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Evaluation of fault diagnosability for nonlinear uncertain systems with multiple faults occurring simultaneously

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Abstract: Up to present, the problem of the evaluation of fault diagnosability for nonlinear systems has been investigated by many researchers. However, no attempt has been done to evaluate the diagnosability of multiple faults occurring simultaneously for nonlinear systems. This paper proposes a method based on differential geometry theories to solve this problem. Then the evaluation of fault diagnosability for affine nonlinear systems with multiple faults occurring simultaneously is achieved. To deal with the effect of control laws on the evaluation results of fault diagnosability, a design scheme of the evaluation of fault diagnosability is proposed. Then the influence of uncertainties on the evaluation results of fault diagnosability for affine nonlinear systems with multiple faults occurring simultaneously is analyzed. The numerical simulation results are obtained to show the effectiveness of the proposed evaluation scheme of fault diagnosability.

Keywords: fault diagnosability, multiple faults occurring simultaneously, uncertain affine nonlinear system.

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1. Introduction

With the increasing complexity of contemporary technological systems, ensuring safety and reliability of industrial processes has been increasingly important. During the last four decades, a huge number of results on fault diagnosis, fault-tolerant control, and their applications in a variety of engineering systems were reported [1 – 4]. The main traditional way to improve the system fault diagnosis ability is to improve the performance of the diagnosis algorithm, e.g., improving the diagnosis accuracy of the algorithm [5] or improving the applicability of the algorithm [6]. However, due to the increasing complexity of the system structure, the design and improvement of the diagnosis algorithms become more difficult. When developing a diagnosis algorithm, the information of achievable diagnosability performance given by the system model is useful. In other words, without such information, it is difficult to achieve the purpose of fault diagnosis by designing or improving the diagnosis algorithm.

Fault diagnosability is an attribute that characterizes the fault diagnosis ability of the control system, which reveals deep insight into fault diagnosis. Evaluation results of fault diagnosability can not only answer the necessary questions such as “whether the fault can be diagnosed”, but also further answer the key questions such as “how difficult is it to diagnose the fault”. The diagnosability of the control system is mainly affected by the system structure. Therefore, for most systems which can be represented by mathematical models, the relationship between fault and system output can be obtained through the structure of the system, and the relationship between the two could be used to study the diagnosability of the system. Patton et al. proposed a qualitative evaluation method for the diagnosability of the general linear system transfer function considering additive faults by analyzing the column elements of the fault matrix [7]. To solve the problem of undetectable faults caused by the influence of uncertainties on the linear time invariant system, a criterion for detectability in the frequency domain was given in [8], that is, a fault is detectable if its impact on the output is greater than the uncertainties in this time period. In fault isolation, an evaluation criterion of fault isolation based on the binary diagnosis matrix was proposed in [9], and the evaluation criterion was applied to the design process of the residual error. Different from the method in [10] that used distance similarity to evaluate the system diagnosability, a novel approach to quantitative evaluation of actual fault diagnosability for dynamic systems was proposed in [11], which could quantify the difficulty level to detect and isolate a fault without designing a diagnosis algorithm. An approach for analyzing the diagnosability on a given continuous system was proposed

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in [12]. This approach mainly used the genetic algorithm to obtain redundancy relations of the system, so as to analyze the fault diagnosability of a given system and optimize the sensor placement problem on the given system that did not fulfill the detectability or isolability properties. Combining signed directed graphs (SDG) with fault temporal information, by using multiple-valued evaluation of measuring variables in the signature matrix to represent different symptoms and time sequences of several typical faults, Liu et al. proposed an approach for fault diagnosability of the spacecraft propulsion system [13]. Trave-Massuyes et al. proposed a model-based method for the fault diagnosability by using the concept of component supported analytical redundancy relation [14]. According to the analysis results, the minimum sensor set that met the actual diagnostic performance requirements was determined, which effectively improved the configuration efficiency, and the method was applied to an industrial smart actuator. Due to the constraints of actual conditions, the system cannot achieve the required diagnosable performance by changing the sensor configuration scheme. An algorithm was proposed in [15] to calculate which sensors could meet the requirements of fault detectability and isolation in a given linear differential-algebraic model, then the optimization algorithm could give the sensor configuration scheme closest to the target, that is, the maximum diagnosable performance could be obtained by optimizing the system configuration. In order to better meet the actual needs of fault diagnosis in semiconductor manufacturing, in addition, Cui et al. also used the on-line analysis results of fault diagnosability to select appropriate diagnosis schemes [16]. Different from the above evaluation of fault diagnosability methods which relied on the diagnosis algorithm, Peng et al. provided the criteria for the detectability and isolation of faults for a class of affine nonlinear systems that did not rely on any diagnostic algorithm, and quantified the diagnosability of the system by using the ratio between the number of faults that could be diagnosed and the total number of faults [17]. In practice, the control system was affected by different types of uncertainties inevitably [18], such as external disturbances and model uncertainties. Uncertainties directly affect the difficulty of fault diagnosis of the system [19]. Xing et al. proposed an evaluation and design scheme of fault diagnosability based on differential geometry theories for a class of uncertain affine nonlinear systems with unknown inputs [20]. To improve the effectiveness of both the real-time monitoring of potential fault components and the cause of the fault, an evaluation of the fault diagnosability scheme considering the rapidity, isolability and robustness was proposed in [21]. The performance of different diagnosis schemes was compared and analyzed with the proposed evaluation criterion, then a better one was selected and applied to the chip wafer production process.

At present, some achievements have been made in the evaluation of fault diagnosability scheme design [22 – 24]. However, most of the research only considers the evaluation of fault diagnosability of the system with a single fault or multiple faults occurring at different time in the system design stage, and the research on the evaluation of fault diagnosability of the system with multiple different faults occurring simultaneously has not yet been seen in the relevant papers. The study of the evaluation of fault diagnosability of the closed-loop systems mainly focuses on detecting and separating faults with various methods according to a certain criterion based on given control laws. However, compared with fault diagnosability of the open-loop systems, there is not yet a complete theory of the evaluation of fault diagnosability of the closed-loop system. In particular, the influence of closed-loop feedback control laws on the evaluation results of fault diagnosability is still at the exploratory stage, which needs to be further studied. The main contributions of this paper are summarized in the following:

(i) A method based on the differential geometry theorem is proposed to solve the evaluation of fault diagnosability for affine nonlinear systems considering multiple faults occurring simultaneously.

(ii) Considering the influence of feedback control laws, a scheme is presented for the evaluation of fault diagnosability for affine nonlinear close-loop systems.

(iii) The influence of uncertainties on the evaluation results of fault diagnosability for affine nonlinear systems with multiple faults occurring simultaneously is analyzed.

This paper is organized as follows. Useful results in relation to the evaluation of fault diagnosability and other preliminaries are presented in Section 2. In Section 3, a method based on differential geometry theories for the evaluation of fault diagnosability for affine nonlinear systems with multiple faults occurring simultaneously is proposed. The influence of uncertainties on the evaluation results of fault diagnosability is analyzed in Section 4. Section 5 provides a validation example for the evaluation of fault diagnosability. Finally, some conclusions are drawn in Section 6.

2. Preliminary definitions and lemmas

Some definitions and theorems are reviewed according to the principle of differential geometry.

Definition 1 [20] Fault diagnosability is defined as the ability to detect, isolate and identify a fault on a component of the system, which is determined by its inherent structure and actual factors.

Consider the following affine nonlinear system with multiple faults:

$$\begin{cases} \dot{x}(t) = f(x(t)) + \sum_{i=1}^{p} p_i(x(t))w_i(t), \quad 1 \leqslant i \leqslant p \\ y_j(t) = h_j(x(t)), \quad 1 \leqslant j \leqslant m \end{cases} \tag{1}$$

where $x(t) \in U^0 \subset \mathbf{R}^n$ is the state vector, U^0 is an open set on the manifold $N = \mathbf{R}^n$, and the vector functions $f(x(t)), p_1(x(t)), p_2(x(t)), \dots, p_p(x(t))$ are smooth, which are defined in U^0. $w_1(t), w_2(t), \dots, w_p(t)$ are system additive faults. $y_1(t), y_2(t), \dots, y_m(t)$ are the system outputs.

Definition 2 [25] Considering system (1), for all $x(t)$ in a neighborhood of $\boldsymbol{x}^0$, if

$$\begin{cases} L_{p_i} L_f^k h_i(x(t)) = 0 \\ L_{p_i} L_f^{s_i^j - 1} h_i(x(t)) \neq 0 \end{cases} \tag{2}$$

where $0 \leqslant k \leqslant s_i^j - 2$, $L_f^k h_i(x(t))$ is the kth Lie derivative of $h_i(x(t))$ (function or form) with respect to $f(x(t))$. Then the fault characteristic index of $w_i(t)$ on the output $y_j(t)$ is s_i^j. If

$$L_{p_i} L_f^k h_i(x(t)) = 0, \quad k \geqslant 0 \tag{3}$$

then $s_i^j = 0$. If $s_i^j > n$, then $s_i^j = \infty$.

Remark 1 The definition of the disturbance characteristic index in [26] is similar as that of the fault characteristic index.

Definition 3 [27] Consider a mapping

$$F(\boldsymbol{z}) : U^0 \to \mathbf{R}^n$$

$$(z_1, z_2, \dots, z_n) \to \phi_{z_1}^{f_1} \circ \phi_{z_2}^{f_2} \circ \dots \circ \phi_{z_n}^{f_n}(\boldsymbol{x}^0) = F(\boldsymbol{z}) = x(t) \tag{4}$$

where $U^0 = \boldsymbol{z} \in \mathbf{R}^n : |z_i| < \varepsilon$ and "$\circ$" denotes composition with respect to the argument $x(t)$. If ε is sufficiently small, this mapping has the following properties:

(i) it is defined for all $\boldsymbol{z} = (z_1, \dots, z_n) \in U^0$ and it is a diffeomorphism onto its image.

