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Fuzzy modeling of multirate sampled nonlinear systems based on multi-model method

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Abstract: Based on the multi-model principle, the fuzzy identification for nonlinear systems with multirate sampled data is studied. Firstly, the nonlinear system with multirate sampled data can be shown as the nonlinear weighted combination of some linear models at multiple local working points. On this basis, the fuzzy model of the multirate sampled nonlinear system is built. The premise structure of the fuzzy model is confirmed by using fuzzy competitive learning, and the conclusion parameters of the fuzzy model are estimated by the random gradient descent algorithm. The convergence of the proposed identification algorithm is given by using the martingale theorem and lemmas. The fuzzy model of the PH neutralization process of acid-base titration for hair quality detection is constructed to demonstrate the effectiveness of the proposed method.

Keywords: multirate sampled data, nonlinear system, fuzzy model, multi-model.

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1. Introduction

In the actual industrial production process, the refreshing time of the input signal and the sampling time of the output signal are different because of the influence of sensor sampling frequencies, physical equipments, manual measuring, environmental factors, network transmission equipment and so on [1 – 7]. Generally speaking, the input signal of the system is fast refreshed, while the output signal of the system is slowly sampled. This kind of system is called the multirate sampled system.

The multirate sampled systems have received extensive attention from the industrial process and academic field. For multirate sampled linear systems, some scholars used the variable lifting technique [8 – 12], the polynomial transformation technique [13 – 17], combined with different model structures, such as the autoregressive moving average model with external input (ARMAX) model, the Box-Jenkins model, the output error model and so on, to propose some new identification algorithms.

The actual industrial systems often have some nonlinearities. For multirate sampled Hammerstein nonlinear systems, the least squares algorithm was proposed to identify them in [18], but the proposed method based on the variable lifting technique often has a large calculation load, and the convergence of the proposed algorithm has not been studied in depth. In [19], for the dual-rate nonlinear systems with colored noise signals, a hierarchical principle based on the key item separation technique was proposed to deal with the modeling of these systems. In [20,21], the Hammerstein-Wiener system and the Wiener system with multirate sampled data were studied by the multi-innovation stochastic gradient algorithm. In [22], based on the hierarchical principle, the identification problem of multirate sampled systems with lifting variables was transformed into the identification problem of several subsystems. The advantage of this method is that it reduces the computational complexity of the algorithm and improves the efficiency of the algorithm. In addition, some researchers gave some research results in [23 – 25]. These results are embodied in the transfer function modeling of multirate sampling data, in numerical operation of the model, or in the simplification of the model structure.

From the above literature, the research of multirate sampled systems generally focuses on linear systems. The physical processes in the real world are generally characterized as nonlinearity, complexity and uncertainty. These processes cannot be represented by linear models derived from conventional system identification. For the simplified model of the multirate sampled nonlinear systems, it is generally described by the combination from static nonlinear modules and dynamic linear system modules, such as Hammerstein system, Wiener system, Hammerstein-Wiener system and so on. When the controlled object has strong nonlinearity, uncertainty and strong time-varying,

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the dynamic process of the actual system cannot be completely described by using the combination from static nonlinear modules and dynamic linear system modules.

In this paper, the identification of the fuzzy model of nonlinear systems with multirate sampled data is studied in accordance with the fuzzy modeling strategy. Firstly, for multirate sampled nonlinear systems, it can be regarded as the nonlinear weighted combination from local linear models at several working points. Secondly, based on the relationship analysis between the nonlinear system and the linear system, the fuzzy model of multirate sampled nonlinear systems is established. Thirdly, the premise structure of the fuzzy model is confirmed by using fuzzy competitive learning, and the conclusion parameters of the fuzzy model are identified by using the stochastic gradient descent algorithm. In addition, the convergence of the proposed identification algorithm is given by lemmas and the martingale theorem. Finally, the PH neutralization process of acid-base titration for hair quality detection is simulated. The simulation results show the effectiveness of the proposed method.

2. Problem description

2.1 Model description of dual-rate sampled linear systems

For multirate sampled systems, the general model of linear discrete systems without noise is described as follows:

$$A(z)y(k) = B(z)u(k) \tag{1}$$

where $u(k)$ is the input signal; $y(k)$ is the output signal; z^{-1} is a unit backward operator, satisfying $z^{-1}u(k) = u(k-1)$; $A(z)$ and $B(z)$ are coefficient polynomials, $A(z) = 1 + a_1z^{-1} + a_2z^{-2} + \cdots + a_nz^{-n}$, $B(z) = b_1z^{-1} + b_2z^{-2} + \cdots + b_nz^{-n}$.

For multirate sampled systems, for example, the dual-rate system, the following data can be obtained:

(i) The input signal $\{u(k): \ k = 0, 1, 2, \ldots\}$ is fast refreshing input data;

(ii) The output signal $\{y(kq): \ k = 0, 1, 2, \ldots\}$ is slowly sampled output data ($q \geqslant 2$).

Based on the above dual-rate data, a polynomial transformation method is used to transform the single-rate sampled model in (1) into the dual-rate sampled model in [14]. We introduce this transformation method as follows.

