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Grey information coverage interaction relational decision making and its application

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Abstract: This study aims to reflect the information coverage grey number and the interaction between attributes in grey relational decision making. Therefore, a multi-attribute decision method based on the grey information coverage interaction relational degree (GIRD) is proposed. Firstly, this paper defines the information coverage grey number, and establishes the GIRD model by using the Choquet fuzzy integral and grey relational principle. It proves that the proposed model not only is the general and unified form of the point relational degree, interval relational degree, mixed relational degree and grey fuzzy integral relational degree, but also can effectively deal with the interaction between attributes. Further, a decision making example of evaluating the industrial operation quality for 14 cities in Hunan province of China is provided to highlight the implementation, availability, and feasibility of the proposed decision model.

Keywords: grey information coverage, interaction effect, fuzzy integral, grey relational degree, industrial quality evaluation.

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1. Introduction

The grey relational decision making model is a multi-attribute decision making method based on the grey relational theory. As an important part of grey decision-making, it plays a vital role in subsequent grey modeling, grey forecasting or grey controlling. With the deepening of the grey relational theory in recent years, there are great advances in grey relational decision making study. The decision-making models and methods with grey information bring about a lot of research results and are widely used in social, economic and engineering fields [1 – 3].

The index values of grey relational decision making appear in the decision matrix as information coverage grey numbers. The information coverage grey number refers to the uncertain information in a certain interval, sets of numbers, languages or whitening functions, including real numbers, interval grey numbers, fuzzy numbers, and morphological grey numbers. At present, many findings are available on the research and application of the information coverage grey number. Deng [4] firstly provided the definition and algebraic operation for the information coverage grey number. Ding et al. [5] defined a new similarity measure, which normalized the similarity calculations of real values, interval numbers, linguistic variables, and uncertain linguistic variables to a unified metric space, thereby presenting a unified quantization processing for different types of attribute values. Yan and Liang [6] proposed a group consistency analysis method to solve the hybrid multi-attribute group decision problem, with the attribute preference information value as the real number, linguistic value, and interval number. Yu et al. [7] proposed a conversion relationship between real numbers, interval numbers, linguistic variables, and intuitionistic fuzzy numbers and used the intuitionistic fuzzy weighted average to aggregate the preference information. Wang [8] proposed a systematic decision-making approach based on the fuzzy Delphi, fuzzy extent analysis, and fuzzy technique for order preference by similarity to an ideal solution (TOPSIS) to evaluate green operations initiatives. Escolar et al. [9] proposed a methodology based on multi-attribute decision-making to rank smart cities through a transversal smartness dimension. The conversion of data types is rendered unnecessary, thus avoiding information loss or distortion of information. Simultaneously, the grey relational analysis method is an example of this type of techniques [10]. Then, the grey information measurement in grey information treated space was researched in [11].

In practical situations, such attributes are usually correlative and dependent [12]. To effectively describe the interactions between attributes, Sugeno [13] proposed the concept of fuzzy measures, and used positive and negative

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values of fuzzy measures to describe positive or negative interactions. Grabisch [14] proposed the fuzzy integration as a nonlinear integration operator to integrate fuzzy measures and decision information, and finally achieved good results. This fuzzy integration operator can deal with the interrelation of attributes and is widely applied in the fuzzy multi-attribute group decision-making problems [15 – 17]. Ding et al. [18] utilized the multivariable grey model with interaction effect and its extended model to reflect the influence of the interaction of different input variables on system behavior variables, but the model was only suitable for two-factor interaction research. Lu et al. [19] used the hierarchical regression analysis to measure the interaction effect of intra- and inter-organizational control. Zhu and Li [20] built a multiple-factor analysis of variance (ANOVA) model with two-way interactions by executing the general linear model to reveal significantly the influence of people's conceptions on the situation of the Chinese. However, a large number of scholars only study the interaction effect for exact numbers and interval numbers.

Up to now, there are few works focusing on the multiple attribute decision making with information coverage grey numbers and interactions between attributes. The main difficulty is how to design a complex algebraic system with information coverage grey numbers, and how to reflect the interaction information in the decision making process. Therefore, this paper proposes a multi-attribute decision making method based on the grey information coverage interaction relational degree (GIRD) by using the difference information principle, Choque fuzzy integral and grey relational theory. The GIRD model can not only deal with different types of information coverage grey numbers, but also avoid its complex algebraic operations. At the same time, it can reflect the interaction between attributes, thus providing a new method of grey relational decision making.

2. Grey information coverage interaction relational degree model

In order to reflect the information coverage grey number and the interaction between attributes in the multiple attribute decision making, this section establishes the GIRD model.

2.1 Information coverage grey number and its difference information

Definition 1 [4] Let $\psi(\theta)$ be the proposition information field of θ, ψ be the number field, $\otimes$ be the uncertainty number under ψ, $\widetilde{\otimes}$ be the white number under $\otimes$, d^0 be the only true value of $\otimes$, and $\widetilde{d}$ be the implicit number of d^0. If $\forall\ \widetilde{\otimes} \in \otimes \Rightarrow \widetilde{\otimes} \in \psi$, $\psi = \{\widetilde{\otimes} \big| \exists \widetilde{\otimes} = d^0, \widetilde{\otimes}$ Apr $\widetilde{d}$ For $\psi(\theta)$, $d^0 \mathrm{Emg} \ \Rightarrow \ \otimes \mathrm{Vns}\}$, then $\otimes$ is called as the information coverage grey number of ρ, it is expressed by $\forall\ \widetilde{\otimes} \in \otimes \Rightarrow \widetilde{\otimes} \in D$, and the simple expression is $\otimes \in D$, where the set D of $\widetilde{\otimes}$ is the information coverage of $\otimes$.

Definition 2 Suppose that D is the information coverage of $\otimes$.

(i) If D is a discrete number set (denoted as D_1), then $\otimes$ is called as the discrete coverage grey number.

(ii) If D is a continuous number set (denoted as D_2), then $\otimes$ is called as the continuous coverage grey number.

(iii) If D is a fuzzy language set (denoted as D_3), then $\otimes$ is called as the language coverage grey number.

(iv) If D is a whitening function set (denoted as D_4), then $\otimes$ is called as the morphological grey number.

In practical applications, the fuzzy language can be written as triangular fuzzy number [21], the grey number generated by the whitening function can be represented by the interval grey number, and the interval grey number belongs to the continuous coverage grey number [22]. Therefore, the information coverage grey number can be unified in three types as follows:

$$\forall\ \widetilde{\otimes} \in \otimes \Rightarrow \widetilde{\otimes} \in \{x(k) \big| k \in U, U = \{1, 2, \ldots, n\}\}$$

$$\forall\ \widetilde{\otimes} \in \otimes \Rightarrow \widetilde{\otimes} \in [x^L(k), x^U(k)]$$

$$\forall\ \widetilde{\otimes} \in \otimes \Rightarrow \widetilde{\otimes} \in \{(x^L(k), x^M(k), x^U(k)) \big| k \in U, U = \{1, 2, \ldots, n\}\}.$$

Any information is different, the measure of information difference between information coverage grey numbers can be defined according to the difference information sequence in the following [23].

