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Belief reliability: a scientific exploration of reliability engineering

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Abstract: This paper systematically introduces and reviews a scientific exploration of reliability called the belief reliability. Beginning with the origin of reliability engineering, the problems of present theories for reliability engineering are summarized as a query, a dilemma, and a puzzle. Then, through philosophical reflection, we introduce the theoretical solutions given by belief reliability theory, including scientific principles, basic equations, reliability science experiments, and mathematical measures. The basic methods and technologies of belief reliability, namely, belief reliability analysis, function-oriented belief reliability design, belief reliability evaluation, and several newly developed methods and technologies are sequentially elaborated and overviewed. Based on the above investigations, we summarize the significance of belief reliability theory and make some prospects about future research, aiming to promote the development of reliability science and engineering.

Keywords: belief reliability, performance margin, reliability experiment, chance measure, uncertainty.

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1. Introduction

Reliability is a significant attribute of products. It is generally regarded as the capability that a product can perform its specified functions under specified conditions over a specified period of time [1]. Improving and verifying reliability have become one of the most crucial aspects in various engineering fields.

Essentially, reliability originated from people's desire for safety, stability, longevity and other positive stuff. In this sense, we can say that reliability is an ancient but still young engineering science. It is ancient because there has been a constant quest for reliable life since the dawn of mankind. From using thicker tree trunks to cross rivers to fighting in heavier armor, people used experiences to pursue and ensure reliability in ancient days. It is still young because it was not until around the 1950s that people began to use quantitative methods to ensure and enhance reliability [2,3]. Compared with mechanics and electricity engineering, the 70-years history of modern reliability theory is still too short.

Since the emergence of quantitative reliability theories in the 1950s, people have used a plenty of techniques to model, analyze and design products' reliability quantitatively [4–6], and obtained a great number of achievements. However, due to the in-depth application of innovative technologies in new products and the continuous compression of product development cycles, many classical reliability theories tend to be unable to adapt to this new trend in real engineering. For example, the success/failure logic-based methods can only provide limited guidance for product function and performance design [7], the small sample size makes the probabilistic statistics method fail to generate a reasonable distribution [8,9], and the new technology brings unknown failure mechanisms and serious epistemic uncertainty [10–12]. These problems have been always challenging the reliability theoretical framework and putting reliability engineers into an embarrassed situation. Where should reliability engineering go from here? Do we have a better theory to solve these problems?

In recent years, Kang has provided an alternative solution, which is called the belief reliability theory. The concept of belief reliability was first proposed in 2013 [13], and then refined by Kang et al. in 2020 [7]. Belief reliability tries to interpret reliability with new scientific perspectives and new mathematical tools, which formulate a new theoretical framework of reliability. Till now, belief reliability has made numerous new developments, and this scientific exploration performs well in various engineering scenarios. To comprehensively demonstrate the basic thoughts, basic principles and basic methods of

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belief reliability theory, in this paper, we will give a systematic overview of the belief reliability theory, including the motivation and origin, the theoretical discourse, the basic methods and technology, so as to show its present results and overall framework.

The remainder of this paper is structured as follows. In Section 2, the background and motivation of belief reliability are first overviewed. This paper then goes on to the theoretical discourse of belief reliability in Section 3, which embeds the most central idea in belief reliability theory. The theoretical framework of belief reliability is also summarized in this section. Section 4 further elaborates the belief reliability methods and technologies. In Section 5, we summarize the contributions and significance of belief reliability theory and make some prospects about future research.

2. Origin of belief reliability

In this section, we will elaborate the basic motivation to develop the belief reliability theory. We will first provide a brief survey of the history of reliability engineering. Then, the problems faced by the traditional reliability theories are discussed. The main contributions of belief reliability are summarized finally.

2.1 Brief history of reliability theory

In this paper, the history of reliability is divided into four stages, which is shown in Fig. 1.

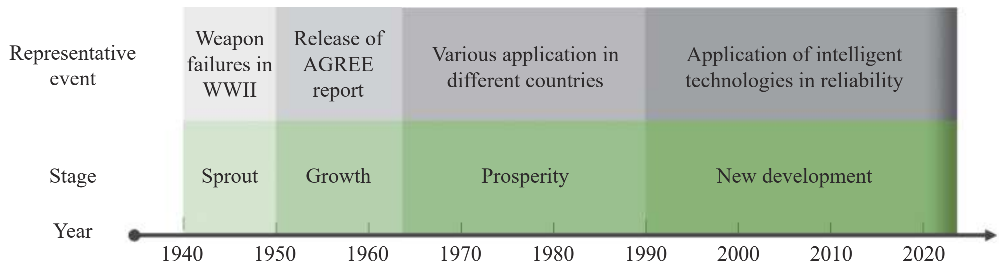

Fig. 1 Stages of reliability history

The concept of reliability first gained prominence in the 1940s, specifically during World War II (WWII) when a large number of United States (U.S.) weapons failed due to unreliable vacuum tubes. Around the same time, reliability ideas began to take shape in Germany during the flight testing of the V1 "flying bomb" (missile) [14]. After the war, Robert Lusser, a German engineer who worked on the V1 missile, formally published his ideas about calculating system reliability using the product of the reliability of each component, with component reliability obtained through statistical methods [3,15]. This marked the first quantitative expression of system reliability and became known as Lusser's Law [16].

During the 1950s and early 1960s, reliability engineering has experienced significant growth and development. One of the key events during this period was the release of a report on the reliability of electronic products in 1957 by the Advisory Group on Reliability of Electronic Equipment (AGREE) [2]. The AGREE report laid the foundation for modern reliability theory and is considered a significant milestone [17]. Based on the report, the U.S. military developed various standards and specifications regarding requirements, outlines, and tests to guide reliability work more systematically [18], such as MIL-STD-785 [19], MIL-HDBK-217 [20], and MIL-STD -781 [21]. In addition, the physics-of-failure (PoF) approach emerged during this period and attracted great attention, which opened a new area to examine failure from a micro perspective and acquire failure time using physical knowledge [22].

After the late 1960s, reliability engineering entered a prosperity stage. Reliability has been applied in various areas and attracted enormous attention all over the world. In the U.S., the design and testing methods were further applied to guarantee the reliability of the F-111 aircraft, the Apollo spacecraft, the Minuteman missiles, and other equipment. Japan applied the reliability technologies to railway, automotive and civilian electronics industry [23,24]. Accompanied with the total quality control, Japan created numerous national brands such as Toyota, Hitachi and NSK. Soviets also contributed greatly to reliability theories when they adopted complex systems like fuel energy complexes and multicomputer complexes [25], in which the redundancy technology, statistics, Markovian process, etc., were further studied. Other countries, such as UK, France and China, also built professional reliability organizations sequentially for reliability research and applications [18,26].

With the development of technology and the increase of system complexity, reliability theory entered a new development stage after the 1990s. Besides the continuous exploration of reliability models for dynamic and complex systems [27] (e.g., dynamic fault tree, Petri net, goal oriented (GO) method), people also started to apply digital and intelligent technologies to reliability engineering. A representative technique is the prognostics and health management (PHM) [28], which is oriented to the product operation and maintenance (O&M) process, and predicts and infers the product status through intelligent methods, thus facilitating condition-based maintenance [29]. In recent years, the digital twin-based reliability technology has gradually emerged and is now mainly used in the product O&M process [30].

2.2 Problems of the present theory

Formally, the reliability is usually defined as the capability that a product can perform its specified functions under specified conditions over a specified period of time. Since the release of the AGREE report, the following expression is widely adopted to quantify reliability [1,2]:

$$R(t) = \Pr\{T > t\} \tag{1}$$

where $R(t)$ is the reliability at the specified time t and is also called the reliability function, T is the time to failure of the product, $\Pr\{\cdot\}$ is the probability measure. This expression indicates that the reliability refers to the probability that a product will function normally over a certain time t.

From the reliability history, we may find that the above concept and expression have been widely applied and adopted in textbooks, standards, and real products. Even though, from the perspective of system science, the present theory is still incomplete in essentially describing the product's ability to be reliable. This section will elaborate three issues.

(i) A query for reliability function

It can be easily figured out that reliability is a kind of ability of products, which is directly related to the specified functions, the specified conditions, and the specified time. The significance of three "specified" issues are stated clearly in the definition, but in the reliability function given by (1), only the "specified time" can be found. Where are the other two "specified" issues?

More specifically, reliability ultimately lies in the "specified function" of a product, which is greatly related to how the product is designed, what environment it is subject to, how long it works, and how people require it. Confusingly, when we quantify reliability, only the time factor is involved, while the influence of other factors is more or less ignored. This indicates the present reliability function is not consistent with the reliability definition.

In fact, this is actually a problem left over by history. If we date back to the era of reliability's birth, it was actually the statistical quality control techniques emerged with mass production in the 1920s [17] and the American data culture formed therefrom [31] that made American scientists more inclined to transform the product reliability problems into statistical analysis problems. Since the failure time reflects the length of time that the product remains functional, the most straightforward method to quantify reliability could be to analyze the products failure time data using statistical methods, and this further derived (1). Since such a form of reliability function was effective at that time, it was not explored in depth whether such a representation covered all the elements of reliability.

With the increase in the complexity of product functions and the diversity of the use scenarios, people seemed to be aware of this problem to some extent, which led to the birth of physical methods of reliability. Unfortunately, the study of reliability physics still comes down to the acquisition of statistical parameters. For example, the covariate models are used to obtain failure rates, and the PoF models can be utilized to obtain statistical properties of the time to failure [32].

Since the idea of "reliability is a statistical problem" is deeply rooted in people's minds, all the studies of reliability engineering have ultimately gone to the description of the inconsistency of product failure time. Clearly, they cannot explain the essential question of "what makes a product reliable". Therefore, a more complete reliability function is urgently needed.