(ii) it is such that, for all $\boldsymbol{z} \in U^0$, the first d columns of the Jacobian matrix

$$\left[\frac{\partial F(z)}{\partial \boldsymbol{z}^{\mathrm{T}}} \right]$$

are linearly independent vectors in $\Delta(F(\boldsymbol{z}))$. To this end, let U^0 denote the image of the mapping $F(\boldsymbol{z})$, and observe that U^0 is indeed an open neighborhood of $\boldsymbol{x}^0$, because $\boldsymbol{x}^0$ is exactly the value of $F(\boldsymbol{z})$ at the point $z = 0$. Since this mapping is a diffeomorphism onto its image (property (i)), the inverse $F^{-1}(x(t))$ exists and is a smooth mapping, defined in U^0. Set

$$\boldsymbol{z} = F^{-1}(x(t)) = \phi(x(t)) = \begin{bmatrix} \phi_1(x(t)) \\ \phi_2(x(t)) \\ \vdots \\ \phi_n(x(t)) \end{bmatrix} \tag{5}$$

where $\phi_1(x(t)), \phi_2(x(t)), \dots, \phi_n(x(t))$ are real-valued functions, defined for all $x(t)$ in U^0.

Theorem 1 [27] Let Δ be a nonsingular involutive distribution of dimension d, suppose $\boldsymbol{\tau}$ is a vector field of Δ. In the new coordinates, this vector field is represented in the form

$$\overline{\boldsymbol{\tau}}(\boldsymbol{z}) = \left[\frac{\partial f(x(t))}{\partial x(t)^{\mathrm{T}}} \boldsymbol{\tau}(x(t)) \right]_{x(t) = \phi^{-1}(\boldsymbol{z})}. \tag{6}$$

Since, by construction, the last $n-d$ rows of the Jacobian matrix of ϕ span $\Delta^{\perp}$, it is immediately deduced that the last $n-d$ entries of the vector on the right-hand side are zero, for all $x(t)$ in the set where the coordinates transformation is defined. We conclude from this that any vector field of Δ, in the new coordinates, has a representation of the form

$$\overline{\boldsymbol{\tau}}(\boldsymbol{z}) = [\overline{\boldsymbol{\tau}}_1(\boldsymbol{z}) \quad \cdots \quad \overline{\boldsymbol{\tau}}_d(\boldsymbol{z}) \quad 0 \quad \cdots \quad 0]^{\mathrm{T}}. \tag{7}$$

Lemma 1 [27] Let Δ be a nonsingular involutive distribution of dimension d and suppose that Δ is invariant under the vector field $f(x(t))$. Then at each point $\boldsymbol{x}^0$ there exist a neighborhood U^0 of $\boldsymbol{x}^0$ and a coordinates transformation $\boldsymbol{z} = \boldsymbol{\tau}(x(t))$ defined in Theorem 1, in which the vector field $f(x(t))$ is represented by a vector of the form as

$$\overline{f}(\boldsymbol{z}) = \begin{bmatrix} \overline{f}(z_1, \dots, z_d, z_{d+1}, \dots, z_n) \\ \vdots \\ \overline{f}_d(z_1, \dots, z_d, z_{d+1}, \dots, z_n) \\ \overline{f}_{d+1}(z_{d+1}, \dots, z_n) \\ \vdots \\ \overline{f}_n(z_{d+1}, \dots, z_n) \end{bmatrix}. \tag{8}$$

Theorem 2 Exact feedback linearization [27]

Suppose a system

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t) \tag{9}$$

where $u(t) \in \mathbf{R}^m$ is the input vector. The state space exact feedback linearization problem is solvable near a point $\boldsymbol{x}^0$, i.e., there exists an "output" function $h(x(t))$ for which the system has relative degree n at $\boldsymbol{x}^0$, if and only if the following conditions are satisfied:

(i) the matrix $[g(\boldsymbol{x}^0) \quad ad_{f(x(t))} g(\boldsymbol{x}^0) \quad \cdots \quad ad_{f(x(t))}^{n-2} g(\boldsymbol{x}^0) \quad ad_{f(x(t))}^{n-1} g(\boldsymbol{x}^0)]$ has rank n.

(ii) the distribution $D = \mathrm{Span}\{g(x(t)), ad_{f(x(t))} g(x(t)), \dots, ad_{f(x(t))}^{n-1} g(x(t))\}$ is involutive near $\boldsymbol{x}^0$.

Theorem 3 Partial feedback linearization [26]

Suppose the system (9) is given. If the distribution $\overline{G}_{r-2}$ which denotes the involutive closure of G_{r-2} has constant rank ρ, which is less than or equal to $n-1$, in a neighborhood U^0 of $\boldsymbol{x}^0$, and there exists an integer r, $2 \leqslant r \leqslant n$, such that

$$ad_{f(x(t))}^{r-1} g(x(t)) \notin \overline{G}_{r-2}(x(t)) =$$
$$\text{inv.cl.Span}\{g(x(t)), \ldots, ad_{f(x(t))}^{n-2} g(x(t))\}, \quad \forall x(t) \in U^0. \tag{10}$$

Then the state space partial feedback linearization problem is solvable.

3. Fault diagnosability for nonlinear systems with multiple faults occurring simultaneously

3.1 Affine nonlinear systems

Theorem 4 [25] Suppose a system

$$\begin{cases} \dot{x}(t) = f(x(t)) + p(x(t))w(t) \\ y(t) = h(x(t)) \end{cases} \tag{11}$$

is given, where $w(t)$ is an unknown fault, and $y(t)$ is the output. The fault $w(t)$ can be diagnosed through the output $y(t)$ if s is strictly less than n, where s is the fault characteristic index of fault $w(t)$ and n is the relative degree.

Lin et al. used the theory of differential geometry to evaluate the fault diagnosability of affine nonlinear systems with only one fault occurring or multiple faults occurring at different time [25]. Based on [25], this paper further studies the problem of the evaluation of fault diagnosability for uncertain affine nonlinear systems with multiple faults occurring simultaneously. The following theorem extends Theorem 4 to the affine nonlinear systems with multiple faults occurring simultaneously.

Theorem 5 For system (1), the fault $w_i(t)$ can be diagnosed through the output $y_i(t)$ if the following two conditions are satisfied:

(i) there exists a distribution Δ, which is nonsingular and involutive on a neighborhood U^0 of $\boldsymbol{x}^0$, such that

$$p_l(x(t)) \in \Delta \subset Ker(h_i(x(t))),$$
$$l = 1, 2, \ldots, i-1, i+1, i+2, \ldots, p \tag{12}$$

where $Ker(h_i(x(t))) := \text{Span}\{X | L_X h = \langle dh, X \rangle = 0\}$ and $h = h_1(x(t)), h_2(x(t)), \ldots, h_m(x(t))$.

(ii) the output $y_i(t)$ is affected by the fault $w_i(t)$, that is,

$$\begin{cases} L_{p_i} L_f^k h_i(x(t)) = 0 \\ L_{p_i} L_f^{s_i^i - 1} h_i(x(t)) \neq 0 \end{cases}, \quad x(t) \in U^0.$$

Proof Let Δ be a nonsingular involutive distribution of dimension d and assume that Δ is invariant under the vector fields $f(x(t)), p_1(x(t)), p_2(x(t)), \ldots, p_p(x(t))$. Moreover, suppose that the codistribution

$$\text{Span}\{p_1(x(t)), p_2(x(t)), \ldots, p_{i-1}(x(t)),$$
$$p_{i+1}(x(t)), p_{i+2}(x(t)), \ldots, p_p(x(t))\}$$

is contained in Δ, and

$$p_i(x(t)) \in \Delta \subset Ker(h_i(x(t))) \tag{13}$$

where $x(t) \in \Delta$ and $Ker(h_i(x(t)))$ is defined as $Ker(h) := \text{Span}X | L_X h| = \langle dh, X \rangle = 0$, the system (1) is represented by equation of the form

$$\dot{\boldsymbol{z}} = \overline{f}(z) + \sum_{i=1}^{p} \overline{p}_i(z) w_i(t). \tag{14}$$

Assuming that the distribution Δ satisfies the assumption of Lemma 1, that is, Δ is a nonsingular involutive distribution, and invariant under the vector field $f(x(t))$. Then the vector field $f(x(t))$ is represented by a vector of the form

$$\overline{f}(\boldsymbol{z}) = \overline{f}(\boldsymbol{\zeta}_1, \boldsymbol{\zeta}_2) = \begin{bmatrix} \overline{f}_1(\boldsymbol{\zeta}_1, \boldsymbol{\zeta}_2) \\ \overline{f}_2(\boldsymbol{\zeta}_2) \end{bmatrix} \tag{15}$$

where $\boldsymbol{\zeta}_1 = (z_1, z_2, \ldots, z_d)$ and $\boldsymbol{\zeta}_2 = (z_{d+1}, z_{d+2}, \ldots, z_n)$.