Definition 3 Let the map be $DI: D \subset \psi \to [0, +\infty)$, if $\forall\ \widetilde{\otimes}_i \in \otimes_i \Rightarrow \widetilde{\otimes}_i \in D_i$, $\forall\ \widetilde{\otimes}_j \in \otimes_j \Rightarrow \widetilde{\otimes}_j \in D_j$, then this map will satisfy the following situations.

(i) $DI(\widetilde{\otimes}_i, \widetilde{\otimes}_i) = 0$;

(ii) $DI(\widetilde{\otimes}_i, \widetilde{\otimes}_j)$ increases with the enlarging of the difference between $\widetilde{\otimes}_i$ and $\widetilde{\otimes}_j$, and it decreases with the diminishing of the difference between $\widetilde{\otimes}_i$ and $\widetilde{\otimes}_j$;

(iii) If $\widetilde{\otimes}_i \succ \widetilde{\otimes}_j$, then $DI(\widetilde{\otimes}_i) < DI(\widetilde{\otimes}_j)$, where $\succ$ means the strength rank of the information coverage grey number $\widetilde{\otimes}$, $\mathrm{potency}(\widetilde{\otimes}_i) = \mathrm{potency}(\widetilde{\otimes}_j)$.

Then $DI(\widetilde{\otimes}_i, \widetilde{\otimes}_j)$ is called as the measure of information difference between $\widetilde{\otimes}_i$ and $\widetilde{\otimes}_j$.

Theorem 1 Let $U = \{1, 2, \ldots, n\}$.

(i) If $D_i = \{x_i(k) \big| k \in U\}$, $D_j = \{x_j(k) | k \in U\}$ and

$$DI(\widetilde{\otimes}_i, \widetilde{\otimes}_j) = \big| x_i(k) - x_j(k) \big|. \tag{1}$$

(ii) If $D_i = [x_i^L(k), x_i^U(k)]$, $D_j = [x_j^L(k), x_j^U(k)]$, and

$$DI(\widetilde{\otimes}_i, \widetilde{\otimes}_j) = \frac{\sqrt{2}}{2} \sqrt{(x_i^L(k) - x_j^L(k))^2 + (x_i^U(k) - x_j^U(k))^2}. \tag{2}$$

(iii) If $D_i = \{(x_i^L(k), x_i^M(k), x_i^U(k)) \big| k \in U\}$, $D_j = \{(x_j^L(k), x_j^M(k), x_j^U(k)) \big| k \in U\}$, and

$$DI(\widetilde{\otimes}_i, \widetilde{\otimes}_j) = \max\{\big|x_i^L(k) - x_j^L(k)\big|, \big|x_i^M(k) - x_j^M(k)\big|, \big|x_i^U(j) - x_j^U(k)\big|\}. \quad (3)$$

Then $DI(\widetilde{\otimes}_i, \widetilde{\otimes}_j)$ is the measure of information difference between $\widetilde{\otimes}_i$ and $\widetilde{\otimes}_j$.

Proof According to the known conditions, $DI(\widetilde{\otimes}_i, \widetilde{\otimes}_j)$ is actually the distance between $\widetilde{\otimes}_i$ and $\widetilde{\otimes}_j$, so $DI(\widetilde{\otimes}_i, \widetilde{\otimes}_j)$ satisfies Definition 3, and is the measure of information difference between $\widetilde{\otimes}_i$ and $\widetilde{\otimes}_j$. □

2.2 GIRD

In order to effectively describe the interaction between attributes, Sugeno [13] proposed the concepts of λ fuzzy measure and ϕ_s transfer function.

Definition 4 Let $U = \{1, 2, \ldots, n\}$ be a finite set of attributes, $P(U)$ be a power set of U, $(U, P(U))$ be a measurable space, and $g : P(U) \to [0, 1]$ be a set of functions. If this measure function g has the following properties:

(i) $g(\varnothing) = 0,\ g(U) = 1$;

(ii) $\forall\, A, B \in P(U)$, if $A \subseteq B$, then $g(A) \leqslant g(B)$;

(iii) $\forall\, A, B \in P(U)$, $A \cap B = \varnothing$, and $\lambda > -1$, there is

$$g(A \cup B) = g(A) + g(B) + \lambda g(A)g(B), \quad (4)$$

then g is called as λ fuzzy measure, where λ indicates the degree of the interaction between attributes. When $\lambda = 0$, all attributes are independent of each other; when $-1 < \lambda < 0$, there are negative cooperation among all attributes; when $\lambda > 0$, there are active cooperation among all attributes.

Definition 5 Assuming $\phi_s : [0, 1] \times [0, 1] \to [0, 1]$, satisfies

$$\phi_s(\xi, w) = \begin{cases} 0, & \xi = 1, w = 0 \\ 1, & \xi = 0, w = 1 \\ 0, & \xi = 0, w < 1 \\ \dfrac{((1-\xi)^2/\xi^2)^w - 1}{((1-\xi)^2/\xi^2) - 1}, & \text{otherwise} \end{cases}, \quad (5)$$

then, ϕ_s is called as a transfer function.

Based on the attribute weights w_k $(k = 1, 2, \ldots, n)$ and the conversion functions ϕ_s, the λ fuzzy measure is determined as follows:

$$g_\xi(T) = \phi_s(\xi, \sum_{k \in T} w_k), \quad \forall\, T \in P(U) \quad (6)$$

where $\xi = 1/(\sqrt{1+\lambda} + 1)$ is the interaction degree of attribute T. In order to effectively deal with the interaction between attributes, the grey relational principle of the Choquet fuzzy integral operator is used to establish the GIRD model. Fig. 1 is the function relationship between ξ, w and $\phi_s(\xi, w)$. When $\xi = 1$ and $w = 0$, $\phi_s(\xi, w)$ is 0. When $\xi = 0$ and $w = 1$, $\phi_s(\xi, w)$ is 1. When $\xi = 0$ and $w < 1$, $\phi_s(\xi, w)$ is 0. Otherwise, $\phi_s(\xi, w)$ is $\dfrac{((1-\xi)^2/\xi^2)^w - 1}{((1-\xi)^2/\xi^2) - 1}$.