(ii) A dilemma of probability

The reliability function clearly shows that reliability is measured by probability. As stated before, this stems from the perception of "reliability is a statistical problem" that arises from mass production. Probability and probabilistic statistics, as basic methods for describing uncertainty in mass production processes, naturally become the mathematical tools for reliability problems. By analyzing the failure time data using probabilistic statistics, the product reliability can be easily calculated [5,33]. However, with time going by, various facts, especially the small sample problem, have shown that these tools are now in an embarrassed situation.

Essentially, the above issue is due to the properties of the mathematical tools. The most original purpose of probability is to characterize the trend of an event frequency over a large number of trials. Consistently, there is a prerequisite for probabilistic statistics called the "law of large numbers", stating that only when the sample size is close to infinity does the frequency tends to probability [34]. When applied to product reliability, the core task will be to acquire the variability of failure times with failure data. Nevertheless, unlike the statistical control in the mass production, failure time data that is oriented by reliability statistics is quite difficult to obtain, and the small sample problem is a common phenomenon in reliability engineering. In this case, the probabilistic statistical methods will fail in some sense. This makes us further reflect on the rationality of using probability theory.

It is undeniable that the inconsistency (uncertainty) of failure time in reliability can be described with probability when the data are sufficient, but the using scenarios of probability are really limited due to its own dilemma explained above. More critically, reliability is not just a problem of uncertainty, so the measurement of failure time inconsistency (uncertainty) is not the whole picture of reliability. Nowadays, products are becoming more and more complex. We are more concerned about how the products can accomplish their functions and avoid failures, while the statistical laws of their failure time are not even that important. Therefore, the identification and quantification of uncertainty for reliability engineering need further discussion and exploration.

(iii) A puzzle of the system reliability

In today's reliability engineering, the Lusser's Law is still widely used in the system basic reliability calculation and the reliability allocation [35]. Let us consider a series system consisting of 50 identical components with reliability 0.9. If these components are working independently, the system reliability will be 0.005. Obviously, this result makes no sense in decision making, because it tells you that the product is totally unreliable. In fact, even when we use a series design, the actual reliability will not be that low. Moreover, let us consider a simplest reliability allocation problem for this system with a high reliability requirement of 0.9999, then each component reliability should be at least 0.999998, which is really hard to meet for the components. Further, according to the property of probability, it is nearly impossible to verify whether the designed component is that highly reliable through tests.

These bad results are essentially caused by the misuse of the product theorem in the probability theory. In fact, to use the product theorem in reliability, we must accept a strong assumption that the components are independent and are working all the time. However, according to the mathematical definition, the independence is usually not able to be acquired until you get the result of system reliability and component reliability. Moreover, the components may not work uninterruptedly in the system in many cases. In other words, the above assumption is usually not valid in real cases, and the way of calculating and allocating the system reliability simply derived from the logical relationship of components using the product theorem is not in line with the system reality. Unfortunately, most of the books or engineering standards are still keen to guide engineers in using this formula for system reliability, which may cause various confusions.

From the perspective of systems engineering and systems science, we may be able to better understand the puzzle of system reliability. On the one hand, a system consists of individual components and their interrelationships [36], that is, the elements of the system are essentially physically interrelated, whether in terms of structure, function, or timing relationships. On the other hand, systems are "emergent" in nature [37], i.e., the characteristics and behaviors exhibited by a system are not simply the sum or product of the characteristics and behaviors of all components. Therefore, the way to use product theorem is debatable. The system reliability calculation is still a puzzle for a lot of reliability engineering scenarios, so it is necessary to start with the physical properties and functional behaviors of the system (including structural and functional correlations, and timing characteristics), rather than only focus on the logic relationships of the components.

2.3 Some efforts made regarding the problems

According to the above discussion, it can be found that these problems have seriously affected the effectiveness of reliability engineering techniques. Facing these issues, researchers do have made some efforts to address these problems. We hereby briefly review them in four aspects.

(i) Structural reliability method

As a branch of reliability research, the structural reliability method provides a different reliability calculation method from (1), i.e., it uses the probability that the limit state function is greater than 0 to characterize the structural reliability [38]. The structural reliability method gives us a great inspiration that in the reliability measurement, various factors affecting the product reliability should be taken into account as much as possible, and it is not necessary to be restrict to the expression of (1), just as stated in the "query of reliability function". However, the structural reliability method is so far dedicated only to structural studies, that is, the limit state function is merely used for the static or kinematic related structural characteristics such as stress, deformation, deflection, and movement accuracy, but has failed to be migrated and applied in other fields, not to mention forming a common expression with the reliability calculation method of (1). This drawback comes to the fore when we face a general system consisting not only of mechanical structures, but also of electronic devices, control systems, etc. Besides the description of structural characteristics, the role of other properties like electrical performance, thermal performance, and control stability, should also be considered in system reliability analysis.

(ii) Small sample method

Small sample problem is usually interpreted as the effect of epistemic uncertainty [39] and people try to solve this problem by introducing different mathematical tools to reliability. The representative methods are previously reviewed and compared by Kang et al. [10], and here we list some important findings in Table 1.

Table 1 Review of representative methods considering epistemic uncertainty

Type	Representative method (mathematical basis)	Existing problem if applied to reliability engineering
Imprecise probability-based method	Bayesian reliability (Bayesian theory)	Due to the inclusion of subjective information, Bayesian probabilities are not probabilities in the sense of frequency, so it is questionable to still follow Kolmogolov's axiomatic system for subsequent calculations. When available information is scarce, the results of Bayesian reliability analysis are greatly sensitive to the prior knowledge and are often not sufficiently complete to support decision making.
	Evidence reliability (evidence theory)	The basic formula of $\mathrm{Bel} \leqslant \mathrm{Pr} \leqslant \mathrm{Pl}$ has not been strictly proved. The results of the interval form will cause interval expansion problems in the system reliability calculation.
	Interval reliability (interval analysis theory)	The results can also bring interval expansion problems. The interval analysis is not self-consistent in mathematics (for two events related to interval analysis with $\Lambda \subset \Delta$, we may find $\pi\{\Lambda\} > \pi\{\Delta\}$, where π is a likelihood measure; one can refer to [40] for an example).
New mathematical measure-based method	Posbist reliability (fuzzy theory)	We can derive results that are not self-consistent, i.e., the sum of reliability and unreliability is not equal to 1.

They do can quantify the epistemic uncertainty associated with small sample problems to some extent, but they still have different problems when applied to reliability engineering. Generally, these methods can be mainly categorized as two types [10]. One is to use different mathematical methods (e.g., Bayesian theory [41], evidence theory [42], interval theory [43]) to model and quantify the epistemic uncertainty brought by small samples, and then use imprecise probabilities, i.e., intervals of probability, to measure the product reliability. Since various subjective information is often involved in these approaches, they are actually contradicted with the original meaning of probability. In other words, the obtained results of these methods may not be strict probabilities but are still interpreted as probabilities and computed using the operational laws of probability theory. This will lead to the interval expansion problem, that is, the epistemic uncertainty will be excessively amplified, making the reliability measurement nearly meaningless in practical engineering [10]. The other type directly discards probability and chooses another mathematical measure to measure reliability. The most representative is the posbist reliability [44], which uses the possibility measure (a measure defined in fuzzy theory) as the basic measure of reliability. However, as discussed by Kang et al., posbist reliability does not satisfy self-duality, i.e., the sum of reliability and unreliability is not equal to one, which will lead to counterintuitive consequences in reliability engineering [10,45]. In addition, posbist reliability ignores the existence of uncertainty with large sample features and only one-sidedly quantifies the epistemic uncertainty, which is also incomplete.

(iii) Quantification of margin and uncertainty (QMU) method

In the 2000s, the three U.S. national laboratories (Los Alamos, Lawrence Livermore, and Sandia) have proposed a QMU method in the course of annual certification of nuclear weapons [46,47]. The core idea is to determine whether the product is reliable by quantifying the designed performance margin M and uncertainty U and calculating the confidence ration of $\mathrm{CR} = M/U$, where $\mathrm{CR} > 1$ means the system is reliable. The QMU approach is indeed very enlightening since the concept of "performance margin" is a more general means of describing system reliability and fits well with the need to construct a scientific system of reliability. However, the QMU method also has significant shortcomings that should be improved. Since QMU is mainly oriented to the annual certification of nuclear weapons, the information provided by CR is usually "pass" or "fail", and other information is very limited, which in turn tends to bring new difficulties to other reliability activities, such as system reliability design. This is because that the QMU method does not emphasize much on the physical relationship between performance margins and system design variables, environmental variables and other factors [48], but focuses more on the integration and evaluation of various types of data reflecting performance margins [49], so it is difficult to guide the design and production of systems based on reliability metrics (or specifically, the CR value).

(iv) Existed scientific exploration

In 2017, Rocchi published a book with a very resounding assertion: Reliability is a New Science [50]. In this book, he attempted to study reliability in a different viewpoint and explain the laws exhibited in reliability engineering from the perspective of fundamental physics using a tool of reliability entropy [51]. In this year, a panel discussion on “Is reliability a new science” was held in the International Conference on Mathematical Methods in Reliability, and a corresponding special issue on journal Applied Stochastic Models in Business and Industry was organized to make a worldwide discussion [52]. Although some scholars believe that there may not be a scientific interpretation of reliability or this discussion is not very meaningful [52–55], some others believe that we should look at reliability from a more scientific perspective rather than limiting it to technology [52,56,57].

In fact, since the 1980s, Chinese scholars have been treating reliability as a science and have explored and discussed the related methodology [58]. Song studied the relationship between reliability and failure study [59], and proposed the idea of constructing a failure science to complement the reliability theory [60]. Zhong et al. studied the philosophical idea of failure science based on the practice of material fatigue and electrical and mechanical systems, and proposed the concept of “failurology”, which means a science for failure [61,62]. In recent years, Sun has claimed that the theoretical basis of reliability analysis may require the assistance of the “applied theoretical physics” [63], and his team tried to interpret some reliability concepts such as time-variant hazard functions using the maximal entropy principle of statistical mechanics [64,65]. Unfortunately, all the above explorations did not result in a complete scientific theory system for reliability engineering till now.

