will have a faster degradation speed, which can be reflected by the increasingly steep degradation curves as shown in Fig. 1. Since imperfect maintenance actions may reduce system uptime, e.g., $E(T_1) > E(T_2) > E(T_3) > E(T_4)$, where $E(\cdot)$ is mathematical expectation, more frequent maintenance actions and longer maintenance duration are required to maintain system reliability and safety.

3. Imperfect maintenance model

3.1 Residual damage model

Evolution of the degradation degree after imperfect maintenance is illustrated in Fig. 2. After imperfect mainte-

nance, $X(R_i^+)$ is somewhere between 0 and D_{PM}.
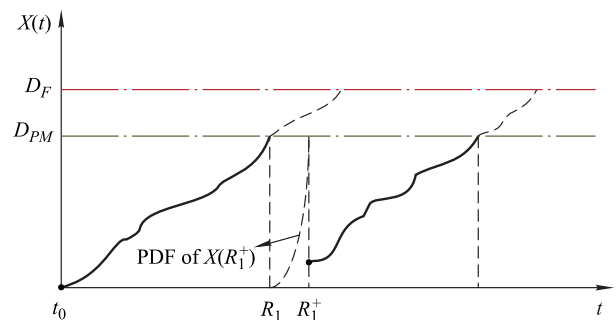

Fig. 2 Illustration of degradation degree evolution after imperfect maintenance

It is assumed that $\{X(R_i^+), i = 1, 2, \ldots, N\}$ follows an exponential distribution, where N denotes the maximum number of maintenance. This assumption is given based on the considerations that (i) from a theoretical point of

view, exponential distribution is easy for analysis and calculation, and (ii) from a practical point of view, $X(R_i^+)$ indeed follows an exponential distribution in many real applications [26]. Then, the cumulative distribution function (CDF) and the probability distribution function (PDF) of $X(R_i^+)$ can be calculated by

$$G_{X(R_i^+)}(x) = \begin{cases} 0, & x \in (-\infty, 0) \\ c_i \left(1 - \exp\left(-\dfrac{x}{1 - \exp(-i\mu)D_{PM}}\right)\right), & x \in [0, D_{PM}] \\ 1, & x \in (D_{PM}, +\infty) \end{cases} \tag{2}$$

$$g_{X(R_i^+)}(x) = \begin{cases} \dfrac{c_i}{(1 - \exp(-i\mu))D_{PM}} \exp\left(-\dfrac{x}{(1 - \exp(-i\mu))D_{PM}}\right), & x \in [0, D_{PM}] \\ 0, & \text{others} \end{cases} \tag{3}$$

where μ is a non-negative real number that can be described as the rectification effort and c_i is a function of i and μ, $c_i = 1/(1 - \exp(-1/(1 - \exp(-i\mu))))$.

It is noted that the maximum likelihood estimation (MLE) method can be utilized to estimate the parameter μ, and the likelihood function L is given by

$$L \propto \prod_{j=1}^{m} \prod_{i=1}^{n} \frac{c_i}{(1 - \exp(-i\mu))D_{PM}^j} \cdot$$

$$\exp\left(-\frac{x_{i,j}}{(1 - \exp(-i\mu))D_{PM}^j}\right) \tag{4}$$

where $x_{i,j}$ is the measured condition value after the ith maintenance for system j; and D_{PM}^j is the maintenance threshold of the system j which is performed by the maintenance department. Then, the parameter μ can be gained by maximizing $\ln L$.

The expectation of $X(R_i^+)$, which implies the degradation degree after maintenance, can be obtained by

$$\mathrm{E}(X(R_i^+)) = \int_{-\infty}^{+\infty} x g_{X(R_i^+)}(x)\mathrm{d}x =$$

$$\int_0^{D_{PM}} x g_{X(R_i^+)}(x)\mathrm{d}x =$$

$$\frac{D_{PM}}{\ln c_i - \ln(c_i - 1)} - D_{PM}(c_i - 1). \tag{5}$$

Remark The expectation of the degradation degree after maintenance is an increasing function of the number of imperfect maintenance actions.

Proof Let $\lambda_i = \dfrac{1}{(1 - \exp(-i\mu))D_{PM}} > 0$, then $\mathrm{E}(X(R_i^+)) = \dfrac{1}{\lambda_i} - \dfrac{D_{PM}}{\exp(\lambda_i D_{PM}) - 1}$.