If the roots of polynomial $A(z) = 0$ are z_i ($i = 1, 2, \ldots$), then polynomial $A(z)$ can be expressed as

$$A(z) = \prod_{i=1}^{n}(1 - z_iz^{-1}).$$

Define

$$\varphi_q(z) = \prod_{i=1}^{n}(1 + z_iz^{-1} + z_i^2z^{-2} + \cdots + z_i^{q-1}z^{-q+1}).$$

$$\prod_{i=1}^{n}\frac{1 - z_i^qz^{-q}}{1 - z_iz^{-1}} = 1 + f_1z^{-1} + f_2z^{-2} + \cdots + f_{n_f}z^{-n_f})$$

where $f_1, f_2, \ldots, f_{n_f}$ are the corresponding coefficients and n_f is the polynomial order. Multiply the left and right sides of (1) by $\varphi_q(z)$, and use the following equation:

$$1 - x^q = (1 - x)(1 + x + x^2 + \cdots + x^{q-1}).$$

We can get the following equation

$$\frac{y(k)}{u(k)} = \frac{B(z)\varphi_q(z)}{A(z)\varphi_q(z)} = \frac{\beta(z)}{\alpha(z)} \tag{2}$$

where $\alpha(z)$ and $\beta(z)$ are shown as

$$\alpha(z) = A(z)\varphi_q(z) = 1 + \alpha_1z^{-q} + \alpha_2z^{-2q} + \cdots + \alpha_nz^{-nq},$$

$$\beta(z) = B(z)\varphi_q(z) = \beta_1z^{-1} + \beta_2z^{-2} + \cdots + \beta_{nq}z^{-nq}.$$

Equation (2) can be written as

$$y(kq) = \frac{\beta(z)}{\alpha(z)}u(kq). \tag{3}$$

In the later discussion of this paper, the model in (3) is taken as the conclusion model of the fuzzy model of the dual-rate sampled nonlinear system.

2.2 Fuzzy model description of dual-rate sampled nonlinear systems

Aiming at the identification of complex nonlinear systems with dual-rate sampled data, this paper attempts to build the model of such systems by the multi-model principle. Based on the decomposition-synthesis principle, the multi-model strategy is suitable for complex industrial processes with strong nonlinearity, uncertainty and multi-working conditions. The multi-model principle establishes local linear models for each working condition interval, and fits the global nonlinear system by using the nonlinear weighted fusion of local linear models.

When the nonlinear system can be linearized at some working points and the linearized system is measurable, the nonlinear system can be regarded as the nonlinear weighted model combined from local linear models at several working points. The nonlinear model is described as

$$y(k) = \sum_{l=1}^{c} f_l[\varphi(k)]g_l[\varphi(k)] \tag{4}$$

where $f_l[\varphi(k)]$ is the lth nonlinear weighted function; $g_l[\varphi(k)]$ is the lth local linear model at the lth working point.

For multirate sampled nonlinear systems, the local conclusion model of each rule of the fuzzy model can be represented by model (3). In this way, the fuzzy model of the nonlinear system can be expressed as follows:

R^l: if $\boldsymbol{\varphi}(kq)$ belongs to F_l (the membership function of F_l is $\mu_l[\boldsymbol{\varphi}(kq)]$), then

$$y(kq) = \frac{\beta_l(z)}{\alpha_l(z)}u(kq), \quad l = 1, 2, \ldots, c \tag{5}$$

where F_l represents the lth fuzzy set, whose corresponding membership degree function is $\mu_l[\boldsymbol{\varphi}(kq)]$, c is the total number of rules, $\boldsymbol{\varphi}(kq)$ is the input information vector,

$$\boldsymbol{\varphi}(kq) = [y(kq-q), y(kq-2q), \ldots, y(kq-nq),$$

$$u(kq-1), u(kq-2), \ldots., u(kq-nq)]^{\mathrm{T}},$$

$$\alpha_l(z) = 1 + \alpha_{l1}z^{-q} + \alpha_{l2}z^{-2q} + \cdots + \alpha_{ln}z^{-nq},$$

$$\beta_l(z) = \beta_{l1}z^{-1} + \beta_{l2}z^{-2} + \cdots + \alpha_{lnq}z^{-nq}, \quad l = 1, 2, \ldots, c.$$

For the fuzzy model in (5), this model is also written as R^l: if $\boldsymbol{\varphi}(kq)$ belongs to F_l (the membership degree function of F_l is $\mu_l[\boldsymbol{\varphi}(kq)]$), then

$$y(kq) = -\alpha_{l1}y(kq-q) - \alpha_{l2}\mu_1(kq)y(kq-2q) - \cdots -$$

$$\alpha_{ln}y(kq-nq) + \beta_{l1}u(kq-1) + \beta_{l2}u(kq-2) + \cdots +$$

$$\beta_{lnq}u(kq-nq), \quad l = 1, 2, \ldots, c. \tag{6}$$

Finally, the total output of the fuzzy model in (6) is

$$y(kq) = \sum_{l=1}^{c} \mu_l(\boldsymbol{\varphi}(kq))[-\alpha_{l1}y(kq-q) - \cdots -$$

$$\alpha_{ln}y(kq-nq) + \beta_{l1}u(kq-1) +$$

$$\beta_{l2}u(kq-2) + \cdots + \beta_{lnq}u(kq-nq)]. \tag{7}$$

After the weighted connection of the membership functions, the local discrete linear models are connected to obtain the global nonlinear model. It can be used to describe the whole dynamic process of the complex nonlinear system.

3. Identification of fuzzy model

3.1 Structure identification of fuzzy competitive learning

The rules of the fuzzy system are always acquired by means of analyzing the experience and knowledge of specialists. For a complex system, the more identifying accuracy of the model is required, the more fuzzy input regions need be partitioned. As a result, the number of fuzzy rules and identified parameters are exponentially increased. To solve these problems, we propose competitive learning to confirm the structure of the fuzzy model and rules.

The competitive learning paradigm adopting a principle of learning according to how well it wins is proposed. The goal of the competitive learning paradigm is to cluster the training patterns into representative groups such that patterns within a cluster are more similar to each other than patterns belonging to different clusters. The fuzzy competitive learning paradigm is that every training pattern belongs to a certain degree of every cluster, depending on its distance to the center vector. The fuzzy competitive learning paradigm was proposed in [26]. Based on the fuzzy competitive learning, the fuzzy space structure of the system is on-line partitioned. The fuzzy competitive learning paradigm is shown as follows:

(i) Select the number c ($2 \leqslant c \leqslant N/2$, and N is the total number of sampled data) of clusters, and initial center vectors $\overline{\boldsymbol{\varphi}}_l (l = 1, 2, \ldots, c)$.