The vector fields $p_1(x(t)), p_2(x(t)), \ldots, p_{i-1}(x(t)), p_{i+1}(x(t)), p_{i+2}(x(t)), \ldots, p_p(x(t))$ are contained in Δ, satisfying the assumptions required by (7). Then, for each point $\boldsymbol{x}^0$ it is possible to find a local coordinates transformation $\boldsymbol{z} = \phi(x(t))$ defined on U^0 such that, in the new coordinates, the vector fields $p_1(x(t)), p_2(x(t)), \ldots, p_p(x(t))$ are represented by a vector of the form

$$\begin{cases} \overline{p}_j(z) = \overline{p}_j(\boldsymbol{\zeta}_1, \boldsymbol{\zeta}_2) = \begin{bmatrix} \overline{p}_{1j}(\boldsymbol{\zeta}_1, \boldsymbol{\zeta}_2) \\ 0 \end{bmatrix}, \\ \qquad j = 1, 2, \ldots, i-1, i+1, i+2, \ldots, p \\ \overline{p}_i(z) = \overline{p}_i(\boldsymbol{\zeta}_1, \boldsymbol{\zeta}_2) = \begin{bmatrix} \overline{p}_{1i}(\boldsymbol{\zeta}_1, \boldsymbol{\zeta}_2) \\ \overline{p}_{2i}(\boldsymbol{\zeta}_2) \end{bmatrix} \end{cases}. \tag{16}$$

Substitute the vector fields $\overline{f}(z), \overline{p}_1(z), \ldots, \overline{p}_p(z)$ into (14), then we have

$$\begin{cases} \dot{\boldsymbol{\zeta}}_1 = \overline{f}_1(\boldsymbol{\zeta}_1, \boldsymbol{\zeta}_2) + \sum_{i=1}^{p} \overline{p}_i(z) w_i(t) \\ \dot{\boldsymbol{\zeta}}_2 = \overline{f}_1(\boldsymbol{\zeta}_1, \boldsymbol{\zeta}_2) + \overline{p}_{2i}(\boldsymbol{\zeta}_2) \end{cases}. \tag{17}$$

Moreover,

$$(dh_i(x(t)), \Delta) = 0 \tag{18}$$

so $h_i(\boldsymbol{z}) = h_i(\boldsymbol{\zeta}_2)$. It is obvious that $w_l(t)$ does not affect $y_i(t)$. Formally we have

$$\frac{\partial h_i(z)}{\partial w_l(t)} = 0, \quad \boldsymbol{z} \in U^0. \tag{19}$$

For system (1), we can formally write the output-fault mapping as

$$h_i(x(t)) = h_i(\boldsymbol{x}^0, w_1(t), w_2(t), \dots, w_p(t)). \tag{20}$$

Now if $w_l(t)$ affects $y_i(t)$ then there exists at least one point $p \in U^0$ such that

$$\left.\frac{\partial h_i(z)}{\partial w_l(t)}\right|_p \neq 0, \quad \boldsymbol{z} \in U^0. \tag{21}$$

This completes the proof of condition (i).

To avoid complication, we assume throughout that the system (1) is analytic. In that case, the effect of the fault $w_i(t)$ on the output $y_i(t)$ is determined by the functions

$$L_{p_i} L_{x_1} \cdots L_{x_k} h_i(x(t)) = 0 \tag{22}$$

where $k \geqslant 0, x_1, \dots, x_k \in \mathrm{Span}\{f(x(t)), p_1(x(t)), \dots, p_p(x(t))\}, x(t) \in U^0$. Consider now the subset of functions of (22) given by

$$L_{p_i} L_f^k h_i(x(t)) = 0, \quad k \geqslant 0; x(t) \in U^0. \tag{23}$$

Clearly when all the functions in (23) are identically zero,

$$L_{p_i} L_f^k h_i(x(t)) \equiv 0, \quad k \geqslant 0; x(t) \in U^0, \tag{24}$$

then also all the functions given in (22) are identically zero, and in no way the fault $w_i(t)$ is going to interact with the output $y_i(t)$. Therefore we assume (24) is not true, that is, the fault characteristic index $s_i^i < \infty$, and condition (ii) is true. Using condition (i) and condition (ii), we have the s_i^ith time derivative of $y_i(t)$:

$$y_i^{(s_i^i)}(t) = L_f^{s_i^i} h_i(x(t)) + L_{p_i} L_f^{s_i^i - 1} h_i(x(t)) w_i(t). \tag{25}$$

The fault $w_i(t)$ instantaneously does influence the output $y_i^{(s_i^i)}(t)$, that is, the output $y_i(t)$ is affected by fault $w_i(t)$. Thus the fault $w_i(t)$ is diagnosable.

This completes the proof of condition (ii). □

3.2 Considering feedback control laws

Considering the following nonlinear affine control system:

$$\begin{cases} \dot{x}(t) = f(x(t)) + g(x(t))u(t) + \\ \qquad \displaystyle\sum_{i=1}^{p} p_i(x(t)) w_i(t), \quad 1 \leqslant i \leqslant p \\ y_j(t) = h_j(x(t)), \quad 1 \leqslant j \leqslant m \end{cases}, \tag{26}$$

there exists a static state feedback control law

$$u(t) = \alpha(x(t)) + \beta(x(t))\lambda. \tag{27}$$

The functions $\alpha(x(t))$ and $\beta(x(t))$ are defined on a suitable open set of $\mathbf{R}^n$, and λ is the external reference input. In fact, the composition of the control law (27) with the system (26) yields a closed-loop system characterized by the similar structure

$$\dot{x}(t) = f(x(t)) + g(x(t))(\alpha(x(t)) + \beta(x(t))\lambda) + \sum_{i=1}^{p} p_i(x(t)) w_i(t) = f(x(t)) + g(x(t))\alpha(x(t)) + g(x(t))\beta(x(t))\lambda + \sum_{i=1}^{p} p_i(x(t)) w_i(t) =$$

$$\widetilde{f}(x(t)) + p_{i+1}(x(t)) w_{i+1}(t) + \sum_{i=1}^{p} p_i(x(t)) w_i(t) \tag{28}$$

where

$$\begin{cases} \widetilde{f}(x(t)) = f(x(t)) + g(x(t))\alpha(x(t)) \\ p_{i+1}(x(t)) = g(x(t))\beta(x(t)) \\ w_{i+1}(t) = \lambda \end{cases}. \tag{29}$$

It can be obtained from the system (28), that its form is the same as the system (1). Therefore, the above results, Theorem 5, can be used to evaluate the fault diagnosability of the nonlinear affine control system when multiple faults occur simultaneously.

4. Influence of uncertainties on evaluation results of fault diagnosability

4.1 Considering only one fault occurring

Considering the influence of uncertainties such as parameter uncertainties or external disturbances, the uncertain affine nonlinear system with one fault can be formed:

$$\begin{cases} \dot{x}(t) = f(x(t)) + p(x(t)) w_i(t) + f(x(t))\theta \\ y(t) = h(x(t)) \end{cases} \tag{30}$$

where θ is the uncertainty parameter of the system (30), and $w_i(t)$ is the actuator fault of the system (30). By analyzing the relationship between $p(x(t))$ and $f(x(t))\theta$, the influence of θ on the evaluation results of diagnosability of the fault $w_i(t)$ can be clarified. For simplicity, we analyze it from four cases.

Case 1 Suppose that the disturbance characteristic index of θ is v and $v < +\infty$. The fault characteristic index of $w_i(t)$ is s and $s < +\infty$.

If $v > s$, it is obvious that θ does not affect the evaluation results of diagnosability of the fault $w_i(t)$. Derivative of the output $y(t)$ produces

$$\begin{cases} y^{(1)}(t) = h(x(t))^{(1)} = \dfrac{\partial h(x(t))}{\partial x(t)^{\mathrm{T}}} \cdot \dfrac{\partial x(t)}{\partial t} = \\ \qquad \dfrac{\partial h(x(t))}{\partial x(t)^{\mathrm{T}}} (f(x(t)) + p(x(t)) w_i(t)) = \\ \qquad L_f h(x(t)) + L_p h(x(t)) w_i(t) = \\ \qquad L_f h(x(t)) \\ y^{(2)}(t) = h(x(t))^{(2)} = \cdots = L_f^2 h(x(t)) \\ \qquad \vdots \\ y^{(s)}(t) = L_f^s h(x(t)) + L_p L_f^{s-1} h(x(t)) w_i(t) \\ \qquad \vdots \end{cases} \tag{31}$$

where we see that $y(t)$ cannot be independent from the fault $w_i(t)$. The s derivative of the output $y(t)$ is first associated with the fault $w_i(t)$, and independent from θ. Therefore, θ has no influence on the evaluation results of diagnosability of the fault $w_i(t)$.

