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Multi-stage attack weapon target allocation method based on defense area analysis

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Abstract: For better reflecting the interactive defense between targets in practical combat scenarios, the basic weapon-target allocation (WTA) framework needs to be improved. A multi-stage attack WTA method is proposed. First, a defense area analysis is presented according to the targets' positions and the radii of the defense areas to analyze the interactive coverage and protection between targets' defense areas. Second, with the coverage status and coverage layer number, a multi-stage attack planning method is proposed and the multi-stage attack objective function model is established. Simulation is conducted with interactive defense combat scenarios, the traditional WTA method and the multi-stage WTA method are compared, and the objective function model is validated with the Monte-Carlo method. The results suggest that if the combat scenario involves interactive coverage of targets' defense areas, it is imperative to analyze the defense areas and apply the multi-stage attack method to weakening the target defense progressively for better combat effectiveness.

Keywords: weapon-target allocation (WTA), defense area analysis, combat effective analysis.

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1. Introduction

Weapon target allocation (WTA) is a key problem for the improvement of combat effectiveness. The main point of WTA is to optimize comprehensive combat effectiveness considering factors including values of different targets, kill probabilities and intercept probabilities between weapons and targets [1 – 3].

Most of the existing research on WTA concentrates on solving the WTA problem. For example, Rezende et al. [4] and Hu et al. [5] emphasized solving the efficiency in reaching the optimality of the WTA problem, and introduced an ant colony algorithm in optimizing. Liu et al. [6] pointed out that traditional exact methods and heuristic algorithms were all capable of solving the WTA problem, but exact methods performed worse when the problem scale increased in WTA, and introduced the artificial bee colony (ABC) algorithm for solution. In [6], different methods were compared including particle swarm optimization (PSO), genetic algorithm (GA) and ABC. Reference [7] is also based on the ABC method. In [8], a tabu search heuristic method was proposed for WTA, and the attack procedure was described with a tree search model. The expansion of the tree is based on the kill probability and the recognization probability. Nonetheless, some research expands the method reference to the geometric method [9], game theory [10], reinforcement learning [11], the PSO method [12,13], the fuzzy logic method [14] and the rule based method [15] to solve the WTA problem. To conclude, the heuristic and hybrid method [16 – 22] and the evolutionary method are the most popular methods for solving the WTA problem [23 – 26]. Other research focuses on other perspectives of the WTA problem, such as distributed algorithms [27,28] and WTA process simulation [29]. Besides, research in [30,31] can also be referenced as comprehensive surveys on the WTA problem.

The existing research provides plenty of solving methods for the WTA problem as references. In most studies, however, the kill probabilities between weapons and targets, the intercept probabilities between targets and weapons are both involved. The kill procedure and intercept procedure are independent of each other and there lacks consideration of the influence on the holistic scenario change by different attack sequences and target distributions. In practical combat scenarios, due to the different importances of different targets, the targets will form closed or reciprocal protection by taking advantage of their own defense capabilities. Thus, a weapon that is allocated to a particular target will possibly be intercepted by multiple targets. Under this condition, the traditional WTA

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method will not completely describe the interaction of defense areas between different targets. Accordingly, a multi-stage attack strategy needs to be introduced to weaken the protection between different targets stage by stage, and after each stage of attack, the weakened areas need to be found out as the attack direction for the next stage of attack.

To conclude, for better describing the practical combat scenarios, the WTA problem needs to be improved fundamentally to reflect the interactive protection between different targets. This work proposes a multi-stage attack WTA method (MM method). The remainder of this work is organized as follows. Section 2 presents the basic problem definition and necessity of a multi-stage attack. Section 3 presents the defense area analysis method, which is to analyze the interactive coverage and protection between the targets according to the targets' positions and radii of the defense areas. Section 4 presents the multi-stage attack planning process, which is to calculate the objective function value and solve the multi-stage attack plan according to the defense area analysis results.

2. Problem definition

The WTA problem can be described as follows. Allocate n_w weapons to n_t targets; the WTA plan is an $n_w \times n_t$ matrix $\boldsymbol{A} = [a_{\omega,\tau}]$, where $a_{\omega,\tau} \in \{0, 1\}$ ($a_{\omega,\tau} = 1$ means a weapon ω is allocated to a target τ), each weapon can only be allocated to one target, namely $\sum_{\tau=1}^{n_t} a_{\omega,\tau} = 1$, and different weapons can be allocated to one target; each target is qualified with a value, and the value vector of all targets is a $1 \times n_t$ vector $\boldsymbol{V} = [v_\tau]$; different weapons have different kill probabilities for different targets, which is an $n_w \times n_t$ matrix $\boldsymbol{P}_k = [p_{k,\omega,\tau}]$, where $p_{k,\omega,\tau} \in [0, 1]$; different targets have different intercept probabilities for different weapons as an $n_w \times n_t$ matrix $\boldsymbol{P}_c = [p_{c,\omega,\tau}]$, where $p_{c,\omega,\tau} \in [0, 1]$. The traditional WTA formulates the objective function H_t as

$$\begin{cases} H_t = \sum_{\tau=1}^{n_t} v_\tau [1 - P_A(\boldsymbol{A})] \\ P_A(\boldsymbol{A}) = \prod_{\omega=1}^{n_w} [1 - a_{\omega,\tau} p_{k,\omega,\tau} (1 - p_{c,\omega,\tau})] \end{cases} \quad (1)$$

where $1 - P_A(\boldsymbol{A})$ is the conditional cumulative kill probability to the target τ and the weapons allocated to the target τ are not intercepted by τ. H_t is the conditional sum of values of all allocated targets.

However, in practical combat scenarios, the defense areas of the targets will overlap with each other as illustrated in Fig. 1. Without loss of generality, all units in this paper are normalized.

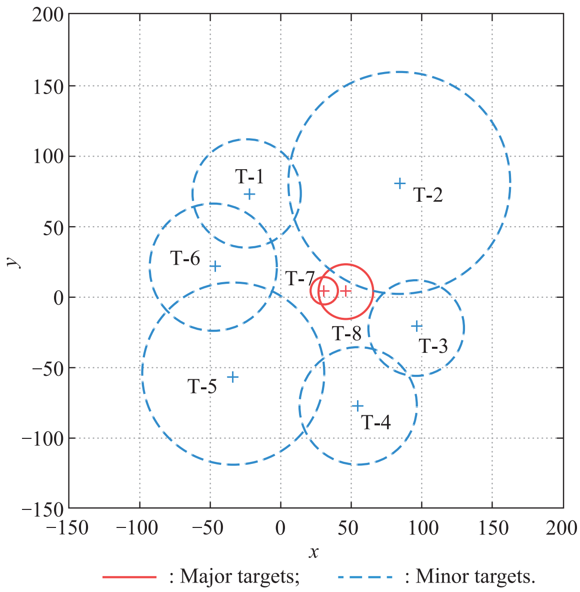


Fig. 1 Typical combat scenario with overlapping defense areas

In Fig. 1, each target has a defense area with a radius ρ_τ centered at its position $\boldsymbol{X}_\tau$. T-7 and T-8 are major targets, and T-1 to T-6 are minor targets. In this scenario, T-1 to T-6 form a closed circle end to end, and every weapon that is allocated to T-7 or T-8 must pass through one or more defense areas of T-1 to T-6. Besides, T-7 is also covered by the defense area of T-8.