(ii) Calculate the membership degree value at time kq:

$$\mu_l(kq) = \left[\sum_{j=1}^{c} \frac{\|\boldsymbol{\varphi}(kq) - \overline{\boldsymbol{\varphi}}_l\|^2}{\|\boldsymbol{\varphi}(kq) - \overline{\boldsymbol{\varphi}}_j\|^2}\right]^{-1},$$

$$l = 1, \ldots, c;\ k = 1, 2, \ldots, N \tag{8}$$

where $\mu_l(kq)$ represents the membership degree value of the lth cluster of the input information vector $\boldsymbol{\varphi}(kq)$ at time kq; $\overline{\boldsymbol{\varphi}}_l$ ($l = 1, 2, \ldots, c$) represents the lth center vector.

(iii) Update center vectors $\overline{\boldsymbol{\varphi}}_l (l = 1, 2, \ldots, c)$ as $\overline{\boldsymbol{\varphi}}_l(t+1) = \overline{\boldsymbol{\varphi}}_l(t) + \eta(\mu_l(kq))^2[\boldsymbol{\varphi}(kq) - \overline{\boldsymbol{\varphi}}_l(t)]$, where η is a learning factor.

(iv) If $\|\overline{\boldsymbol{\varphi}}_l(t+1) - \overline{\boldsymbol{\varphi}}_l(t)\| \leqslant \lambda$, stop, else $t = t + 1$, return (ii).

The fuzzy input space of the fuzzy model can be determined by using the above algorithm.

3.2 Parameter identification of fuzzy model

For the fuzzy model in (6), the identification purpose is to confirm the conclusion parameters α_{lj} and β_{lj} of the fuzzy model by using multirate sampled data $\{u(k), y(kq), k = 1, 2, \ldots\}$.

The overall parameter vector $\boldsymbol{\theta}$ and the sub-parameter vectors $\boldsymbol{\theta}_i$ ($i = 1, 2, \ldots, c$) of the fuzzy model are defined as

$$\boldsymbol{\theta} = [\boldsymbol{\theta}_1^{\mathrm{T}}, \boldsymbol{\theta}_2^{\mathrm{T}}, \ldots, \boldsymbol{\theta}_c^{\mathrm{T}}]^{\mathrm{T}} \in \mathbf{R}^{cn_0},$$

$$\boldsymbol{\theta}_1 = [\alpha_{11}, \alpha_{12}, \ldots, \alpha_{1n}, \beta_{11}, \beta_{12}, \ldots, \beta_{1nq}]^{\mathrm{T}} \in \mathbf{R}^{n_0},$$

$$\boldsymbol{\theta}_2 = [\alpha_{21}, \alpha_{22}, \ldots, \alpha_{2n}, \beta_{21}, \beta_{22}, \ldots, \beta_{2nq}]^{\mathrm{T}} \in \mathbf{R}^{n_0},$$

$$\vdots$$

$$\boldsymbol{\theta}_c = [\alpha_{c1}, \alpha_{c2}, \ldots, \alpha_{cn}, \beta_{c1}, \beta_{c2}, \ldots, \beta_{cnq}]^{\mathrm{T}} \in \mathbf{R}^{n_0},$$

$$n_0 = n + nq.$$

The overall input vector $\boldsymbol{\psi}(kq)$ and the sub-input vectors $\boldsymbol{\psi}_i(kq)$ $(i=1,2,\dots,c)$ of the fuzzy model are indicated as follows:

$$\boldsymbol{\psi}(kq)=[\boldsymbol{\psi}_1^{\mathrm{T}}(kq),\boldsymbol{\psi}_2^{\mathrm{T}}(kq),\dots,\boldsymbol{\psi}_c^{\mathrm{T}}(kq)]^{\mathrm{T}},$$

$$\begin{aligned}\boldsymbol{\psi}_1(kq)=[&-\mu_1(kq)y(kq-q),-\mu_1(kq)y(kq-2q),\dots,\\&-\mu_1(kq)y(kq-nq),\mu_1(kq)u(kq-1),\dots,\\&\mu_1(kq)u(kq-nq)]^{\mathrm{T}},\\
\boldsymbol{\psi}_2(kq)=[&-\mu_2(kq)y(kq-q),-\mu_2(kq)y(kq-2q),\dots,\\&-\mu_2(kq)y(kq-nq),\mu_2(kq)u(kq-1),\dots,\\&\mu_2(kq)u(kq-nq)]^{\mathrm{T}},\\
&\vdots\\
\boldsymbol{\psi}_c(kq)=[&-\mu_c(kq)y(kq-q),-\mu_c(kq)y(kq-2q),\dots,\\&-\mu_c(kq)y(kq-nq),\mu_c(kq)u(kq-1),\dots,\\&\mu_c(kq)u(kq-nq)]^{\mathrm{T}}.\end{aligned}$$

In this way, (7) can be written as a linear regression form. Since the system always contains noise, the overall output of the fuzzy model can be expressed as follows:

$$y(kq)=\boldsymbol{\psi}^{\mathrm{T}}(kq)\boldsymbol{\theta}+v(kq) \tag{9}$$

where $\boldsymbol{\theta}=[\boldsymbol{\theta}_1^{\mathrm{T}},\boldsymbol{\theta}_2^{\mathrm{T}},\dots,\boldsymbol{\theta}_c^{\mathrm{T}}]^{\mathrm{T}}\in\mathbf{R}^{cn_0}$; $v(kq)$ is a statistic independent noise with variance σ^2 and zero mean.

