

Direct solution for fixed source location using well-posed TDOA and FDOA measurements

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Abstract: Based on the time differences of arrival (TDOA) and frequency differences of arrival (FDOA) measurements of the given planar stationary radiation source, the joint TDOA/FDOA location algorithm which solves the location of the target directly is proposed. Compared with weighted least squares (WLS) methods, the proposed algorithm is also suitable for well-posed conditions, and gets rid of the dependence on the constraints of Earth's surface. First of all, the solution formulas are expressed by the radial range. Then substitute it into the equation of the radial range to figure out the radial range between the target and the reference station. Finally use the solution expression of the target location to estimate the location of the target accurately. The proposed algorithm solves the problem that WLS methods have a large positioning error when the number of observation stations is not over-determined. Simulation results show the effectiveness of the proposed algorithm, including effectively increasing the positioning accuracy and reducing the number of observatories.

Keywords: passive location, joint time differences of arrival/ frequency differences of arrival (TDOA/FDOA) location, direct solution, stationary emitter, well-posed equations.

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1. Introduction

In recent years, the passive localization technology has attracted more and more attention. According to the number of observing stations, passive localization can be divided into two parts [1]: the first part is the single-station passive localization, with simple equipment and using less measured information to locate the target; the other part is the multi-station passive localization, which is an effective method to realize fast and high-precision localization of radiation source [2]. Relative to the single-station passive location, the multi-station passive localization can always locate the target with high-precision in short time.

Compared with the stationary multi-station passive localization, it is more flexible and able to move to specific areas according to the needs of the task.

The localization method is usually determined by the observed measurements, in which the time differences of arrival (TDOA) information is one of the most basic measurements, and it has a higher localization accuracy under the current observed conditions [3]. Therefore, it is meaningful to research the passive localization algorithm based on the TDOA measurement. When there is a relative motion between the source and the observing stations, the location accuracy can be improved by joint TDOA and frequency differences of arrival (FDOA). Moreover, with the large number of applications of mobile observations and their maneuvering advantages, estimating the location of an emitter from a combination of TDOA and FDOA has been the focus of current research [4–11]. The mobile emitter localization and tracking can be obtained by using TDOA and FDOA measurements in [5]. At the same time, He et al. proposed a new method about a mobile target localization with joint TDOA/FDOA by multidimensional scaling analysis in [6]. Then, in the second stage of the two stage weighted least squares (TSWLS), the relationship among the nuisance parameters and the unknown position and velocity of the source are utilized to refine the solution and improve the performance of the method [7]. When the target radiation source is stationary, an iterative algorithm is proposed based on the TDOA and FDOA united location using moving observations [8]. However, the joint TDOA/FDOA location method mentioned above can only be applied when the number of observation stations is over-determined. If the number of stations is well-posed, there would be low positioning accuracy which can not meet the actual needs [4,7,11].

Therefore, a joint TDOA/FDOA algorithm is proposed, without relying on the Earth constraint equation, to solve the problem of low positioning accuracy under well-posed conditions. Based on measured time difference and fre-

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quency difference equations, the proposed method solves the target location directly. First of all, the solution formulas are expressed by the radial range. Then we substitute the target location expression into the equation of the radial range to figure out the radial range between the target and the reference station. Finally use the solution expression of the target location to obtain the target location accurately. Simultaneously, the simulation analysis is used to examine the algorithm's correctness and performance.

2. Positioning principle

The joint TDOA/FDOA location of dual-station is a combined localization technology. Based on the TDOA and FDOA united location of stationary source using moving dual observations, the range difference between observations and the target is related to TDOA, while the range difference rate is related to FDOA. First, transform the TDOA and FDOA equations to a set of linear equations by introducing nuisance parameters, then use the other constraint equations to obtain the nuisance parameters, and finally estimate the target location parameters by the linear equations directly.

In a three-dimensional (3-D) scenario, each TDOA measurement defines a region of possible target locations around two unique hyperbola. In the same way, a single FDOA measurement defines an area of possible target locations, whose shape is two closed semi-elliptic surfaces. Therefore, at least three moving observations are needed to achieve the TDOA and FDOA united location for space targets. However, in practical applications, such as locating the targets of sea surface and ground, since their altitude information is known, only the longitude and latitude parameters need to be estimated, namely, the 2-dimensional (2-D) plane localization problem in 3-D space. At this point, two moving observations can meet the most basic localization requirements. The dual-station localization system requires no networking and requires communication between the two stations. Thus, the system is simple to implement and the operation is flexible, and it has become the focus of current passive localization research.