Assume a salvo attack with four weapons W-1 to W-4, and the value vector $\boldsymbol{V}$, the kill probabilities matrix $\boldsymbol{P}_k$ and the intercept probabilities matrix $\boldsymbol{P}_c$ are shown in Table 1 – Table 3. The WTA plan that is solved with the traditional method is in Table 4.

Because the weapons allocated to T-7 and T-8 will pass through defense areas that belong to other targets, which will increase the probability being intercepted, and the practical objective function value is 47.26.

Table 1 Value vector

Target	T-1	T-2	T-3	T-4
Value	18.67	20.30	6.29	20.44
Target	T-5	T-6	T-7	T-8
Value	15.38	5.76	53.01	35.84

Table 2 Kill probabilities matrix

Weapon	Target			
	T-1	T-2	T-3	T-4
W-1	0.79	0.79	0.68	0.73
W-2	0.79	0.70	0.78	0.61
W-3	0.63	0.76	0.76	0.77
W-4	0.79	0.63	0.79	0.79
Weapon	Target			
	T-5	T-6	T-7	T-8
W-1	0.74	0.73	0.66	0.74
W-2	0.75	0.63	0.61	0.66
W-3	0.75	0.74	0.62	0.79
W-4	0.68	0.61	0.76	0.61

Table 3 Intercept probabilities matrix

Weapon	Target			
	T-1	T-2	T-3	T-4
W-1	0.28	0.17	0.38	0.36
W-2	0.25	0.30	0.40	0.17
W-3	0.41	0.28	0.21	0.15
W-4	0.42	0.36	0.37	0.30
Weapon	Target			
	T-5	T-6	T-7	T-8
W-1	0.48	0.40	0.46	0.16
W-2	0.24	0.20	0.48	0.20
W-3	0.33	0.30	0.32	0.44
W-4	0.19	0.38	0.16	0.20

Table 4 Traditional WTA plan

Weapon	W-1	W-2	W-3	W-4
Target	T-8	T-1	T-4	T-7

Instead, if the salvo attack is conducted stage by stage with the same WTA plan, then, (i) the first stage is W-2 to T-1 and W-3 to T-4; (ii) the second stage is W-1 to T-8 passing through T-1; and (iii) the third stage is W-4 to T-7 passing through T-4 and T-8. With a multi-stage strategy, the objective function value can reach 72.17. The objective function value is calculated with the Monte-Carlo method in Section 5.1.

Obviously, in this combat scenario with overlapping defense areas, the traditional WTA method will not be capable of describing the interaction between targets and their defense areas, and it is harder still to acquire a solution with optimality. Thus, a multi-stage attack strategy and its corresponding solving method are imperative.

To conclude, multi-stage attack WTA has two main problems.

(i) Defense area analysis. Analyze the interrelationships between defense areas, including overlapping and coverage according to each defense area's position and radius, for further objective function value estimation and multi-stage attack planning.

(ii) Objective function formulation of multi-stage attack and multi-stage planning. Generate the multi-stage plan with defense area analysis results, and simultaneously calculate the objective function value of a multi-stage attack.

Besides, the multi-stage WTA problem will become more complicated considering path planning. For simplicity, this study focuses on the WTA problem without path planning, and assumes that the initial positions of weapons can be arbitrarily configured outside the defense areas of targets.

3. Defense area analysis

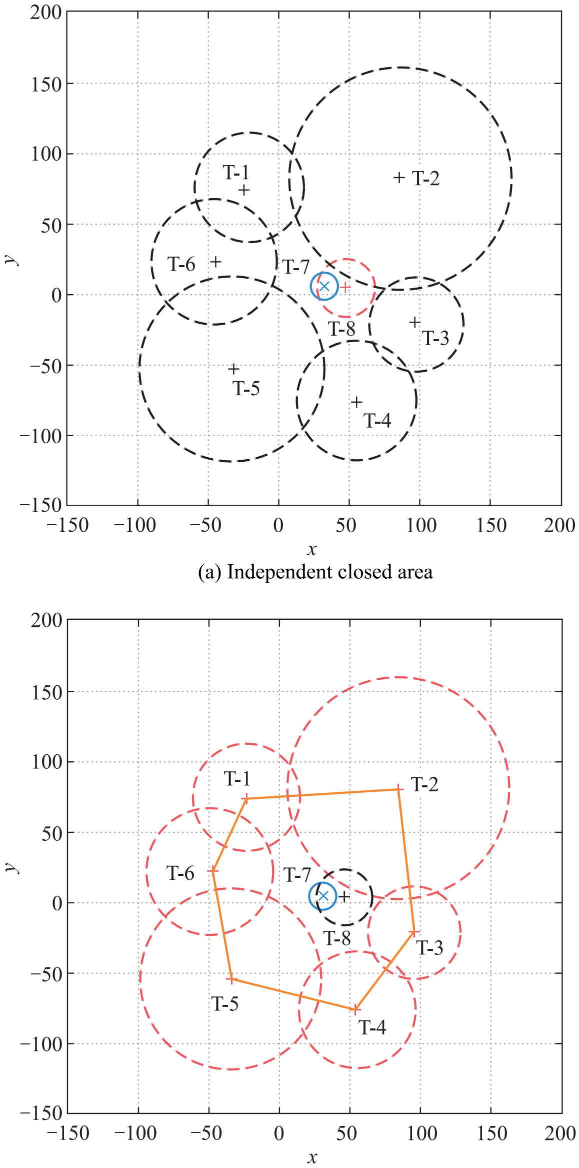
The distribution of targets and their corresponding defense areas forms a reciprocal protecting combat scenario. It is necessary to analyze the defense areas for finding out the weak points for improving attack effectiveness.

3.1 Fundamentals

Definition 1 Closed area.

A closed area is formed with one or multiple defense areas. A closed area separates the space into two parts (internal part and external part). Any curve connecting the internal part and the external part will pass through the defense area(s) that form(s) this closed area.

A closed area can protect other targets. By Definition 1, there are two basic forms of closed areas: (i) independent closed area, which is formed with single defense areas, as shown in Fig. 2(a); (ii) circular closed area, which is formed with multiple defense areas connected with each other end to end, as shown in Fig. 2(b).



(b) Circular closed area
: Covered targets; : Closed area.

Fig. 2 Basic forms of closed area

Besides these two basic forms, any union of basic closed areas is still a close area.

Theorem 1 The union of any closed areas is a closed area.

Definition 2 Coverage.

A target covered by a closed area is that the target is in the internal part of the closed area, and that any curve connecting the target with any point in the external part must pass through the defense area of this closed area.

It is obvious that by finding out all closed areas, whether a target is covered or not can be further judged. If a target is in the internal part of a closed area, any weapon allocated to this target will pass through the defense area of this closed area.

Considering that a closed area is formed with one or multiple defense areas, we can introduce the $1 \times n_t$ vector $\boldsymbol{R}_s = [r_\tau]$ to describe which defense areas are selected in the closed area. $r_\tau \in \{0, 1\}$, and that r_τ equals 1 means the τth target is selected, and 0 means not.

With $\boldsymbol{R}_s$, we can further define the following operation

$$\boldsymbol{R}_s = \boldsymbol{R}_{s,1} \wedge \boldsymbol{R}_{s,2} \tag{2}$$

is the logical AND operation, and $\boldsymbol{R}_s$ is the intersection of defense areas of $\boldsymbol{R}_{s,1}$ and $\boldsymbol{R}_{s,2}$.

$$\boldsymbol{R}_s = \boldsymbol{R}_{s,3} \vee \boldsymbol{R}_{s,4} \tag{3}$$

is the logical OR operation, and $\boldsymbol{R}_s$ is the union of defense areas of $\boldsymbol{R}_{s,3}$ and $\boldsymbol{R}_{s,4}$.

$$\|\boldsymbol{R}_s\| = \sum_{\tau=1}^{n_t} r_\tau \tag{4}$$

is the sum of 1-elements in $\boldsymbol{R}_s$, or the number of defense areas in $\boldsymbol{R}_s$. $\|\boldsymbol{R}_{s,1} \wedge \boldsymbol{R}_{s,2}\| > 0$ means $\boldsymbol{R}_{s,1}$ and $\boldsymbol{R}_{s,2}$ share common defense areas.