The derivative of $\mathrm{E}(X(R_i^+))$ with respect to λ_i can be written as

$$\frac{\partial \mathrm{E}(X(R_i^+))}{\partial \lambda_i} = \frac{(D_{PM}\lambda_i)^2 \exp(D_{PM}\lambda_i) - \exp(2D_{PM}\lambda_i) + 2\exp(D_{PM}\lambda_i) - 1}{\lambda_i^2 (\exp(D_{PM}\lambda_i) - 1)^2}.$$

By defining

$$h(t) = t^2 \exp(t) - \exp(2t) + 2\exp(t) - 1, \quad t > 0,$$

then one has

$$h'(t) = \exp(t)(2t + t^2 - 2\exp(t) + 2).$$

By defining

$$s(t) = 2t + t^2 - 2\exp(t) + 2,$$

one can get

$$s'(t) = 2 + 2t - 2\exp(t),$$

$$s''(t) = 2 - 2\exp(t).$$

For $t > 0$, $s''(t) = 2 - 2\exp(t) < 0$. Therefore, the following can be obtained:

$$\because \quad s'(t) < s'(0) = 0, \quad s(t) < s(0) = 0,$$

$$\therefore \quad h'(t) = \exp(t)s(t) < 0, \quad h(t) < h(0) = 0.$$

Thus,

$$\frac{\partial E(X(R_i^+))}{\partial \lambda_i} = \frac{h(D_{PM}\lambda_i)}{\lambda_i^2 (\exp(D_{PM}\lambda_i) - 1)^2} < 0,$$

which indicates that $\mathrm{E}(X(R_i^+))$ is a monotonic decreasing function with respect to $\lambda_i > 0$. For $0 < i_1 < i_2$, one can have $\lambda_{i_1} > \lambda_{i_2} > 0$ and $\mathrm{E}(X(R_{i_1}^+)) < \mathrm{E}(X(R_{i_2}^+))$. Therefore, $\mathrm{E}(X(R_i^+))$ has an increasing tendency with the increasing number of imperfect maintenance actions. $\square$

It should be noted that this conclusion is consistent with that of [26]. The difference is that this paper proves it under the Gamma process, while Guo et al. proved it under the Winner process. This means that both the Gamma process

and the Winner process can describe the system degradation process. In practice, it is necessary to first determine whether the system degradation process is more consistent with the Gamma process or the Winner process.

Similarly, it can be proved that $\mathrm{E}(X(R_i^+))$ is also an increasing function with respect to μ. When $\mu \to 0$, $\mathrm{E}(X(R_i^+)) \to 0$, it means perfect maintenance. On the other hand, when $\mu > 0$, $0 < \mathrm{E}(X(R_i^+)) < D_{PM}$, it means imperfect maintenance. The sensitivity analysis to the parameter μ will be discussed in Section 5.2.

3.2 Accelerated degradation model

Evolution of the degradation speed after imperfect maintenance is illustrated in Fig. 3. The grey model [37], GM(1,1), is utilized to model the degradation speed because of the sparse maintenance data samples available in practice. The model is given as below.
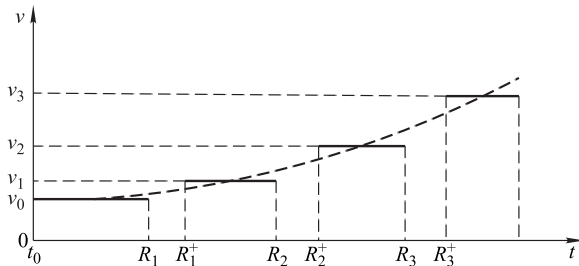

Fig. 3 **Illustration of degradation speed evolution after imperfect maintenance**

$$v_i = \left(v_0 + \frac{\lambda}{\gamma}\right)(\exp(\gamma) - 1)\exp(\gamma(i - 1)),$$
$$i = 1, 2, \dots, N \quad (6)$$

where v_0 is the initial degradation speed of the system when it is put into use; $\lambda > 0$ and $\gamma > 0$ are two grey parameters. According to the GM(1,1) theory, the parameters, λ and γ, can be estimated by the least square method and are given by

$$[-\gamma, \lambda] = (\boldsymbol{B}^{\mathrm{T}}\boldsymbol{B})^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{Y} \quad (7)$$

where

$$\boldsymbol{B} = \begin{bmatrix} -\frac{1}{2}\left(v_0 + \sum_{k=0}^{1} v_i\right) & 1 \\ -\frac{1}{2}\left(\sum_{k=0}^{1} v_i + \sum_{k=0}^{2} v_i\right) & 1 \\ \vdots & \vdots \\ -\frac{1}{2}\left(\sum_{k=0}^{N-1} v_i + \sum_{k=0}^{N} v_i\right) & 1 \end{bmatrix}, \quad \boldsymbol{Y} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}.$$

Obviously, the degradation speed depends on the values of λ and γ. The sensitivity analysis of the parameters λ and γ will be discussed in Section 5.2.

4. Condition-based maintenance optimization

4.1 Maintenance duration and uptime

Replacement of the worn parts means the end of maintenance actions in a life cycle. Maintenance duration, M_i, is bounded by the expected replacement duration Q. If the system condition $X(R_{i-1}^+)$ after maintenance is becoming higher, the system needs longer maintenance duration M_i. It can be modeled by

$$\mathrm{E}(M_i|X(R_{i-1}^+)) = \eta\left(\frac{Q}{\eta}\right)^{\left(\frac{x}{D_F}\right)^k} \quad (8)$$

where η is the duration for the first maintenance, i.e., $\eta = \mathrm{E}(M_1|x = 0)$, and k is a non-negative real number. Obviously, the function (8) is incremental, and its value range is between η and Q, which means that it is reasonable to model the maintenance duration using (8). Besides, k is used to describe the shape of function (8), e.g., concave or convex. In reality, the value of k can be determined by using statistical methods.