As shown in Fig. 1, considering a 3-D scenario where two moving observations are used to determine the position \mathbf{u} of a given planar stationary emitter using the TDOA and FDOA united location. The observation positions $\mathbf{s}_i = (x_i, y_i, z_i)^T$ and velocities $\dot{\mathbf{s}}_i = (\dot{x}_i, \dot{y}_i, \dot{z}_i)^T$ where $i = 1, 2$, are assumed known. If the given target plane is $z = z_T$, then there is the target position $\mathbf{u} = (x, y, z_T)^T$. Assuming that the first observation is a reference receiver, the TDOA Δt_{21} and the FDOA Δf_{21} about signal arriving at the second station with respect to the reference receiver can be expressed as

$$\Delta t_{21} = \frac{1}{c} (|\mathbf{u} - \mathbf{s}_2| - |\mathbf{u} - \mathbf{s}_1|) = \frac{1}{c} (\sqrt{(x - x_2)^2 + (y - y_2)^2 + (z_T - z_2)^2} - \sqrt{(x - x_1)^2 + (y - y_1)^2 + (z_T - z_1)^2})$$

and

$$\Delta f_{21} = \frac{f_c}{c} \left(\frac{-\dot{\mathbf{s}}_2^T (\mathbf{u} - \mathbf{s}_2)}{|\mathbf{u} - \mathbf{s}_2|} - \frac{-\dot{\mathbf{s}}_1^T (\mathbf{u} - \mathbf{s}_1)}{|\mathbf{u} - \mathbf{s}_1|} \right) = \frac{f_c}{c} \left(-\frac{\dot{x}_2(x - x_2) + \dot{y}_2(y - y_2) + \dot{z}_2(z_T - z_2)}{\sqrt{(x - x_2)^2 + (y - y_2)^2 + (z_T - z_2)^2}} + \frac{\dot{x}_1(x - x_1) + \dot{y}_1(y - y_1) + \dot{z}_1(z_T - z_1)}{\sqrt{(x - x_1)^2 + (y - y_1)^2 + (z_T - z_1)^2}} \right)$$

where f_c is the center frequency of the signal radiated by the stationary source, and c represents the speed of signal propagation.

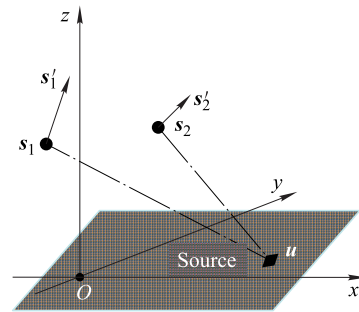


Fig. 1 3-D dual-station localization geometry diagram

In order to more clearly illustrate the principle of joint TDOA/FDOA location, the 2-D dual-station localization is taken as an example and the corresponding localization principle diagram is shown in Fig. 2. The two localization equations are determined by TDOA and FDOA measurements from the two observing stations. On a 2-D plane, these two equations can determine two localization lines, whose intersection is the source position.

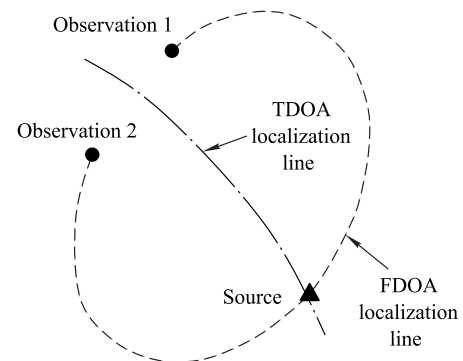


Fig. 2 2-D dual-station localization principle diagram

3. Proposed solution

According to the previously assumed localization scenario, the distance between the source and the receiver i is

$$r_i = |\mathbf{u} - \mathbf{s}_i| = \sqrt{(\mathbf{u} - \mathbf{s}_i)^T(\mathbf{u} - \mathbf{s}_i)}, \quad i = 1, 2. \quad (1)$$

Assuming that the time difference and range difference of the signal arriving at observing station 2 and station 1 are Δt_{21} and r_{21} respectively, the signal propagation speed is c , then the relationship between r_{21} and Δt_{21} is

$$r_{21} = c\Delta t_{21} = r_2 - r_1. \quad (2)$$

Rewrite (2) as $r_{21} + r_1 = r_2$, square both sides, and substitute (1) for r_1^2 and r_2^2 , then the TDOA equation is

$$r_{21}^2 + 2r_{21}r_1 = \mathbf{s}_2^T \mathbf{s}_2 - \mathbf{s}_1^T \mathbf{s}_1 - 2(\mathbf{s}_2 - \mathbf{s}_1)^T \mathbf{u}. \quad (3)$$

Note that (3) is nonlinear with respect to \mathbf{u} since r_1 is related to \mathbf{u} through (1).