The authors of this paper have also participated in the scientific exploration of reliability [56]. Though we agree with the viewpoint of Prof. Rocchi that reliability is a science, we still differ from his basic ideas in sight of how to interpret the reliability science. The entropy-based reliability theory is gorgeous, but it seems to be limited in guiding engineering practices. In our opinion, reliability originates from human’s practice, which indicates that reliability science should be extracted from engineering to better solve engineering problems; in other words, reliability should be an engineering science. We philosophically analyze the validity of reliability science from the perspective of human practice and subject-object relationship, and draw the conclusion that “reliability science is the unity of certainty and uncertainty” [56], based on which we have started our new exploration of the science for reliability engineering—the belief reliability theory.

2.4 Belief reliability theory: innovation in China

As discussed earlier, people have attempted to solve the problems in reliability engineering from various perspectives, but still have not fully scientifically resolved “the query of reliability function”, “the dilemma of probability” and “the puzzle of system reliability” that we mentioned.

From the perspective of constructing reliability science, the existing reliability theory and methods are still in a fragmented state and are not strong enough to support the scientific edifice of reliability. Delightfully, these attempts have given us great encouragement and inspiration, and have promoted the birth of belief reliability.

The concept of “belief reliability” was proposed by Kang and his team from Beihang University in 2013 [13]. Later, a relatively complete theoretical and methodology framework of belief reliability was formulated in around 2020 [7]. Belief reliability theory is an innovative theory, which aims to provide a scientific way to measure, model, design, and verify products’ reliability. Briefly speaking, belief reliability theory addresses the three problems mentioned above from the following three perspectives:

(i) Belief reliability theory proposed three scientific principles and three basic equations of reliability, based on which the causal relationship between product reliability and specified functions, specified conditions, and specified time can be constructed, thus enabling a more reasonable reliability measurement.

(ii) Belief reliability theory extends the measurement of uncertainty. The theory adopts a new axiomatic mathematical theory called uncertainty theory [66] to measure epistemic uncertainty, and continues to employ probability theory for aleatory uncertainty, thus forming a comprehensive uncertainty quantification framework.

(iii) Belief reliability theory systematically considers the emergence property of systems. The modeling of reliability is performed based on the functional principles of the system and the correlations of the constituent elements, thus providing a more scientific description and characterization of the system reliability.

The first one makes belief reliability be able to organically incorporates all the reliability-related elements into the reliability function and thus answer the query for reliability function. The second one makes belief reliability be able to better cope with small sample problems with uncertainty theory and reliefs the dilemma of probability. The third one makes belief reliability be able to describe product reliability at system level with a systematic characterization of the system function and physical dependence, thus overcoming the puzzle of system reliability.

To summarize, the belief reliability theory attempts to explore a more scientific method for reliability engineering. This theory is inspired by the philosophical thinking of Chinese scientist and engineers and embraces people's good vision of reliability science. In the following part of this paper, we will elaborate the foundations of belief reliability theory and review the present research and applications of it.

3. Theoretical discourse of belief reliability

3.1 Philosophical reflection

The research of belief reliability begins with some philosophical reflections about reliability. Zhang et al. claimed that reliability is definitely an engineering science oriented to the world of failures, where the failures are studied and resolved to achieve system optimization [56]. Since failure comes from the cognition and experience of people, it possesses a nature of subordination to human beings. To be further, the ultimate reason why the system generates failures is that humans are practical beings with the unity of certainty and uncertainty [7]. Therefore, the meaning of reliability should be re-examined from the perspective of certainty and uncertainty.

The certainty of reliability is reflected in two aspects. From the perspective of the object, the product reliability has its objective laws that are usually deterministic and can be grasped, such as the function of products and the trend of performance degradation. It is easy to know that these deterministic laws of reliability are closely related to the product's own properties (inner factors) and external environment or conditions (external factors). Different influencing factors can result in different functions, performances, and evolution processes of a product when being used. From the perspective of the subject, human thought is with sovereign [67], i.e., human beings can always understand and grasp the laws of objective existence and make correct practical choices under certain historical conditions, such as to avoid, locate and repair possible failures. Moreover, whether a product is reliable or not often depends on the subject's experience and practice. Specifically, reliability depends to a large extent on how much the subject (people) require and expects the object (product) when using it. This is a certainty issue that should and must be clarified in reliability.

The uncertainty of reliability is also reflected in two aspects. From the perspective of the object, the real world is filled with various forms of risks, which create uncertainty in the product performance and failure process. This kind of uncertainty is usually manifested as aleatory uncertainty. From the perspective of the subject, human thought is also with non-sovereign [67], i.e., people's understanding of the objective world is always limited, which may generate a systematic bias between practice purposes and results, thereby causing failures. This kind of uncertainty can be interpreted as epistemic uncertainty. Generally, the certainty parts of reliability (including the internal and external influencing factors from the object aspects, and the use requirements from the subject aspects) are often affected by uncertainty to a greater or smaller extent. In practical problems, depending on different product characteristics and use scenarios, the kinds of uncertainty of these factors may be either aleatory or epistemic.

Actually, the three "specified" elements have shown us some key points. First, the core of reliability is the specified function, which is determined by some internal mechanisms and people's requirements. This is a certainty result or an external appearance after the product is designed and manufactured according to specific influencing factors. Second, the time scale requirement for reliability is given by the specified time, in which the product should not be fundamentally changed. It implicitly reflects a certainty evolutionary process and rules of the product in a certain time period. Of course, there is also uncertainty in the evolutionary process because of the variability of different factors. Third, the use scenarios embedded in reliability is the specified condition, showing the various internal and external influences. This reflects that the product is always in an uncertain environment with dynamic changes. We should say the failures that emerge in people's practice are essentially caused by the contradiction between certainty and uncertainty.

According to the above analysis, reliability should be regarded as an integration of certainty and uncertainty, and the certainty part may be even more important in reliability engineering, because the certainty laws are the basis for generating uncertainty. In this sense, the reliability theory should be used to optimize certainty and resolve uncertainty of products. Only through exploring the certainty and understanding uncertainty can we better quantify reliability. Based on the thinking of integrating certainty and uncertainty, belief reliability theory presented three scientific principles, which are shown in Subsection 3.2.

3.2 Scientific principles

In belief reliability theory, reliability is first a problem of certainty, i.e., whether a product is reliable or not depends on the margin (the distance between performance and its requirement) and the degradation law of the product. The greater the margin is and the slower the degradation is, the more reliable the product is. Reliability is also a problem of uncertainty, i.e., the various uncertain factors will jointly affect the magnitude of margin and the trend of degradation. Therefore, Kang et al. summarized the following scientific principles of reliability [7,45]:

Principle 1 Margin-based reliable principle: the performance margin determines how reliable the product is.

Principle 2 Eternal degradation principle: the margin of the product always undergoes degradation along the time.

Principle 3 Uncertainty principle: the margin of the product is uncertain.

The margin-based reliable principle points out the most basic demand of people to the products. A larger margin implies that the product can perform its specified function more effectively (the product is stronger and harder to fail). The eternal degradation principle highlights the objective law of product operation. Products inevitably undergo an irreversible degradation process over an infinitely long time scale. The uncertainty principle points out another element of reliability practice, i.e., the uncertainty. It is not enough to simply understand the certainty laws of margin and degradation; we must also consider the impact of different types of uncertainty.

Essentially, reliability is regarded as a kind of engineering science guided by the above three scientific principles. As Qian stated in Engineering Cybernetics [68], "an engineering science aims to organize the design principles used in engineering practice into a discipline and thus to exhibit the similarities between different areas of engineering practice and to emphasize the power of fundamental concepts", the reliability scientific principles are derived using inductive method based on extensive experiences gained during the product design and manufacturing processes. The aim is to capture the essence of what makes products reliable. This process is depicted in Fig. 2.

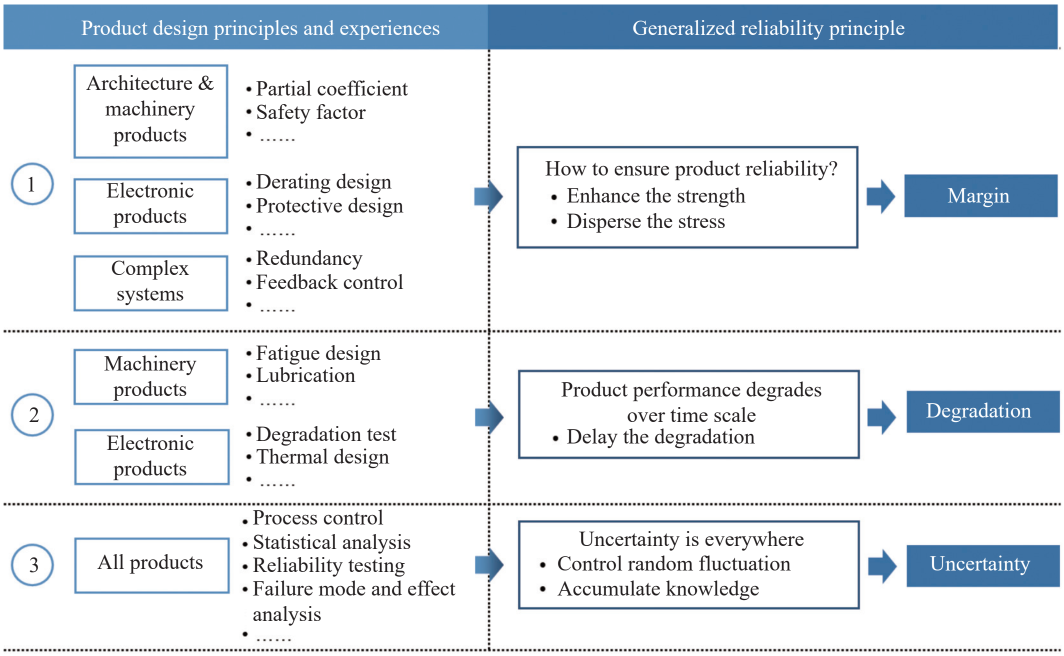

Fig. 2 Inductive process of reliability scientific principles

First, various design principles and experiences have taught us that we can make products stronger or safer by drawing on related disciplinary knowledge. Some tools commonly used to ensure or enhance product reliability are listed in the first row of Fig. 2. For example, many structural reliability related standards or codes specify partial coefficient or safety factors for different structures to increase their design strength [69]. For electronic products, derating and protective design are often used to reduce or isolate the impact of external environments on components [26,70]. For more complex products, redundancy or feedback methods are often employed to increase their ability to resist internal and external disturbances [71]. By examining the underlying principles of these tools, we can see that they either enhance the strength of the product or disperse the stress that the product is experiencing. In this sense, reliability is essentially determined by the margin, which characterize the distance between the product current performance status and its functional boundaries. The above thinking forms the first principle.