Due to the influences of imperfect maintenance actions on the degradation degree and speed, the system cannot be repaired unlimitedly. There exists a maximum number N of the possible maintenance actions in a life cycle. N depends on the desired availability, which will be discussed in Section 5.1.

Therefore, the unconditional expectation of maintenance duration M_i can be calculated by

$$\mathrm{E}(M_i) = \mathrm{E}(\mathrm{E}(M_i|X(R_{i-1}^+))) =$$

$$\begin{cases} \eta, & i = 1 \\ \int_0^{D_{PM}} \eta\left(\frac{Q}{\eta}\right)^{\left(\frac{x}{D_F}\right)^k} g_{X(R_{i-1}^+)}(x)\mathrm{d}x, & i = 2, 3, \dots, N \end{cases}.$$
$$(9)$$

Moreover, based on the Gamma degradation process, the unconditional expectation of T_i can be calculated by

$$\mathrm{E}(T_i) = \mathrm{E}(\mathrm{E}(T_i|X(R_{i-1}^+))) =$$

$$\begin{cases} \int_0^{+\infty}\left(1 - \frac{\Gamma(\alpha_{i-1}t), \beta D_{PM}}{\Gamma(\alpha_{i-1}t)}\right)\mathrm{d}t, & i = 1 \\ \int_0^{D_{PM}}\int_0^{+\infty}\left(1 - \frac{\Gamma(\alpha_{i-1}t, \beta(D_{PM} - x))}{\Gamma(\alpha_{i-1}t)}\right) \\ \quad g_{X(R_{i-1}^+)}(x)\mathrm{d}t\mathrm{d}x, & i = 2, 3, \dots, N \end{cases}. \quad (10)$$

4.2 Optimization objective

The optimization objective of the proposed CBM strategy includes the average maintenance cost rate and the availa-bility. There are two kinds of availability, the short-run and the long-run. Between the two successive maintenance actions, the short-run availability (SA) is defined as

$$\text{SA}(i) = \frac{\text{Expected uptime per maintenance}}{(\text{Expected uptime} + \text{Expected downtime}) \text{ per maintenance}} = \frac{\text{E}(T_{i+1})}{\text{E}(T_{i+1}) + \text{E}(M_i)}. \tag{11}$$

It should be noted that, N, the maximum number of the possible maintenance actions in a life cycle, can be obtained by pre-setting the SA threshold ξ. The threshold ξ is specified by the customers to achieve their expected system availability. Once the SA after the Nth maintenance is below the threshold, replacement is carried out. Mathe-matically, N can be defined as

$$N = \inf_{i \in \mathbf{Z}^+} \left\{ i : \left| \frac{\text{E}(T_{i+1})}{\text{E}(T_{i+1}) + \text{E}(M_i)} \right| < \xi \right\}. \tag{12}$$

Then, long-run availability LA of the system can be defined as

$$\text{LA} = \frac{\text{Expected uptime per cycle}}{(\text{Expected uptime} + \text{Expected downtime}) \text{ per cycle}} = \frac{\text{E}\left(\sum_{i=1}^{N+1} T_i\right)}{\text{E}\left(\sum_{i=1}^{N+1} T_i + \sum_{i=1}^{N} M_i + Q\right)}. \tag{13}$$

The average maintenance cost rate (CR) of the system is defined as

$$\text{CR} = \frac{\text{Expected cost per cycle}}{(\text{Expected uptime} + \text{Expected downtime}) \text{ per cycle}} = \frac{\text{E}\left(c_{ins} \sum_{i=1}^{N+1} T_i + c_p \sum_{i=1}^{N} M_i + R + c_r Q\right)}{\text{E}\left(\sum_{i=1}^{N+1} T_i + \sum_{i=1}^{N} M_i + Q\right)} \tag{14}$$

where c_{ins} is the inspection cost per unit time; c_p is the preventive maintenance cost per unit time. The replacement cost consists of two parts, one is the basic replacement cost R, and the other is proportional to the replacement time Q at cost per unit time c_r.

Maintenance cost and availability are two contradictory indices [38]. To balance the maintenance cost and availability, a new CBM optimization problem is formulated, as shown in (15). Specifically, minimizing CR and maximizing LA are considered as the objective criterion. The preventive maintenance threshold is the decision variable. Constraint conditions include the preventive maintenance threshold and the SA. The SA constraint is designed to avoid too low SA by determining the maximum number of maintenance.

$$\text{Min CR}(D_{PM})$$

$$\text{Max LA}(D_{PM}) \tag{15}$$

$$\text{s.t.} \begin{cases} \text{SA}(i) \geqslant \xi, & i = 1, 2, \ldots, N-1 \\ \text{SA}(N) < \xi \\ 0 < D_{PM} \leqslant D_F \end{cases}.$$

For the single-objective optimization problem, there exists usually one unique optimal solution. However, multi-objective optimizations have no unique optimum because the objectives often conflict. Thus the key is to find the Pareto optimal solutions that meet all the conditions.