The time derivative of (1)

$$\dot{r}_i = \frac{(-\dot{\mathbf{s}}_i)^T(\mathbf{u} - \mathbf{s}_i)}{r_i}, \quad i = 1, 2 \quad (4)$$

gives the relationship between the range rate and target location parameters.

The range rate corresponds to the Doppler frequency, and the range rate difference corresponds to the Doppler frequency difference. In order to use the frequency difference information, take the time derivative of (3) and obtain

$$2(\dot{r}_{21}r_{21} + \dot{r}_{21}r_1 + r_{21}\dot{r}_1) = 2(\dot{\mathbf{s}}_2^T \mathbf{s}_2 - \dot{\mathbf{s}}_1^T \mathbf{s}_1 - (\dot{\mathbf{s}}_2 - \dot{\mathbf{s}}_1)^T \mathbf{u}) \quad (5)$$

where \dot{r}_{21} is the range rate difference, which has a definite relationship with the Doppler frequency difference, $\dot{r}_{21} = \lambda \Delta f_{21}$, and λ is the signal wavelength.

The observation equations of TDOA and FDOA united location

$$\begin{cases} r_{21}^2 - \mathbf{s}_2^T \mathbf{s}_2 + \mathbf{s}_1^T \mathbf{s}_1 = -2(\mathbf{s}_2 - \mathbf{s}_1)^T \mathbf{u} - 2r_{21}r_1 \\ \dot{r}_{21}r_{21} - \dot{\mathbf{s}}_2^T \mathbf{s}_2 + \dot{\mathbf{s}}_1^T \mathbf{s}_1 = -(\dot{\mathbf{s}}_2 - \dot{\mathbf{s}}_1)^T \mathbf{u} - \dot{r}_{21}r_1 - r_{21}\dot{r}_1 \end{cases} \quad (6)$$

can be got from (3) and (5). For the first equation in (6), substitute $\mathbf{u} = (x, y, z_T)^T$ and obtain

$$(x_2 - x_1)x + (y_2 - y_1)y = m - r_{21}r_1 \quad (7)$$

where

$$m = -\frac{1}{2}[r_{21}^2 - \mathbf{s}_2^T \mathbf{s}_2 + \mathbf{s}_1^T \mathbf{s}_1 + 2(z_2 - z_1)z_T]. \quad (8)$$

Substituting $\mathbf{u} = (x, y, z_T)^T$, $\mathbf{s}_i = (x_i, y_i, z_i)^T$, $\dot{\mathbf{s}}_i = (\dot{x}_i, \dot{y}_i, \dot{z}_i)^T$ ($i = 1, 2$) and (4) into the second equation in (6), there is

$$\begin{aligned} [r_{21}\dot{x}_1 - (\dot{x}_2 - \dot{x}_1)r_1]x + [r_{21}\dot{y}_1 - (\dot{y}_2 - \dot{y}_1)r_1]y = \\ \dot{r}_{21}r_1^2 + n \cdot r_1 + l, \end{aligned} \quad (9)$$

where

$$\begin{cases} n = \dot{r}_{21}r_{21} - \dot{\mathbf{s}}_2^T \mathbf{s}_2 + \dot{\mathbf{s}}_1^T \mathbf{s}_1 + (\dot{z}_2 - \dot{z}_1)z_T \\ l = r_{21}[\dot{x}_1x_1 + \dot{y}_1y_1 - (z_T - z_1)\dot{z}_1] \end{cases} \quad (10)$$

For simultaneous (7) and (9), the equations related to the TDOA and FDOA are

$$\begin{cases} (x_2 - x_1)x + (y_2 - y_1)y = m - r_{21}r_1 \\ [r_{21}\dot{x}_1 - (\dot{x}_2 - \dot{x}_1)r_1]x + \\ [r_{21}\dot{y}_1 - (\dot{y}_2 - \dot{y}_1)r_1]y = \dot{r}_{21}r_1^2 + n \cdot r_1 + l \end{cases} \quad (11)$$