3.2 Circular closed area solving

Obviously, the independent closed area is the corresponding defense area itself, and in $\boldsymbol{R}_s$, there is only one element equal to 1. It is relatively difficult to solve the circular closed area.

Considering that the circular closed area involves multiple overlapping defense areas, we introduce an $n_t \times n_t$ matrix $\boldsymbol{C} = [c_{ij}]$ as a connection matrix to describe it. $c_{ij} \in \{0, 1\}$, and that c_{ij} equals 1 means the defense area i overlaps with the defense area j, and 0 means not, that is

$$c_{ij} = \begin{cases} 1, & |\boldsymbol{X}_i - \boldsymbol{X}_j| \leqslant \rho_i + \rho_j \\ 0, & |\boldsymbol{X}_i - \boldsymbol{X}_j| > \rho_i + \rho_j \end{cases}. \tag{5}$$

For all defense areas, only some parts of them can form a circular closed area, and thus we introduce the transformation operator S:

$$S(\boldsymbol{C}|\boldsymbol{R}_s) = \boldsymbol{M}(\boldsymbol{R}_s)\boldsymbol{C}\boldsymbol{M}(\boldsymbol{R}_s)^{\mathrm{T}} \tag{6}$$

where $\boldsymbol{M}(\boldsymbol{R}_s)$ is formulated with $\boldsymbol{R}_s$, $\boldsymbol{M}(\boldsymbol{R}_s)$ is an $n_s \times n_t$ matrix, n_s is the number of 1-elements in $\boldsymbol{R}_s$ or $n_s = \sum_{\tau=1}^{n_t} r_\tau$. $\boldsymbol{M}(\boldsymbol{R}_s) = [m_{i_m,j_m}]$, and we have

$$m_{i_m,j_m} = \begin{cases} 1, & i_m = \sum_{\tau=1}^{\tau^*} r_\tau, \quad j_m = \tau^* \\ 0, & \text{otherwise} \end{cases}. \tag{7}$$

Simply speaking, the operator S transforms $\boldsymbol{C}$ into a new square matrix by selecting columns and rows corresponding to 1-elements in $\boldsymbol{R}_s$.

Theorem 2 For a circular closed area, the transformed connection matrix $S(\boldsymbol{C}|\boldsymbol{R}_s) = [s_{i_s,j_s}]$ has the property that for $\forall j_s$, there is $\sum_{i_s=1}^{n_s} s_{i_s,j_s} = 2$.

Proof Since in a circular closed area, the defense areas connect with each other end by end, and for any defense area, there are two other defense areas connected with it, that is $\sum_{i_s=1}^{n_s} s_{i_s,j_s} = 2$. □

Theorem 3 For a given $S(\boldsymbol{C}|\boldsymbol{R}_s) = [s_{i_s,j_s}]$, if for $\forall j_s$, there is $\sum_{i_s=1}^{n_s} s_{i_s,j_s} = 2$, and the corresponding defense areas form a circular closed area.

Proof We prove this theorem with reduction to absurdity.

Assume that for $\forall j_s$, there is $\sum_{i_s=1}^{n_s} s_{i_s,j_s} = 2$, and the defense area selected by $\boldsymbol{R}_s$ cannot form a circular closed area.

Select a defense area in $\boldsymbol{R}_s$ marked as i_s, with $\sum_{i_s=1}^{n_s} s_{i_s,j_s} = 2$, and there are two other defense areas in $\boldsymbol{R}_s$ connected with i_s, marked as $i_{s,1}$ and $i_{s,2}$.

For $i_{s,1}$, two defense areas connect with it, namely i_s and $i_{s,3}$.

Further, we can have $i_{s,n}$ that connects with $i_{s,n-1}$. Notice that $\sum_{i_s=1}^{n_s} s_{i_s,j_s} = 2$, and there are in total four possibilities.

(i) $i_{s,n}$ connects with $i_{s,2}$, but with assumption, the defense areas selected by $\boldsymbol{R}_s$ cannot form a circular closed area, so $i_{s,n}$ cannot connect with $i_{s,2}$, which contradicts the assumption.

(ii) $i_{s,n}$ connects with $i_{s,p}$, where $p \in \{3, \dots, n-2\}$, so $i_{s,p}$ connects with $i_{s,p-1}$, $i_{s,p+1}$ and $i_{s,n}$, which contradicts $\sum_{i_s=1}^{n_s} s_{i_s,j_s} = 2$.

(iii) $i_{s,n}$ only connects with $i_{s,n-1}$, which contradicts $\sum_{i_s=1}^{n_s} s_{i_s,j_s} = 2$.

(iv) A new defense area $i_{s,n+1}$ connects with $i_{s,n}$. Notice that the number of defense areas is finite, $n_s < \infty$, which will terminally reach cases (i) – (iii).

To conclude, all possibilities contradict the assumption. □

From Theorem 3, for $S(\boldsymbol{C}|\boldsymbol{R}_s)$ that satisfies $\sum_{i_s=1}^{n_s} s_{i_s,j_s} = 2$, the corresponding defense areas can form a circular closed area, but the number of the formed circular closed areas is not regulated. In fact, when $\sum_{i_s=1}^{n_s} s_{i_s,j_s} = 2$ is satisfied, if $S(\boldsymbol{C}|\boldsymbol{R}_s)$ is a partitioned matrix as

$$S(\boldsymbol{C}|\boldsymbol{R}_s) = \begin{bmatrix} 0 & 1 & 1 & & & \\ 1 & 0 & 1 & & & \\ 1 & 1 & 0 & & & \\ & & & 0 & 1 & 1 \\ & & & 1 & 0 & 1 \\ & & & 1 & 1 & 0 \end{bmatrix}, \tag{8}$$

$\boldsymbol{R}_s$ corresponds to two disjoint circular closed areas. Therefore, $\boldsymbol{R}_s$ which satisfies $\sum_{i_s=1}^{n_s} s_{i_s,j_s} = 2$ may contain multiple circular closed areas.

With the analysis above, we can find out all circular defense areas with Algorithm 1 and Algorithm 2. First, Algorithm 1 gives all circular closed areas and their union sets. Second, with Algorithm 2, all union sets are deleted, and only circular closed areas remain.

Algorithm 1 Circular closed areas and their union sets.

Input: a connection matrix $\boldsymbol{C}$.

Step 1 Formulate a mapping form $\boldsymbol{R}_s$ to a binary integer $n_{\boldsymbol{R}}$, and every bit of $n_{\boldsymbol{R}}$ corresponds to an element of $\boldsymbol{R}_s$ sequentially, where $n_{\boldsymbol{R}} \in \{1, \dots, 2^{n_t} - 1\}$.

Step 2 Traverse $n_{\boldsymbol{R}}$ in its value range, and for each $n_{\boldsymbol{R}}$, there is $\boldsymbol{R}_s(n_{\boldsymbol{R}})$. Judge whether $S(\boldsymbol{C}|\boldsymbol{R}_s)$ satisfies that for $\forall j_s$, there is $\sum_{i_s=1}^{n_s} s_{i_s,j_s} = 2$.