4.3 Optimization algorithm

Evolutionary algorithms such as the genetic algorithm (GA) and particle swarm optimization (PSO) can solve the multi-objective optimization problems. However, they are not suitable for the developed CBM problem due to the complexity. For simplification, a value iteration algorithm [26,30,33] is used to find the optimal solution set. This algorithm searches over the range $(0, D_F]$ to determine a set of optimal preventive maintenance thresholds that can balance the maintenance cost and the availability under all constraints. The optimization algorithm is given as follows.

Step 1 Start with a small value of D_{PM} within the $(0, D_F]$;

Step 2 Set $i = 1$;

Step 3 Calculate $E(M_i)$ by (9) and $E(T_{i+1})$ by (10);

Step 4 Calculate SA(i) by (11);

Step 5 If SA(i) $< \xi$, then $N = i$; otherwise, $i = i + 1$ and go to Step 3;

Step 6 Calculate LA by (13) and CR by (14) for current D_{PM};

Step 7 Adjust D_{PM} by a small increment unless $D_{PM} > D_F$ and repeat Steps 2 – 6;

Step 8 Choose the optimal preventive maintenance threshold D_{PM}^{CR} with the minimization of CR and the optimal preventive maintenance threshold D_{PM}^{LA} with the maximization of LA;

Step 9 Return the optimal solution set $\{D_{PM}^* | D_{PM}^* \in [D_{PM}^{\text{CR}}, D_{PM}^{\text{LA}}] \text{ or } D_{PM}^* \in [D_{PM}^{\text{LA}}, D_{PM}^{\text{CR}}]\}$.

5. A numerical example

In this section, a numerical example [36] will be introduced to show how the proposed maintenance strategy can be used in maintenance optimization under imperfect maintenance actions.

Suppose that the degradation process of the system follows a Gamma process with $\alpha_0 = 1$ and $\beta = 1$. When the degradation of the system exceeds the failure threshold $D_F = 20$, the system is failed. In the degradation degree and speed models, $\mu = 0.5$, $\lambda = 0.02$ and $\gamma = 1.3$. For the maintenance duration, $\eta = 0.2$, $Q = 2$ and $k = 2$. In addition, the maintenance cost availability indices are $c_{ins} = 5$, $c_p = 50$, $R = 850$, $c_r = 20$ and $\xi = 0.95$.

5.1 Optimal CBM strategy

According to (12), the maximum number of maintenance, N, can be obtained. As an illustration, the increment of D_{PM} is taken as 1 for the optimization algorithm. Due to the limitation of space, Table 1 reports the SA evolution after multiple maintenance actions when the increment of D_{PM} is 2. The cases when $D_{PM} \leqslant 6$ are excluded because the corresponding SAs are always less than the threshold $\xi = 0.95$. Taking $D_{PM} = 16$ as an example, Fig. 4 shows the evolution of the SA with the number of maintenance. It can be clearly observed that multiple maintenance actions reduce the SA of the system. In Table 1, when the SA constraint is violated, N can be obtained as follows: $N = 3$ for $D_{PM} = 8$, $N = 5$ for $D_{PM} = 10$, $N = 9$ for $D_{PM} = 12$, and so on.
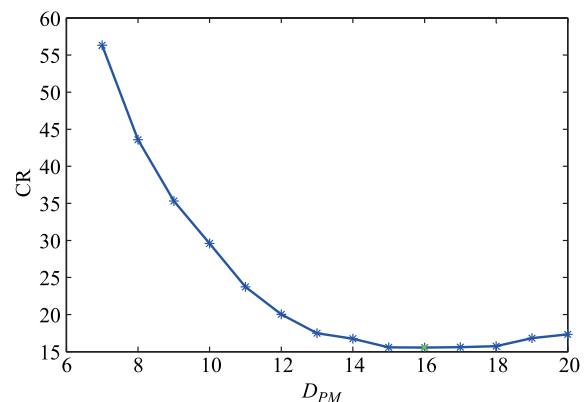
Table 1 SA vs. D_{PM} for the maintained system ($\xi = 0.95$)