Second, engineering practices have consistently shown that over a long enough time scale, any system will inevitably deteriorate. This means that products are subject to degradation processes that significantly impact their lifetime and user experience. To address this, engineers use various methods, such as fatigue design, lubrication, and thermal design, to delay the degradation process. For more refined designs, degradation testing is necessary to explore the degradation characteristics of products, such as their degradation path and rate. The above discussion highlights the origin of the second principle of reliability.

Third, as Leibniz once said, "no two leaves in the world are exactly the same", and the same applies to the production of products. According to our philosophical understanding of reliability, uncertainty permeates every aspect of reliability practice. Engineering experiences have also taught us this fact. For instance, random fluctuations can cause stress to exceed the product strength at some point, or an unknown failure mechanism can occur under new use conditions. Therefore, in product design and production processes, various measures and statistical methods are used to quantify variability and accumulate knowledge, with the aim of controlling the various uncertainties. As a major enemy of reliability, uncertainty constitutes a reliability principle.

3.3 Basic equations

On the basis of the scientific principles, some basic equations of belief reliability theory are further proposed. The equations are first given by Zhang et al. [56] in the philosophical thinking of the theoretical discourse of reliability. Later, they are refined by Kang et al. [7,45]. The specified time, the specified condition and the specified function are all included in these equations. In this paper, we would like to give some more details and explanations about these equations. The equations are expressed as follows:

(i) Equation No.1—Margin equation:

$$M = d(P, P_{\mathrm{th}}).$$

(ii) Equation No.2—Degradation equation:

$$P = f_{\boldsymbol{t}}(\boldsymbol{X}, \boldsymbol{Y}, t).$$

(iii) Equation No.3—Measurement equation:

$$R = \mu\{\tilde{M} > 0\}.$$

In these equations, three quantities are the core points, i.e., the performance P, the margin M and the reliability R. These quantities are related to three functions, i.e., the distance function $d(\cdot)$, the degradation function $f_{\boldsymbol{t}}(\cdot)$, and the measure function $\mu\{\cdot\}$. The influencing factors are also listed within the functions, where P_{th} is the threshold of P, $\boldsymbol{X}$ is the vector of product internal variables (e.g., material, size, layout), $\boldsymbol{Y}$ is the vector of external variables (e.g., operating and environment conditions), t is the reversible time (or Newton time), $\boldsymbol{t}$ is the irreversible time (or degradation time) with specific direction, and is also usually written as $\overrightarrow{t}$ to emphasize its directional properties, and the wave line above the $\tilde{M}$ means a quantification of uncertainty. For convenience, these equations are labeled with numbers according to their usage.

Equation No.1 is called the margin equation, which corresponds to the margin-based reliable principle. It characters a distance between the performance and its threshold. In the equation, P describes the specific form and exhibited features of a product to perform its function, while P_{th} is determined according to our requirements of the specified function. According to the characteristic of performance, the distance function may have different forms [10,45]. This equation has implied a truth of reliability, that is, a greater value of margin indicates a more reliable product.

Equation No.2 is called the degradation equation, which corresponds to the eternal degradation principle. It shows a quantitative description of the degradation process of the performance, whose degradation path is related to $\boldsymbol{X}$, $\boldsymbol{Y}$ and t. The subscript $\boldsymbol{t}$ of the degradation function $f_{\boldsymbol{t}}(\cdot)$ is a time vector [50], which is essentially different with the reversible time t (the difference will be explained later). It means the degradation process is directional and irreversible, indicating that degradation equation only describes the system behavior with the time flows positively (e.g., from 0 to $\boldsymbol{t}$) and the performance status cannot go back to any former status without any external energy input or intervention in the degradation process.

Equation No.3 is called the measurement equation, which corresponds to the uncertainty principle. It is essentially an uncertainty quantification of the event that the margin is greater than 0, since a positive margin means the product can accomplish the specified function. According to different types of uncertainty, different mathematical measures should be used. For example, probability measure, uncertain measure and chance measure are all candidate measures.

By integrating the three basic equations, we can recognize that R in Equation No.3 is ultimately an abbreviated form of $R(\boldsymbol{X}, \boldsymbol{Y}, t, \boldsymbol{t}, P_{\mathrm{th}})$, which indicates that the reliability metrics is determined by all related factors from Equation No.1 and No.2. From the perspective of the reliability definition, it can be found that the margin requirement P_{th} of performance P reflects the "specified function", the degradation time $\boldsymbol{t}$ reflects the "specified time", and the internal and external variables, i.e., $\boldsymbol{X}$ and $\boldsymbol{Y}$, reflect the "product character" and "specified condition". Therefore, the reliability function based on the four equations can truly cover all the elements in the definition of reliability.

For these three basic equations, we must further make several remarks of their connotations.

Remark 1 About the irreversible time

The irreversible time (also called the degradation time) $\boldsymbol{t}$ is essentially different from the reversible time (also called the Newton time) t. The irreversible time can be illustrated by the second law of thermodynamics, that is, the system is always moving to a state of disorder irreversibly, which means the performance degradation has a specific direction. We can only figure out the degradation process as the time flows from 0 to $\boldsymbol{t}$, but cannot go back from $\boldsymbol{t}$ to 0. Correspondingly, the reversible time t applies to the classical Newtonian mechanics, that is, the associated equation will be always applicable to the system no matter the time flows positively (e.g., from 0 to t) or negatively (e.g., from t to 0). For example, the time t in the Newton's second law, which can be written as $F = \mathrm{d}p/\mathrm{d}t$, is reversible, where p is the momentum and F is the force.

A small case of the slider-crank mechanism (SCM) can better show the difference of the two kinds of time. Since the main purpose of an SCM is to use the rotation of the crank to drive the linear movement of the slider, the displacement of the slider is the critical performance of the SCM, and this performance may degrade because of the wear in joints. When establishing the basic equations of SCM, $\boldsymbol{t}$ exists in the wear process model (e.g., an Archard model) where the time is irreversible, and t will show up in the geometric relationship of the movement related to angular velocity where the time is reversible. Further, if we want to get the degradation rate, we should find the partial derivative of $\boldsymbol{t}$, rather than t.

In real engineering, the two kinds of time often exist simultaneously. However, when the product is designed, the irreversible time is usually not considered completely or even ignored; when the product reliability is analyzed, the reversible time is not considered well. These treatments may cause imprecise evaluation of the product reliability and lifetime. Therefore, the belief reliability theory will consider both kinds of time by utilizing these basic equations.

Remark 2 About the interdisciplinary equation

If the degradation effect of the performance is not considered, Equation No.2 will degenerate to a special case. It is called as the interdisciplinary equation, which is denoted as $P = f_{\boldsymbol{t}=0}(\boldsymbol{X},\boldsymbol{Y},t)$. This equation is determined by fundamental laws in different discipline areas that the product will obey. We advocate that the interdisciplinary equation should be given by scientists and engineers in every discipline related with the designed product, such as mechanics, electricity, and electromagnetism, and thus forming the inputs of reliability issues. Actually, before we establish the margin equation and degradation equation, we usually should first figure out the interdisciplinary equation. Therefore, the interdisciplinary equation is also labeled as Equation No.0, which means it is the basis of the three basic equations.

A perfect interdisciplinary equation should include not only the physical laws of the system operation corresponding to its design factors, but also the system interaction with the environment. Take a simple resistance-capacitor (RC) filter circuit for example. Basically, the circuit is dominated by electrical property equations such as the Kirchhoff's equation, but when establishing the interdisciplinary equation, we should also figure out how the resistance and capacitance change under different electrical stresses and temperatures.

Remark 3 About the hierarchy

The internal variable $\boldsymbol{X}$ and the external variable $\boldsymbol{Y}$ in the three basic equations can also be functions, which is especially true for systems with multiple levels. For $\boldsymbol{X}$, the internal factors in the system equations may be determined by the internal factors and performance of the subsystems, which is further dependent on the internal factors and performance of the components. For example, the landing distance of an aircraft is related to internal variables such as the mass of the aircraft, brake pressure, and spoiler counterthrust, which are further related to all subsystem masses, brake subsystem deceleration rates and response times, and spoiler profiles and angles. For $\boldsymbol{Y}$, the operating or environmental stresses experienced at the system level may be the result of the transmission of components from bottom levels to the top level layer by layer, and the stresses at the component level may also be the result of the decomposition of stresses at the system level downward. For example, the thermal performance of a computer depends on the external temperature and the computer housing temperature, which originates from the thermal stress generated by the heat of chips and power devices. Another example is that the vibration stress in the steering subsystem of a car is transmitted from the vibration that the whole car is subjected to. In this sense, the above property of the basic equations enables belief reliability theory to cope with the complex systems with different levels.

3.4 Experimental verification

The basic equations have quantitively described the scientific principles of reliability. Apparently, these equations are very abstract expressions. For a particular reliability problem of a product, we need to first answer the certainty issues in reliability, i.e., what are the specific structures of the functions $f_{\boldsymbol{t}=0}$, $f_{\boldsymbol{t}}$, and d, which are the foundations of calculating margin. In essence, we can transform the above problems into the following two general problems:

(i) What are the laws, over a "certain range", that govern the variation of the product performance with physical properties, external stresses, and degradation time?