Step 3 For all satisfying $\boldsymbol{R}_s$, mark it as $\boldsymbol{R}^*_{s,B}$ and construct a set $\Omega^*_B = \{\boldsymbol{R}^*_{s,B}\}$.

Output: All circular closed areas and their union sets $\Omega^*_B = \{\boldsymbol{R}^*_{s,B}\}$.

With Algorithm 2, the union sets will be deleted and we can reach a set with only single circular closed areas.

Algorithm 2 Circular closed areas.

Input: $\Omega^*_B = \{\boldsymbol{R}^*_{s,B}\}$.

Step 1 Mark $\Omega_{B,k_B}|_{k_B=0} = \Omega^*_B$.

Step 2 In Ω_{B,k_B}, for the k_sth $\boldsymbol{R}^*_{s,B,k_s}$, if $\exists k^*_s \neq k_s$ and

$$\|\boldsymbol{R}^*_{s,B,k_s} \wedge \boldsymbol{R}^*_{s,B,k^*_s}\| > 0, \tag{9}$$

it means there exists other closed areas marked as $\boldsymbol{R}^*_{s,B,k^*_s}$ containing defense areas in $\boldsymbol{R}^*_{s,B,k_s}$, and $\boldsymbol{R}^*_{s,B,k^*_s}$ is a union set of multiple circular closed areas.

Delete $\boldsymbol{R}^*_{s,B,k^*_s}$ from Ω_{zB,k_B} as

$$\Omega_{B,k_B+1} = \Omega_{B,k_B} / \boldsymbol{R}^*_{s,B,k^*_s}. \tag{10}$$

Step 3 Repeat Step 2 until for Ω_{B,k_B}, $\forall k^*_s \neq k_s$, there is

$$\|\boldsymbol{R}^*_{s,B,k_s} \wedge \boldsymbol{R}^*_{s,B,k^*_s}\| = 0 \tag{11}$$

which means Ω_{B,k_B} contains only single circular closed areas. Mark Ω_{B,k_B} as Ω_B.

Output: $\Omega_B = \{\boldsymbol{R}_{s,B}\}$.

3.3 Coverage judge

After finding out all independent closed areas and circular closed areas, we can judge whether a target is covered by any closed areas.

(i) Coverage judge by independent closed areas.

Assume the center of the k_tth independent closed area is $\boldsymbol{X}_{k_t}$, the radius of the defense area is ρ_{k_t}, and the target $\boldsymbol{X}_\tau$ is in its internal part if

$$|\boldsymbol{X}_\tau - \boldsymbol{X}_{k_t}| \leqslant \rho_{k_t}. \tag{12}$$

(ii) Coverage judge by circular closed areas.

Since circular closed areas are formed with connected defense areas, and whether a target is in each single defense area is judged in (i), here we only need to judge whether the target is in the polygon area formed by connecting the centers of intersecting defense areas. There are well researched methods to judge whether a point is in the internal part of a polygon.

3.4 Pivotal node vector, closed node vector and coverage layer number

With coverage judging, for each target τ, we can present a set that contains all closed areas covering it, marked as $\Psi_\tau = \{\boldsymbol{R}_{s,\tau,q}\}$. That is, when a weapon is allocated to τ, it will definitely pass through some defense areas designated by Ψ_τ. In these $\boldsymbol{R}_{s,\tau,q}$, some defense areas are avoidable while others are not, we can introduce a pivotal node vector to describe the inevitable defense areas.

Definition 3 Pivotal node vector $\boldsymbol{R}_{se,\tau}$.

For $\Psi_\tau = \{\boldsymbol{R}_{s,\tau,q}\}$, there exists $\boldsymbol{R}_{se,\tau}$ that separates Ψ_τ into two disjoint sets namely $\Psi_{se,\tau}(\boldsymbol{R}_{se,\tau})$ and $\overline{\Psi}_{se,\tau}(\boldsymbol{R}_{se,\tau})$, and

$$\Psi_\tau = \Psi_{se,\tau}(\boldsymbol{R}_{se,\tau}) + \overline{\Psi}_{se,\tau}(\boldsymbol{R}_{se,\tau}). \tag{13}$$

For $\forall \boldsymbol{R}_{s,\tau,q_a} \in \Psi_{se,\tau}(\boldsymbol{R}_{se,\tau})$ and $\forall \boldsymbol{R}_{s,\tau,q_b} \in \overline{\Psi}_{se,\tau}(\boldsymbol{R}_{se,\tau})$, there is

$$\|\boldsymbol{R}_{s,\tau,q_a} \wedge \boldsymbol{R}_{s,\tau,q_b}\| = 0. \tag{14}$$

In $\Psi_{se,\tau}(\boldsymbol{R}_{se,\tau})$, for $\forall \boldsymbol{R}_{s,\tau,q_{e1}}$, $\boldsymbol{R}_{s,\tau,q_{e2}} \in \Psi_{se,\tau}(\boldsymbol{R}_{se,\tau})$, where q_a and q_b are indices, there is

$$\|\boldsymbol{R}_{s,\tau,q_{e1}} \wedge \boldsymbol{R}_{s,\tau,q_{e2}}\| > 0. \tag{15}$$

Thus, we have the pivotal node vector as

$$\boldsymbol{R}_{se,\tau} = \bigcap_{\boldsymbol{R}_{s,\tau,q_e} \in \Psi_{se,\tau}(\boldsymbol{R}_{se,\tau})} \boldsymbol{R}_{s,\tau,q_e} \tag{16}$$

where q_e is an index. Obviously, any weapon assigned to τ will definitely pass through at least one defense area in $\boldsymbol{R}_{se,\tau}$.

Correspondingly, there is a union set in $\Psi_{se,\tau}(\boldsymbol{R}_{se,\tau})$ that describes all defense areas connected with $\boldsymbol{R}_{se,\tau}$.

Definition 4 Closed node vector $\boldsymbol{R}_{sa,\tau}$.

The closed node vector $\boldsymbol{R}_{sa,\tau}$ is represented by

$$\boldsymbol{R}_{sa,\tau} = \bigcup_{\boldsymbol{R}_{s,\tau,q_e} \in \Psi_{se,\tau}(\boldsymbol{R}_{se,\tau})} \boldsymbol{R}_{s,\tau,q_e}. \tag{17}$$

Here we solve $\boldsymbol{R}_{se,\tau}$ and $\boldsymbol{R}_{sa,\tau}$ in $\Psi_\tau = \{\boldsymbol{R}_{s,\tau,q}\}$ with Algorithm 3.

Algorithm 3 Solve $\boldsymbol{R}_{se,\tau}$ and $\boldsymbol{R}_{sa,\tau}$.

Input: $\Psi_\tau = \{\boldsymbol{R}_{s,\tau,q}\}$.

Step 1 Note the number of elements in Ψ_τ as n_q, and assume $\boldsymbol{R}_{se,\tau,k_{rs}}|_{k_{rs}=1} = \boldsymbol{R}_{s,\tau,q}|_{q=1}$ and $\boldsymbol{R}_{sa,\tau,k_{rs}}|_{k_{rs}=1} = \boldsymbol{R}_{s,\tau,q}|_{q=1}$.

Step 2 From the second element in Ψ_τ, traverse q if

$$\|\boldsymbol{R}_{se,\tau,k_{rs}} \wedge \boldsymbol{R}_{s,\tau,q}\| > 0 \tag{18}$$

which means $\boldsymbol{R}_{se,\tau,k_{rs}}$ and $\boldsymbol{R}_{s,\tau,q}$ share common defense areas.