Taking r_1 as a nuisance variable, the purpose of introducing the nuisance variables is to make (11) become a set of linear equations. Solve the equations

$$\begin{cases} x = \frac{a_1r_1^2 + b_1r_1 + c_1}{d_1r_1 + e_1} \\ y = \frac{a_2r_1^2 + b_2r_1 + c_2}{d_2r_1 + e_2} \end{cases} \quad (12)$$

where

$$\begin{cases} a_1 = r_{21}(\dot{y}_2 - \dot{y}_1) - \dot{r}_{21}(y_2 - y_1) \\ b_1 = -[r_{21}^2\dot{y}_1 + m(\dot{y}_2 - \dot{y}_1) + n(y_2 - y_1)] \\ c_1 = mr_{21}\dot{y}_1 - l(y_2 - y_1) \\ d_1 = (y_2 - y_1)(\dot{x}_2 - \dot{x}_1) - (x_2 - x_1)(\dot{y}_2 - \dot{y}_1) \\ e_1 = [(x_2 - x_1)\dot{y}_1 - (y_2 - y_1)\dot{x}_1]r_{21} \end{cases} \quad (13)$$

and

$$\begin{cases} a_2 = r_{21}(\dot{x}_2 - \dot{x}_1) - \dot{r}_{21}(x_2 - x_1) \\ b_2 = -[r_{21}^2\dot{x}_1 + m(\dot{x}_2 - \dot{x}_1) + n(x_2 - x_1)] \\ c_2 = mr_{21}\dot{x}_1 - l(x_2 - x_1) \\ d_2 = (x_2 - x_1)(\dot{y}_2 - \dot{y}_1) - (y_2 - y_1)(\dot{x}_2 - \dot{x}_1) \\ e_2 = [(y_2 - y_1)\dot{x}_1 - (x_2 - x_1)\dot{y}_1]r_{21} \end{cases} \quad (14)$$

When $i = 1$, rewrite (1) as

$$r_1^2 = (x - x_1)^2 + (y - y_1)^2 + (z_T - z_1)^2. \quad (15)$$

Substituting (12) into (15), we obtain

$$\begin{aligned} \left(\frac{a_1r_1^2 + b_1r_1 + c_1}{d_1r_1 + e_1} - x_1 \right)^2 + \\ \left(\frac{a_2r_1^2 + b_2r_1 + c_2}{d_2r_1 + e_2} - y_1 \right)^2 + \\ (z_T - z_1)^2 - r_1^2 = 0 \end{aligned} \quad (16)$$

Simplify (16), then we obtain

$$k_6r_1^6 + k_5r_1^5 + k_4r_1^4 + k_3r_1^3 + k_2r_1^2 + k_1r_1 + k_0 = 0 \quad (17)$$

where

$$k_0 = c_1^2 e_2^2 + c_2^2 e_1^2 + e_1^2 e_2^2 [x_1^2 + y_1^2 + (z_T - z_1)^2] - 2e_1 e_2 (x_1 c_1 e_2 + y_1 c_2 e_1), \quad (18)$$

$$k_1 = 2c_1^2 d_2 e_2 + 2b_1 c_1 e_2^2 + 2c_2^2 d_1 e_1 + 2b_2 c_2 e_1^2 + 2e_1 e_2 (d_1 e_2 + d_2 e_1) [x_1^2 + y_1^2 + (z_T - z_1)^2] - 2x_1 e_2 (2c_1 d_2 e_1 + c_1 d_1 e_2 + b_1 e_1 e_2) - 2y_1 e_1 (2c_2 d_1 e_2 + c_2 d_2 e_1 + b_2 e_1 e_2), \quad (19)$$

$$k_2 = c_1^2 d_2^2 + 4b_1 c_1 d_2 e_2 + 2a_1 c_1 e_2^2 + b_1^2 e_2^2 + c_2^2 d_1^2 + 4b_2 c_2 d_1 e_1 + 2a_2 c_2 e_1^2 + b_2^2 e_1^2 + [x_1^2 + y_1^2 + (z_T - z_1)^2] (d_2^2 e_1^2 + 4d_1 d_2 e_1 e_2 + d_1^2 e_2^2) - e_1^2 e_2^2 - 2y_1 (2c_2 d_1 d_2 e_1 + b_2 d_2 e_1^2 + c_2 d_1^2 e_2 + 2b_2 d_1 e_1 e_2 + a_2 e_1^2 e_2) - 2x_1 (2c_1 d_1 d_2 e_2 + b_1 d_1 e_2^2 + c_1 d_2^2 e_1 + 2b_1 d_2 e_1 e_2 + a_1 e_1^2 e_2), \quad (20)$$