(ii) What is the "certain range" of a product?

To answer these questions, it is necessary to start with the "scientific method". In other words, we need to determine and organize the characteristics of a phenomenon by observations and experiments, after which the logic of the human mind should be utilized to formulate conjectures about the causes and conditions of the phenomenon, and then to apply further experiments to test these conjectures and formulate theories through logical deductions based on the verified conjectures [37]. Correspondingly, we must discover and verify the forms of the equations, especially the interdisciplinary, margin, and degradation equations, through science experiments, which is called reliability experiment in belief reliability.

Reliability experiment is regarded as a kind of experiment that explores the product reliable boundary constructed by $\boldsymbol{X}$, $\boldsymbol{Y}$, $\boldsymbol{t}$ and P_{th}, and degradation laws characterized by the causal relationship among P, $\boldsymbol{X}$, $\boldsymbol{Y}$ and $\boldsymbol{t}$ [72]. Generally, reliability experiments are categorized as two types: reliable boundary experiment (RBE) and degradation law experiment (DLE) [72,73]. The RBE is conducted to investigate the boundaries where the product can function well and how the product functions within these boundaries, so as to verify the margin magnitude and Equation No.1. Mathematically, the reliable boundary is constituted by the value ranges of $\boldsymbol{X}$, $\boldsymbol{Y}$, $\boldsymbol{t}$ and P_{th}, denoted as C_X, C_Y, C_t and $C_{P_{\mathrm{th}}}$, respectively. The DLE is designed to explore the time varying degradation path of P and sensitivity factors of $\boldsymbol{X}$, $\boldsymbol{Y}$ over the value space formulated by C_X, C_Y and C_t, thus verifying the Equation No.2 [74,75].

In general, reliability experiments are performed starting from a deterministic theoretical model or hypothesis. The experiments are used to verify the accuracy of the model or hypothesis and to make corrections accordingly. In the actual implementation of the reliability experiments, the means of stress applying in some tests such as reliability enhancement tests or degradation tests (e.g., stepwise applying) can be used to assist in obtaining the reliable boundaries and degradation processes. As mentioned earlier, if we are dealing with a complex system with multiple levels, i.e., when both $\boldsymbol{X}$ and $\boldsymbol{Y}$ become a function, then reliability experiments need to be carried out layer by layer, thus making verifications and analysis of the equations at each level. While exploring certainty laws, the reliability experiment can also cope with uncertainties embraced in the certainty laws, including the uncertainty of parameters (i.e., $\boldsymbol{X}$, $\boldsymbol{Y}$, and P_{th}) and the uncertainty of the functions themselves (i.e., $f_{t=0}$ and f_{t},). These uncertainties can be analyzed and quantified using statistical inference methods in real application, which involves the sample size problem and corresponding statistical techniques.

It also needs to be noted that reliability experiments are different from the traditional reliability testing. Reliability experiments aim to explore the evolution laws of function and performance of products under different design schemes and environmental factors considering various uncertainties, while the reliability testing can only provide a quantification of the stochastic character of product lifespan under a determined condition [2,33]. Therefore, the traditional reliability testing, such as reliability qualification testing and reliability enhancement testing, is actually special cases of reliability experiments [72]. Furthermore, if there is a small sample problem in the reliability testing, it will face the problem of epistemic uncertainty, which is actually not compatible with the reliability testing under the principle of probability sampling. In this sense, only reliability experiments can help us truly understand the product and construct the basic scientific equations.

3.5 Chance prediction

Through theoretical analysis and experiment verification, the certainty laws can be constructed and verified, and the data for uncertainty quantification can also be collected. A crucial issue that arises thereafter is how to predict the reliability of a product by integrating the three basic equations. This raises a vital question: how do we choose the mathematical measure in Equation No. 3? Since the measure is a tool to describe and quantify uncertainty, we must get back to uncertainties.

The previous explorations in reliability engineering have shown that uncertainties are typically classified into aleatory uncertainty and epistemic uncertainty. Aleatory uncertainty refers to the inherent randomness of the physical world [39], and it is widely believed that probability can effectively describe this type of uncertainty [39,76]. Epistemic uncertainty, on the other hand, arises from our lack of knowledge or information [39], and there is still no consensus on the best method to represent it [8]. As we argued earlier, the various existing theories for mathematically expressing epistemic uncertainty still have limitations, but the emergence of uncertainty theory has provided us with a brand new alternative. The uncertainty theory was founded to describe the uncertain phenomenon related to humans [66]. It is based on four axioms and can be regarded as a new branch of mathematics parallel to probability theory [77]. Liu was the first scholar to use uncertainty theory in reliability analysis [78]. Later it was thought to be a good tool for describing epistemic uncertainty and was formally introduced to reliability engineering [13]. Kang et al. have compared it to existing non-probabilistic reliability methods, highlighting its theoretical advantages [10]. Numerous real applications have also demonstrated its superiority in the precision of evaluation, the conformity to actual data, and the stability under small sample cases [75,79]. Therefore, the uncertainty theory is chosen to represent epistemic uncertainty in belief reliability.

Generally, the real products are subject to the combined effects of both aleatory and epistemic uncertainties. Therefore, in belief reliability theory, the chance prediction is utilized, where μ is chosen as a general measure called the chance measure [80]. This measure is essentially a mixture of probability measure (for aleatory uncertainty) and uncertain measure (for epistemic uncertainty). From this perspective, Zhang et al. have identified three cases of the chance prediction [81]:

(i) When we can collect enough data and information of the product, the chance measure may degenerate to the probability measure, and the product can be regarded as a random system that is mainly affected by aleatory uncertainty.

(ii) When we face with small sample data or we have some subjective information from expert inference, the chance measure may degenerate to the uncertainty measure, and the product can be regarded as an uncertain system that is mainly affected by epistemic uncertainty.

(iii) When the product contains the above two cases simultaneously, i.e., some parts have enough data and information while the other parts do not, the chance measure will be used to integrate the two types of uncertainty comprehensively, and the product will be called an uncertain random system.

Based on the above discussions and Equation No.3, the belief reliability is given as the chance that the system margin is greater than 0 [81]. Mathematically, it is written as

$$R_B = \mathrm{Ch}\{\tilde{M} > 0\} \tag{2}$$

where R_B is called the belief reliability, $\mathrm{Ch}\{\cdot\}$ is the chance measure, and $\tilde{M}$ is the system margin. Particularly, for random systems or uncertain systems, we may use special mathematical measures, which are written as

$$\begin{cases} R_B = \Pr\{\tilde{M} > 0\} \\ R_B = \mathcal{M}\{\tilde{M} > 0\} \end{cases} \tag{3}$$

where $\Pr\{\cdot\}$ and $\mathcal{M}\{\cdot\}$ are probability measure and uncertain measure, respectively. According to the three basic equations of belief reliability, the belief reliability function can be more specifically written as

$$R(\boldsymbol{X},\boldsymbol{Y},t,\boldsymbol{t},P_{\mathrm{th}}) = \mathrm{Ch}\{\mathrm{d}(f_t(\boldsymbol{X},\boldsymbol{Y},t),P_{\mathrm{th}}) > 0\},$$

showing that all the factors related to reliability are considered and the chance prediction is more complete.

Researchers have identified two special cases for different usages of prediction framework [7]. Firstly, the margin can be narrowly defined as the performance margin, denoted as m_p, such as electrical or mechanical performance margin, representing the distance between physical performance and its threshold. The corresponding belief reliability is called the performance margin-based belief reliability, which was first proposed by Zeng et al. [82,83] and later refined by Zhang et al. [81] and Kang [7]. Secondly, the margin can also be defined as the time margin denoted as m_T, such as failure time margin or service time margin, representing the gap between a featured time and the specified time threshold. The corresponding belief reliability is called the time margin-based belief reliability and was modified from Zeng's [13,84] definition by Zhang [85]. In most cases, the performance margin-based belief reliability is used for reliability analysis, design and experiment, while the time margin-based belief reliability is mostly used for reliability evaluation. In fact, these two special cases provide different perspectives on prediction based on the basic equations. The first one predicts directly using the performance margin magnitude, while the second one predicts based on the reliability boundary of degradation time. Of course, we can also provide prediction methods based on reliability boundaries of environmental stress or product physical properties accordingly, when other factors affecting reliability are fixed. This means that the prediction of reliability in belief reliability theory can be more flexible. It can adapt to most of the reliability engineering scenarios and information sources, demonstrating the powerful capabilities of belief reliability theory.

It is important to note that chance prediction is carried out on the basis of deterministic equations with the consideration of uncertainty. Therefore, a key aspect of prediction is to identify and quantify the uncertainty in the equations. As stated before, the analysis and quantification of uncertainty are usually achieved by conducting reliability experiments to collect relevant information. From the perspective of the three basic equations, sources of uncertainty include the variables such as $\boldsymbol{X}$, $\boldsymbol{Y}$, P_{th}, and the structure of the equations themselves [85]. Therefore, when applying reliability experiments, we must use the related data to model and analyze different kinds of uncertainties through statistical methods. In general, probability distributions are used to describe variables/parameters with aleatory uncertainty, such as the dispersion of shape variables and material parameters; and uncertainty distributions are used to model the epistemic uncertainty, for example, the design variables without sufficient data, environment parameters with lacking knowledge about application scenarios, and incompleteness of the model itself. Moreover, after making a chance prediction of the product reliability, we can also feed the result back to the basic equations and reliability experiments, making further updates or validations.

What is discussed above has formed a relatively complete framework of belief reliability metrics. We hereby summarize it in Fig. 3. Under this framework, several variants related to belief reliability metrics have emerged, including uncertain random reliability index [86,87], belief availability [88], and uncertain connectivity reliability [89].