In this paper, the stochastic gradient descent algorithm is used to estimate the conclusion parameters of the fuzzy model. The algorithm is shown as follows:

$$\widehat{\boldsymbol{\theta}}(kq)=\widehat{\boldsymbol{\theta}}(kq-q)+\frac{\boldsymbol{\psi}(kq)}{r(kq)}e(kq), \tag{10}$$

$$\widehat{\boldsymbol{\theta}}(kq+i)=\widehat{\boldsymbol{\theta}}(kq),\quad i=1,2,\dots,q-1, \tag{11}$$

$$e(kq)=y(kq)-\boldsymbol{\psi}^{\mathrm{T}}(kq)\widehat{\boldsymbol{\theta}}(kq-q), \tag{12}$$

$$r(kq)=r(kq-q)+\|\boldsymbol{\psi}(kq)\|^2,\quad r(0)=1 \tag{13}$$

where $\widehat{\boldsymbol{\theta}}(kq)$ represents the estimated vector of the conclusion parameters at time kq; $\widehat{\boldsymbol{\theta}}(0)$ is a very small real vector; the norm of matrix $\boldsymbol{X}$ is defined as $\|\boldsymbol{X}\|^2=\mathrm{tr}(\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}})$.

Next, we summarize the proposed identification algorithm as follows:

(i) Select the initial values, including c, $\overline{\boldsymbol{\varphi}}_l$ $(l=1,2,\dots,c)$, and $\widehat{\boldsymbol{\theta}}(0)$;

(ii) The input information vectors $\boldsymbol{\varphi}(kq)$ $(k=1,2,\dots,N)$ are partitioned in different clusters to acquire center vectors $\overline{\boldsymbol{\varphi}}_1$, $\overline{\boldsymbol{\varphi}}_2$, $\dots$, $\overline{\boldsymbol{\varphi}}_c$ and membership function values $\mu_l(kq)$ $(l=1,2,\dots,c;\ k=1,2,\dots,N)$ by using the fuzzy competitive learning algorithm;

(iii) Construct the overall input vector $\boldsymbol{\psi}(kq)$, and estimate the overall parameter vector $\widehat{\boldsymbol{\theta}}(kq)$ by using the algorithm in (10)–(13);

(iv) Let $k=k+1$, if $k\leqslant N$, return (ii), else go to (v);

(v) Return (ii), the above calculation process is repeated until satisfactory results are obtained.

4. Performance analysis

Before the performance analysis, first of all, some mathematical notations are given:

(i) $\lambda_{\max}[\boldsymbol{X}]$ and $\lambda_{\min}[\boldsymbol{X}]$ represent the maximum and minimum eigenvalues of matrix $\boldsymbol{X}$, respectively;

(ii) $f(t)=o(g(t))$ represents $\lim\limits_{t\to\infty}f(t)/g(t)=0$;

(iii) For $g(t)\geqslant 0$, if there exists a positive constant δ_1 such that $|f(t)|\leqslant\delta_1 g(t)$, then there is $f(t)=O(g(t))$ or $f(t)\sim g(t)$;

(iv) $|\boldsymbol{X}|=\det[\boldsymbol{X}]$ represents the determinant of matrix $\boldsymbol{X}$;

(v) $\|\boldsymbol{X}\|^2=\mathrm{tr}[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}]$ represents the trace of matrix $\boldsymbol{X}$.

In order to illustrate the convergence of the algorithm in (10)–(13), the following lemma is given.

Lemma 1 The following inequalities hold:

(i) $\displaystyle\sum_{i=1}^{\infty}\frac{\|\boldsymbol{\psi}(iq)\|^2}{r(iq)^2}<\infty$, a.s.;

(ii) $\displaystyle\sum_{i=1}^{\infty}\frac{\|\boldsymbol{\psi}(iq)\|^2}{r(iq)}<\ln r(kq)$, a.s..

Proof According to the definition of $r(kq)$ in (13), $r(kq)$ is monotonically increased, so we have

$$\sum_{i=1}^{\infty}\frac{\|\boldsymbol{\psi}(iq)\|^2}{r(iq)^2}\leqslant\sum_{i=1}^{\infty}\frac{\|\boldsymbol{\psi}(iq)\|^2}{r(iq-q)r(iq)}=$$
$$\sum_{i=1}^{\infty}\left[\frac{1}{r(iq-q)}-\frac{1}{r(iq)}\right]=\frac{1}{r(0)}-\frac{1}{r(\infty)}<\infty,\quad\text{a.s.},$$
$$\sum_{i=1}^{k}\frac{\|\boldsymbol{\psi}(iq)\|^2}{r(iq)}\leqslant\sum_{i=1}^{k}\int_{r(iq-q)}^{r(iq)}\frac{\mathrm{d}x}{r(iq)}\leqslant$$
$$\sum_{i=1}^{k}\int_{r(iq-q)}^{r(iq)}\frac{\mathrm{d}x}{x}=\sum_{i=1}^{k}\int_{r(0)}^{r(iq)}\frac{\mathrm{d}x}{x}\leqslant\ln r(kq),\quad\text{a.s.}.$$

Lemma 1 is proved. □

Theorem 1 Martingale convergence theorem [27,28]

Let $\{T(t)\}$, $\{\alpha(t)\}$, $\{\beta(t)\}$ be time sequences of non-negative random variables. These time series variables are adapted to the incremental σ-algebraic sequence F_t, i.e., they are F_t measurable, and satisfy

$$\mathrm{E}[T(t+1)|F_t]\leqslant T(t)-\alpha(t)+\beta(t). \tag{14}$$

If $\displaystyle\sum_{t=1}^{\infty}\beta(t)<\infty$, a.s., then $T(t)$ almost necessarily converges to a finite random variable T_0, $T(t)\to T_0$, and

$\sum_{t=1}^{\infty}\alpha(t)<\infty$.