$$k_3 = 2b_2 c_2 d_1^2 + 2(2a_2 c_2 + b_2^2) d_1 e_1 + 2a_2 b_2 e_1^2 + 2b_1 c_1 d_2^2 + 2(2a_1 c_1 + b_1^2) d_2 e_2 + 2a_1 b_1 e_2^2 + 2\{[x_1^2 + y_1^2 + (z_T - z_1)^2] d_1 d_2 - e_1 e_2\} (d_2 e_1 + d_1 e_2) - 2y_1 (c_2 d_1^2 d_2 + 2b_2 d_1 d_2 e_1 + a_2 d_2 e_1^2 + b_2 d_1^2 e_2 + 2a_2 d_1 e_1 e_2) - 2x_1 (c_1 d_2^2 d_1 + 2b_1 d_1 d_2 e_2 + a_1 d_1 e_2^2 + b_1 d_2^2 e_1 + 2a_1 d_2 e_1 e_2), \quad (21)$$

$$k_4 = (2a_1 c_1 + b_1^2) d_2^2 + 4a_1 b_1 d_2 e_2 + a_1^2 e_2^2 + (2a_2 c_2 + b_2^2) d_1^2 + 4a_2 b_2 d_1 e_1 + a_2^2 e_1^2 + [x_1^2 + y_1^2 + (z_T - z_1)^2] d_1^2 d_2^2 - 4d_1 d_2 e_1 e_2 - d_1^2 e_2^2 - d_2^2 e_1^2 - 2x_1 b_1 d_1 d_2^2 - 2x_1 a_1 d_2^2 e_1 - 4x_1 a_1 d_1 d_2 e_2 - 2y_1 b_2 d_2 d_1^2 - 2y_1 a_2 d_1^2 e_2 - 4y_1 a_2 d_1 d_2 e_1, \quad (22)$$

$$k_5 = 2a_1 b_1 d_2^2 + 2a_1^2 d_2 e_2 + 2a_2 b_2 d_1^2 + 2a_2^2 d_1 e_1 - 2d_1 d_2^2 e_1 - 2d_1^2 d_2 e_2 - 2a_1 d_1 d_2^2 x_1 - 2a_2 d_1^2 d_2 y_1 \quad (23)$$

and

$$k_6 = a_1^2 d_2^2 + a_2^2 d_1^2 - d_1^2 d_2^2. \quad (24)$$

Let \hat{r}_{21} and $\hat{\dot{r}}_{21}$ be the noisy range difference and range rate difference respectively. Assuming that the TDOA and FDOA measurements can be described by the additive noise model as

$$\begin{cases} \hat{r}_{21} = r_{21} + \hat{n}_{21} \\ \hat{\dot{r}}_{21} = \dot{r}_{21} + \hat{\dot{n}}_{21} \end{cases} \quad (25)$$

where \hat{n}_{21} and $\hat{\dot{n}}_{21}$ are TDOA and FDOA noises. Use the measured values \hat{r}_{21} and $\hat{\dot{r}}_{21}$ in the noise model of the above formula to replace the corresponding values r_{21} and \dot{r}_{21} in (12), then use (16) or (17) with the measured values

to solve the range r_1 from reference observation 1 to the target. Finally, use the solved range to estimate the target location by (16).

Solving (17) yields six roots, the positive real root is the value of r_1 . Therefore, there may be a problem of localization ambiguity based on the method of solving r_1 first and then solving the target positions x and y , that is, a fuzzy problem of r_1 . In this case, it is necessary to judge according to a priori information and remove the false values, so as to realize the localization of the radiation source. The simplest method to solve this problem is to increase observing times and make cluster analysis. Because the estimated values of the target location must be concentrated near the actual value, fuzzy estimates can be eliminated.

The following summarizes the specific steps for solving the target position using the proposed algorithm.

Step 1 Use (13) and (14) to obtain the coefficients a_1, b_1, c_1, d_1, e_1 and a_2, b_2, c_2, d_2, e_2 related to the target positions x and y in the noise environment.

Step 2 Solve the radial distance r_1 .

(i) Search method

Construct the equation shown in (16), and use the direct search method to search the solution of the equation that meets a certain search step and a certain search threshold.

(ii) Root method

i) According to (18) to (24), the corresponding coefficients $k_0 - k_6$ in (17) are solved successively.

ii) Construct (17), and use the direct root method to find the roots that satisfy the equation.

Step 3 Solve the radial distance r_1 ambiguity. Remove false values based on a priori information.

Step 4 Solve the target position. Substitute the solution \hat{r}_1 into (12) to solve the target positions \hat{x} and \hat{y} .