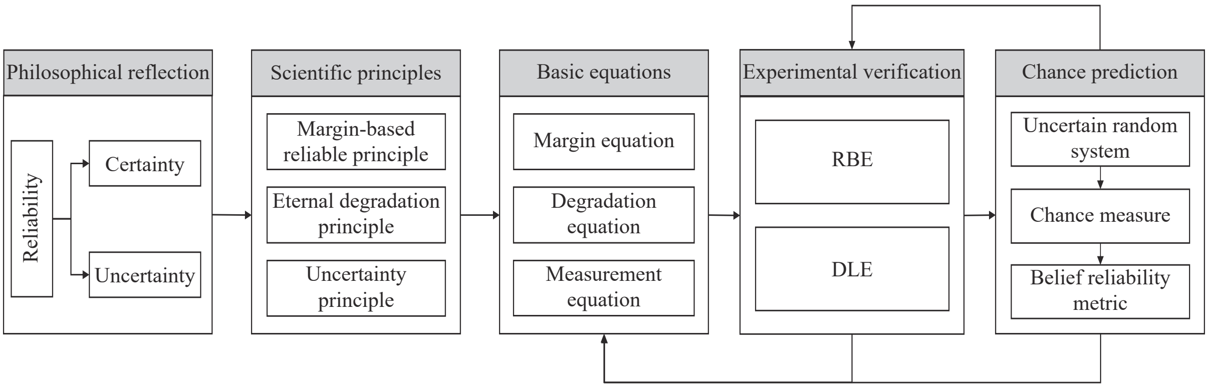

Fig. 3 Framework of belief reliability

4. Belief reliability methods and technologies

Under the theoretical discourse and framework of belief reliability, critical methods and technologies are sequentially developed. In literature, scholars mostly focus on three aspects, namely, belief reliability modeling and analysis, function-oriented belief reliability design, and belief reliability evaluation. Some other methods and technologies, namely maintainability and supportability, risk analysis, and PHM, are also being researched under the framework of belief reliability in recent years. In this section, we tend to introduce some current results from the above studies, with the aim of highlighting the advantages and advancements of belief reliability theory.

4.1 Belief reliability modeling and analysis

Belief reliability modeling and analysis is the most important and fundamental technology in belief reliability theory, for which it receives the most attention in recent research. The main topics about belief reliability modeling and analysis are summarized in Table 2.

Table 2 Research topics and representative papers about belief reliability modeling and analysis

Topic	Representative paper
Methodology	Basic model and analysis procedure [7,13,83,85], adjustment factor model [82], structure reliability index method [86,90]
Belief reliability analysis based on the basic equations	Belief reliability analysis of different products: hydraulic servo actuator [82,85], aircraft lock mechanism [91], gear [92], electrical connector [93], harmonic reducer [90,94], DC power supply [95], filter circuit [73], turbine disk of aircraft engine [96]
Uncertainty propagation	Propagation formula and algorithm [85,97], propagation model in different stages [96]
Belief reliability analysis of different system configurations	Series systems [13,78,81,84,98–101], parallel systems [13,81,84,98–101], series-parallel hybrid systems [81,99–102], cold standby systems [99,101,103–105], warm standby systems [103], k-out-of-n systems [84,100,106–109], bridge systems [85,110], general configurations and algorithms [78,81,100,110]
Fault tree analysis	Minimal cut set theorem [13,84], analysis algorithm [84]
Importance index	Uncertain system [108,111], uncertain random system [112]
Belief reliability analysis of systems with degradation-shock dependency	Uncertain degradation with random shock arrival time and uncertain shock size [113,114], uncertain degradation with uncertain shock [115–118], degradation-shock dependency with a change point [119] and uncertain failure threshold [120], belief reliability analysis with different shock modes [121], belief reliability analysis for systems with failure trigger effect [122]
Network system belief reliability	Connectivity belief reliability of transportation network [89] and crude oil maritime network [123], belief reliability of traffic network [124,125], service reliability analysis of cloud data centers [126], cascading failure modeling for circuit network [127]
Belief reliability analysis of multi-state system	Modified universal generating function technique with uncertain measure [128], multi-state k-out-of-n system [108], uncertain state transition chain method considering degradation [129]

Basically, the belief reliability modeling and analysis is conducted under the theoretical framework shown in Section 3 [7,45]. More specifically, we firstly, in most cases, need to establish the various Equation No.0 according to the specific disciplines that the product's critical performance parameters belong to. Later, the Equations No.1 and No.2 need to be obtained considering the characters of the performance parameters and the effect of internal/external factors. Various uncertainty effects should be then quantified and propagated using reasonable mathematical tools, and Equation No.3 can thus be constructed. Inevitably, the construction of the four equations needs to be verified by reliability experiments, including the form of equations, the sensitivity of influencing factors, and the uncertainty character of important variables. The above procedure is mainly summarized in some methodology papers involving belief reliability, where several models are also put forward (see Table 2).

As explained before, a vital problem of the method is the uncertainty quantification and propagation, which means to quantify the uncertainty characters of input variables within the Equations No.1 and No.2 and to calculate the uncertainty character (e.g. distribution, moment) of the performance margin based on them. Accounting for the amount of available data and information, the variables are usually classified into different uncertainty categories, where aleatory uncertainties are described by random variables and epistemic uncertainties are described by uncertain variables [85]. For random variables, we can often collect enough data to make estimates of their probability distributions, and the commonly used probability statistics methods can all be used for uncertainty quantification. For uncertain variables, there is often only sparse data that we can collect, so uncertain statistical methods are usually used to obtain their uncertainty distributions. Commonly used uncertain statistical methods include two types: parametric and nonparametric methods. The parametric methods include the method of moments [130] and the method of graduation formula [131]; the nonparametric methods include the maximum entropy method [132], the distribution average based on the Delphi method [133], and the interpolation method of expert empirical data [40]. To verify whether the obtained uncertainty quantification results match the actual sparse data, the hypothesis testing method under uncertainty measure can also be used [134]. Based on the result of uncertainty quantification, different uncertainty propagation methods based on chance theory, uncertainty theory and probability theory have been proposed, and thus different belief reliability formulas in terms of performance margin are put forward [85,90].

In addition to the previous issues, scholars also studied the belief reliability modeling and analysis according to the system structures and compositions using reliability logic models. It should be noted that some of these studies are still under the assumptions of independent situation of components. Although it does not make sense in the context of reliability science, these studies make important contributions to the thinking of the "puzzle of system reliability" and is therefore presented in this paper. Among these research, the product axiom of uncertain measure called "minimal event measure" [135] is widely utilized, and a lot of them have concentrated on the system belief reliability analysis of different system configurations with both aleatory and epistemic uncertainty, including series system, parallel system, k-out-of-n system, and cold standby system. The concepts of uncertain systems, random systems and uncertain random systems are put forward to describe the different uncertainty categories embraced by the systems (see [81]), and considerable belief reliability formulas and importance measure are developed by using uncertainty theory, probability theory and chance theory, respectively. In addition, some researches have also attempted to model the dependence in reliability at the system level using system simulation and surrogate models [124], semi-Markov chain models [125], petri net models [126], and cascading failure models [127].

Another hotspot of belief reliability modeling and analysis is oriented to the system with degradation-shock dependency. Considering the epistemic uncertainty, the system degradation process is usually modeled as an uncertain process (e.g., the Liu process [135]) and the shock process is modeled by a Poisson process or an uncertain renewal process [135]. Several models are proposed to account for different interact effects of degradation and shock. The belief reliability analysis method also involves network systems and multi-state systems, in which the probability theory and uncertainty theory are flexibly utilized for different uncertainties and have shown their effectiveness.

In general, the significance of belief reliability modeling and analysis methods are manifested in two aspects. First, by using the basic equations to model products, the reliability can be truly linked to the value spaces of variables (i.e., the internal and external factors) related to the function of products. This makes it possible to improve the reliability of products by changing design variables and controlling the external environment, so as to achieve a function-oriented reliability design (see Subsection 4.2). Second, the existing results show that the "puzzle of system reliability" can be solved to some extent by introducing uncertainty theory and some system reliability models to analyze reliability, thus the over-amplification of epistemic uncertainty and the excessive risk-taking in the decision-making process can be avoided [85,96].

4.2 Function-oriented belief reliability design

Function-oriented belief reliability design, regarded as another important technique of belief reliability theory, refers to a reliability design process that utilizes methods of belief reliability analysis and optimization to acquire the design options and control strategies under the constraints of reliability requirements related to each "specified function" of a product. The ultimate target of function-oriented belief reliability design is to make a product achieve its "specified function" as well as possible. Since the quantitative relationships between reliability indexes and the internal/external factors can be established and various uncertainties can be reasonably described, the function-oriented belief reliability design method can better embed reliability into the product function and performance design process (especially for the development of innovative or new products with great epistemic uncertainty) and provide a more convenient and persuasive design approach.

Essentially, the function-oriented belief reliability design is conducted according to the basic equations of belief reliability. In this paper, we summarize the main procedure in Fig. 4. Basically, it contains mainly four parts, namely the requirement input process, the belief reliability analysis process, the iterative optimization process, and the specifications output process.