Theorem 2 For (9), it is assumed that the noise $v(kq)$ is a martingale difference sequence defined in probability space (Ω, F, P), which is an incremental σ algebraic sequence $\{F_{kq}, k\in\mathbf{N}\}$. It also satisfies the following conditions:

(i) $\mathrm{E}[v(kq)|F_{kq-q}]=0$, a.s.;

(ii) $\mathrm{E}[v^2(kq)|F_{kq-q}]=\sigma_v^2(kq)\leqslant\overline{\sigma}_v^2\leqslant\infty$, a.s..

Let $\boldsymbol{R}(kq)=\sum_{i=1}^{k}\boldsymbol{\psi}(iq)\boldsymbol{\psi}^{\mathrm{T}}(iq)$. If the input vector $\{\boldsymbol{\psi}(iq),\ i=1,2,\ldots,k\}$ in (9) is continuously stimulated and $r(kq)=O(\lambda_{\min}[\boldsymbol{R}(kq)])$ exists, the parameter estimation error given by the algorithm in (10)–(13) converges to zero uniformly, that is,

$$\|\widehat{\boldsymbol{\theta}}(kq)-\boldsymbol{\theta}\|^2=O\left(\frac{r(kq)}{\lambda_{\min}[\boldsymbol{R}(kq)]}\right)\to 0,\ k\to\infty;\ \text{ a.s..}$$

Proof Define parameter error vector $\widetilde{\boldsymbol{\theta}}(kq)$,

$$\widetilde{\boldsymbol{\theta}}(kq)=\widehat{\boldsymbol{\theta}}(kq)-\boldsymbol{\theta}.$$

Let

$$\widetilde{Y}(kq)=\boldsymbol{\psi}^{\mathrm{T}}(kq)\widehat{\boldsymbol{\theta}}(kq-q)-\boldsymbol{\psi}^{\mathrm{T}}(kq)\boldsymbol{\theta}= \boldsymbol{\psi}^{\mathrm{T}}(kq)\widetilde{\boldsymbol{\theta}}(kq-q). \tag{15}$$

By subtracting $\boldsymbol{\theta}$ from both sides of (10) and using (9), (12) and (13), we can get the following result:

$$\widetilde{\boldsymbol{\theta}}(kq)=\widetilde{\boldsymbol{\theta}}(kq-q)+ \frac{\boldsymbol{\psi}(kq)}{r(kq)}[\boldsymbol{\psi}^{\mathrm{T}}(kq)\boldsymbol{\theta}+v(kq)-\boldsymbol{\psi}^{\mathrm{T}}(kq)\widehat{\boldsymbol{\theta}}(kq-q)]= \widetilde{\boldsymbol{\theta}}(kq-q)+\frac{\boldsymbol{\psi}(kq)}{r(kq)}[-\widetilde{Y}(kq)+v(kq)]= \widetilde{\boldsymbol{\theta}}(kq-q)+\Delta\widetilde{\boldsymbol{\theta}}(kq) \tag{16}$$

where $\Delta\widetilde{\boldsymbol{\theta}}(kq)=\dfrac{\boldsymbol{\psi}(kq)}{r(kq)}[-\widetilde{Y}(kq)+v(kq)]$.

Taking norm for the left and right sides of (16) at the same time, we can get

$$\|\widetilde{\boldsymbol{\theta}}(kq)\|^2=\|\widetilde{\boldsymbol{\theta}}(kq-q)\|^2+2\frac{\widetilde{Y}(kq)}{r(kq)}[-\widetilde{Y}(kq)+v(kq)]+ \frac{\|\boldsymbol{\psi}(kq)\|^2}{r^2(kq)}[-\widetilde{Y}(kq)+v(kq)]^2= \|\widetilde{\boldsymbol{\theta}}(kq-q)\|^2+2\frac{\widetilde{Y}(kq)}{r(kq)}[-\widetilde{Y}(kq)+v(kq)]+ \frac{\|\boldsymbol{\psi}(kq)\|}{r^2(kq)}[\widetilde{Y}^2(kq)+v^2(kq)-2\widetilde{Y}(kq)v(kq)]=$$

$$\|\widetilde{\boldsymbol{\theta}}(kq-q)\|^2-\left[\frac{2}{r(kq)}-\frac{\|\boldsymbol{\psi}(kq)\|^2}{r^2(kq)}\right]\widetilde{Y}^2(kq)+ 2\left[\frac{1}{r(kq)}-\frac{\|\boldsymbol{\psi}(kq)\|^2}{r^2(kq)}\right]\widetilde{Y}(kq)v(kq)+\frac{\|\boldsymbol{\psi}(kq)\|^2}{r^2(kq)}v^2(kq)= \|\widetilde{\boldsymbol{\theta}}(kq-q)\|^2-\frac{r(kq)+r(kq-q)}{r^2(kq)}\widetilde{Y}^2(kq)+ \frac{\|\boldsymbol{\psi}(kq)\|^2}{r^2(kq)}v^2(kq)+\frac{2r(kq-q)}{r^2(kq)}\widetilde{Y}(kq)v(kq)\leqslant \|\widetilde{\boldsymbol{\theta}}(kq-q)\|^2-\frac{1}{r(kq)}\widetilde{Y}^2(kq)+\frac{\|\boldsymbol{\psi}(kq)\|^2}{r^2(kq)}v^2(kq)+ \frac{2r(kq-q)}{r^2(kq)}\widetilde{Y}(kq)v(kq). \tag{17}$$

Because $\widetilde{Y}(kq)$, $\|\boldsymbol{\psi}(kq)\|^2$, $r(kq)$ and $v(kq)$ are independent and F_{kq-q} is measurable, we can get the conditional expectation of F_{kq-q} on both sides of the above equation by using the hypotheses (i) and (ii) of Theorem 2.