Number of maintenance	D_{PM}									
	2	4	6	8	10	12	14	16	18	20
1				0.957 7	0.965 3	0.970 6	0.974 5	0.977 5	0.979 8	0.981 7
2				0.950 1	0.957 5	0.962 1	0.965 0	0.966 5	0.966 8	0.965 8
3				**0.946 4**	0.953 7	0.958 1	0.960 4	0.961 0	0.960 1	0.957 3
4					0.951 5	0.955 8	0.957 9	0.958 3	0.956 8	0.953 2
5					**0.949 9**	0.954 2	0.956 3	0.956 5	0.954 7	0.950 7
6		SA < 0.95				0.952 9	0.955 0	0.955 1	0.953 2	**0.948 9**
7						0.951 8	0.953 8	0.953 9	0.951 9	
8						0.950 7	0.952 8	0.952 8	0.950 7	
9						**0.949 7**	0.951 8	0.951 8	**0.949 6**	
10							0.950 8	0.950 8		
11							**0.949 8**	**0.949 8**		
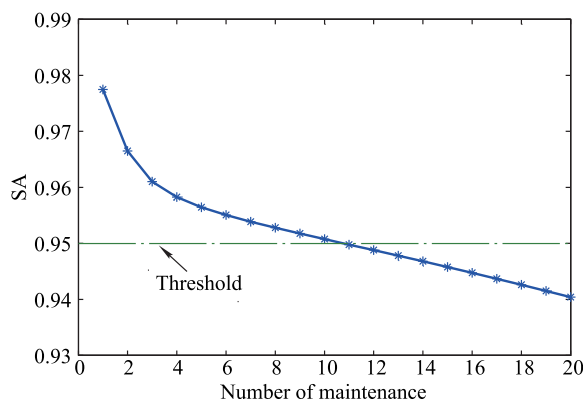
Fig. 4 Evolution of the SA with number of maintenance for $D_{PM} = 16$

With the obtained N, the evolution of evaluation indicators, i.e., the average maintenance CR and the LA, with D_{PM}, is shown in Fig. 5.

(a) Average maintenance CR

(b) LA
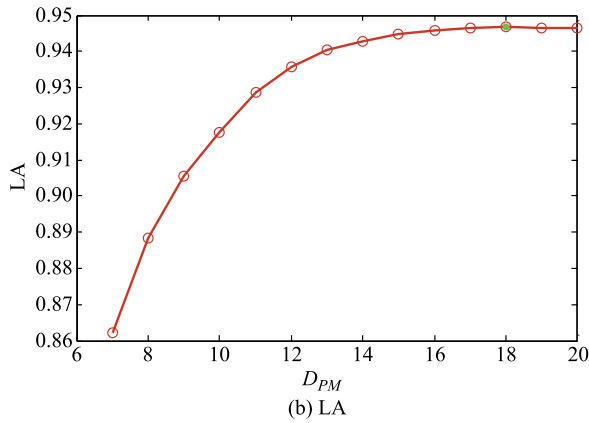
Fig. 5   Evolution of the evaluation indicators with the changing D_{PM}


(b) LA

: $\mu=0.5$;   : $\mu=1.0$;   : $\mu=1.5$;
: $\mu=2.0$;   : $\mu=2.5$.

Fig. 6   Sensitivity analysis of μ

From Fig. 5(a), when $D_{PM} = 16$, the average mainte-nance CR is the lowest. From Fig. 5(b), when $D_{PM} = 18$, the LA is the highest. It is clear that it is impossible for a D_{PM} to obtain the optimal CR and LA at the same time. To balance CR and LA, the optimal solution, D_{PM}^*, should be in the interval [16,18]. In this paper, the midpoint of the interval is considered as a compromise, and the deci-sion parameter is $D_{PM}^* = 17$ with CR$^* = 15.5892$ and LA$^* = 0.9464$.

5.2   Sensitivity analysis of the optimal strategy

In practice, inaccurate imperfect maintenance parameters may affect the optimal maintenance strategy. To this end, sensitivity of the maintenance strategy with respect to the imperfect maintenance parameters is investigated in this section, by changing one parameter to be studied and fix-ing the remaining parameters.

When μ varies from 0.5 to 2.5 as shown in Fig. 6(a), the average maintenance CR increases with the increasing of μ. It can be found that the minimum average maintenance CR of the system increases from 15.5349 to 15.8272.
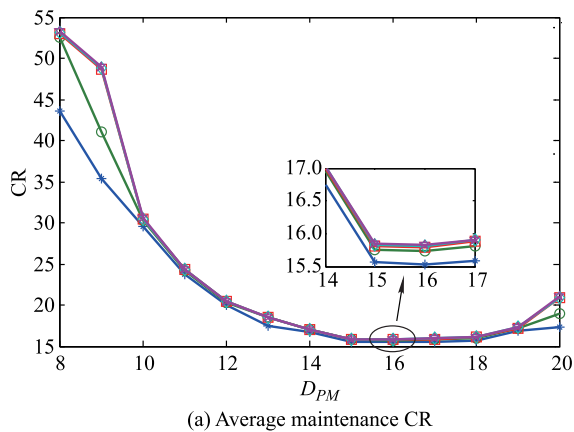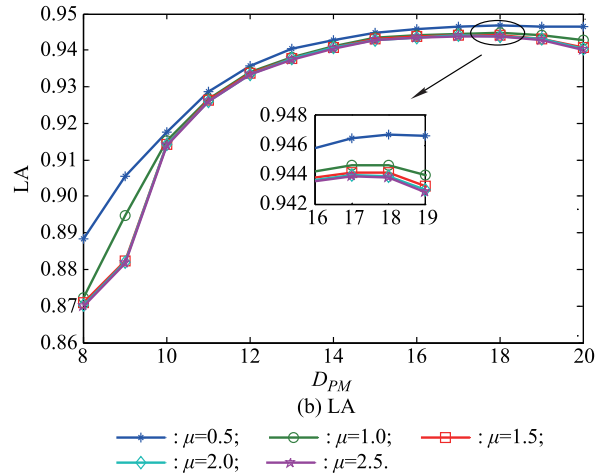
Fig. 6(b) shows the evolution of the LA as μ varies from 0.5 to 2.5. From Fig. 6(b), the LA of the system decreases from 0.9468 to 0.9439. This is because that the degrada-tion degree of the system increases with the increasing of μ, indicating that it will take more time for maintenance. Consequently, the average maintenance CR increases and the LA decreases as μ increases.