We have

$$\boldsymbol{R}_{se,\tau,k_{rs}+1} = \boldsymbol{R}_{se,\tau,k_{rs}} \wedge \boldsymbol{R}_{s,\tau,q}, \tag{19}$$

$$\boldsymbol{R}_{sa,\tau,k_{rs}+1} = \boldsymbol{R}_{sa,\tau,k_{rs}} \vee \boldsymbol{R}_{s,\tau,q}. \tag{20}$$

Step 3 After the traverse, we have the final $\boldsymbol{R}_{se,\tau,k_{rs}}$ and $\boldsymbol{R}_{sa,\tau,k_{rs}}$, marked as $\boldsymbol{R}_{se,\tau,l}$ and $\boldsymbol{R}_{sa,\tau,l}$. Delete all $\boldsymbol{R}_{s,\tau,q}$ in Ψ_τ that satisfies

$$\|\boldsymbol{R}_{se,\tau,l} \wedge \boldsymbol{R}_{s,\tau,q}\| > 0. \tag{21}$$

If

$$\Psi_\tau - \{\boldsymbol{R}_{s,\tau,q} | \|\boldsymbol{R}_{se,\tau,l} \wedge \boldsymbol{R}_{s,\tau,q}\| > 0\} \neq \varnothing,$$

the remaining set contains closed areas that do not intersect with $\boldsymbol{R}_{se,\tau,l}$ and $\boldsymbol{R}_{sa,\tau,l}$.

By inputting

$$\Psi_\tau - \{\boldsymbol{R}_{s,\tau,q} | \|\boldsymbol{R}_{se,\tau,l} \wedge \boldsymbol{R}_{s,\tau,q}\| > 0\}$$

into Step 1 and repeating all procedures, we have $\boldsymbol{R}_{se,\tau,l+1}$ and $\boldsymbol{R}_{sa,\tau,l+1}$.

Output: all $\boldsymbol{R}_{se,\tau,l}$ and $\boldsymbol{R}_{sa,\tau,l}$.

With Algorithm 3, all $\boldsymbol{R}_{se,\tau,l}$ and $\boldsymbol{R}_{sa,\tau,l}$ in Ψ_τ are found out, and we can form $\Psi_{se,\tau} = \{\boldsymbol{R}_{se,\tau,l}\}$ and $\Psi_{sa,\tau} = \{\boldsymbol{R}_{sa,\tau,l}\}$. For $l_1 \neq l_2$, there is $\boldsymbol{R}_{se,\tau,l_1} \cap \boldsymbol{R}_{se,\tau,l_2} = \varnothing$, so different pivotal node vectors do not intersect with each other. Thus, for any weapon assigned to the target τ, it is inevitable to pass through at least $n_{se,\tau}$ defense areas, where $n_{se,\tau}$ is the number of $\boldsymbol{R}_{se,\tau,l}$, and $n_{se,\tau}$ can be noted as the coverage layer number.

4. Multi-stage attack planning process

With the defense area analysis, the traditional WTA plan matrix $\boldsymbol{A} = [a_{\omega,\tau}]$ is insufficient to describe the holistic attack plan. We need to find an attack path for those weapons allocated to targets that are protected by other closed areas.

Besides, with the following Theorem 4, when the allocation matrix $\boldsymbol{A}$ is given, an optimal path can be determined analytically, and thus, it is still viable to use $\boldsymbol{A}$ as a parameter for optimization in the multi-stage attack WTA problem.

4.1 Optimal attack path

The optimal attack path is comprised of 1-elements in each pivotal node vector covering the target τ, which means the weapon allocated to τ needs to pass through those defense areas above sequentially.

Theorem 4 For a weapon ω that is allocated to the target τ, the optimal attack path is comprised of $n_{se,\tau}$ nodes, and each node corresponds to the pivotal node vector $\boldsymbol{R}_{se,\tau,l}$ covering the target τ whose defense area has the lowest intercept probability to the weapon ω.

Proof For each layer of the pivotal node vector $\boldsymbol{R}_{se,\tau,l}$ covering the target τ, the corresponding closed node vector is $\boldsymbol{R}_{sa,\tau,l}$, and we have

$$\boldsymbol{R}_{su,\tau,l} = \boldsymbol{R}_{sa,\tau,l} - \boldsymbol{R}_{se,\tau,l}. \tag{22}$$

$\boldsymbol{R}_{su,\tau,l}$ designates defense areas that are not pivotal but still cover the target τ. If a weapon passes through some defense areas in $\boldsymbol{R}_{su,\tau,l}$, it still needs to pass through at least one defense area in $\boldsymbol{R}_{se,\tau,l}$ to penetrate this layer of close areas.

Assume in the lth layer of closed areas, $p^{\min}_{c,\omega,\tau_l,se}$ is the lowest intercept probability in $\boldsymbol{R}_{se,\tau,l}$, and $p^{\min}_{c,\omega,\tau_l,su}$ is the lowest intercept probability in $\boldsymbol{R}_{su,\tau,l}$.

$$p^{\min}_{c,\omega,\tau_{l,\min},se} = \min\{p_{c,\omega,\tau_l,se} | \tau_l \in \boldsymbol{R}_{se,\tau,l}\} \tag{23}$$

$$p^{\min}_{c,\omega,\tau_{l,\min},su} = \min\{p_{c,\omega,\tau_l,su} | \tau_l \in \boldsymbol{R}_{su,\tau,l}\} \tag{24}$$

We have

$$p^{\min}_{c,\omega,\tau_{l,\min},se} < 1-(1-p^{\min}_{c,\omega,\tau_{l,\min},su})(1-p^{\min}_{c,\omega,\tau_{l,\min},se}) \tag{25}$$

and

$$p^{\min}_{c,\omega,\tau_{l,\min},se} < p_{c,\omega,\tau_l,se}, \tau_l \in \{\boldsymbol{R}_{se,\tau,l}/\tau_{l,\min}\}. \tag{26}$$

Thus, for all closed areas covering the target τ, we have the optimal (minimal) cumulative intercept probability for the weapon ω as

$$p_{o,c} = 1-\prod_{l=1}^{n_{se,\tau}}(1-p^{\min}_{c,\omega,\tau_{l,\min},se}) \tag{27}$$

where $\tau_{l,\min}$ designates the defense area with the lowest intercept probability in the lth layer. □

4.2 Multi-stage attack objective function calculation principle

Theoretically, an optimal multi-stage attack plan is companioned with a path parameter, compared with the $n_w \times n_t$ allocation matrix $\boldsymbol{A}$, and the dimension of the parameter with a path is $\max\{n_{se,\tau}\} \times n_w \times n_t$, which complicates the multi-stage attack WTA problem. However, with Theorem 4, when $\boldsymbol{A}$ is given, the attack path can also be determined analytically. Thus we can still use $\boldsymbol{A}$ for optimization. Moreover, in the optimization procedure, we need to calculate the objective function value in the order from the target with a small coverage layer number to those with a large coverage layer number. The reason for this is as follows.

For two targets τ_1 and τ_2, the weapon ω_1 is allocated to τ_1 and ω_2 to τ_2. If $n_{se,\tau_1} < n_{se,\tau_2}$, the coverage layer number of τ_1 is smaller than τ_2's.