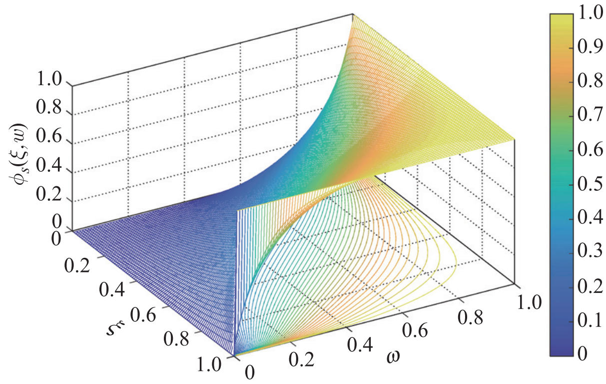

Fig. 1 Function relationship between ξ, w and $\phi_s(\xi, w)$

Definition 6 Assuming $X = \{x_0, x_1, x_2, \ldots, x_m\}$ is the information coverage grey relational factor set, where x_0 is the reference sequence, $x_1, x_2, \ldots, x_m$ are comparison sequences, $x_i = (\widetilde{\otimes}_i(1), \widetilde{\otimes}_i(2), \ldots, \widetilde{\otimes}_i(n))$, $i = 0, 1, 2, \ldots, m$, $\rho \in (0, 1)$ is the resolution coefficient, let $u_j = j$. The information coverage grey relational coefficient and the information coverage grey relational coefficient matrix are shown respectively as follows:

$$\gamma_{0i}(u_k) \triangleq \gamma(\widetilde{\otimes}_0(k), \widetilde{\otimes}_i(k)) = \frac{DI_{\min} + \rho\, DI_{\max}}{DI_{0i}(\widetilde{\otimes}_0(k), \widetilde{\otimes}_i(k)) + \rho\, DI_{\max}} \quad (7)$$

$$\boldsymbol{R} = \begin{bmatrix} \gamma_{01}(u_1) & \gamma_{01}(u_2) & \cdots & \gamma_{01}(u_n) \\ \gamma_{02}(u_1) & \gamma_{02}(u_2) & \cdots & \gamma_{02}(u_n) \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{0m}(u_1) & \gamma_{0m}(u_2) & \cdots & \gamma_{0m}(u_n) \end{bmatrix} \quad (8)$$

where $DI_{0i}(\widetilde{\otimes}_0(k), \widetilde{\otimes}_i(k))$ is the information difference between $\widetilde{\otimes}_0(k)$ and $\widetilde{\otimes}_i(k)$, $DI_{\min}$ and $DI_{\max}$ are the environmental parameters shown as follows:

$$DI_{\min} = \min_i \min_k DI_{0i}(\widetilde{\otimes}_0(k), \widetilde{\otimes}_i(k)),$$

$$DI_{\max} = \max_i \max_k DI_{0i}(\widetilde{\otimes}_0(k), \widetilde{\otimes}_i(k)). \quad (9)$$

If $\gamma_{0i}(u_k)$ is ranked from small to large, it gets the subscript (j):

$$r_{0i}(u_{(1)}) \leqslant r_{0i}(u_{(2)}) \leqslant \cdots \leqslant r_{0i}(u_{(n)}),$$

$$U_{(k)} = \{u_{(k)}, u_{(k+1)}, \ldots, u_{(n)}\}, \gamma_{0i}(u_{(0)}) = 0. \quad (10)$$

Then the GIRD of the reference sequence x_0 to the comparison sequence x_i with respect to λ fuzzy measure g is defined as follows:

$$\int \gamma(x_0, x_i)\mathrm{d}g = \sum_{k=1}^{n}[\gamma_{0i}(u_{(k)}) - \gamma_{0i}(u_{(k-1)})]g(U_{(k)}). \tag{11}$$

According to Definition 6, the GIRD is actually integrated with the Choquet fuzzy integral operator of the information coverage grey relational coefficient.

Theorem 2 The above GIRD $\int \gamma(x_0, x_i)\mathrm{d}g$ satisfies the following four conditions (four axioms of grey relation analysis [4,24]).

(i) Normalization

$$0 \leqslant \int \gamma(x_0, x_i)\mathrm{d}g \leqslant 1$$

$$\int \gamma(x_0, x_i)\mathrm{d}g = 0 \Leftrightarrow x_0, x_i \in \varnothing$$

$$\int \gamma(x_0, x_i)\mathrm{d}g = 1 \Leftrightarrow x_0 = x_i$$

(ii) Symmetry

$$\int \gamma(x_0, x_i)\mathrm{d}g = \int \gamma(x_i, x_0)\mathrm{d}g \Leftrightarrow X = \{x_0, x_i\}$$

(iii) Wholeness

$$x_k, x_i \in X = \{x_j | j = 0, 1, 2, \dots, n, n \geqslant 2\},$$

$$\int \gamma(x_k, x_i)\mathrm{d}g \overset{\text{often}}{\neq} \int \gamma(x_i, x_k)\mathrm{d}g.$$

(iv) Approachability

The smaller the difference information $DI_{0i}(\widetilde{\otimes}_0(k), \widetilde{\otimes}_i(k))$ is, the greater the value of $\int \gamma(x_0, x_i)\mathrm{d}g$ is.

Proof Wholeness and approachability are proved apparently.

(i) Normalization

According to $\gamma_{0i}(u_{(1)}) \leqslant \gamma_{0i}(u_{(2)}) \leqslant \dots \leqslant \gamma_{0i}(u_{(n)})$, we can get $\gamma_{0i}(u_{(k)}) - \gamma_{0i}(u_{(k-1)}) \geqslant 0$. For fuzzy measure g, $0 \leqslant g(U_{(k)}) \leqslant 1$. Therefore,

$$\int \gamma(x_0, x_i)\mathrm{d}g = \sum_{k=1}^{n}[\gamma_{0i}(u_{(k)}) - \gamma_{0i}(u_{(k-1)})]g(U_{(k)}) \geqslant 0.$$

According to Definition 4, for any $k \in U$, $g(U_{(k)}) \leqslant g(U_{(1)}) = g(U) = 1$, therefore,

$$\int \gamma(x_0, x_i)\mathrm{d}g = \sum_{k=1}^{n}[\gamma_{0i}(u_{(k)}) - \gamma_{0i}(u_{(k-1)})]g(U_{(k)}) \leqslant$$

$$\sum_{k=1}^{n}[\gamma_{0i}(u_{(k)}) - \gamma_{0i}(u_{(k-1)})]g(U_{(1)}) =$$

$$\sum_{k=1}^{n}[\gamma_{0i}(u_{(k)}) - \gamma_{0i}(u_{(k-1)})] =$$

$$\gamma_{0i}(u_{(1)}) - \gamma_{0i}(u_{(0)}) + \gamma_{0i}(u_{(2)}) - \gamma_{0i}(u_{(1)}) + \dots +$$

$$\gamma_{0i}(u_{(n)}) - \gamma_{0i}(u_{(n-1)}) =$$

$$\gamma_{0i}(u_{(n)}) - \gamma_{0i}(u_{(0)}) = \gamma_{0i}(u_{(n)}) \leqslant 1.$$