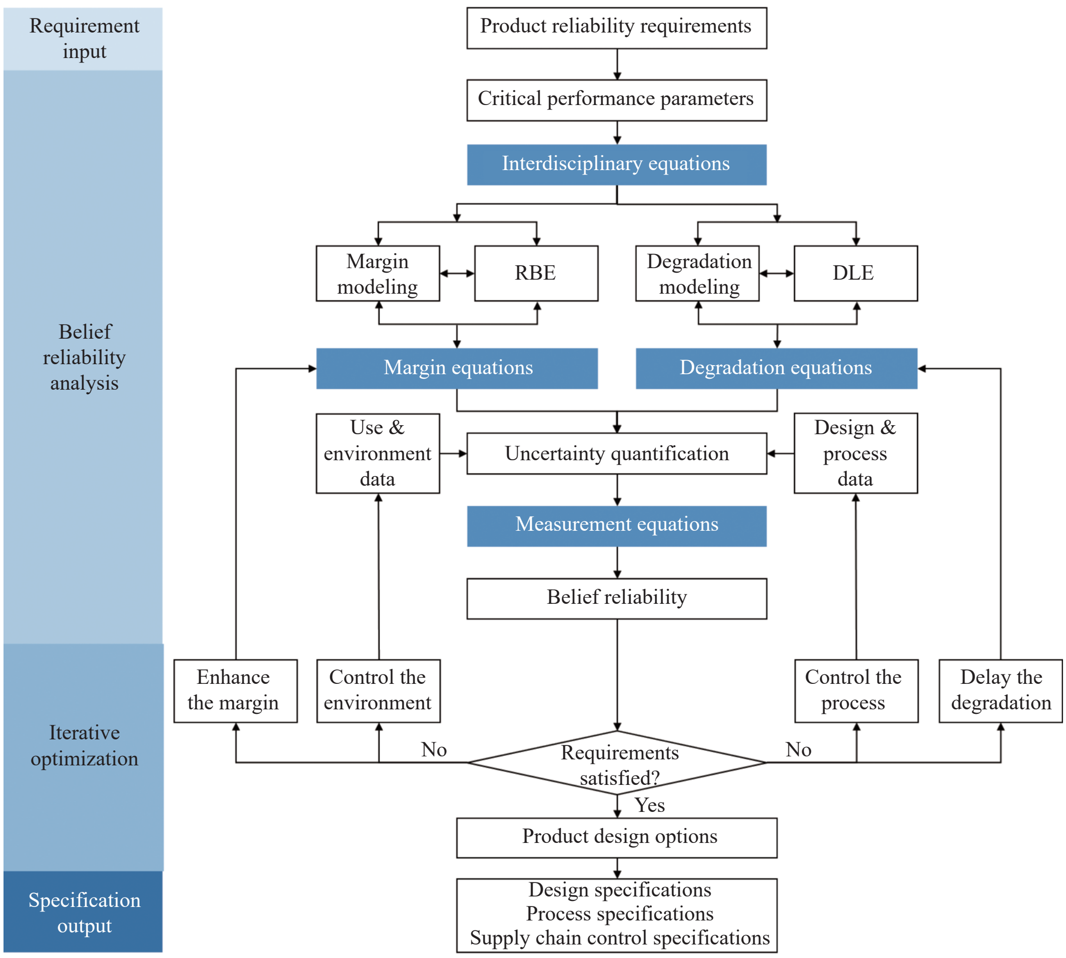

Fig. 4 Flow chart of function-oriented belief reliability design

The reliability requirements, which are the input of the function-oriented belief reliability design process, are usually determined comprehensively after balancing the user demands and costs [136]. For products with hierarchical relationships, the belief reliability allocation method is usually used to decompose the system reliability requirements into individual subsystems and components [137,138]. It is noted that the reliability requirements may vary for different products. For example, the reliability requirements of an aircraft usually fall on the total lifetime and mean time to failure (MTTF), but for an electronic product, the requirements may be the initial reliability.

The belief reliability analysis process has been previously reviewed in Subsection 4.1. It is regarded as a fundamental part of the function-oriented belief reliability design because it will provide the objective for the following optimization process. In this part, the determination of critical performance parameters and the usage of reliability experiments are two important parts that should be further discussed:

(i) The determination of critical performance parameters. In the function-oriented belief reliability design, a formalized method called "function, performance and margin analysis" (FPMA) is usually utilized to obtain the critical performance parameters thus avoiding overly burdensome analysis and design work [85,139]. FPMA is developed to figure out the key factors affecting product reliability by analyzing the functions, corresponding performances, and margins step by step. Through FPMA, we can not only obtain the performance parameters that affect product reliability significantly, but also identify the core design variables that we should concentrate on in reliability analysis and design optimization.

(ii) The use of reliability experiments. Reliability experiments are indispensable to better understand the product and can help designers choose better design options according to margin magnitude and degradation path. For a newly developed product, as pointed out previously, the RBE and DLE are effective tools to verify the margin and degradation, respectively. Another vital role of the reliability experiments is to embrace the internal factors (design and process variables) and external factors (use and environmental variables) in the credible Equations No.1 and No.2, making it possible to increase product reliability by controlling these variables, which is also the initial intention of function-oriented belief reliability design.

The iterative optimization process is to ensure that we can get the optimal design options. In this part, we need to first compare the calculated belief reliability index with the reliability requirements. If the requirements are not satisfied, the designers should try to increase the product reliability by various feasible means. The most direct strategy is to enhance the margin and delay the degradation by adjusting the value combination of design variables according to the Equations No.1 and No.2. Another effective method to achieve this strategy is to use redundancy techniques [140–142], which may change the form of the first two basic equations but will work well at the same value combination of design variables. As the uncertainty is another important factor of belief reliability, we can also try to decrease the effect of uncertainty by controlling the environment and manufacturing processes, thereby increasing the product reliability. Because the internal factors, the external factors, and their uncertainty characters are all involved in the four equations, we can choose and make trade-offs within the above measures under the constraints of cost and resources. Different optimization methods can be used in this part to achieve the reliability goals according to different demands and product situation [73,143,144].

The specifications output process is a summary of all the above processes by the designer. For different products, there are usually different means of designing for reliability. In the function-oriented belief reliability design, the designers should organize the practiced reliability design methods into several specifications according to the characteristic of the product. The output specifications should include, but not limited to, the design specification, the process specification, and the supply chain control specification. These specifications can improve the efficiency of research and development processes, and the reliability of similar products can be enhanced by referring to them. Of course, the specifications also need to be updated after accumulating the actual using data, maintenance feedback, and other information.

4.3 Belief reliability evaluation

Belief reliability evaluation refers to a technique that aims to calculate the system belief reliability using statistical methods with the various data that provide information of reliability. From the perspective of the scientific principle of reliability, the product reliability data is essentially the margin data of the product, because it directly reflects how reliable the product is according to the margin-based reliable principle. Referring to the belief reliability function, the information of reliability data (margin data) is then shown by the character of the data for $\boldsymbol{X}$, $\boldsymbol{Y}$, $\boldsymbol{t}$, P and P_{th}. In this paper, we call the data of these factors as the reliability metadata. Different combinations of reliability metadata will derive different information about reliability (or margin), thus different belief reliability evaluation methods should be used.

It is an undeniable fact that the more adequate reliability metadata we can collect, the more accurate the belief reliability evaluation will be. According to the amount of data, belief reliability evaluation methods usually choose different mathematical tools for statistical analysis. In existing literature, traditional probability statistical methods are effective when a large sample of data is available. Once we can only get a small sample of data, uncertain statistical methods based on uncertainty theory should be utilized because the probability statistical methods are based on the assumption of big sample size. As elaborated in the "dilemma of probability", there are few cases with a large sample size. Therefore, this paper focuses on belief reliability evaluation methods based on small sample data.

To better elaborate the methods and models of belief reliability evaluation, in this paper, we establish a "metadata tree" (see Fig. 5) to show the method that is suggested to be chosen for different combinations of collected metadata. This metadata tree is rooted in the metadata set that we obtain in real engineering practices.

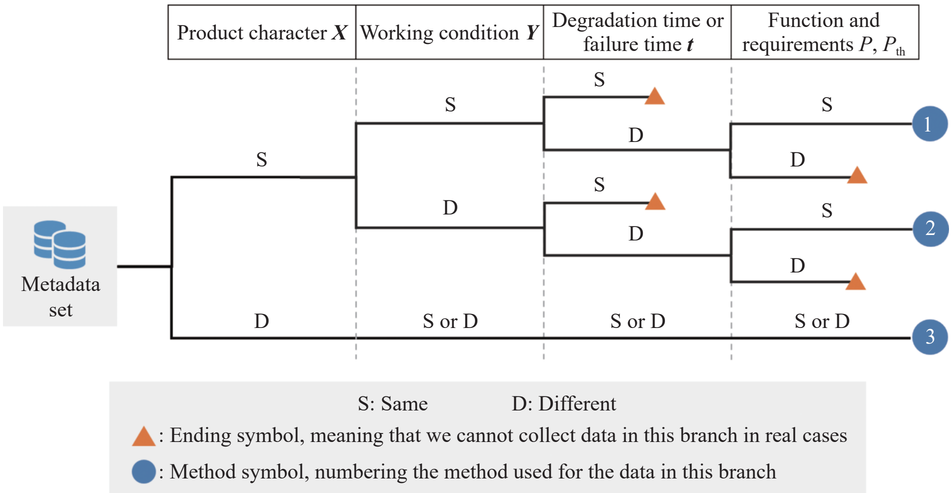

Fig. 5 Metadata tree for belief reliability evaluation

By sequentially figuring out whether the elements described by the collected metadata are same or not according to the criteria given in Table 3, a numbered method symbol can be obtained. Then, the belief reliability evaluation methods or models can be found according to Table 4, where the main research of belief reliability evaluation are listed. For example, if we collect metadata in an accelerated degradation testing for a product with several samples, then the metadata will correspond to the products with same $\boldsymbol{X}$, accelerated stress $\boldsymbol{Y}$ with different levels, different $\boldsymbol{t}$, and same P and P_{th}. According to the metadata tree, we need to find the methods with symbol 2 in Tbale 4. Further, since the data mainly embrace the information of degradation, the uncertain process model and time-variant distribution model should be chosen.