If $v = s$, θ and the fault $w_i(t)$ occur simultaneously in $y^{(s)}(t)$, then (31) can be reduced to

$$\begin{cases} y^{(s)}(t) = L_f^s h(x(t)) + L_{\Delta f} L_f^{s-1} h(x(t))\theta + L_p L_f^{s-1} h(x(t)) w_i(t) \\ y^{(s+1)}(t) = \dfrac{\partial y^v}{\partial x(t)^{\mathrm{T}}} \cdot \dfrac{\partial x(t)}{\partial t} = \dfrac{\partial (L_f^s h(x(t)) + L_p L_f^{s-1} h(x(t)) w_i(t) + L_{\Delta f} L_f^{s-1} h(x(t))\theta)}{\partial x(t)^{\mathrm{T}}} \cdot \dfrac{\partial x(t)}{\partial t} = \\ \qquad \dfrac{\partial L_f^s h(x(t))}{x(t)^{\mathrm{T}}} \cdot (f(x(t)) + p(x(t)) w_i(t) + \Delta f(x(t))\theta) + \dfrac{\partial L_p L_f^{s-1} h(x(t))}{x(t)^{\mathrm{T}}} \cdot (f(x(t)) + p(x(t)) w_i(t) + \\ \qquad \Delta f(x(t))\theta) w_i(t) + \dfrac{\partial L_{\Delta f} L_f^{s-1} h(x(t))}{x(t)^{\mathrm{T}}} \cdot (f(x(t)) + p(x(t)) w_i(t) + \Delta f(x(t))\theta)\theta = \\ \qquad L_f^{v+1} h(x(t)) + L_p L_f^v h(x(t)) w_i(t) + L_{\Delta f} L_f^v h(x(t))\theta + L_f L_p L_f^{v-1} h(x(t)) w_i(t) + \\ \qquad L_p L_p L_f^{v-1} h(x(t)) w_i(t)^2 + L_{\Delta f} L_p L_f^{v-1} h(x(t)) w_i(t)\theta + L_f L_{\Delta f} L_f^{v-1} h(x(t))\theta + \\ \qquad L_p L_{\Delta f} L_f^{v-1} h(x(t))\theta w_i(t) + L_{\Delta f} L_{\Delta f} L_f^{v-1} h(x(t))\theta^2 \\ \qquad \vdots \end{cases} . \tag{32}$$

Without loss of generality, from (32), at least one of $L_p L_f^v h(x(t))$, $L_{\Delta f} L_f^v h(x(t))$, $L_f L_p L_f^{v-1} h(x(t))$, $L_p L_p L_f^{v-1} h(x(t))$, $L_{\Delta f} L_p L_f^{v-1} h(x(t))$, $L_f L_{\Delta f} L_f^{v-1} h(x(t))$, $L_p L_{\Delta f} L_f^{v-1} h(x(t))$ and $L_{\Delta f} L_{\Delta f} L_f^{v-1} h(x(t))$ is not identity zero. θ and $w_i(t)$ can be decoupled by the solution of (32) simultaneously. A clear relationship between the fault $w_i(t)$ and the output $y^{(s)}(t)$ can be obtained. Therefore, the fault $w_i(t)$ is diagnosable. If all of $L_p L_f^v h(x(t))$, $L_{\Delta f} L_f^v h(x(t))$, $L_f L_p L_f^{v-1} h(x(t))$, $L_p L_p L_f^{v-1} h(x(t))$, $L_{\Delta f} L_p L_f^{v-1} h(x(t))$, $L_f L_{\Delta f} L_f^{v-1} h(x(t))$, $L_p L_{\Delta f} L_f^{v-1} h(x(t))$ and $L_{\Delta f} L_{\Delta f} L_f^{v-1} h(x(t))$ are identity zero, then

$$\begin{cases} y^{(s+1)}(t) = L_f^{v+1} h(x(t)) \\ y^{(s+2)}(t) = L_f^{v+3} h(x(t)) \\ \quad \vdots \end{cases} . \tag{33}$$

As can be seen from (33), no corresponding expression can be found for simultaneous solution. Therefore, θ and $w_i(t)$ cannot be decoupled. The addition of θ affects the relationship between the fault $w_i(t)$ and the output $y(t)$, so the evaluation results of diagnosability of the fault $w_i(t)$ is affected by the uncertainty parameter θ.

If $s < v$, it is obvious that there is a clear relationship between θ and $y^{(v)}(t)$. The v derivative of the output $y(t)$ is

$$y^{(v)}(t) = L_f^v h(x(t)) + L_{\Delta f} L_f^{v-1} h(x(t))\theta, \tag{34}$$

it follows that

$$\theta = \frac{y^{(v)}(t) - L_f^v h(x(t))}{L_{\Delta f} L_f^{v-1} h(x(t))}. \tag{35}$$

For the fault $w_i(t)$, there is an expression

$$y^{(s)}(t) = \mathrm{E}\left(x(t), \frac{y^v - L_f^v h(x(t))}{L_{\Delta f} L_f^{v-1} h(x(t))}, w_i(t)\right) \tag{36}$$

which is an expression of θ, $w_i(t)$, and $x(t)$. Therefore, there is also a clear relationship between the fault $w_i(t)$ and the output $y(t)$. Thus the fault $w_i(t)$ affects the output $y(t)$, that is, the fault $w_i(t)$ is diagnosable.

Case 2 If $v = +\infty, s < +\infty$, then

$$y^{(s)}(t) = L_f^s h(x(t)) + L_p L_f^{s-1} h(x(t)) w_i(t). \tag{37}$$

There is a clear relationship between the fault $w_i(t)$ and the output $y^{(s)}(t)$. Therefore, θ does not affect the evaluation result of diagnosability of the fault $w_i(t)$.

Case 3 If $s = +\infty$, $v < +\infty$, then

$$y^{(v)}(t) = L_f^v h(x(t)) + L_{\Delta f} L_f^{v-1} h(x(t))\theta. \tag{38}$$

The relationship between the fault $w_i(t)$ and the output $y(t)$ could not be found. The fault $w_i(t)$ is still undiagnosed. Therefore, θ does not affect the evaluation results of diagnosability of the fault $w_i(t)$.

Case 4 If $s = v = +\infty$, θ and $w_i(t)$ do not affect the output $y(t)$ simultaneously, the fault $w_i(t)$ is not diagnosable. Therefore, θ does not affect the evaluation results of diagnosability of the fault $w_i(t)$.

4.2 Considering multiple faults occurring simultaneously

Considering the uncertain affine nonlinear system with multiple faults:

$$\begin{cases} \dot{x}(t) = f(x(t)) + \sum\limits_{i=1}^{p} p_i(x(t)) w_i(t) + \Delta f(x(t))\theta \\ y_j(t) = h_j(x(t)), \quad 1 \leqslant j \leqslant m \end{cases} \tag{39}$$

where $1 \leqslant i \leqslant p$. Without loss of generality, assuming that the faults $w_i(t)$ and $w_j(t)$ can be diagnosed through the outputs $y_i(t)$ and $y_j(t)$ respectively. It can be concluded that the output $y_i(t)$ is only affected by the fault $w_i(t)$ and the output $y_j(t)$ is only affected by the fault $w_j(t)$. The influence of $\Delta f(x(t))\theta$ on the evaluation results of fault diagnosability is discussed below. For simplicity, we analyze it from two cases.

Case 5 The evaluation of fault diagnosability for the affine nonlinear system under the condition that the outputs $y_i(t)$ and $y_j(t)$ are not affected by $\Delta f(x(t))\theta$.

Define

$$\begin{gathered}\{x_1, \ldots, x_m\} \in F(x(t)) := \\ \{f(x(t)), p_1(x(t)), \ldots, p_p(x(t)), \Delta f(x(t))\}.\end{gathered} \tag{40}$$

If $\Delta f(x(t))\theta$ does not affect the output $y_i(t)$, if and only if for any $m > 0$, there is

$$\begin{cases} L_{\Delta f} L_{x_1} \cdots L_{x_m} h_i(x(t)) = 0 \\ L_{\Delta f} L_{x_1} \cdots L_{x_m} h_j(x(t)) = 0 \end{cases}. \tag{41}$$

The proof process can relate to the proof process of Theorem 5. The outputs $y_i(t)$ and $y_j(t)$ are not affected by θ, that is, θ cannot be found in any derivative of the outputs $y_i(t)$ and $y_j(t)$. Assuming that the faults $w_i(t)$ and $w_j(t)$ are diagnosable before $\Delta f(x(t))\theta$ is added, it indicates that the fault $w_i(t)$ maintains a one-to-one correspondence with the output $y_i(t)$, and the fault $w_j(t)$ maintains a one-to-one correspondence with the output $y_j(t)$. And then the addition of $\Delta f(x(t))\theta$ does not affect the relationship between them. Therefore, the faults $w_i(t)$ and $w_j(t)$ can still be diagnosed through the outputs $y_i(t)$ and $y_j(t)$ respectively, that is, the faults $w_i(t)$ and $w_j(t)$ are diagnosable.

Case 6 The evaluation of fault diagnosability for the affine nonlinear system under the condition that the outputs $y_i(t)$ and $y_j(t)$ are affected by $\Delta f(x(t))\theta$.

Assuming that the disturbance characteristic index of θ on the output $y_i(t)$ is v_j^i, the disturbance characteristic index of θ on the output $y_j(t)$ is v_j^j, the fault characteristic index of the fault $w_i(t)$ on the output $y_i(t)$ is s_j^i, and the fault characteristic index of the fault $w_j(t)$ on the output $y_j(t)$ is s_j^j. Then the expressions are established:

$$\begin{cases} L_{\Delta f} L_f^k h_i(x(t)) = 0, & 0 \leqslant k \leqslant v_i^i - 2 \\ L_{\Delta f} L_f^{v_i^i - 1} h_i(x(t)) \neq 0, & x(t) \in U^0 \end{cases}, \tag{42}$$

$$\begin{cases} L_{\Delta f} L_f^k h_j(x(t)) = 0, & 0 \leqslant k \leqslant v_j^j - 2 \\ L_{\Delta f} L_f^{v_j^j - 1} h_j(x(t)) \neq 0, & x(t) \in U^0 \end{cases}. \tag{43}$$

If $v_i^i > s_i^i$, $v_j^j > s_j^j$, $i \neq j$, then the following expressions are established:

$$y_i^{s_i^i}(t) = L_f^{s_i^i} h_i(x(t)) + L_{p_i} L_f^{s_i^i - 1} h_i(x(t)) w_i(t), \tag{44}$$

$$y_j^{s_j^j}(t) = L_f^{s_j^j} h_j(x(t)) + L_{p_j} L_f^{s_j^j - 1} h_j(x(t)) w_j(t). \tag{45}$$

It indicates that the relationship between the fault $w_i(t)$ and the output $y_i(t)$, the fault $w_j(t)$ and the output $y_j(t)$ remains unchanged, and the faults $w_i(t)$ and $w_j(t)$ are still diagnosable.