When $x_0 = x_i$, for any $k \in U$, $\widetilde{\otimes}_0(k) = \widetilde{\otimes}_i(k)$, $DI_{0i}(\widetilde{\otimes}_0(k), \widetilde{\otimes}_i(k)) = 0$, $DI_{\min} = 0$,

$$\gamma_{0i}(u_k) = \gamma(\widetilde{\otimes}_0(k), \widetilde{\otimes}_i(k)) =$$

$$\frac{DI_{\min} + \rho\, DI_{\max}}{DI_{0i}(\widetilde{\otimes}_0(k), \widetilde{\otimes}_i(k)) + \rho\, DI_{\max}} = 1$$

and

$$\int \gamma(x_0, x_i)\mathrm{d}g = \sum_{k=1}^{n}[\gamma_{0i}(u_{(k)}) - \gamma_{0i}(u_{(k-1)})]g(U_{(k)}) =$$

$$[\gamma_{0i}(u_{(1)}) - \gamma_{0i}(u_{(0)})]g(U_{(1)}) +$$

$$\sum_{k=2}^{n}[\gamma_{0i}(u_{(k)}) - \gamma_{0i}(u_{(k-1)})]g(U_{(k)}) = 1.$$

Therefore, $\int \gamma(x_0, x_i)\mathrm{d}g$ satisfies the normalization.

(ii) Symmetry

Because $X = \{x_0, x_i\}$, according to Theorem 1, we have $DI_{0i}(\widetilde{\otimes}_0(k), \widetilde{\otimes}_i(k)) = DI_{i0}(\widetilde{\otimes}_i(k), \widetilde{\otimes}_0(k))$, $\gamma_{0i}(u_k) = \gamma_{i0}(u_k)$. Thus,

$$\int \gamma(x_0, x_i)\mathrm{d}g = \sum_{k=1}^{n}[\gamma_{0i}(u_{(k)}) - \gamma_{0i}(u_{(k-1)})]g(U_{(k)}) =$$

$$\sum_{k=1}^{n}[\gamma_{i0}(u_{(k)}) - \gamma_{i0}(u_{(k-1)})]g(U_{(k)}) =$$

$$\int \gamma(x_i, x_0)\mathrm{d}g. \qquad \square$$

Theorem 3 In the GIRD model,

(i) If $\lambda \to +\infty$, then $\int \gamma(x_0, x_i)\mathrm{d}g \to 0$;

(ii) If $\lambda \to -1$, then $\int \gamma(x_0, x_i)\mathrm{d}g \to \gamma_{0i}(u_{(n)})$;

(iii) If $\lambda = 0$, then $\int \gamma(x_0, x_i)\mathrm{d}g = \sum_{k=1}^{n} w_k \gamma_{0i}(u_{(k)})$.

Proof

(i) If $\lambda \to +\infty$, then $\xi = 1/(\sqrt{1+\lambda} + 1) \to 0$, $s = (1-\xi)^2/\xi^2 \to +\infty$, and

$$\lim_{\lambda\to+\infty}\phi_s(\xi,w)=\lim_{\xi\to 0}\frac{((1-\xi)^2/\xi^2)^w-1}{((1-\xi)^2/\xi^2)-1}=$$

$$\lim_{s\to+\infty}\frac{s^w-1}{s-1}=\lim_{s\to+\infty}\frac{w\ s^{w-1}}{1}=0,$$

$$g(U_{(k)})=\phi_s\left(\xi,\sum_{k\in U}w(u_{(k)})\right)\to 0.$$

According to Definition 6, we can get that if $\lambda\to+\infty$, then

$$\int\gamma(x_0,x_i)\mathrm{d}g=\sum_{k=1}^{n}[\gamma_{0i}(u_{(k)})-\gamma_{0i}(u_{(k-1)})]g(U_{(k)})\to 0.$$

(ii) If $\lambda\to-1$, then $\xi\to 1$, $s\to 0$, and

$$\lim_{\lambda\to-1}\phi_s(\xi,w)=\lim_{\xi\to 1}\frac{((1-\xi)^2/\xi^2)^w-1}{((1-\xi)^2/\xi^2)-1}=$$

$$\lim_{s\to 0}\frac{s^w-1}{s-1}=1,$$

$$g(U_{(k)})=\phi_s(\xi,\sum_{k\in U}w(u_{(k)}))\to 1.$$

Therefore,

$$\int\gamma(x_0,x_i)\mathrm{d}g=\sum_{k=1}^{n}[\gamma_{0i}(u_{(k)})-\gamma_{0i}(u_{(k-1)})]g(U_{(k)})=$$

$$\sum_{k=1}^{n}[\gamma_{0i}(u_{(k)})-\gamma_{0i}(u_{(k-1)})]=\gamma_{0i}(u_{(n)})-\gamma_{0i}(u_{(0)})=$$

$$\gamma_{0i}(u_{(n)}).$$

(iii) If $\lambda=0$, then $\xi=0.5$, $s=1$, and

$$\lim_{\lambda\to 0}\phi_s(\xi,w)=\lim_{\xi\to 0.5}\frac{((1-\xi)^2/\xi^2)^w-1}{((1-\xi)^2/\xi^2)-1}=$$

$$\lim_{s\to 1}\frac{s^w-1}{s-1}=\lim_{s\to 1}\frac{w\ s^{w-1}}{1}=w,$$

$$g(U_{(k)})=\phi_s(\xi,\sum_{k\in U}w(u_{(k)})=$$

$$\sum_{k\in U}w(u_{(k)})=\sum_{k=1}^{n}w(u_{(k)})$$

where $w(u_{(k)})$ is the weight of $u_{(k)}$. Thus,

$$\int\gamma(x_0,x_i)\mathrm{d}g=\sum_{k=1}^{n}[\gamma_{0i}(u_{(k)})-\gamma_{0i}(u_{(k-1)})]g(U_{(k)})=$$