$$\mathrm{E}[\|\widetilde{\boldsymbol{\theta}}(kq)\|^2|F_{kq-q}]\leqslant\|\widetilde{\boldsymbol{\theta}}(kq-q)\|^2- \frac{\widetilde{Y}^2(kq)}{r(kq)}+\frac{\|\boldsymbol{\psi}(kq)\|^2}{r^2(kq)}\sigma_v(kq)\leqslant \|\widetilde{\boldsymbol{\theta}}(kq-q)\|^2-\frac{\widetilde{Y}^2(kq)}{r(kq)}+\frac{\|\boldsymbol{\psi}(kq)\|^2}{r^2(kq)}\overline{\sigma}_v \tag{18}$$

Using Lemma 1, the calculation value of the sum of the last term from $k=1$ to $k=\infty$ on the right of (18) is bounded, namely $\sum_{k=1}^{\infty}\dfrac{\|\boldsymbol{\psi}(kq)\|\overline{\sigma}_v}{r^2(kq)}<\infty$. By using Theorem 1, we can know that $\|\widetilde{\boldsymbol{\theta}}(kq)\|^2$ almost certainly converges to bounded random variable C_0 and we have

$$\|\widetilde{\boldsymbol{\theta}}(kq)\|^2\to C_0<\infty,\quad \text{a.s.}; k\to\infty$$

and

$$\sum_{k=1}^{\infty}\frac{\widetilde{Y}^2(kq)}{r(kq)}<\infty,\quad \text{a.s..}$$

Since $k\to\infty$, $r(kq)\to\infty$, the following equation can be obtained by using the lemma:

$$\lim_{k\to\infty}\frac{1}{r(kq)}\sum_{i=1}^{k}\widetilde{Y}^2(iq)=0,\quad \text{a.s..} \tag{19}$$

Using (16), we can get

$$\widetilde{\boldsymbol{\theta}}(kq-iq-q)=\widetilde{\boldsymbol{\theta}}(kq)-\sum_{j=0}^{i}\Delta\widetilde{\boldsymbol{\theta}}(kq-jq). \tag{20}$$

If kq in (15) is replaced by $kq-iq$, there exists

$$\widetilde{Y}(kq-iq)=\boldsymbol{\psi}^{\mathrm{T}}(kq-iq)\widetilde{\boldsymbol{\theta}}(kq-iq-q).$$

By using (20), the following equation can be established

$$\widetilde{Y}(kq-iq)=\boldsymbol{\psi}^{\mathrm{T}}(kq-iq)[\widetilde{\boldsymbol{\theta}}(kq)-\sum_{j=0}^{i}\Delta\widetilde{\boldsymbol{\theta}}(kq-jq)],$$

or

$$\boldsymbol{\psi}^{\mathrm{T}}(kq-iq)\widetilde{\boldsymbol{\theta}}(kq)=$$

$$\widetilde{Y}(kq-iq)+\boldsymbol{\psi}^{\mathrm{T}}(kq-iq)\sum_{j=0}^{i}\widetilde{\boldsymbol{\theta}}(kq-jq).$$

The above equation is squared on both sides, using inequality $(a+b)^2 \leqslant 2(a^2+b^2)$, we can get the following result:

$$\|\boldsymbol{\psi}^{\mathrm{T}}(kq-iq)\widetilde{\boldsymbol{\theta}}(kq)\|^2 \leqslant 2\widetilde{Y}^2(kq-iq)+$$

$$2\left\|\boldsymbol{\psi}^{\mathrm{T}}(kq-iq)\sum_{j=0}^{i}\Delta\widetilde{\boldsymbol{\theta}}(kq-jq)\right\|^2=$$

$$2\widetilde{Y}^2(kq-iq)+2\|\boldsymbol{\psi}^{\mathrm{T}}(kq-iq)\|^2\cdot$$

$$\left\|\sum_{j=0}^{i}\frac{\boldsymbol{\psi}(kq-iq)}{r(kq-jq)}[-\widetilde{Y}(kq-jq)+v(kq-jq)]\right\|^2.$$

Because $\widetilde{Y}(kq-iq)$, $\boldsymbol{\psi}(kq-iq)$, $r(kq-iq)$ and $v(kq-iq)$ are independent and F_{kq-q} is measurable, we can get the conditional expectation of F_{kq-q} on both sides of the above equation by using the hypotheses (i) and (ii) of Theorem 2.

$$\mathrm{E}\left[\|\boldsymbol{\psi}^{\mathrm{T}}(kq-iq)\widetilde{\boldsymbol{\theta}}(kq)\|^2|F_{kq-q}\right] \leqslant 2\widetilde{Y}^2(kq-iq)+$$

$$2\|\boldsymbol{\psi}(kq-iq)\|\sum_{j=0}^{i}\frac{\|\boldsymbol{\psi}(kq-jq)\|^2}{r^2(kq-jq)}[\widetilde{Y}^2(kq-iq)+\overline{\sigma}_v^2]$$

By the sum from $i=0$ to $i=k-1$ on both sides of the upper inequality divided by $r(kq)$, we can get the following result:

$$\frac{\mathrm{E}[\|\boldsymbol{\theta}^{\mathrm{T}}(kq)\boldsymbol{R}(kq)\boldsymbol{\theta}(kq)\||F_{kq-q}]}{r(kq)} \leqslant$$

$$\frac{2}{r(kq)}\sum_{i=0}^{k-1}\widetilde{Y}^2(kq-iq)+2\sum_{i=0}^{k-1}\frac{\|\boldsymbol{\psi}(kq-iq)\|^2}{r(kq)}\cdot$$

$$\sum_{j=0}^{i}\frac{\|\boldsymbol{\psi}(kq-iq)\|^2}{r^2(kq-iq)}\widetilde{Y}^2(kq-iq)+$$

$$2\sum_{i=0}^{k-1}\frac{\|\boldsymbol{\psi}(kq-iq)\|^2}{r(kq)}\cdot\sum_{j=0}^{i}\frac{\|\boldsymbol{\psi}(kq-iq)\|^2}{r^2(kq-iq)}\overline{\sigma}_v^2.$$

