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Design and pricing of maintenance service contract based on Nash non-cooperative game approach

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Abstract: Nowadays manufacturers are facing fierce challenge. Apart from the products, providing customers with multiple maintenance options in the service contract becomes more popular, since it can help to improve customer satisfaction, and ultimately promote sales and maximize profit for the manufacturer. By considering the combinations of corrective maintenance and preventive maintenance, totally three types of maintenance service contracts are designed. Moreover, attractive incentive and penalty mechanisms are adopted in the contracts. On this basis, Nash non-cooperative game is applied to analyze the revenue for both the manufacturer and customers, and so as to optimize the pricing mechanism of maintenance service contract and achieve a win-win situation. Numerical experiments are conducted. The results show that by taking into account the incentive and penalty mechanisms, the revenue can be improved for both the customers and manufacturer. Moreover, with the increase of repair rate and improvement factor in the preventive maintenance, the revenue will increase gradually for both the parties.

Keywords: maintenance service contract, Nash game, incentive and penalty mechanism, corrective maintenance, preventive maintenance.

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1. Introduction

For engineering products, such as wind turbine and construction machinery, their performance will degrade with the increase of service time. If the maintenance service is not timely and reasonable, it may result in failures. Meanwhile, due to lack of specialized knowledge of the products as well as professional maintenance devices, it is usually more economic for customers to outsource maintenance activities to the manufacturer or service agent, rather than maintaining the products by themselves [1]. To simplify the expression, here the term of "service agent" is used to refer to the manufacturer or maintenance service provider.

Commonly used maintenance options include corrective maintenance (CM) and preventive maintenance (PM). Among them, CM is used to restore failed items to operational state; and PM is done in advance to prevent from excessive degradation or possible failures, either based on the age or performance of the items. Su et al. [2] proposed a type of two-dimensional PM strategy, where the implementation of imperfect PM activities was dependent on both the age and usage of the products. With the aim to minimize total maintenance cost, Cheng et al. [3] proposed a joint optimization model by considering the production lot sizing and condition-based maintenance, moreover economical dependency among the components was considered when performing CM. Selecting single-unit systems as the objective of study, Ma et al. [4] proposed a joint optimization policy of PM and spare ordering, with the aim to minimize the long-run expected cost rate, optimize the inspection and age-replacement intervals, and maximize the number of imperfect maintenance respectively. For customers, they need to optimize the items in the contract according to the product's availability requirement, and so as to achieve maximum benefit. Thus, as a scientific maintenance service contract, it should be beneficial for both the service agent and customers.

Actually, the two parties (i.e. the service agent and customers) have obvious benefit gambling in the maintenance contract. Thus, game theory attracts much attention in this area, and it can help them to share expected profits in a bargaining way. Murthy et al. [5] formulated two types of models for maintenance decision-making, and the optimal strategies were derived with game theory for both parties respectively. Murthy et al. and Ashgarizadeh et al. [6,7] developed a Stackelberg game model to optimize the strategies for both customers and service agent. In their model, the duration of failures was

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assumed to obey exponential distribution, and the penalty terms were adopted in the maintenance service contract. Kim et al. [8] constructed a multi-task principal agent model, where service provider was the agent and customers was the principal. Moreover, the aim was to minimize total maintenance cost, including fixed payment, customer's shared maintenance cost, and penalty payment for not satisfying the required performance metric, etc. Jackson et al. [9] applied Nash non-cooperative game theory to negotiate the pricing of the service contracts, and both the parties shared expected profits in a bargaining way. Iskandar et al. [10] designed three types of maintenance service contracts, and non-cooperative game theory was applied to optimize the pricing mechanism of the contract. Jin et al. [11] proposed a multi-party, multi-criterion and multi-purpose service contract delivery mechanism, where game theory was used to optimize the inventory of spare parts and maintenance activities, and so as to guarantee the availability and maximize the interest for both customers and agent. Chagas et al. [12] provided maintenance service contract with different priorities for two types of customers, and the optimal results were obtained for each type of customers by using Stackelberg game and queuing theory. Feng et al. [13] analyzed the process of making decision on condition-based maintenance (CBM) for military fleets, a heuristic hybrid game approach was proposed by integrating competition game and cooperative game with the heuristic rule.