$$[\gamma_{0i}(u_{(1)})-\gamma_{0i}(u_{(0)})]g(U_{(1)})+$$

$$[\gamma_{0i}(u_{(2)})-\gamma_{0i}(u_{(1)})]g(U_{(2)})+\cdots+$$

$$[\gamma_{0i}(u_{(n)})-\gamma_{0i}(u_{(n-1)})]g(U_{(n)})=$$

$$\gamma_{0i}(u_{(1)})[g(U_{(1)})-g(U_{(2)})]+$$

$$\gamma_{0i}(u_{(2)})[g(U_{(2)})-g(U_{(3)})]+\cdots+$$

$$\gamma_{0i}(u_{(n-1)})[g(U_{(n-1)})-g(U_{(n)})]+$$

$$\gamma_{0i}(u_{(n)})g(U_{(n)})-\gamma_{0i}(u_{(0)})g(U_{(1)})=$$

$$\sum_{k=1}^{n-1}\gamma_{0i}(u_{(k)})[g(U_{(k)})-g(U_{(k+1)})]+\gamma_{0i}(u_{(n)})g(U_{(n)})=$$

$$\sum_{k=1}^{n-1}\gamma_{0i}(u_{(k)})[\sum_{k=1}^{n}w(u_{(k)})-\sum_{k=1}^{n}w(u_{(k+1)})]+$$

$$\gamma_{0i}(u_{(n)})w(u_{(n)})=$$

$$\sum_{k=1}^{n-1}\gamma_{0i}(u_{(k)})w(u_{(k)})+\gamma_{0i}(u_{(n)})w(u_{(n)})=$$

$$\sum_{k=1}^{n}\gamma_{0i}(u_{(k)})w(u_{(k)})=\sum_{k=1}^{n}w_k\gamma_{0i}(u_k).\qquad\square$$

Theorem 3 shows that $\int\gamma(x_0,x_i)\mathrm{d}g$ approaches the minimum value when attributes have the most positive interaction ($\lambda\to+\infty$), $\int\gamma(x_0,x_i)\mathrm{d}g$ approaches the maximum value when attributes have the most negative interaction ($\lambda\to-1$), $\int\gamma(x_0,x_i)\mathrm{d}g$ is the same as the classic grey relational degree when attributes are independent ($\lambda=0$).

Corollary 1 In the GIRD model,

(i) When $\lambda=0$, and information coverage $D_2=D_3=D_4=\varnothing$, then $\int\gamma(x_0,x_i)\mathrm{d}g$ is the classic grey relational degree [4].

(ii) When $\lambda=0$, and information coverage $D_1=D_3=D_4=\varnothing$, then $\int\gamma(x_0,x_i)\mathrm{d}g$ is the interval grey relational degree [25].

(iii) When $\lambda=0$, and information coverage $D_1=D_2=D_4=\varnothing$, then $\int\gamma(x_0,x_i)\mathrm{d}g$ is the fuzzy grey relational degree [26].

(iv) When $\lambda=0$, and information coverage $D_4=\varnothing$, then $\int\gamma(x_0,x_i)\mathrm{d}g$ is the mixed sequence grey relational degree [24].

(v) When $\lambda\neq 0$, and information coverage $D_2=D_3=D_4=\varnothing$, then $\int\gamma(x_0,x_i)\mathrm{d}g$ is the grey fuzzy integral relational degree [27].

Theorem 3 and Corollary 1 show that the GIRD is the general and unified form of the point relational degree, interval sequence relational degree, mixed sequence relational degree and grey fuzzy integral relational degree. The characteristics and differences of various models are clearly reflected in the information coverage D_i $(i = 1,2,3,4)$ and parameter λ, in which D_i reflects the types of grey information coverage, and λ can effectively embody the interaction between attributes.

3. Grey information coverage interaction relational analysis

Let $Y = \{x_1, x_2, \dots, x_m\}$ be a scheme set for the multiple attribute decision making, $U = \{u_1, u_2, \dots, u_n\}$ be an attribute set, $\boldsymbol{W} = [w_1, w_2, \dots, w_n]^{\mathrm{T}}$ be an attribute weight vector, the scheme x_i for the attribute u_k is information coverage grey number $\widetilde{\otimes}a_i(k)$, the decision matrix is defined as follows:

$$\boldsymbol{A} = \begin{bmatrix} \widetilde{\otimes}a_1(1) & \widetilde{\otimes}a_1(2) & \cdots & \widetilde{\otimes}a_1(n) \\ \widetilde{\otimes}a_2(1) & \widetilde{\otimes}a_2(2) & \cdots & \widetilde{\otimes}a_2(n) \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{\otimes}a_m(1) & \widetilde{\otimes}a_m(2) & \cdots & \widetilde{\otimes}a_m(n) \end{bmatrix}. \tag{12}$$

The types of the grey information coverage are discrete coverage D_1, continuous coverage D_2 and language coverage D_3. In addition, the types of grey information coverage are identical if they have identical attributes.

In order to transform the decision matrix $\boldsymbol{A} = (\widetilde{\otimes}a_i(k))_{m\times n}$ into a normalized matrix $\boldsymbol{X} = (\widetilde{\otimes}x_i(k))_{m\times n} \triangleq (\widetilde{\otimes}_i(k))_{m\times n}$, at the same time, considering the contradiction and incommensurability between multiple attributes, it is necessary to carry out the standardized processing to transform the raw data into the same scale because different data have different measuring units described by Dong et al. [28]. Let the attribute type be divided into the benefit type and the cost type, their attribute subscript sets are P_1 and P_2, where $U = \{1, 2, \dots, n\} = P_1 \cup P_2$, $P_1 \cap P_2 = \varnothing$.

For discrete coverage grey number processing,

$$\widetilde{\otimes}_i(k) = \frac{\widetilde{\otimes}a_i(k) - \min\limits_k \widetilde{\otimes}a_i(k)}{\max\limits_k \widetilde{\otimes}a_i(k) - \min\limits_k \widetilde{\otimes}a_i(k)},\ k \in P_1, \tag{13}$$

$$\widetilde{\otimes}_i(k) = \frac{\max\limits_k \widetilde{\otimes}a_i(k) - \widetilde{\otimes}a_i(k)}{\max\limits_k \widetilde{\otimes}a_i(k) - \min\limits_k \widetilde{\otimes}a_i(k)},\ k \in P_2, \tag{14}$$

$$\widetilde{\otimes}_0(k) = \max_i \widetilde{\otimes}x_i(k). \tag{15}$$

For continuous coverage grey number processing, when $k \in P_1$,

$$\widetilde{\otimes}_i^L(k) = \frac{\widetilde{\otimes}a_i^L(k) - \min\limits_k \widetilde{\otimes}a_i^L(k)}{\max\limits_k \widetilde{\otimes}a_i^L(k) - \min\limits_k \widetilde{\otimes}a_i^L(k)} \tag{16}$$

$$\widetilde{\otimes}_i^U(k) = \frac{\widetilde{\otimes}a_i^U(k) - \min\limits_k \widetilde{\otimes}a_i^U(k)}{\max\limits_k \widetilde{\otimes}a_i^U(k) - \min\limits_k \widetilde{\otimes}a_i^U(k)}; \tag{17}$$