By using the second inequality of Lemma 1, we can get

$$\frac{\mathrm{E}[\|\widetilde{\boldsymbol{\theta}}^{\mathrm{T}}(kq)\boldsymbol{R}(kq)\widetilde{\boldsymbol{\theta}}(kq)\||F_{kq-q}]}{r(kq)} \leqslant$$

$$\frac{2}{r(kq)}\sum_{i=1}^{k}\widetilde{Y}^2(iq)+\frac{2}{r(kq)}\sum_{i=2}^{k}\widetilde{Y}^2(iq)+\frac{2\ln r(kq)}{r(kq)}\overline{\sigma}_v^2 \rightarrow 0.$$

Let $k\rightarrow\infty$, using (19), the following conclusion can be drawn:

$$\frac{\mathrm{E}[\|\widetilde{\boldsymbol{\theta}}^{\mathrm{T}}(kq)\boldsymbol{R}(kq)\widetilde{\boldsymbol{\theta}}(kq)\|^2|F_{kq-q}]}{r(kq)} \rightarrow 0, \quad \text{a.s.}$$

Hence, the following inequality hold:

$$\frac{\lambda_{\min}[\boldsymbol{R}(kq)]\|\widetilde{\boldsymbol{\theta}}(kq)\|^2}{r(kq)} \leqslant \frac{\widetilde{\boldsymbol{\theta}}^{\mathrm{T}}(kq)\boldsymbol{R}(kq)\widetilde{\boldsymbol{\theta}}(kq)}{r(kq)}.$$

Since $r(kq)=O(\lambda_{\min}[\boldsymbol{R}(kq)])$, we have

$$\|\widetilde{\boldsymbol{\theta}}(kq)\|^2=\|\widehat{\boldsymbol{\theta}}(kq)-\boldsymbol{\theta}\|^2=O\left(\frac{r(kq)}{\lambda_{\min}[\boldsymbol{R}(kq)]}\right)\rightarrow 0.$$

Theorem 2 is proved. □

5. Simulation

In this paper, the hair quality detection data set of the certain quality inspection institute is employed to study its modeling problem. Hair quality detection should include three titration processes, namely redox titration, complexometric titration and acid-base titration. For the proposed three titration methods, because the chemical reactions in each titration process are very complex, many chemical equations are needed to calculate, so each titration process needs to rely on visual observation of indicator discoloration to build the model of the titration process, which causes some problems: (i) the detection process is slow, and it cannot meet the real-time requirement; (ii) manual detection is prone to cause detection error and data omission; (iii) high economic cost occurs.

Therefore, it is important that the acidity and basicity of three titration methods can be automatically tracked and forecasted by the automatic detection technique. Therefore, the automatic prediction can be achieved instead of manual detection by using the automation detection system combined from PH meter measurements, A/D sampling circuits and the fuzzy identification algorithm. In this paper, the dynamic process of acid-base titration is studied by using fuzzy modeling.

In the study of the dynamic process of acid-base titration, three solutions of HNO_3, NaOH and $NaHCO_3$ are simultaneously poured into the chemical reactor, and the PH neutralization value of the complex solution is detected by intelligent sensors. The dynamic process system is shown in Fig. 1.

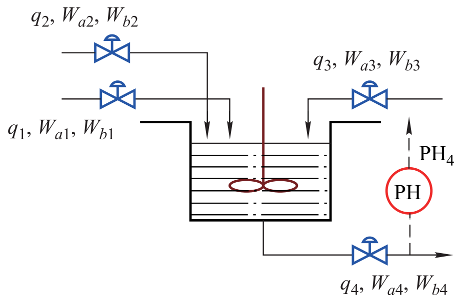

Fig. 1 Dynamic process of PH neutralization value for hair quality detection

In Fig. 1, the flow rates of HNO_3, NaOH and $NaHCO_3$ solutions are represented by F_a, F_b and F_{bf}, respectively, and the following chemical reaction equations are given:

$$W_a = [H^+] - [OH^-] - [HCO_3^-] - 2[CO_3^{2-}],$$

$$W_b = [H_2CO_3] + [HCO_3^-] + [CO_3^{2-}],$$

$$K_w = [H^+][OH^-], \quad K_{a1} = \frac{[HCO_3^-][H^+]}{[H_2CO_3]},$$

$$K_{a2} = \frac{[CO_3^{2-}][H^+]}{[HCO_3^-]}, \quad pk_1 = -\lg K_{a1},$$

$$pk_2 = -\lg K_{a2}, \quad \mathrm{PH} = -\lg[H^+].$$

In addition, the equation of the static titration curve is shown as follows:

$$W_a - 10^{-\mathrm{PH}} + 10^{\mathrm{PH}-14} +$$
$$W_b \frac{1 + 2 \times 10^{\mathrm{PH}-pk_2}}{1 + 10^{pk_2-\mathrm{PH}} + 10^{\mathrm{PH}-pk_2}} = 0.$$

The system dynamic mechanism model is

$$\begin{cases} \dfrac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = f(\boldsymbol{x}) + g(\boldsymbol{x})u + p(\boldsymbol{x})\mathrm{d}c(\boldsymbol{x}, y) = 0 \\ c(\boldsymbol{x}, y) = 0 \end{cases}$$

where $\boldsymbol{x} = [h, W_{a4}, W_{b4}]^{\mathrm{T}}$, $u = F_b$, $d = F_{bf}$, $y = \mathrm{PH}$, $c = c_1$,

$$f(\boldsymbol{x}) = \begin{bmatrix} -\dfrac{c_V x_1^{1/2}}{A} & 0 & 0 \end{bmatrix}^{\mathrm{T}},$$

$$g(\boldsymbol{x}) = \begin{bmatrix} \dfrac{1}{A} & \dfrac{1}{Ax_1}(W_{ba} - x_2) & \dfrac{1}{Ax_1}(W_{bb} - x_3) \end{bmatrix}^{\mathrm{T}},$$

$$p(\boldsymbol{x}) = \begin{bmatrix} \dfrac{1}{A} & \dfrac{1}{Ax_1}(W_{bfa} - x_2) & \dfrac{1}{Ax_1}(W_{bfb} - x_3) \end{bmatrix}^{\mathrm{T}}.$$

$$c_1(\boldsymbol{x}, y) = y_1 - x_1, \quad c_2(\boldsymbol{x}, y) = 0.$$

Table 1 gives the normal operating parameters of the PH neutralization process.