(i) If τ_1 belongs to the pivotal node vector of τ_2, the attack to τ_1 should be conducted in advance, so the intercept probability from τ_1 to ω_2 will be

$$p^*_{c,\omega_2,\tau_1} = p_{c,\omega_2,\tau_1}(1-p_{k,\omega_1,\tau_1}). \tag{28}$$

(ii) On the contrary, if the attack to τ_2 is conducted ahead of τ_1,

$$p_{c,\omega_2,\tau_1} \geqslant p_{c,\omega_2,\tau_1}(1-p_{k,\omega_1,\tau_1}) \tag{29}$$

which will reduce the penetrate probability for the following weapons.

(iii) If τ_1 does not belong to the pivotal node vector of τ_2, the order to attack these two targets will not affect the penetrate probability.

4.3 Multi-stage attack objective function model and multi-stage planning

With the analysis above, note the multi-stage attack objective function as H_m, and the calculation of H_m is as follows.

Step 1 Set the initial value as

$$n_{se,k_m}|_{k_m=0} = 0 \tag{30}$$

$$H_{m,k_m}|_{k_m=0} = 0 \tag{31}$$

$$\boldsymbol{P}_{c,k_m}|_{k_m=0} = [p_{c,\omega,\tau,k_m}]|_{k_m=0} = \boldsymbol{P}_c \tag{32}$$

and $\boldsymbol{Q}$ is an $n_w \times \max\{n_{se,\tau}\}$ zero matrix to mark the optimal penetration path.

Step 2 For all targets τ satisfying $n_{se,\tau} = n_{se,k_m}$, calculate p_{m,τ,k_m} as

$$p_{m,\tau,k_m} = 1-\prod_{\omega=1}^{n_w}[1-a_{\omega,\tau}g_{m,\tau,k_m}p_{k,\omega,\tau}(1-p_{c,\omega,\tau,k_m})]$$

where

$$g_{m,\tau,k_m} = \begin{cases} 1, & n_{se,\tau} = n_{se,k_m} \\ 0, & n_{se,\tau} \neq n_{se,k_m} \end{cases}. \tag{33}$$

Find the optimal path. If $n_{se,k_m} > 0$, for a target τ allocated with the weapon ω, sequentially (l from 1 to $n_{se,\tau}$) select τ^*_m from $\boldsymbol{R}_{se,\tau,l} = [r_{se,\tau,\tau_m,l}]$, where τ_m is the τ_mth element of $\boldsymbol{R}_{se,\tau,l}$, and we have

$$p_{c,\omega,\tau^*_m} = \min\{p_{c,\omega,\tau_m}|r_{se,\tau,\tau_m,l} = 1\}. \tag{34}$$

Set $\boldsymbol{Q} = [q_{\omega,l}]$ as

$$q_{\omega,l} = \tau^*_m. \tag{35}$$

Update $\boldsymbol{P}_{c,k_m}$, for each column of $\boldsymbol{P}_{c,k_m}$, we have

$$p_{c,\omega,\tau,k_m+1} = p_{c,\omega,\tau,k_m}(1-p_{m,\tau,k_m}). \tag{36}$$

Update H_{m,k_m} as

$$H_{m,k_m+1} = H_{m,k_m} + \sum_{\tau=1}^{n_t} v_\tau p_{m,\tau,k_m}. \tag{37}$$

Repeat Step 2 until $n_{se,k_m} = \max\{n_{se,\tau}\}$, and output H_m and $\boldsymbol{Q}$.

4.4 Multi-stage attack planning flow

To conclude, we can present the complete flow for multi-stage attack planning as follows.

(i) According to the target position $\boldsymbol{X}_\tau$ and its defense area radii ρ_τ, solve all independent closed areas, and solve all circular closed areas with Algorithm 1 and Algorithm 2.

(ii) For each target τ, traverse all closed areas, judge whether the closed area covers this target, and form the closed area set covering the target τ as $\varPsi_\tau = \{\boldsymbol{R}_{s,\tau,q}\}$.

(iii) For each target τ, solve all pivotal node vectors $\varPsi_{se,\tau} = \{\boldsymbol{R}_{se,\tau,l}\}$, and cover the layer number $n_{se,\tau}$.

(iv) For a given WTA plan matrix $\boldsymbol{A}$, calculate the multi-stage attack objective function value H_m and penetrate the path $\boldsymbol{Q}$.

(v) Optimize the matrix $\boldsymbol{A}$ with a particular optimization algorithm to maximize H_m, and output the corresponding $\boldsymbol{A}^*$ and $\boldsymbol{Q}^*$.

5. Numerical results

5.1 Methods and Monte-Carlo objective function value calculation

To validate the MM method, we present the traditional WTA method and calculate the objective function of both methods with the Monte-Carlo method.

(i) MM method.

The multi-stage attack plan is solved according to Section 4.4.

The path of any weapon is expressed as T-X_1→T-X_2→ T-X_N, and the path length of the node number is equal to the attack stage or the attack order. For an attack stage larger than 1, the weapon will pass through T-X_1, T-X_2 and T-(X_{N-1}), where there are $N-1$ defense areas of targets in total. Therefore, the Monte-Carlo objective function value calculation should be in order from the small attack stage to the large attack stage.

In each attack stage, for the weapon ω, when it passes through the defense area of the k_tth target τ_{k_t}, it generates a random number $p_{c,r}$, where $p_{c,r} \in [0, 1]$, and if

$$p_{c,r} \leqslant p_{c,\omega,\tau_{k_t}}, \tag{38}$$

the weapon ω is intercepted by the target τ_{k_t}, and calculate another weapon. If not, continue the intercept judge of the next target.

If this weapon is not intercepted by the defense areas passing through, and reaches the allocated target τ, it generates a random number $p_{c,r}$, and if

$$p_{c,r} \leqslant p_{c,\omega,\tau}, \tag{39}$$

the weapon ω is intercepted by target τ, and calculate another weapon. If not, it generates a random number $p_{k,r}$. If

$$p_{k,r} \leqslant p_{k,\omega,\tau}, \tag{40}$$

the weapon ω destroys the target τ successfully. Add the value of the target τ to the objective function value, and set the intercept probability of the target τ to 0.

(ii) Traditional WTA method with single attack stage (TS method).

Solve the WTA plan with the traditional WTA method. In the Monte-Carlo procedure, if the allocated target is covered by other defense areas, select a defense area randomly from the closed node vector for this weapon to pass through, until this weapon reaches the target allocated.

In the TS method, $\Psi_{se,\tau} = \{\boldsymbol{R}_{se,\tau,l}\}$ and $\Psi_{sa,\tau} = \{\boldsymbol{R}_{sa,\tau,l}\}$ are both used in the Monte-Carlo objective function value calculation for that the TS method cannot provide an attack path.

For each weapon ω, acquire $\Psi_{se,\tau} = \{\boldsymbol{R}_{se,\tau,l}\}$ and $\Psi_{sa,\tau} = \{\boldsymbol{R}_{sa,\tau,l}\}$ of the target τ allocated. Sequentially select a defense area from $\boldsymbol{R}_{sa,\tau,l}$, and conduct the intercept judge. If the defense area selected is also in $\boldsymbol{R}_{se,\tau,l}$ and the weapon is not intercepted, this weapon will pass through this layer of coverage, until this weapon reaches the target τ.