4. Existence of solutions

The joint TDOA/FDOA location algorithm proposed in this paper is based on the measurement equations of TDOA and FDOA. First, the solution formula of the target position is expressed by the radial range. Then, substitute the target location expression into the radial range equation to figure out the radial range between the target and the reference station. Finally, use the target location expression to estimate the target position accurately. In order to prove the correctness and validity of the proposed algorithm, the following simulations are performed.

The information of two stations and target radiation source is shown in Table 1.

Table 1 Positions and velocities of receivers and target (single scenario)

Name	Position/km			Velocity/(m/s)		
	x	y	z	v_x	v_y	v_z
Observation 1	0	0	6	120	180	0
Observation 2	8	10	5	100	150	0
Target	80	120	1	0	0	0

When the TDOA and FDOA measurement errors are both 10^{-5} times the size of their true values, the search method is now used to search for the solution of (16), and the root of (17) is solved by the root method. The searching range of the search method is 80 – 200 km, the search step length is 5 km, and the resolution threshold is 25 km^2 . Using the two different methods to solve the nuisance variables r_1 related to the position of the far-field target as shown in Fig. 3.

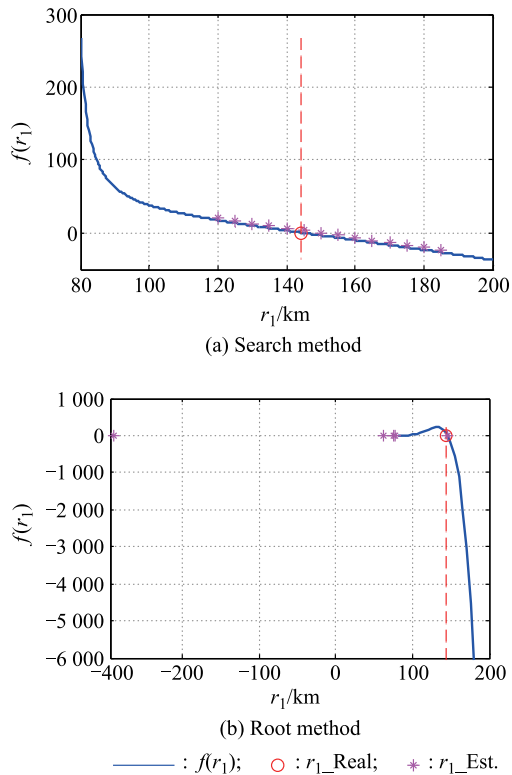


Fig. 3 Radial range solving by dual-station location

As can be seen from Fig. 3, whether the search method or the root method is used to solve the nuisance variables, the two methods will produce multiple solution results, namely the fuzzy problem of r_1 . In this case, it is necessary to judge according to a priori information and remove the false values, so as to realize the localization of the radiation source. The simplest method to solve this problem is to increase observing times and make cluster analysis. The estimated values of the target position must be concentrated near the actual values. Therefore, fuzzy estimates

can be eliminated. Increasing the number of stations can also improve the estimates.

When r_1 is solved, use the target location expression (12) to estimate the target position. It can be known from (12), the condition of the target position solution is that the denominator in (12) is not zero.

5. Simulation results

According to the joint TDOA/FDOA location principle, it can be seen that the two observations on the single platform move in the same direction and velocity, and on a straight line. No matter using the TDOA localization or the FDOA localization, the error of the localization curve is always the smallest on the mid-line perpendicular to the baseline. Therefore, choose the observations with parallel configuration for analysis, but here the two observations are not lying on the same line.

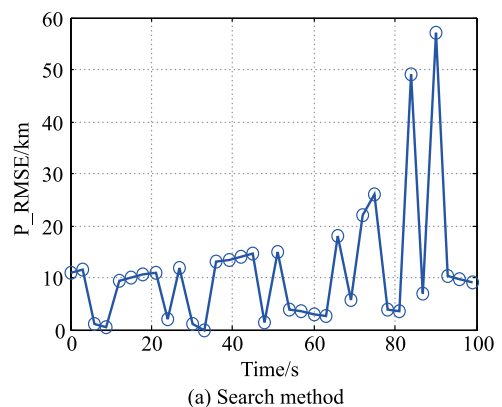
Since the two stations are configured in parallel, the observing stations and the target details are as follows.

Let the TDOA and FDOA measurement errors be 10^{-5} times the size of their true values, and continuously observe for 100 s with the observation time interval $\Delta = 3 \text{ s}$, then the TDOA and FDOA united method is used to solve the target location under the following conditions.