when $k \in P_2$,

$$\widetilde{\otimes}_i^L(k) = \frac{\max\limits_k \widetilde{\otimes}a_i^L(k) - \widetilde{\otimes}a_i^L(k)}{\max\limits_k \widetilde{\otimes}a_i^L(k) - \min\limits_k \widetilde{\otimes}a_i^L(k)}, \tag{18}$$

$$\widetilde{\otimes}_i^U(k) = \frac{\max\limits_k \widetilde{\otimes}a_i^U(k) - \widetilde{\otimes}a_i^U(k)}{\max\limits_k \widetilde{\otimes}a_i^U(k) - \min\limits_k \widetilde{\otimes}a_i^U(k)}, \tag{19}$$

$$\widetilde{\otimes}_0^L(k) = \max_i \widetilde{\otimes}_i^L(k),\ \widetilde{\otimes}_0^U(k) = \max_i \widetilde{\otimes}_i^U(k). \tag{20}$$

For language coverage grey number processing, when $k \in P_1$, (16) and (17) hold, and

$$\widetilde{\otimes}_i^M(k) = \frac{\widetilde{\otimes}a_i^M(k) - \min\limits_k \widetilde{\otimes}a_i^M(k)}{\max\limits_k \widetilde{\otimes}a_i^M(k) - \min\limits_k \widetilde{\otimes}a_i^M(k)}; \tag{21}$$

when $k \in P_2$, (18) and (19) hold, and

$$\widetilde{\otimes}_i^M(k) = \frac{\max\limits_k \widetilde{\otimes}a_i^M(k) - \widetilde{\otimes}a_i^M(k)}{\max\limits_k \widetilde{\otimes}a_i^M(k) - \min\limits_k \widetilde{\otimes}a_i^M(k)}, \tag{22}$$

$$\widetilde{\otimes}_0^L(k) = \max_i \widetilde{\otimes}_i^L(k), \widetilde{\otimes}_0^M(k) = \max_i \widetilde{\otimes}_i^M(k),$$
$$\widetilde{\otimes}_0^U(k) = \max_i \widetilde{\otimes}_i^U(k). \tag{23}$$

The optimal value of various attributes can constitute the ideal scheme of the industrial operation quality. Let x_0 be the reference sequence, the scheme after normalization x_i be the comparison reference, we finally calculate the GIRD of the λ fuzzy measure g. The greater $\int \gamma(x_0, x_i)\mathrm{d}g$ is, the better the scheme is.

The information coverage grey numbers include real numbers, interval grey numbers, fuzzy numbers, and morphological grey numbers and so on. The multi-source heterogeneous data can also be converted into information coverage grey numbers according to certain rules, and the interaction between attributes is ubiquitous. Therefore, the grey information coverage interaction relational analysis can be widely used in multi-attribute decision making of complex systems.

Above all, the flowchart of the grey information coverage interaction relational analysis is shown in Fig. 2. The operation steps are shown as follows.

Step 1 Establish the decision index system.

Step 2 Normalize the decision index matrix and determine the ideal scheme.

Step 3 Determine the weight of indicators.

Step 4 Calculate the information coverage grey relational coefficients.

Step 5 Calculate the λ fuzzy measures and the GIRD.

Step 6 Rank the decision schemes.

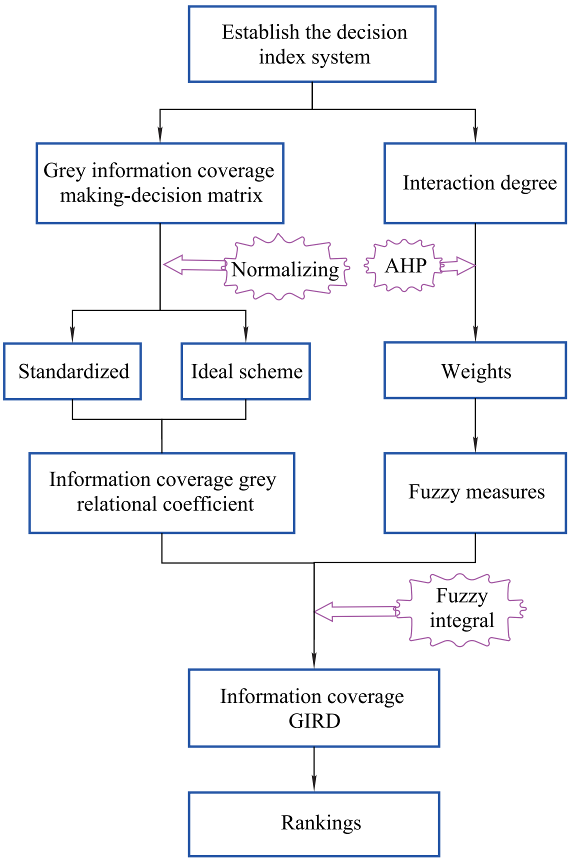

Fig. 2 Flowchart of the grey information coverage interaction relational analysis

4. Case study of industrial operation quality for Hunan

This section establishes the evaluation system for the industrial operation quality of Hunan province in accordance with the scientific and systematic qualities, operability, combined qualitative and quantitative data, and data accessibility of the index system.

4.1 Data source and interaction test

This study selected 22 evaluation indexes of industrial operation quality of 14 cities in Hunan province in 2016. u_1 stands for the growth rate of industrial added value (%). u_2 stands for the growth rate of total investment of industrial enterprises above a designated scale (%). u_3 stands for the growth rate of the total profit of industrial enterprises above a designated scale (%). u_4 stands for the industrial labor productivity (yuan/person). u_5 stands for the industrial value-added rate (%). u_6 stands for the sale profit rate (%). u_7 stands for the energy consumption per unit of the industrial added value (tones). u_8 stands for the proportion of the industrial export value in total investment (%). u_9 stands for the proportion of the scale industrial export delivery value in the national export delivery value (%). u_{10} stands for the mechanical and electrical products export growth rate (%). u_{11} stands for the proportion of added value of all industrial parks in the industrial added value of one region (%). u_{12} stands for the proportion of newly added industrial enterprises in local industrial enterprises (%). u_{13} stands for the proportion of the income from the main business of large and medium-sized industrial enterprises in main business income of local-scale industrial enterprises (%). u_{14} stands for the proportion of the value added of strategic emerging industries in gross domestic product (GDP) (%). u_{15} stands for the industrial R&D investment intensity. u_{16} stands for the increment of industrial enterprise invention patent authorization (Piece). u_{17} stands for the new product output growth rate (%). u_{18} stands for the integration of industrial and enterprise development index (%). u_{19} stands for the water consumption per unit of the industrial added value (m^2/10 000 yuan). u_{20} stands for the electricity consumption per unit of the industrial added value (kW/10 000 yuan). u_{21} stands for the key pollutant emission intensity per unit industrial added value (kg/10 000 yuan). u_{22} stands for the utilization rate of the industrial solid waste (%).