Table 3 Meaning of "same" and "different" for each element in metadata set

Element of metadata	Meaning of being "same"	Meaning of being "different"
$\boldsymbol{X}$	The products corresponding to the collected metadata are from the same population, i.e., the types and design values of $\boldsymbol{X}$ are same for each product	The products corresponding to the collected metadata are from the different populations, i.e., the types and design values of $\boldsymbol{X}$ for each product may be different, but similar, for example, the digital simulation prototype, improved version, etc
$\boldsymbol{Y}$	The type and size of the stress that the products bear during working are all the same	The type of the stress that the products bear during working is same, but the size is different, for example, different stress level in tests, stress in both tests and actual use, etc
$\boldsymbol{t}$	—	The degradation time or lifetime/failure time obtained are usually different due to the uncertainty of products
P, P_{th}	The products corresponding to the collected metadata are designed for same function with same performance parameters and requirements	(i) The products corresponding to the collected metadata are designed for same function with different performance parameters and requirements (ii) The products corresponding to the collected metadata are designed for different functions

Table 4 Belief reliability evaluation methods or models for different metadata combinations

Method symbol	Method or model	
1	Graduation formula method [131], the kth moment method based on maximal entropy principle [132,145], order statistics method with Bernard approximation [146]	
2	Metadata with information of degradation	(i) Uncertain process-based models [75,147,148] $P(t)=(e(s)+\sigma_s\xi_s)\cdot t^{\beta}+\sigma C(t^{\beta})$, $C(t^{\beta})\sim \mathrm{N}(0,t^{\beta}),\ e(s)\sim \mathrm{N}(\mu_e(s),\sigma_e(s)),\sigma_s\xi_s\sim \mathrm{N}(0,\sigma_s s)$ (ii) Time variant uncertainty distribution model [149] $\Phi_p(x\|t)=\left(1+\exp\left(\frac{\pi(e(t)-x)}{\sqrt{3}\sigma(t)}\right)\right)^{-1}$ (iii) Uncertain differential equation-based model [150] $\mathrm{d}P(t)=\beta_0\gamma\exp(\beta_1\phi(s))t^{\gamma-1}\mathrm{d}t+\sigma\gamma\exp(\beta_1\phi(s))t^{\gamma-1}\mathrm{d}C(t)$, $C(t)\sim \mathrm{N}(0,t),\quad P(0)=0$ (iv) Performance and health status margin degradation framework for belief reliability evaluation [151]
	Metadata with information of lifetime/failure time	Data fusion method: consistent belief degree method (data equivalence method and constant coefficient of variation method) [79], maximal cross entropy method [152]
3	Similarity fusion method [152]	

Note the research in existing literature tabulated in Table 4, a significant point is to determine an uncertainty distribution through the small sample data collected by different ways, especially the various data obtained in the whole product developing process, including the simulation and testing in design phase, and the actual operation in use phase. The proposed methods are concentrated on the identification of the belief degree of each data point and the expansion of data set, thus forming a reasonable uncertainty distribution with as much data information as possible for belief reliability evaluation. Researchers have also made some comparisons between these methods and other methods to cope with small sample data. For example, by randomly selecting small sample data from the whole data set, the reliability prediction results of the proposed belief reliability evaluation models using uncertainty theory are more stable than those obtained by Bayesian methods [79,147]. Besides, the predicted degradation paths acquired with these methods have shown better fitness to the real data [148], which indicates the proposed belief reliability evaluation process may be more convincing under the cases with only small sample data.

4.4 Other methods and technologies

In recent years, some new belief reliability-based methods and technologies have gradually come into being in the face of real engineering scenarios with great epistemic uncertainty. In this paper, we summarize these researches in Table 5.

Table 5 Some new belief-reliability-related methods and technologies

Field	Representative paper
Belief reliability centered maintenance and supportability optimization	(i) Maintenance optimization model: maintenance indexes and analysis for repairable systems [153], optimization of the level of repair considering spare stocks [154] (ii) Spare parts optimization: variety optimization [155,156], quantity optimization [45,157], depot location optimization [158–160]
Risk analysis	Uncertainty representation and propagation in risk analysis [161–163], aviation risk assessment [164,165], transportation risk assessment and avoidance [89]
Prognostics and health management	Failure prognostics with scarce data [166], remaining useful life prediction with degradation data [167]
Software belief reliability assessment	Software belief reliability growth model using uncertain differential equation with perfect [168] or imperfect [169] debug processes
Others	Belief reliability analysis of supply chain [170], soil slopes design [171], assembly line analysis and optimization [172], path selection in maritime transportation [173]

The belief reliability centered maintenance and supportability optimization plays an important role in the belief reliability framework. It is strongly related to the availability of products, aiming to obtain the optimal maintenance strategy or spare parts plan. In existing literature, these problems are usually transformed to multiattribute decision problems with reliability constraints or objectives, thus the data envelopment analysis (DEA) method is widely utilized, especially in the spare parts optimization. For the widely existing cases with small samples in maintenance and supportability optimization, the uncertainty theory is introduced to DEA (i.e., the uncertain DEA [174,175]). Due to the lack of real data, there may be some human influence on the determination of variable distributions or solution preferences, but the uncertain DEA method can eliminate as much as possible the influence of subjective human tendencies on decision making, thereby giving a reasonable solution for the maintenance strategy or spare parts plan.

Risk analysis and PHM are also performed under the frame of belief reliability theory. Similarly, the uncertainty theory and probability theory are used to model the epistemic uncertainty and aleatory uncertainty, respectively. It is found that the risk analysis results based on belief reliability show better duality and robustness than the Bayesian methods when the epistemic uncertainty cannot be ignored [161,162], and the results in PHM under belief reliability framework can better fit the degradation data and provide a more precise prediction of remaining useful life [166,167]. Software belief reliability assessment is a new application of belief reliability theory. Due to the complex failure mechanisms and the strong relationship with coders, the software is always full of epistemic uncertainty. Thus, the belief reliability theory is utilized to assess the software reliability under the process of debugging based on uncertain differential equations [168,169].

5. Summary and prospects

Belief reliability is a scientific theory aiming to solve theoretical and practical problems in reliability engineering. Compared with the present reliability engineering theory that resorts to “phenomenological theory”, belief reliability constructs the “first principle” of reliability science, based on which the basic theories are systematically explored, and the methodological research is carried out, so that it can better serve reliability engineering. In this paper, we have comprehensively elaborated the related issues about belief reliability, including its origin, theoretical discourse and basic methods and technologies. As overviewed in this paper, the belief reliability theory is now also applied in different areas, such as aeronautics, astronautics, automobile, and electronic products.

It should also be noted that the current research regarding belief reliability is mainly conducted by Chinese researchers. To show the whole research picture of belief reliability, this paper compiles relevant research from all over the world as far as possible. We hope to convey the latest thoughts of Chinese researchers to all over the world, and we believe this is one of the contributions to the world reliability community.

In sight of these research and practices, we hereby provide a broad summary about the significance of belief reliability theory:

(i) Belief reliability reveals that reliability science is an integration of certainty and uncertainty. By constructing a theoretical discourse system consisting of philosophical reflection, scientific principles, mathematical expressions, experimental verification, and chance prediction, the belief reliability theory provides a new paradigm for studying reliability science.

(ii) Belief reliability theory realizes the complete expression of reliability function, expands the connotation of uncertainty, and enriches the way of reliability research of complex systems. This enables the “query of reliability function”, the “dilemma of probability”, and the “puzzle of system reliability” in reliability engineering to be fully solved.

(iii) Belief reliability have provided affluent methods and techniques oriented to various problems in reliability engineering, including measurement, analysis, design, verification, and evaluation, which have shown great advantages. This indicates a good adaptability of belief reliability theory to the reliability engineering practice and provide an important direction for future reliability research.

In the long run, the current research of belief reliability is still in the growth stage. Numerous details of belief reliability methods and technologies still need to be enriched. For example, the dependency of performance margin that involves different disciplines should be further studied in belief reliability analysis; more distribution types should be considered in belief reliability evaluation; more optimization models and algorithms need to be developed, etc. More generally, to better promote the development of reliability science and engineering, it is convinced that further work on reliability theory is needed in the following areas.

(i) About the theoretical system

The present belief reliability theoretical discourse, especially the description of margin and degradation in the basic equations tend to be relatively conceptual. Will there be a uniform representation of margin that can integrate different disciplines and interpret the evolution of systems? For this question, we may need to resort to system sciences, from which a more comprehensive operating behaviors and laws of products can be explored.

In the framework of belief reliability, a central element is the system function and its associated performance. In terms of system characteristics, the relationships between system components are usually nonlinear and have dynamic characters. In this case, the exhibition of system performance is essentially due to the continuous flow, conversion, transfer, and dissipation of energy within the system [176]. Therefore, if the system operation process can be studied from the perspective of energy generation and dissipation, it could be possible to give a general expression of the system performance emergence, and thus obtain some common representations of the margin.

(ii) About the application

From the perspective of application, though the belief reliability has been tested in practice, it is still far from the standard of "engineering science" mentioned by Xuesen Qian.

When applied to the general industrial systems, there is still an urgent need for more individualized belief reliability methods and techniques regarding different disciplines. For example, for electronic products, there are multiple characteristics to consider such as the matching of component performance, the ability of the circuit to withstand stress (temperature, vibration, etc.), and the depletion of the circuit board structure (e.g., solder joint fatigue). The reliability equations corresponding to these characteristics are all different. For another example, many performances of control systems are constrained by differential equations and there is usually no explicit solution, so the reliability equation in this case should also be expressed differently. Besides, for different types of products, the uncertainty factors are also different. Further exploration is needed in practical applications.

Furthermore, we are convinced that the belief reliability theory should be applied to a wider range of systems besides engineering systems, such as financial systems, social systems, human body systems, artificial intelligence systems, etc., thereby help reliability morph into a common science that can guide the development of technology in various fields.

(iii) About the reliability tools

From the perspective of serving humanity and transforming the world, the methodology which attempts to discover the laws of "product to be reliable" in belief reliability theory is still an idealized framework. In real engineering practice, it is necessary not only to conduct detailed modeling for products, but also to carry out reliability experiments as much as possible to verify the relevant laws and accumulate models and experiences. In this process, we also need to use more digital and intelligent tools. For example, by building a digital twin model based on belief reliability, we can better guide design, production and use. In addition, to achieve higher product reliability at a relative lower cost, we also need to organize and develop various standards, databases, and software related to belief reliability, making the theory more practical and instrumental.

In summary, reliability research should intersect with other disciplines more extensively and deal with practical problems in reliability engineering more flexibly, so as to construct a more scientific system of reliability. With the rapid theoretical research and numerous practical applications of belief reliability, we believe this new theory can contribute more to reliability engineering.

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