When λ varies from 0.01 to 0.05 as shown in Fig. 7, the average maintenance CR increases with the increasing of λ, while the LA decreases with the increasing of λ. This is because that the degradation speed of the system increases with the increasing of λ, allowing the working condition to reach preventive maintenance thresholds earlier. Con-sequently, the uptime of the system is reduced, which in-creases the average maintenance cost and reduces the LA.

Fig. 8 shows the evolution of the average maintenance CR and the LA as γ varies from 1.30 to 1.50. Obviously, Fig. 8 is similar to Fig. 7. This is because a larger λ or γ can make the system have a larger degradation speed.
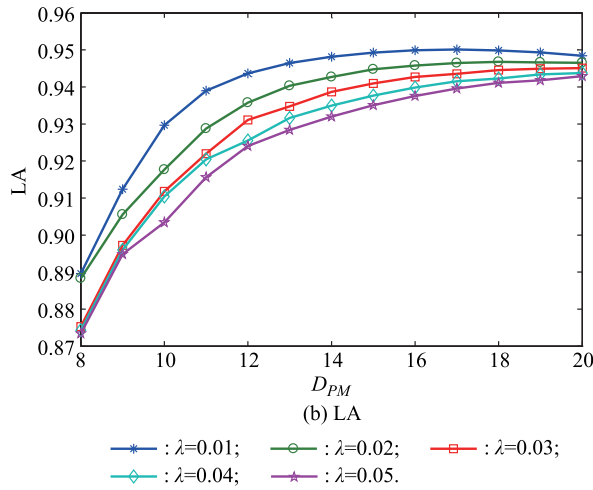

(a) Average maintenance CR
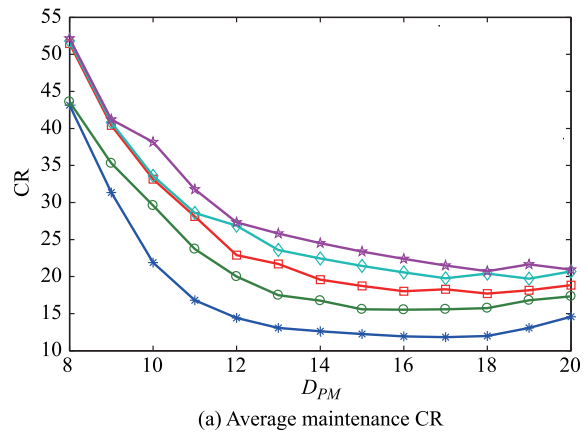
(a) Average maintenance CR

Fig. 7   Sensitivity analysis of λ on evaluation indicators
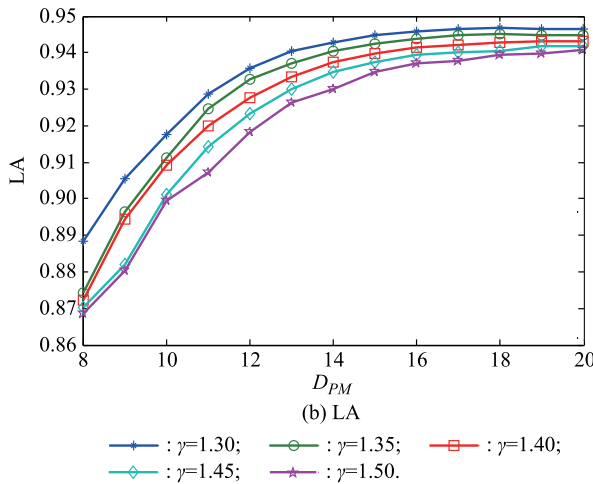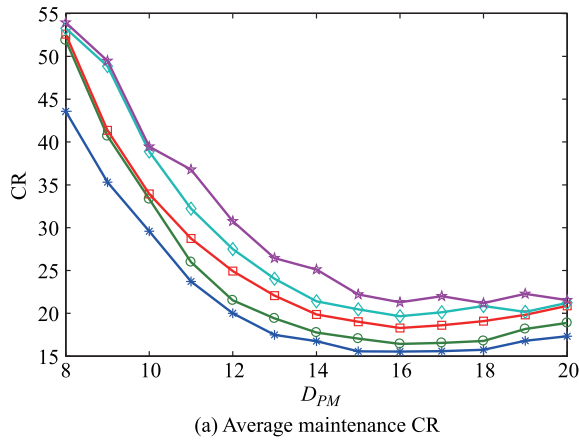