All data are obtained from Hunan Provincial Economic and Information Commission. Among them, u_{10} and u_{17} are represented by continuous coverage grey numbers, and u_{15} is represented by the language coverage grey number, others are discrete coverage grey numbers. In addition, u_7, u_{19}, u_{20} and u_{21} are cost-type indicators, and others are benefit-type indicators.

To verify the validity of the new model, the interaction test is conducted on the discrete coverage grey number. The interaction test indicates that the "industrial value-added growth rate" indicator u_1 and the "growth rate of the total investment of industrial enterprises above a designated scale" indicator u_2 exhibit a certain degree of repeatability. If they can be combined into one attribute, then such attribute will further highlight the role of the said indicators in the evaluation process. To objectively verify the interaction between the indicators, a single-factor analysis of variance is performed on the two indicators, and the analysis results are displayed in Table 1.

We assign the 2016 GDP of the 14 cities in Hunan province as the dependent variable. The results are sig<0.01 for the three indicators including the growth rates of the industrial added value, the growth rate of the total investment of industrial enterprises above a designated scale, and the industrial added value multiple growth rate of the total investment of industrial enterprises above a designated scale.

Table 1 Interaction test

Soruce	Three types sum of squares	Degree of freedom	Square of mean	F	sig
Modified model	57 418 591.785*	7	8 202 655.969	14.988	0.002
Intercept	83 779 746.887	1	83 779 746.887	153.087	0.000
u_1	36 460 783.438	2	18 230 391.719	33.312	0.001
u_2	12 660 102.319	2	6 330 051.159	11.567	0.009
$u_1 \times u_2$	27 706 643.714	3	9 235 547.905	16.876	0.002
Error	3 283 616.777	6	547 269.463	–	–
Statistic	140 860 195.013	14	–	–	–
Total after adjustment	60 702 208.561	13	–	–	–

Note: Dependent variable in this test is GDP, $R^2 = 0.946$ (adjusted $R^2 = 0.883$).

Thus, an interaction exists between the growth rates of the industrial added value and the growth rate of the total investment of industrial enterprises above a designated scale.

4.2 Modeling process and calculation

Firstly, the evaluation index matrix needs to be standardized. Then, we use the analytic hierarchy process (AHP) to determine the subjective weight, the results are shown in Table 2. Moreover, we calculate the λ fuzzy measures and GIRD, and rank them according to the integral values. We select eight interaction values if $-0.99 \leqslant \lambda \leqslant 500$, and analyze the GIRD of each decision when the decision attributes are expressed as the negative cooperation, independent and positive cooperation respectively. The calculation results are shown in Table 3.

Table 2 Attribute weights

Indicator	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}
AHP weight	3.56	5.98	6.23	5.17	3.96	5.08	2.15	3.38	5.76	6.98	4.87
Indicator	u_{12}	u_{13}	u_{14}	u_{15}	u_{16}	u_{17}	u_{18}	u_{19}	u_{20}	u_{21}	u_{22}
AHP weight	3.60	4.27	5.09	3.58	4.81	4.60	5.97	2.90	4.07	3.95	3.77

Table 3 GIRD of industrial operation quality of 14 cities

ξ	λ	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}
1	−0.99	0.957	0.904	0.814	0.705	0.611	0.714	0.911	0.789	0.676	0.781	0.697	0.620	0.760	0.646
0.580	−0.50	0.798	0.722	0.633	0.556	0.493	0.558	0.704	0.598	0.536	0.644	0.549	0.510	0.571	0.506
0.500	0	0.752	0.676	0.595	0.525	0.469	0.524	0.654	0.556	0.506	0.610	0.519	0.486	0.534	0.478
0.414	1	0.705	0.630	0.558	0.495	0.447	0.491	0.604	0.517	0.478	0.576	0.492	0.463	0.499	0.452
0.232	10	0.597	0.527	0.484	0.438	0.403	0.424	0.498	0.437	0.422	0.502	0.438	0.415	0.431	0.400
0.123	50	0.523	0.460	0.440	0.406	0.378	0.385	0.434	0.391	0.390	0.454	0.408	0.386	0.393	0.370
0.090	100	0.498	0.437	0.427	0.396	0.371	0.373	0.414	0.377	0.380	0.439	0.400	0.377	0.381	0.361
0.040	500	0.455	0.400	0.404	0.382	0.360	0.355	0.383	0.356	0.365	0.413	0.386	0.362	0.364	0.348

Note: x_1-Changsha (Cs), x_2-Zhuzhou (Zz), x_3-Xiangtan (Xt), x_4-Hengyang (Hy), x_5-Shaoyang (Sy), x_6-Yueyang (Yy), x_7-Changde (Cd), x_8-Zhangjiajie (Zjj), x_9-Yiyang (Yiy), x_{10}-Chenzhou (Cz), x_{11}-Yonghua (Yh), x_{12}-Huaihua (Hh), x_{13}-Loudi (Ld), and x_{14}-West Hunan (WH)

Fig. 3 describes the change trend of the GIRD under different cooperation relationships for 14 cities in Hunan province. According to the relationship of λ and ξ mentioned before, it is easily to know that the attributes have negative relationship, independence and positive relationship to each other when $0.5 < \xi < 1$, $\xi = 0.5$ and $0 < \xi < 0.5$ respectively. Fig. 4 is the ranking change for industrial operation quality of 14 cities under different interaction degrees λ.

4.3 Results and discussion

The Hunan Provincial Economic and Information Commission established an industrial quality evaluation index in 2012. We use the proposed method to evaluate the industrial operation quality of 14 cities in Hunan province and conduct a comparative analysis for a comprehensive assessment of the results.

In Fig. 3, the GIRD of Changsha can be the maximum whatever value ξ is. At the same time, with the decrease of ξ, the interaction degree λ increases, Zhuzhou, Changde, Chenzhou and Xiangtan have the higher GIRD and rankings than other cities. On the contrary, West Hunan is gradually reduced to the minimum when increasing the interaction degree. In addition, Shaoyang, Huaihua and Yiyang also have relatively low GIRD.