From the above analysis, it can be seen that the faults $w_i(t)$ and $w_j(t)$ are decoupled, so the influence of θ on the evaluation results of diagnosability of the faults $w_i(t)$ and $w_j(t)$ can be analyzed separately. It can be analyzed by using the conclusion of Subsection 4.1.

5. Simulation example

Considering an affine nonlinear control system in the following form:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = x_1(t)x_4^2(t) - \theta_1 \dfrac{1}{x_1^2(t)} + \theta_2 u_1(t) \\ \dot{x}_3(t) = x_4(t) + w_2(t) \\ \dot{x}_4(t) = -\dfrac{2x_2(t)x_4(t)}{x_1(t)} + \theta_2 \dfrac{1}{x_1(t)} u_2(t) + \dfrac{\theta_2}{x_1(t)} w_1(t) \\ h_1(x(t)) = x_3(t) \\ h_2(x(t)) = x_4(t) \end{cases} \tag{46}$$

where

$$f(x(t)) = \begin{pmatrix} x_2(t) \\ x_1(t)x_4^2(t) - \theta_1 \dfrac{1}{x_1^2(t)} \\ x_4(t) \\ -\dfrac{2x_2(t)x_4(t)}{x_1(t)} \end{pmatrix},$$

$$g_1(x(t)) = p_1(x(t)) = \begin{pmatrix} 0 \\ \theta_2 \\ 0 \\ 0 \end{pmatrix},$$

$$g_2(x(t)) = p_2(x(t)) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \theta \dfrac{1}{x_1(t)} \end{pmatrix},$$

$$y(x(t)) = \begin{pmatrix} h_1(x(t)) \\ h_2(x(t)) \end{pmatrix} = \begin{pmatrix} x_3(t) \\ x_4(t) \end{pmatrix}.$$

5.1 Evaluation of fault diagnosability without the control law

For the system (46), the fault characteristic indexes of the fault $w_1(t)$ and the fault characteristic indexes of the fault $w_2(t)$ can be calculated respectively. Take the derivative of the outputs $h_1(x(t))$ and $h_2(x(t))$ respectively:

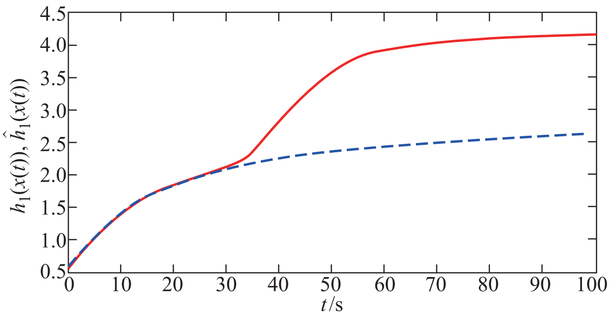
$$\left\{\begin{array}{l} h_1^{(2)}(x(t)) = \dfrac{\mathrm{d}\dot{x}_3}{\mathrm{d}t} = \dot{x}_4(t) + \dot{w}_2(t) = \\ \quad -\dfrac{2x_2(t)x_4(t)}{x_1(t)} + \theta_2 \dfrac{1}{x_1(t)} u_2(t) + \\ \quad \dfrac{\theta_2}{x_1(t)} w_1(t) + \dot{w}_2(t) \\ h_2^{(1)}(x(t)) = \dfrac{\partial h_2(x(t))}{\partial x(t)^{\mathrm{T}}} \cdot \dfrac{\partial \dot{x}(t)}{\partial t} = \dot{x}_4(t) = \\ \quad -\dfrac{2x_2(t)x_4(t)}{x_1(t)} + \theta_2 \dfrac{1}{x_1(t)} u_2(t) + \dfrac{\theta_2}{x_1(t)} w_1(t) \\ \qquad\qquad \vdots \end{array}\right. . \tag{47}$$

From (47), it can be concluded that $h_1^{(2)}(x(t))$ is affected by the faults $w_1(t)$ and $w_2(t)$ simultaneously, and the fault characteristic index of the fault $w_1(t)$ on the output $h_1(x(t))$ is $s_1^1 = 2$. $h_2^{(1)}(x(t))$ is obviously affected by the fault $w_1(t)$, and the fault characteristic index of the fault $w_1(t)$ on the output $h_2(x(t))$ is $s_1^2 = 1$. There is a clear relationship between the fault $w_2(t)$ and the output $h_1(x(t))$, and the fault characteristic index of the fault $w_2(t)$ on output $h_1(x(t))$ is $s_2^1 = 1$. Similarly, the fault characteristic index of the fault $w_2(t)$ on the output $h_2(x(t))$ is $s_2^2 = +\infty$. According to Theorem 5, the fault $x_1(t)$ is diagnosability.

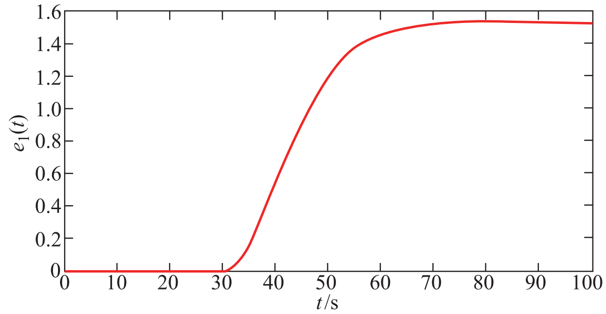
Numerical simulation is carried out for the system (46) with $u_1(t) = u_2(t) = 0$. To get the changes of the outputs $h_1(x(t))$ and $h_2(x(t))$ before and after the addition of the fault, the output errors are defined as

$$\left\{\begin{array}{l} e_1(t) = h_1(x(t)) - \widehat{h}_1(x(t)) \\ e_2(t) = h_2(x(t)) - \widehat{h}_2(x(t)) \end{array}\right. \tag{48}$$

where $\widehat{h}_1(x(t))$ and $\widehat{h}_2(x(t))$ are the outputs of the system (46) when the faults are free (i.e., $w_1(t) = 0, w_2(t) = 0$). Within the time of 30 s to 50 s, only injecting fault $w_1(t) = 0.2$ into the system (46), the output errors are shown in Fig. 1 and Fig. 2, respectively.

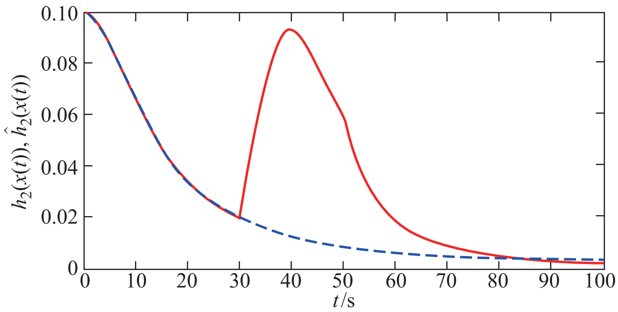


(a) Outputs $h_1(x(t))$ and $\hat{h}_1(x(t))$

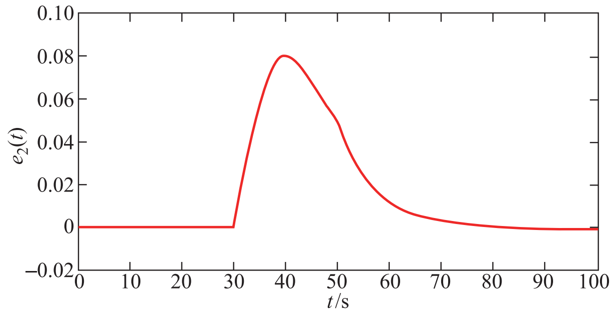


(b) Error

Fig. 1 Outputs $h_1(x(t))$, $\widehat{h}_1(x(t))$ and their errors when $w_1(t) = 0.2, w_2(t) = 0$



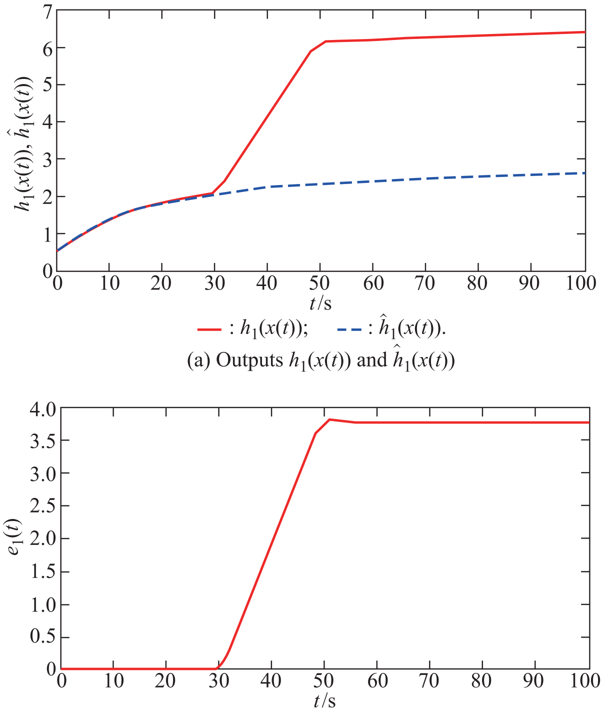
(a) Outputs $h_2(x(t))$ and $\hat{h}_2(x(t))$



(b) Error

Fig. 2 Outputs $h_2(x(t))$, $\widehat{h}_2(x(t))$ and their errors when $w_1(t) = 0.2, w_2(t) = 0$