(a) Average maintenance CR

Fig. 8   Sensitivity analysis of γ on evaluation indicators

The parameters μ, λ and γ have different effects on D_{PM} as shown in Fig. 9. When μ varies from 0.5 to 2.5 to avoid longer miantenance duration, D_{PM} decreases from 17 to 16.5. When λ varies from 0.01 to 0.05 or γ from 1.30 to 1.50, D_{PM} exhibits an increasing trend to get more uptime. As a consequence, the parameters μ, λ and γ have

important effects on the proposed maintenance strategy. In practice, accurate estimation of those parameters are very important.
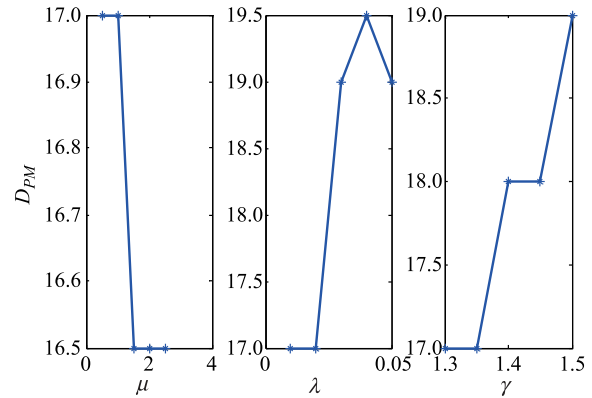

Fig. 9   Effects of varied μ, λ and γ on D_{PM}

6. Conclusions

In this work, a CBM strategy for continuously monitored degrading systems under imperfect maintenance actions is developed. The influences of imperfect maintenance actions on the degradation degree and degradation speed are investigated. A novel application-oriented optimization objective is proposed. Moreover, thanks to the availability constraint, the proposed maintenance strategy can obtain the maximum number of maintenance. Finally, the performance of the proposed maintenance strategy is illustrated and discussed through a numerical example. Besides, sensitivity analysis to the key parameters is also investigated to show the flexibility of the proposed maintenance strategy. It is noteworthy that this work can be also applied to the system with other degradation behaviors, such as the Wiener process. One of the limitation of the proposed maintenance strategy is that only continuousstate degrading systems under continuous monitoring are considered. Future work will focus on the situation that a multi-component system undergoes a multi-state degrading phase.

References

[1] LU C, XU T X, WANG H, et al. Joint optimization decision of equipment condition-based maintenance and spare parts inventory. Systems Engineering and Electronics, 2019, 41(7): 1560 – 1567. (in Chinese)

[2] DING S H, KAMARUDDIN S. Maintenance policy optimization — literature review and directions. The International Journal of Advanced Manufacturing Technology, 2015, 76(5 – 8): 1263 – 1283.

[3] SU G S, PENG L F, HU L H. A Gaussian process-based dynamic surrogate model for complex engineering structural reliability analysis. Structural Safety, 2017, 68: 97 – 109.

[4]   CASTRO I T, CABALLE N C, PEREZ C J. A condition-based maintenance for a system subject to multiple degradation processes and external shocks. International Journal of Systems Science, 2015, 46(9): 1692 – 1704.

[5]   CHEN J Y, LI Z H. An extended extreme shock maintenance model for a deteriorating system. Reliability Engineering & System Safety, 2008, 93(8): 1123 – 1129.

[6]   RAFIEE K, FENG Q M, COIT D W. Condition-based maintenance for repairable deteriorating systems subject to a generalized mixed shock model. IEEE Trans. on Reliability, 2015, 64(4): 1164 – 1174.

[7]   BOUSDEKIS A, MAGOUTAS B, APOSTOLOU D, et al. A proactive decision making framework for condition-based maintenance. Industrial Management & Data Systems, 2015, 115(7): 1225 – 1250.

[8]   ALASWAD S, XIANG Y S. A review on condition-based maintenance optimization models for stochastically deteriorating system. Reliability Engineering & System Safety, 2017, 157: 54 – 63.

[9]   LIU X, LI J R, AL-KHALIFA K N, et al. Condition-based maintenance for continuously monitored degrading systems with multiple failure modes. IIE Transactions, 2013, 45(4): 422 – 435.

[10] YOU M Y, LI L, MENG G, et al. Cost-effective updated sequential predictive maintenance policy for continuously monitored degrading systems. IEEE Trans. on Automation Science and Engineering, 2010, 7(2): 257 – 265.

[11] BASURKO O C, URIONDO Z. Condition-based maintenance for medium speed diesel engines used in vessels in operation. Applied Thermal Engineering, 2015, 80: 404 – 412.

[12] MERIGAUD A, RINGWOOD J V. Condition-based maintenance methods for marine renewable energy. Renewable and Sustainable Energy Reviews, 2016, 66: 53 – 78.

[13] MOURTZIS D, VLACHOU E. A cloud-based cyber-physical system for adaptive shop-floor scheduling and condition-based maintenance. Journal of Manufacturing Systems, 2018, 47: 179 – 198.

[14] TANG D Y, SHENG W B, YU J S. Dynamic condition-based maintenance policy for degrading systems described by a random-coefficient autoregressive model: a comparative study. Eksploatacja i Niezawodnosc — Maintenance and Reliability, 2018, 20(4): 590 – 601.