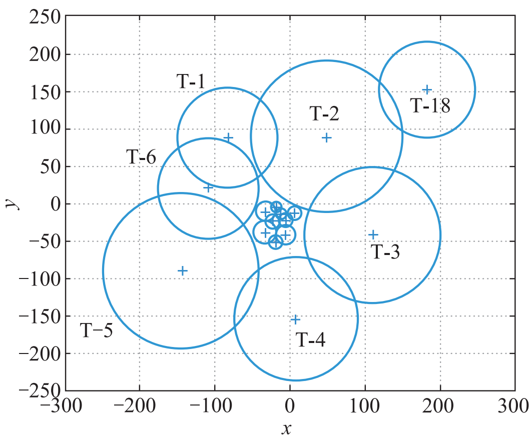
When the weapon ω reaches the target τ, conduct the intercept judge and the kill judge. If the kill judge passes, add the value of the target τ to the objective function value and set the intercept probability of the target τ to 0.

5.2 Methods comparison

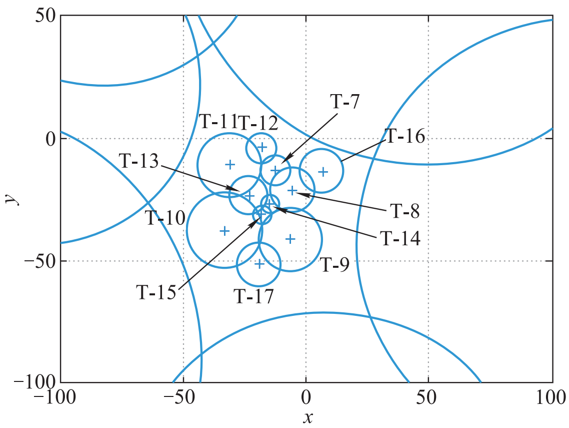
In the following combat scenario, 24 weapons launch an attack to 18 targets. The value vector $\boldsymbol{V}$ in this scenario is shown in Table 5 and the target distribution is in Fig. 3.

Table 5 Value vector in the combat scenario

Target	T-1	T-2	T-3	T-4	T-5	T-6
Value	20.46	14.83	21.97	14.99	18.95	8.29
Target	T-7	T-8	T-9	T-10	T-11	T-12
Value	39.46	41.18	41.37	43.41	40.44	35.41
Target	T-13	T-14	T-15	T-16	T-17	T-18
Value	64.13	53.05	64.76	35.44	33.17	14.69



(a) External target distribution



(b) Internal target distribution

Fig. 3 Target distribution

With the method given above, the WTA plan is solved as shown in Table 6 and the multi-stage attack plan is given in Table 7.

In Table 6, the expression of the WTA plan is the same as that of the traditional method, while in Table 7, the path of each weapon is given.

Table 6 WTA plan

Method	W-1	W-2	W-3	W-4	W-5	W-6	W-7	W-8	W-9	W-10	W-11	W-12
MM	T-9	T-13	T-8	T-9	T-11	T-14	T-10	T-1	T-3	T-17	T-17	T-7
TS	T-1	T-4	T-18	T-9	T-11	T-10	T-10	T-13	T-14	T-12	T-17	T-15

Method	W-13	W-14	W-15	W-16	W-17	W-18	W-19	W-20	W-21	W-22	W-23	W-24
MM	T-16	T-5	T-15	T-2	T-7	T-14	T-15	T-16	T-13	T-3	T-8	T-12
TS	T-3	T-5	T-15	T-16	T-7	T-13	T-16	T-14	T-8	T-17	T-8	T-12

Table 7 Multi-stage attack plan

Attack stage	W-1	W-2	W-3	W-4	W-5	W-6
1	/	/	/	/	/	/
2	T-3→T-9	/	T-3→T-8	T-3→T-9	T-3→T-11	/
3	/	T-2→T-8→T-13	/	/	/	T-3→T-9→T-14

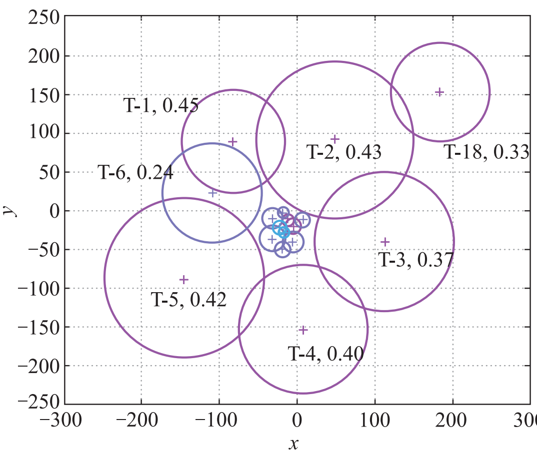
Attack stage	W-7	W-8	W-9	W-10	W-11	W-12
1	/	T-1	T-3	/	/	/
2	T-3→T-10	/	/	T-3→T-17	T-3→T-17	T-1→T-7
3	/	/	/	/	/	/

Attack stage	W-13	W-14	W-15	W-16	W-17	W-18
1	/	T-5	/	T-2	/	/
2	T-3→T-16	/	/	/	T-2→T-7	/
3	/	/	T-3→T-9→T-15	/	/	T-3→T-8→T-14

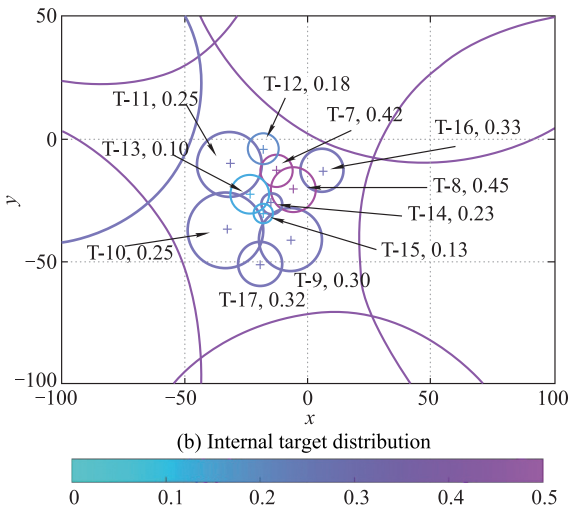
Attack stage	W-19	W-20	W-21	W-22	W-23	W-24
1	/	/	/	T-3	/	/
2	/	T-3→T-16	/	/	T-5→T-8	T-3→T-12
3	T-3→T-9→T-15	/	T-3→T-7→T-13	/	/	/

After each stage of attack, the intercept probabilities to weapons will be changed. Take W-6 as an example (W-6 is in the third attack stage), the changed intercept probabilities and the target distribution are in Figs. 4–6. The number behind the target label is the intercept probability to W-6, and the changed intercept probabilities are marked with the red font.

The color of the circles around the target indicates the intercept probability. A higher intercept probability tends to be red, while a lower one blue.