Scenario 1 Change the position coordinates of observing station 2 to observation 2 (Scenario 1) as shown in Table 2, and observe stations 1 and 2 moving in parallel on the same horizontal plane. In this scenario, the proposed algorithm is used to estimate the given planar stationary target, and the root-mean-square error (RMSE) of the target position estimation is shown in Fig. 4.

Table 2 Positions and velocities of receivers and target (multiple scenarios)

Name	Position/km			Velocity/(m/s)		
	x	y	z	v_x	v_y	v_z
Observation 1	0	0	6	120	180	0
Observation 2 (Scenario I)	8	10	6	100	150	0
Observation 2 (Scenario II)	8	10	5	100	150	0
Observation 2 (Scenario III)	1	1	8	100	150	0
Target	80	120	1	0	0	0



(a) Search method

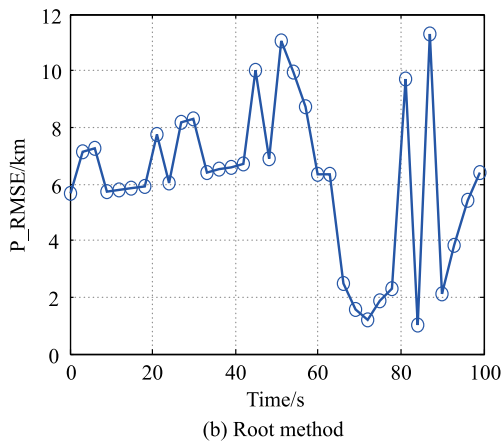


Fig. 4 RMSE of target position estimates (Scenario 1)

Scenario 2 When observations 1 and 2 (Scenario 2) neither move in parallel at the same horizontal plane nor in the vertical plane, the target is stationary. Use the proposed algorithm to locate the target, and its RMSE is shown in Fig. 5.

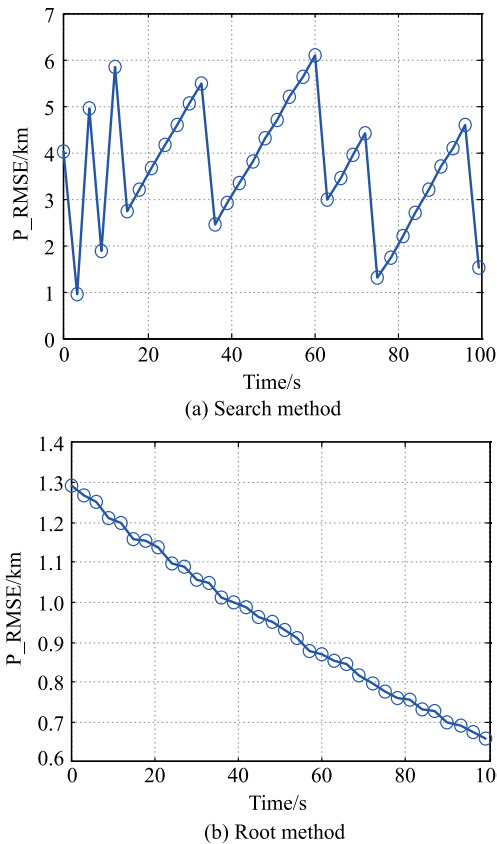


Fig. 5 RMSE of target position estimates (Scenario 2)

Scenario 3 When observations 1 and 2 (Scenario 3) do not move in parallel on the same horizontal plane, but move in the same vertical plane, the target is still stationary. In this scenario, the RMSE of estimates solved by the algorithm in this paper is shown in Fig. 6.

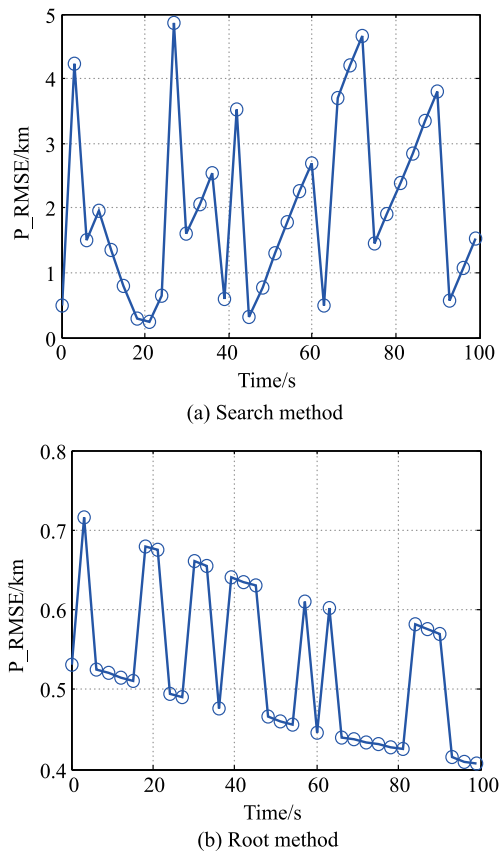


Fig. 6 RMSE of target position estimates (Scenario 3)