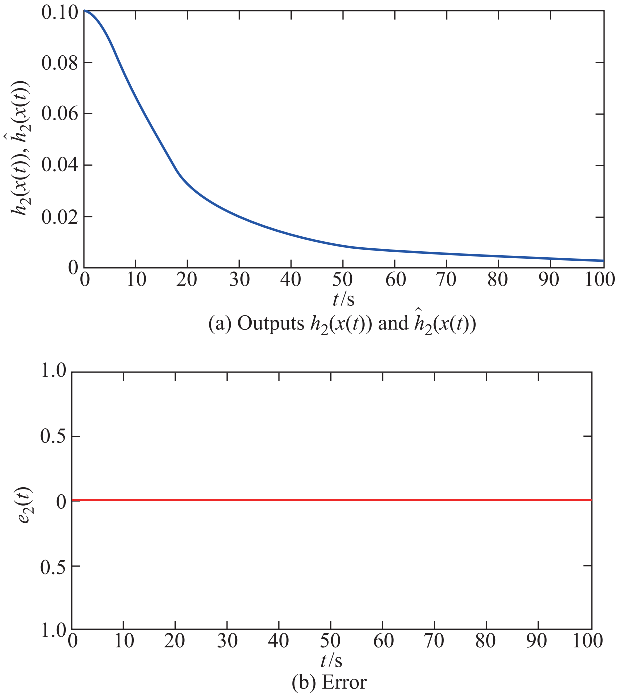
According to Fig. 1 and Fig. 2, it can be noted that the fault $w_1(t)$ is diagnosable. When the fault $w_2(t) = 0.2$ is only injected into the system (46) within the time of 30 s to 50 s, the output errors are shown in Fig. 3 and Fig. 4, respectively. According to Fig. 3 and Fig. 4, it can be noted that the fault $w_2(t)$ cannot be diagnosed through the output $h_2(x(t))$. Fig. 5 and Fig. 6 are simulation results of the output errors when the faults $w_1(t)$ and $w_2(t)$ are injected into the system simultaneously within the time of 30 s to 50 s. By comparing Fig. 1(b), Fig. 3(b) and Fig. 5(b), it can be found that the error $e_1(t)$ in Fig. 5(b) is the result of the superposition of Fig. 1(b) and Fig. 3(b). It can be shown that the output $h_1(x(t))$ is affected by the faults $w_1(t)$ and $w_2(t)$ simultaneously. By comparing Fig. 2(b), Fig. 4(b) and Fig. 6(b), it can be found that the error $e_2(t)$ in Fig. 2(b) and that in Fig. 6(b) are exactly the same. It can be shown that the output $h_2(x(t))$ is only affected by the fault $w_1(t)$.



(a) Outputs $h_1(x(t))$ and $\hat{h}_1(x(t))$

(b) Error

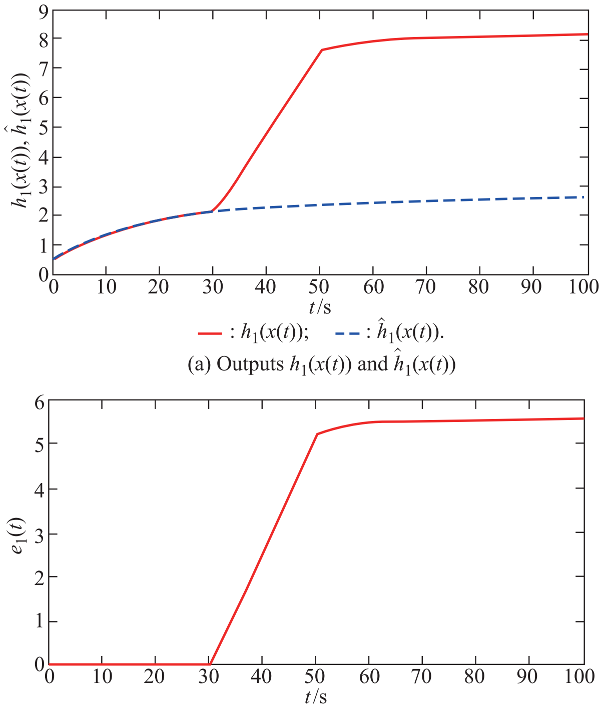
Fig. 3 Outputs $h_1(x(t))$, $\widehat{h}_1(x(t))$ and their errors when $w_1(t) = 0, w_2(t) = 0.2$



(a) Outputs $h_2(x(t))$ and $\hat{h}_2(x(t))$

(b) Error

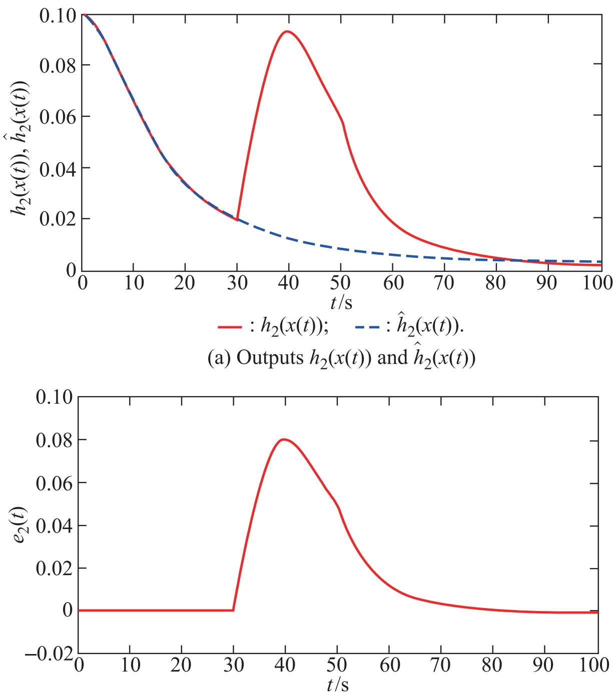
Fig. 4 Outputs $h_2(x(t))$, $\widehat{h}_2(x(t))$ and their errors when $w_1(t) = 0, w_2(t) = 0.2$



(a) Outputs $h_1(x(t))$ and $\hat{h}_1(x(t))$

(b) Error

Fig. 5 Outputs $h_1(x(t))$, $\widehat{h}_1(x(t))$ and their errors when $w_1(t) = 0.2, w_2(t) = 0.2$



(a) Outputs $h_2(x(t))$ and $\hat{h}_2(x(t))$

(b) Error

Fig. 6 Outputs $h_2(x(t))$, $\widehat{h}_2(x(t))$ and their errors when $w_1(t) = 0.2, w_2(t) = 0.2$

According to Theorem 5, when the faults $w_1(t)$ and $w_2(t)$ occur simultaneously, only fault $w_1(t)$ can be diagnosed in the system (46), and the simulation results once again verify the correctness of the theorem.

5.2 Evaluation of fault diagnosability considering the control law

We consider the static state feedback linearization of the system (46). Assume the point $x_1(t) = 0$ is not included in any open set, that is, excluding the singular points of the system (46). Then the system (46) can be linearized by static state feedback. The relative degree of the system (46) is $\{2, 2\}$, then we have

$$\begin{cases} ad_{f(x(t))}g(x(t)) = -2x_4(t)\theta_2\dfrac{\partial f(x(t))}{\partial x_2(t)} + \dfrac{x_2(t)\theta_2}{x_1^2(t)}\dfrac{\partial f(x(t))}{\partial x_4(t)} - \dfrac{\theta_2}{x_1(t)}\dfrac{\partial f(x(t))}{\partial x_3(t)} \\ [g(x(t)), ad_{f(x(t))}g(x(t))] = -\dfrac{2\theta_2^2}{x_1(t)}\dfrac{\partial f(x(t))}{\partial x_2(t)} \end{cases}. \tag{49}$$

Due to

$$[g(x(t)), ad_{f(x(t))}g(x(t))] \notin G_1 = \text{Span}\{g(x(t)), ad_{f(x(t))}g(x(t))\}, \tag{50}$$

the distribution G_1 is not an involutional distribution. Therefore, according to Theorem 2, the system (46) is not exactly feedback linearized. Moreover, because of $ad^2_{f(x(t))}g(x(t)) \notin \overline{G}_1$ and G_1 has constant rank 3, it can be seen from Theorem 3 that the system (46) is partially feedback linearized. Actually, choosing new coordinates

$$\begin{cases} z_1 = x_1(t) \\ z_2 = x_2(t) \\ z_3 = x_1(t)x_4^2(t) - \dfrac{\theta_1}{x_1^2(t)} \\ z_4 = x_3(t) \end{cases}, \tag{51}$$

one obtains a system which contains a linear subsystem of dimension 3. Then the system (46) becomes the following expression:

$$\begin{cases} \dot z_1 = z_2 \\ \dot z_2 = z_3 \\ \dot z_3 = -3z_2\dfrac{z_3}{z_1} + \dfrac{2\theta_1 z_2}{z_1^3} + 2\theta_2\sqrt{\dfrac{z_3}{z_1} + \dfrac{\theta_1}{z_1^3}}u_2(t) + \\ \qquad 2\theta_2\sqrt{\dfrac{z_3}{z_1} + \dfrac{\theta_1}{z_1^3}}w_1(t) \\ \dot z_4 = \sqrt{\dfrac{z_3}{z_1} + \dfrac{\theta_1}{z_1^3}} + w_2(t) \end{cases}. \tag{52}$$

Let

$$u_2(t) = \frac{1}{2\theta_2\sqrt{\dfrac{z_3}{z_1} + \dfrac{\theta_1}{z_1^3}}}\left(\left(3z_2\frac{z_3}{z_1} - \frac{2\theta_1 z_2}{z_1^3}\right) - k_1(z_1 - z_{1r}) - k_2 z_2 - k_3 z_3\right). \tag{53}$$

Numerical simulation is carried out for the system (52) with the controller (53). Simulation results are shown in Fig. 7, and it can be seen that z_1 is finally stable at the given value z_{1r}. The fault characteristic indexes of the faults $w_1(t)$ and $w_2(t)$ are calculated respectively by the same method.