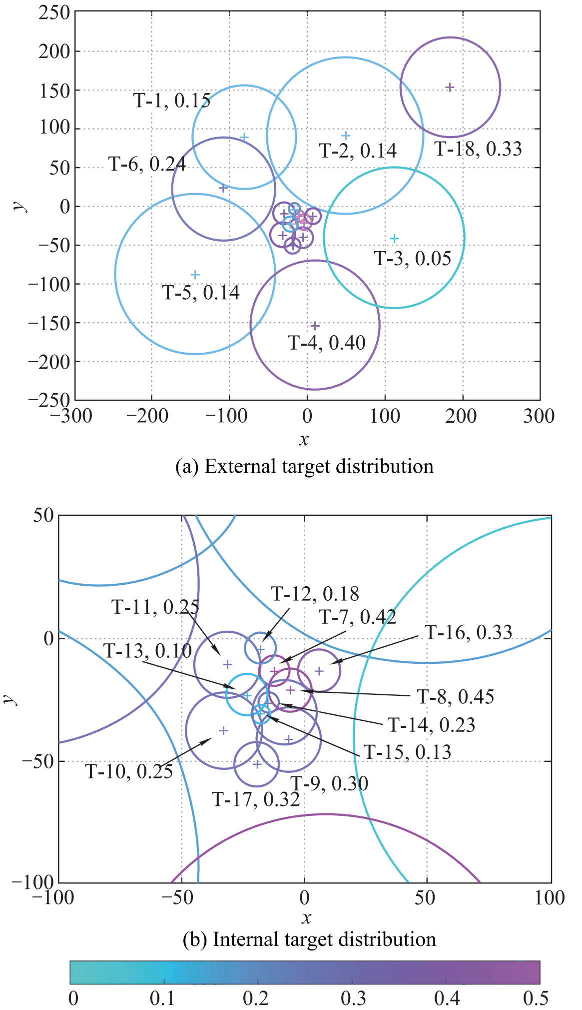
(a) External target distribution



(b) Internal target distribution

Fig. 4 Intercept probabilities to W-6 (non attack)

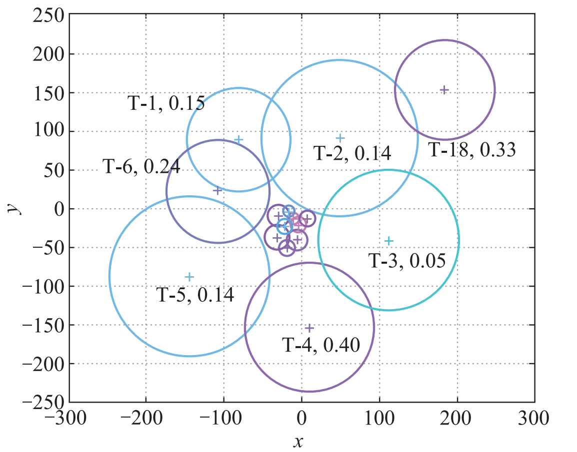
In Fig. 4 to Fig. 6, the intercept probabilities of the outer targets are weakened after each stage of attack. The circles of defense areas of the outer targets are changed into blue. The objective function value calculated with the Monte-Carlo method is shown in Fig. 7, where the objective function value is calculated with the Monte-Carlo experiment. In each experiment, a certain objective function value is obtained as an example. With a large amount of experiments, all examples are counted into a value range, and the probability of the value falling in a value range is computed. It is obvious that the statistical results of the MM method fall into higher value ranges. That is, the MM method is likely to reach high combat performance.



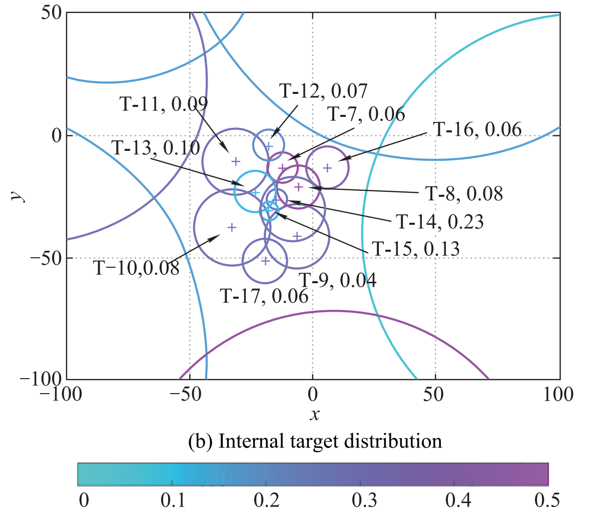
(a) External target distribution

(b) Internal target distribution

Fig. 5 Intercept probabilities to W-6 (after the first attack stage)



(a) External target distribution



(b) Internal target distribution

Fig. 6 Intercept probabilities to W-6 (after the second attack stage)

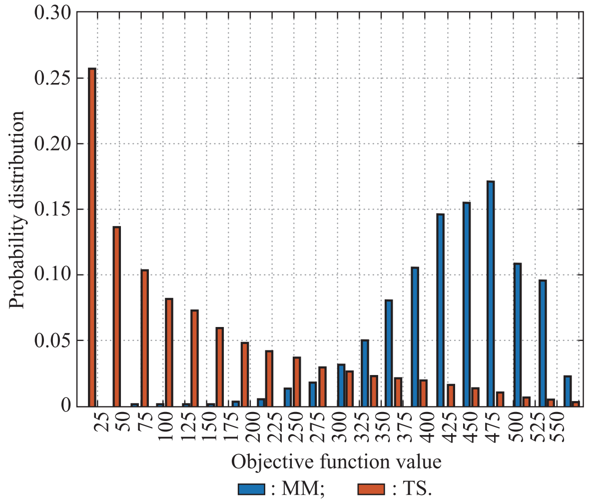


Fig. 7 Ojbective function value distribution with Monte-Carlo method

5.3 Result analysis

To further compare the multi-stage attack method and the traditional WTA method, the objective function value is calculated with the Monte-Carlo method. The scenarios include the complex scenario (in Section 5.2) and the simple scenario (in Section 2).

The mean value and confidence interval of the Monte-Carlo objective function value and the corresponding analytical value are shown in Table 8 and Table 9. The confidence coefficient is 0.95.

Table 8 Objective function value comparison (simple scenario)

Method	Mean value of Monte-Carlo	Confidence interval of Monte-Carlo	Analytical value
MM	72.17	[72.09,72.27]	72.11
TS	47.26	[47.18,47.35]	/

From the results we have the following conclusions.

Table 9 Objective function value comparison (complex scenario)

Method	Mean value of Monte-Carlo	Confidence interval of Monte-Carlo	Analytical value
MM	439.55	[438.13, 440.96]	439.85
TS	129.83	[128.68, 130.98]	/

(i) The analytical value is consistent with the mean value by the Monte-Carlo method, and falls into the confidence interval of 0.95 confidence coefficient.

(ii) The MM method is better than the TS method. The more complex the combat scenario is, the larger the gap between MM and TS is. According to the objective value distribution of the Monte-Carlo method, the value of the MM method distributes mainly in the high value range, while the value of the TS method distributes mainly in the low value range. It suggests that in complicated combat scenarios, it is imperative to analyze the defense areas of targets and use the multi-stage attack strategy to weaken the interactive defense between targets progressively.

(iii) If there is no interrelationship between the targets' defense areas, then the MM method is equivalent to the TS method, for in this case, the MM method only calculates the first stage of attack that involves all weapons, and the solving procedure is equal to the TS method. That means the MM method includes the TS method.

(iv) With Figs. 4 – 6, after each stage of attack of the MM method, the intercept probabilities will be updated, and therefore the MM method can also be applied to online or real-time combat scenario estimation for weapons with communication capabilities. After each stage of attack, a re-allocation can also be conducted for better further combat effectiveness. Besides, the MM method may also be used as a framework of scenario analysis for commanders to find out the weak area in further attack.

6. Conclusions

First, if the combat scenario involves interactive coverage of the targets' defense areas, it is imperative to analyze the defense areas and use the multi-stage attack method to weaken the target defense progressively for better combat effectiveness. Second, if there is no interrelationship between the targets' defense areas, the multi-stage WTA method is equivalent to the traditional WTA method, and the traditional method can be regarded as a special case in the multi-stage WTA problem. Finally, the multi-stage WTA framework including defense area analysis and defense scenario update can be used as a reference for decision-making.

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