[15] TIAN Z G, JIN T D, WU B R, et al. Condition based maintenance optimization for wind power generation systems under continuous monitoring. Renewable Energy, 2011, 36(5): 1502 – 1509.

[16] WU B R, TIAN Z G, CHEN M Y. Condition-based maintenance optimization using neural network-based health condition prediction. Quality & Reliability Engineering International, 2013, 29(8): 1151 – 1163.

[17] MERCIER S, CASTRO I T. On the modelling of imperfect repairs for a continuously monitored Gamma wear process through age reduction. Journal of Applied Probability, 2013, 50(4): 1057 – 1076.

[18] PHAM H, WANG H Z. Imperfect maintenance. European Journal of Operational Research, 1996, 94(3): 425 – 438.

[19] GRALL A, DIEULLE L, BERENGUER C, et al. Asymptotic failure rate of a continuously monitored system. Reliability Engineering & System Safety, 2006, 91(2): 126 – 130.

[20] HOSSEINI M M, KERR R M, RANDALL R B. An inspection model with minimal and major maintenance for a system with deterioration and Poisson failures. IEEE Trans. on Reliability, 2000, 49(1): 88 – 98.

[21] TOMASEVICZ C L, ASGARPOOR S. Optimum maintenance policy using semi-Markov decision processes. Electric Power Systems Research, 2009, 79(9): 1286 – 1291.

[22] KIJIMA M. Some results for repairable systems with general repair. Journal of Applied Probability, 1989, 26(1): 89 – 102.

[23] NAKAGAWA T. Sequential imperfect preventive maintenance policies. IEEE Trans. on Reliability, 1988, 37(3): 295 – 298.

[24] ZHOU X J, XI L F, LEE J. Reliability-centered predictive maintenance scheduling for a continuously monitored system subject to degradation. Reliability Engineering & System Safety, 2007, 92(4): 530 – 534.

[25] VAN P D, BERENGUER C. Condition-based maintenance with imperfect preventive repairs for a deteriorating production system. Quality & Reliability Engineering International, 2012, 28(6): 624 – 633.

[26] GUO C M, WANG W B, GUO B, et al. A maintenance optimization model for mission-oriented systems based on Wiener degradation. Reliability Engineering & System Safety, 2013, 111: 183 – 194.

[27] ZHANG M M, GAUDOIN O, XIE M. Degradation-based maintenance decision using stochastic filtering for systems under imperfect maintenance. European Journal of Operational Research, 2015, 245(2): 531 – 541.

[28] GRALL A, DIEULLE L, BERENGUER C, et al. Continuous-time predictive-maintenance scheduling for a deteriorating system. IEEE Trans. on Reliability, 2001, 51(2): 141 – 150.

[29] KLUTKE G A, YANG Y J. The availability of inspected systems subject to shocks and graceful degradation. IEEE Trans. on Reliability, 2002, 51(3): 371 – 374.

[30] LIAO H T, ELSAYED E A, CHAN L Y. Maintenance of continuously monitored degrading systems. European Journal of Operational Research, 2006, 175(2): 821 – 835.

[31] HUYNH K T, BARROS A, BERENGUER C, et al. A periodic inspection and replacement policy for systems subject to competing failure modes due to degradation and traumatic events. Reliability Engineering & System Safety, 2011, 96(4): 497 – 508.

[32] TAN L, CHENG Z J, GUO B, et al. Condition-based maintenance policy for gamma deteriorating systems. Journal of Systems Engineering and Electronics, 2010, 21(1): 57 – 61.

[33] CHEN Y X, GONG W J, XU D, et al. Imperfect maintenance policy considering positive and negative effects for deteriorating systems with variation of operating conditions. IEEE Trans. on Automation Science and Engineering, 2017, 15(2): 872 – 878.

[34] HUYNH K T, CASTRO I T, BARROS A, et al. On the use of mean residual life as a condition index for condition-based maintenance decision-making. IEEE Trans. on Systems, Man, and Cybernetics: Systems, 2014, 44(7): 877 – 893.

[35] WANG Y B, ZHAO J M, CHEN Z H, et al. Integrated decision on spare parts ordering and equipment maintenance under condition based maintenance strategy. Eksploatacja i Niezawodnosc — Maintenance and Reliability, 2015, 17(4): 591 – 599.

[36] DO P, VOISIN A, LEVRAT E, et al. A proactive condition-based maintenance strategy with both perfect and imperfect maintenance actions. Reliability Engineering & System Safety, 2015, 133: 22 – 32.

[37] XIA T B, JIN X N, XI L F, et al. Operating load based real-time rolling Grey forecasting for machine health prognosis in dynamic maintenance schedule. Journal of Intelligent Manu-

facturing, 2015, 26(2): 269 – 280.

[38] TIAN Z G, LIN D M, WU B R. Condition based maintenance optimization considering multiple objectives. Journal of Intelligent Manufacturing, 2012, 23(2): 333 – 340.

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