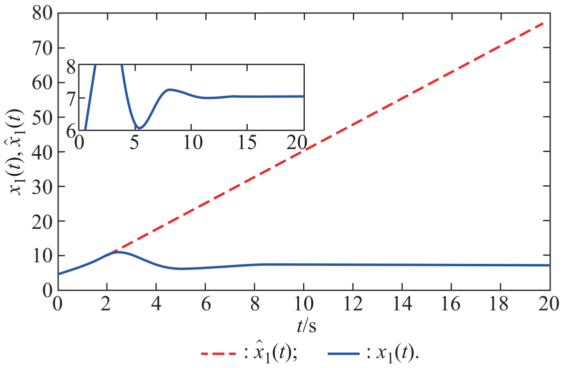
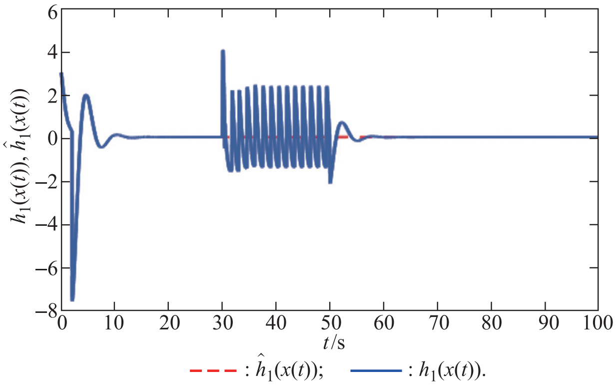


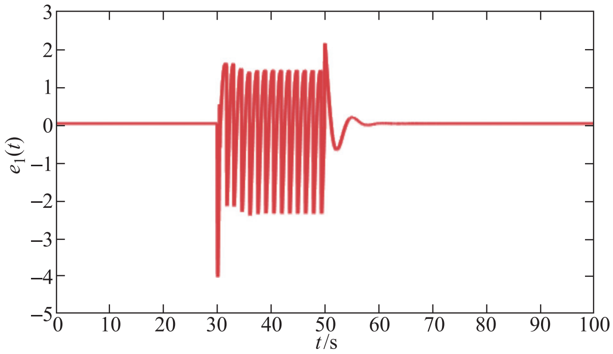
Fig. 7 Partial feedback linearization based on the controller (53)

The fault characteristic index of the fault $w_1(t)$ on the output $h_1(x(t))$ is $s_1^1 = 1$, and the fault characteristic index of the fault $w_1(t)$ on the output $h_2(x(t))$ is $s_1^2 = 2$, and the fault characteristic index of the fault $w_2(t)$ on the output $h_1(x(t))$ is $s_2^1 = 1$, and the fault characteristic index of the fault $w_2(t)$ on the output $h_2(x(t))$ is $s_2^2 = \infty$. Due to the fault characteristic index values being changed after the control law (53) is added, the fault diagnosability should be reevaluated.

Corresponding to the theoretical analysis in Section 3, this part mainly focuses on the numerical simulation on the system (52) with the fault added when $u_1(t) = 0$, $z_{1r} = 7$. Within the time of 30 s to 50 s, only injecting the fault $w_1(t) = 200$ into the system (52), the output errors are shown in Fig. 8 and Fig. 9, respectively. According to Fig. 8(a) and Fig. 9(a), it can be noted that the outputs $h_1(x(t))$ and $h_2(x(t))$ are both affected by the fault $w_1(t)$. Similarly, the simulation results as shown in Fig. 10 and Fig. 11 can be obtained with $w_1(t) = 0$ and $w_2(t) = 200$. Fig. 12 and Fig. 13 are simulation results of the output errors when the faults $w_1(t)$ and $w_2(t)$ are injected into the system simultaneously within the time of 30 s to 50 s. Comparing Fig. 9(b) and Fig. 11(b), it can be shown that the output $h_2(x(t))$ is affected by the faults $w_1(t)$ and $w_2(t)$ simultaneously. Comparing Fig. 8(b) and Fig. 10(b), it can be shown that the output $h_1(x(t))$ is only affected by the fault $w_1(t)$.

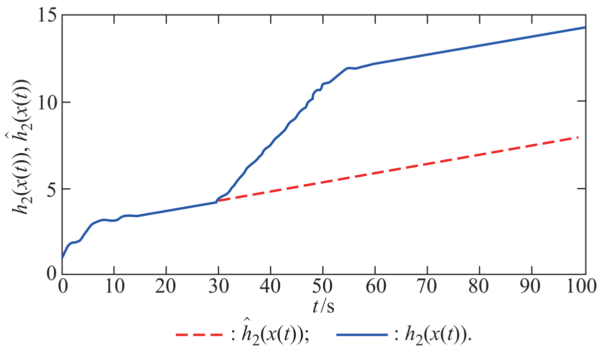


(a) Outputs $h_1(x(t))$ and $\hat{h}_1(x(t))$

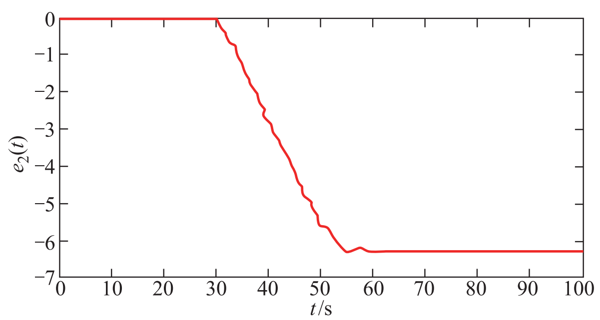


(b) Error

Fig. 8 Outputs $h_1(x(t))$, $\hat{h}_1(x(t))$ and their errors when $w_1(t) = 200, w_2(t) = 0$

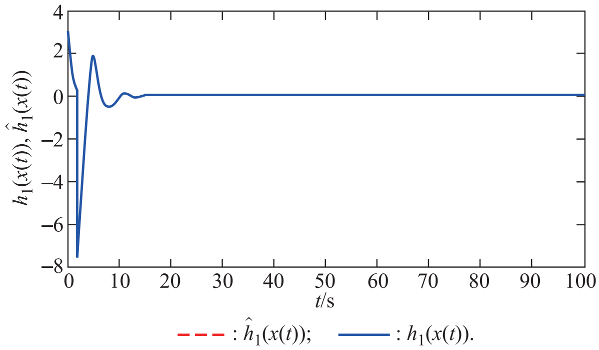


(a) Outputs $h_2(x(t))$ and $\hat{h}_2(x(t))$

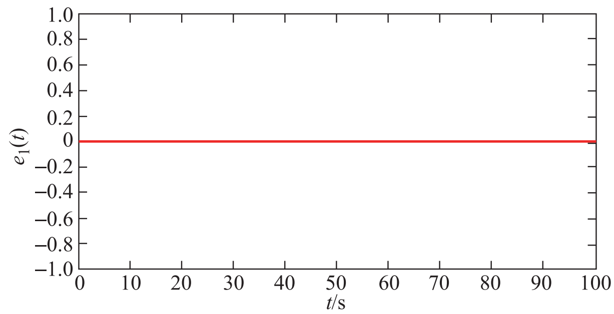


(b) Error

Fig. 9 Outputs $h_2(x(t))$, $\hat{h}_2(x(t))$ and their errors when $w_1(t) = 200, w_2(t) = 0$

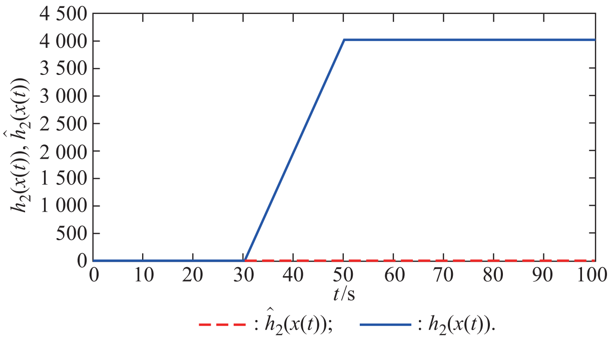


(a) Outputs $h_1(x(t))$ and $\hat{h}_1(x(t))$

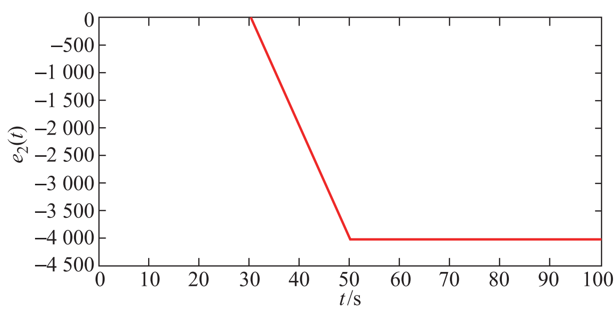


(b) Error

Fig. 10 Outputs $h_1(x(t))$, $\hat{h}_1(x(t))$ and their errors when $w_1(t) = 0, w_2(t) = 200$