In Fig. 4, each row represents different cities on the same

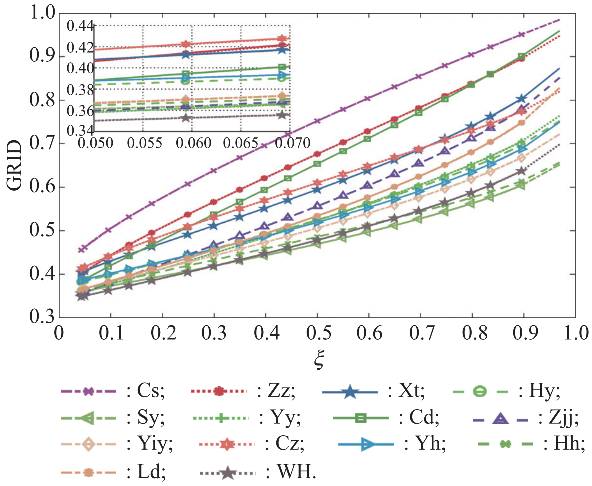

Fig. 3 GIRD trend of industrial operation quality for 14 cities

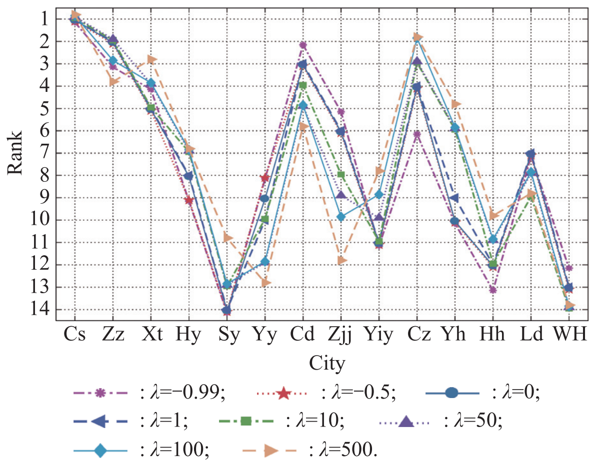

Fig. 4 Ranking for industrial operation quality of 14 cities

rankings with change of λ, each column means different rankings of the same city with the change of λ. Changsha always ranks in the first place when decision attributes have different cooperation relationships. Thus, Changsha has a strong adaptability and stability. The suboptimal city is Chenzhou, because Chenzhou's sorting rising is the fastest, with the rise from the 6th to the 2nd, and finally the complementarity reaches saturation. Xiangtan fills the third, the ranking increases from the 5th to the 3rd. In addition, Yonghua ranks from the 10th to the 5th with the increase in interaction. Therefore, the decision-making attributes of Yonghua have a strong complementarity, and it ranks in the 5th place. Similar cities also include Yiyang and Hengyang, and their rankings change greatly. On the contrary, the city with the poorest industrial operation quality is West Hunan, because the ranking of West Hunan is gradually reduced to the minimum, from the 12th to the 14th, with the increase in interaction. It shows that West Hunan is less complementary. Cities with rapid declines include Zhangjiajie and Yueyang, whose rankings are dropped by 7 and 5 points respectively, thereby indicating a poor complementarity of the two cities. Similar cities also include Changde, Loudi, and Zhuzhou. Moreover, it is easy to find that Changsha, Zhuzhou, and Xiangtan have higher rankings and stability compared with other cities, because the change range of rankings is relatively small. At the same time, West Hunan, Huaihua and Shaoyang have lower rankings and stability. The other cities in the middle undergo significant change when decision attributes include negative cooperation, independence, and active cooperation. Finally, according to Fig. 3 and Fig. 4, the optimal sorting results for industrial operation quality when considering strong interaction are as follows: Changsha $>$ Chenzhou $>$ Xiangtan $>$ Zhuzhou $>$ Yonghua $>$ Changde $>$ Hengyang $>$ Yiyang $>$ Loudi $>$ Huihua $>$ Shaoyang $>$ Zhangjiajie $>$ Yueyang $>$ West Hunan.

The optimal sorting is the same as the government's expectation. At the same time, we also select two methods for a multi-attribute decision-making for hybrid data comparison with the grey information coverage interaction relational analysis. Liu [29] suggested a new method for the interval number, which ranks the superiority of decision objects. By defining the dominant relationship, the ordering problem of the interval number decision object is transformed into the comparison of the sizes of the real numbers. Fu [30] aimed at a hybrid number and presented a method through which the weight of each performance index was determined by the entropy weight coefficient method, and then the scheme was ranked by calculating the distance and relative closeness of each scheme.

The sorting results are summarized in Table 4. The results from Fu's method indicates that the comprehensive industrial operation qualities of Changsha, Xiangtan, and Zhuzhou rank in the top three, whereas the industrial operation qualities of Shaoyang, Huaihua, and West Hunan rank in the bottom three. The rest of the cities are placed in the middle developmental level. The ranking order in Fu has a slight difference from that in the present study. However, the ranking of West Hunan is higher with Liu's method, but Changde's ranking is regressive, which is far from the actual situation in the present study. The main reason for the disparity is that both methods only consider the hybrid information of attributes and disregard interaction between attributes. By contrast, the proposed method considers the interaction between attributes, and the results are consistent with the actual situation. Thus, the multi-attribute decision-making method based on the grey information coverage interaction relational analysis has an effective and favorable promotion effect in industrial operation quality evaluation.

Table 4 Ranking results of three methods

Liu's method [29]	$x_1 > x_{14} > x_2 > x_5 > x_{11} > x_{10} > x_3 > x_{13} > x_8 > x_6 > x_9 > x_4 > x_{12} > x_7$
Fu's method [30]	$x_1 > x_3 > x_2 > x_{13} > x_7 > x_8 > x_4 > x_{11} > x_6 > x_9 > x_{10} > x_5 > x_{12} > x_{14}$
Proposed method	$x_1 > x_{10} > x_3 > x_2 > x_{11} > x_7 > x_4 > x_9 > x_{13} > x_{12} > x_5 > x_8 > x_6 > x_{14}$

5. Conclusions

Aiming at the information coverage grey number and the interaction between attributes in the multiple attribute decision making, a multi-attribute decision-making method based on the GIRD is proposed.

This paper provides the mathematical definition and classification of the information coverage grey number, and fuses different information coverage in the grey relational model using the difference information. At the same time, it can effectively avoid the complex algebraic operation and ordering of the information coverage grey number.

Based on fuzzy measure λ obtained by the transfer function ϕ_s, this paper establishes the GIRD model by using the Choquet fuzzy integral and grey relational principle. At the same time, this paper proves that the model not only is the popularization form of the point relational degree, interval relational degree, mixed relational degree and grey fuzzy integral relational degree, but also can effectively deal with the interaction between attributes. In addition, it enriches and develops the theory of grey information coverage.

Based on the grey information coverage interaction relational analysis, a decision-making example of evaluating the industrial operation quality for 14 cities in Hunan province of China is provided to highlight the implementation, availability, and feasibility of the proposed decision model.

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