Compare Fig. 4(a), Fig. 5(a) and Fig. 6(a). When using the search method to solve the radial range r_1 and then estimate the target, in Scenario 1, most of the estimates RMSE float around 10 km, as shown in Fig. 4(a), and the RMSE is slightly larger. When the two observing stations move in Scenario 2, it can be seen from Fig. 5(a), that the RMSE is up to 6 km, which is slightly smaller than that in Scenario 1. When the observations move in Scenario 3, the RMSE is up to 5 km, as shown in Fig. 6(a), that is slightly smaller than that in Scenario 2.

Compare Fig. 4(b), Fig. 5(b), and Fig. 6(b). When using the root method to solve the radial range r_1 and then estimate the target, in Scenario 1, the estimates RMSE change irregularly with the observing time. As shown in Fig. 4(b), it basically moves up and down at 5 km, and the RMSE is slightly larger. When the two observing stations move in Scenario 2, it can be seen from Fig. 5(b) that the RMSE gradually changes with the observation time from 1 300 m to 650 m, which is obviously smaller than that in Scenario 1. When the observations move in Scenario 3, the RMSE fluctuates as the observing time increases, and shows a downward trend from 700 m to 400 m, as shown in Fig. 6(b). It is obviously smaller than that in Scenario 2.

In summary, when using the proposed algorithm to estimate the location of a given planar stationary source in

the actual war, the situation in which two observing stations move in parallel in the same horizontal plane should be avoided, because in the case of configuration, the localization error in this case is the greatest compared with other dual-station configurations. Thus, in order to improve the localization accuracy, the two observations should not move in the same horizontal plane, but they are still in parallel configuration. When the two stations are not in the same horizontal plane, but they are configured in parallel in the same vertical plane, the RMSE is the smallest and the localization effect is the best.

As shown in Table 2, Scenario 2 is selected so that the two observing stations are neither parallel in the same horizontal plane nor in the same vertical plane. As for the root method, 1 000 Monte Carlo simulation experiments are carried out to compare the RMSE and Cramer-Rao lower bound (CRLB) of the localization, as shown in Fig. 7. When the error $10 \lg(c^2 \sigma_d^2) < 0$, the RMSE is closer to CRLB, which proves the proposed algorithm positioning performance is better. If the TDOA and FDOA errors are smaller, the positioning accuracy is higher.

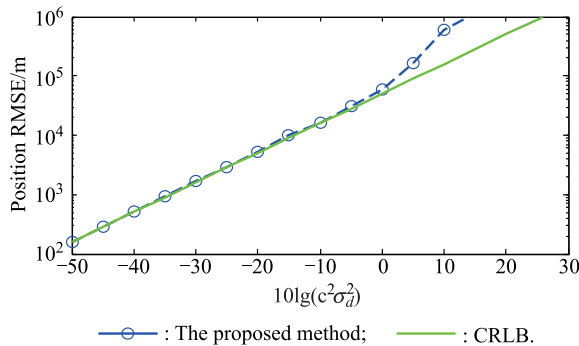


Fig. 7 CRLB and RMSE source positions comparison

6. Conclusions

When the height of the plane where the stationary radiation source is located is known, based on the joint TDOA/FDOA location of the source using two moving observations, the joint TDOA/FDOA location algorithm which solves the location of the target directly is proposed, which overcomes the difficulty in solving the joint TDOA/FDOA location algorithm. The time complexity of the proposed method is only one-dimensional searching for the stationary source or solving the root of a 6th degree equation in one variable. The method has fast localization speed, high localization accuracy and certain practicality. Moreover, when the two observations move in parallel, the situation that two stations are in the same horizontal plane should be avoided. When the two stations are not in the same horizontal plane, but they are configured in parallel in the same vertical plane, the localization error is the

smallest and the localization effect is the best. When the measurement error is small, the root method can be used to solve the radial range and then to estimate the target position. When the measurement error is large, the search method can be used to estimate the target position.

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