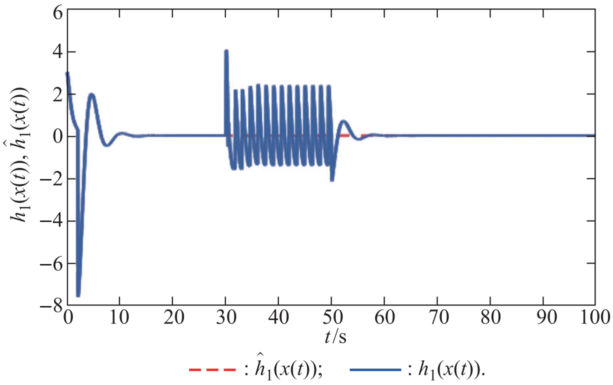


(a) Outputs $h_2(x(t))$ and $\hat{h}_2(x(t))$

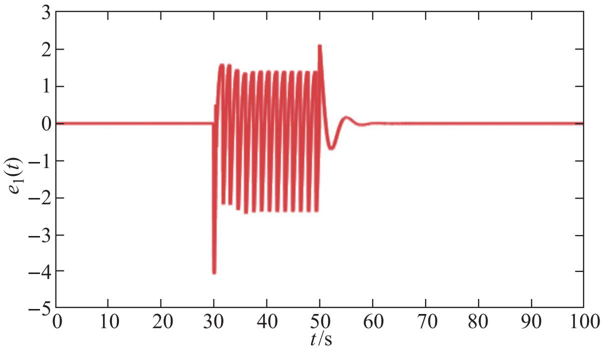


(b) Error

Fig. 11 Outputs $h_2(x(t))$, $\hat{h}_2(x(t))$ and their errors when $w_1(t) = 0, w_2(t) = 200$

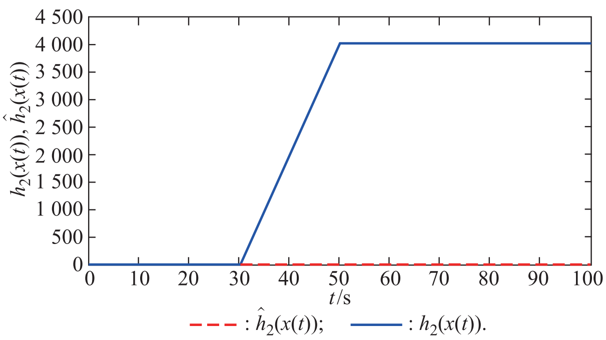


(a) Outputs $h_1(x(t))$ and $\hat{h}_1(x(t))$

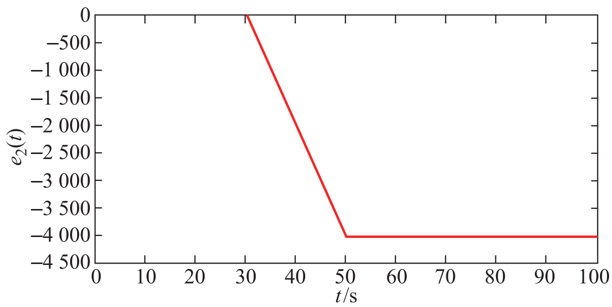


(b) Error

Fig. 12 Outputs $h_1(x(t))$, $\hat{h}_1(x(t))$ and their errors when $w_1(t) = 200, w_2(t) = 200$



(a) Outputs $h_2(x(t))$ and $\hat{h}_2(x(t))$



(b) Error

Fig. 13 Outputs $h_2(x(t))$, $\hat{h}_2(x(t))$ and their errors when $w_1(t) = 200, w_2(t) = 200$

According to Section 3.2, when the faults $w_1(t)$ and $w_2(t)$ occur simultaneously, only fault $w_1(t)$ can be diagnosed in the system (53), and the simulation results once again verify the correctness of Theorem 5.

6. Conclusions

In this paper, we present the evaluation of fault diagnosability for a class of uncertain affine nonlinear systems with multiple faults occurring simultaneously by the theory of differential geometry. By using the method of system reconfiguration, a design scheme of fault diagnosability has been provided, which solves the influence of control laws on the results of fault diagnosability evaluation. The effectiveness of the developed evaluation of fault diagnosability has been demonstrated by an illustrative simulation example. For the future research, the evaluation of fault diagnosability for other nonlinear uncertain systems will be under investigation.

References

[1] KELIRIS C, POLYCARPOU M, PARISINI T. An integrated learning and filtering approach for fault diagnosis of a class of nonlinear dynamical systems. IEEE Trans. on Neural Networks and Learning Systems, 2017, 28(4): 988 – 1004.

[2] GAO Z, DING S X, CECATI C. Real-time fault diagnosis and fault-tolerant control. IEEE Trans. on Industrial Electronics, 2015, 62(6): 3752 – 3756.

[3] JMAL A, NAIFAR O, BEN MAKHLOUF A, et al. Sensor fault estimation for fractional-order descriptor one-sided Lipschitz systems. Nonlinear Dynamics, 2017, 91(3): 1713 – 1722.

[4] BISWAL M, BRAHMA S, CAO H. Supervisory protection and automated event diagnosis using PMU data. IEEE Trans. on Power Delivery, 2016, 31(4): 1855 – 1863.

[5] WANG T Z, QI J, XU H, et al. Fault diagnosis method based on FFT-RPCA-SVM for cascaded-multilevel inverter. ISA Transactions, 2016, 60: 156 – 163.

[6] YIN S, ZHU X. Intelligent particle filter and its application on fault detection of nonlinear system. IEEE Trans. on Industrial Electronics, 2015, 62(6): 3852 – 3861.

[7] PATTON R J, CHEN J. Observer-based fault detection and isolation: robustness and applications. Control Engineering Practice, 1997, 5(5): 671 – 682.

[8] GOBBO D D, NAPOLITANO M R. Issues in fault detectability for dynamic systems. Proc. of the American Control Conference, 2000: 3203 – 3207.

[9] KOSCIELNY J M, SYFERT M, ROSTEK K, et al. Fault isolability with different forms of the faults-symptoms relation. International Journal of Applied Mathematics and Computer Science, 2016, 26(4): 815 – 826.

[10] ERIKSSON D, FRISK E, KRYSANDER M. A method for quantitative fault diagnosability analysis of stochastic linear descriptor models. Automatica, 2013, 49(6): 1591 – 1600.

[11] LI W B, WANG D Y, LIU C R. Quantitative evaluation of actual fault diagnosability for dynamic systems. Acta Automatica Sinica, 2015, 41(3): 497 – 507. (in Chinese)

[12] LEAL R, AGUILAR J, TRAVE-MASSUYES L, et al. An approach for diagnosability analysis and sensor placement for continuous processes based on evolutionary algorithms and analytical redundancy. Applied Mathematical Sciences, 2015,

9(43): 2125 – 2146.

[13] LIU J, HUA Y, LI Q, et al. Fault diagnosability qualitative analysis of spacecraft based on temporal fault signature matrix. Proc. of the IEEE Chinese Guidance, Navigation and Control Conference, 2016: 1496 – 1500.

[14] TRAVE-MASSUYES L, ESCOBET T, OLIVE X. Diagnosability analysis based on component-supported analytical redundancy relations. IEEE Trans. on Systems, Man and Cybernetics — Part A: Systems and Humans, 2006, 36(6): 1146 – 1160.

[15] FRISK E, KRYSANDER M, SLUND J. Sensor placement for fault isolation in linear differential-algebraic systems. Automatica, 2009, 45(2): 364 – 371.

[16] CUI Y Q, SHI J Y, WANG Z L. System-level operational diagnosability analysis in quasi real-time fault diagnosis: the probabilistic approach. Journal of Process Control, 2014, 24(9): 1444 – 1453.

[17] PENG X F, LIN L X, ZHONG X Y, et al. Methods for fault diagnosability analysis of a class of affine nonlinear systems. Mathematical Problems in Engineering, 2015: 409184.

[18] ZHANG B, JIA Y M, MATSUNO F, et al. Task-space synchronization of networked mechanical systems with uncertain parameters and communication delays. IEEE Trans. on Cybernetics, 2017, 47(8): 2288 – 2298.

[19] PARK S, PARK Y, PARK Y S. Degree of fault isolability and active fault diagnosis for redundantly actuated vehicle system. International Journal of Automotive Technology, 2016, 17(6): 1045 – 1053.

[20] XING Z R, XIA Y Q. Evaluation and design of actuator fault diagnosability for nonlinear affine uncertain systems with unknown indeterminate inputs. International Journal of Adaptive Control and Signal Processing, 2016, 31(1): 122 – 137.

[21] NGUYEN D T, DUONG Q B, ZAMAI E, et al. Fault diagnosis for the complex manufacturing system. Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability, 2016, 230(2): 178 – 194

[22] DING Y, KIM P, CEGLAREK D, et al. Optimal sensor distribution for variation diagnosis in multistation assembly processes. IEEE Trans. on Robotics and Automation, 2003, 19(4): 543 – 556.

[23] WANI M F, GANDHI O P. Diagnosability evaluation of systems using bipartite graph and matrix approach. Artificial Intelligence for Engineering Design Analysis and Manufacturing, 2000, 14(3): 193 – 206.

[24] LIU Z T, AHMED Q, ZHANG J Y, et al. Structural analysis based sensors fault detection and isolation of cylindrical lithium-ion batteries in automotive applications. Control Engineering Practice, 2016, 52: 46 – 58.

[25] LIN L X, LIU C R, ZHONG X Y, et al. Fault diagnosis for nonlinear systems based on differential geometry approach. Journal of Huanzhong University of Science and Technology (Natural Science Edition), 2015, 43(9): 24 – 28. (in Chinese)

[26] RICCARDO M, PATRIZIO T. Nonlinear control design: geometric, adaptive and robust. Hertfordshire: Prentice Hall International (UK) Ltd., 1996.

[27] ISIDORI A. Nonlinear control systems. 3rd ed. New York: Springer-Verlag World Publishing Corp., 1999.

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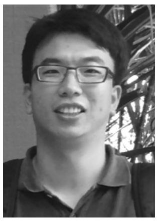

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