Table 1 Normal operating parameters of PH neutralization process

Parameter	Setting value	Parameter	Setting value
A/cm^2	207	W_{a3}/(moL·L^{-1})	-3.06×10^{-3}
H/cm	14.0	W_{b1}/(moL·L^{-1})	0
C_0/(mL·cm$^{-0.5}$·s)	8.75	W_{b2}/(moL·L^{-1})	3×10^{-2}
q_1/(mL·S^{-1})	16.6	W_{b3}/(moL·L^{-1})	5×10^{-5}
q_2/(mL·S^{-1})	0.55	pK_1	6.35
q_3/(mL·S^{-1})	16.2	pK_2	10.25
W_{a1}/(moL·L^{-1})	3×10^{-3}	τ	0.5
W_{a2}/(moL·L^{-1})	-3×10^{-2}	PH_4	7

In the process of PH neutralization of acid-base titration for hair quality detection, the flow rate F_b of alkali solution is used as the control variable $u(t)$, and the PH measuring value of output liquid is used as the output $y(t)$. The refreshing time of the input liquid is 1 s, and the sampled time of the output liquid is 3 s. In the detection, we collect 180 sampling points of flow rate and 60 measuring points of PH.

Select $y(k-3)$, $y(k-6)$, $u(k-1)$, $u(k-2)$, $u(k-3)$, $u(k-4)$, $u(k-5)$ and $u(k-6)$ to construct input information vector $\varphi(kq)$. The total number of fuzzy rules is taken as 6. The identification method in this paper is used to build the fuzzy model of the PH neutralization process of hair quality detection. In order to illustrate the effectiveness of the proposed method, we use two clustering methods, including the fuzzy clustering method (FCM) in [29] and the weighted fuzzy clustering method (WFCM) in [30], to determine the premise structure of the fuzzy model, and use the gradient descent method to identify the parameters of the conclusion part. Fig. 2 shows the comparison result among the output of the fuzzy model, the output of WFCM, the output of FCM and the actual output.

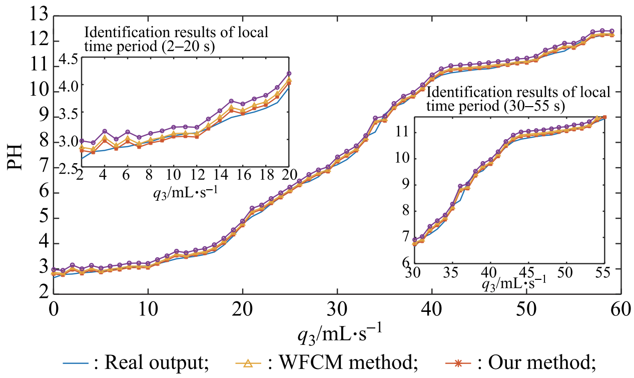

Fig. 2 Comparison of the output of the fuzzy model and the actual output

Fig. 3 is the identification error diagram of these methods. Table 2 shows the mean square errors (MSEs) of different methods. By comparing and analyzing Table 2, Fig. 2 and Fig. 3, it is shown that the fuzzy model established by the proposed method can well fit the non-linear dynamical characteristics of PH neutralization. The MSE of our method is better than that of the WFCM method and that of the FCM method.

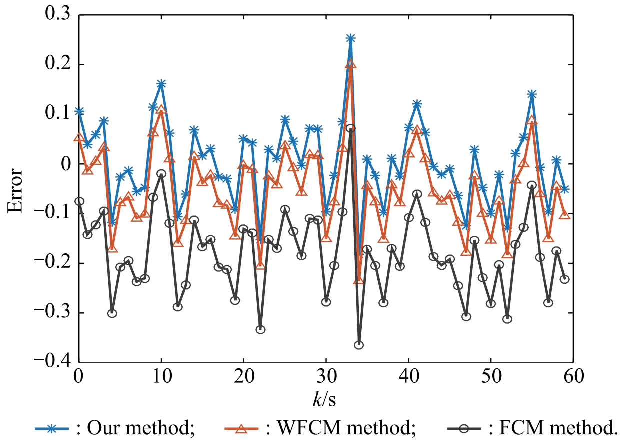

Fig. 3 Identification error diagram

Table 2 Comparison of several modeling methods

Modeling method	Number of rules	MSE
FCM	6	0.217 0
WFCM	6	0.105 0
Our method	6	0.063 2

6. Conclusions

Aiming at the modeling issues of the multi-rate sampled nonlinear system, a novel identification method based on the fuzzy model is proposed. The basic idea is to expand the nonlinear system into the model combined from local linear systems of working points. According to the multi-model modeling principle, the premise structure of the fuzzy model is determined by using fuzzy competitive learning, and the conclusion parameters are identified by using the stochastic gradient descent algorithm. The algorithm convergence of the modeling algorithm is proved by some lemmas and theorems. Modeling problems of the multi-input and multi-output (MIMO) multi-rate system are very difficult problems. In the future, the fuzzy modeling of MIMO multi-rate systems is our research direction.

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