As a whole, the maintenance contracts can be classified into two types, i.e. time and material maintenance contract (T&MMC) and performance- based maintenance contract (PBMC) [14]. In T&MMC, customer pays for the resources (including the time and materials, etc.) consumed by the service agent. On the contrary, PBMC focuses on the outcome or performance of maintenance actions. Recently, the design of PBMC has received much attention [15]. Under the condition of required availability level and fixed budget, Wang [16] provided three types of options in the maintenance service contracts including PBMC, and the parameters in the contract were optimized for the customer under each option. To optimize the interval and degree of PM, Iskandar et al. [17] studied maintenance service contract for the products with two-dimensional warranty period. Su et al. [18] investigated an availability-based warranty policy for the products with availability requirement, where besides providing repairs or replacements upon failures, the manufacturer also guaranteed a negotiated availability level for the products within the warranty period. With considering stochastic dependency and economical dependency among the components, Vijayan et al. [19] proposed a type of maintenance optimization model, where the penalty function was formulated to optimize maintenance interval and associated cost benefits.

Studies indicated that PBMC can prompt the service agent to optimize the maintenance process and improve the service quality. For service agents, their reward from PBMC is heavily dependent on the availability of the product. Thus, to maximize the profit rate, the service agent needs to improve the availability. Jing et al. [14] proposed a framework to establish a risk-based PBMC model from the view of the service provider, where multiple performance measures were taken into account. Xiang et al. [20] analyzed the impact of PBMC on maintenance decisions for repairable systems. Li et al. [21] presented a multiple-component maintenance policy to maximize the operating revenue under the requirement of shipborne antenna's availability. It was concluded that compared with T&MMC, PBMC can improve the product reliability by about 25%−40%. Li et al. [22] studied the availability of periodic inspection system under mixed maintenance policy, an analytical model was derived for the system availability. To maximize the discounted profit over the product lifecycle for each option, Darghouth et al. [23] designed four types of maintenance service contracts by considering CM and PM, where the manufacturer was allowed to set the optimal combination of product reliability, sale price and warranty period. To ensure system availability and maximize profit for the manufacturer, Wang et al. [24] proposed a maintenance strategy on the basis of PBMC, where CM, PM and opportunistic maintenance were adopted concurrently. Wang et al. [25] dealt with reliability and condition-based maintenance modeling for the system under PBMC, where the long-run maintenance cost rate and average system availability were evaluated, and optimal preventive replacement strategy was obtained.

Up to now, penalty cost and punishment mechanism have been adopted in some studies, while incentive measures were often ignored. However, if no incentive mechanism is considered in the contract, the quality of maintenance service may be reduced. Mirzahosseinian et al. [26] and Husniah et al. [27] studied maintenance service contract with incentive terms, where the service agent was rewarded if the performance of the product exceeded its stated target. By applying the mechanism design theory, Hong et al. [28] presented an approach to design and optimize the maintenance service contract, where the contract including incentives was used to trade off the interests of the service provider and customers. Husniah et al. [29] studied two types of PBMC for a fleet of dump trucks, where the truck's performance as well as penalty and incentive mechanisms were considered. Li et al. [30] analyzed the bilateral negotiation of service contracts between the customer and repair service supplier, and three types of service contracts were considered. Moreover, the incentives under each contract were investigated. Existing studies show that to enhance the competitiveness and promote customer's satisfaction, punishment and reward mechanism should be taken into account concurrently in the maintenance service contract.

In this paper, the concepts of PBMC and Nash non-cooperative game are adopted to negotiate the pricing mechanism in the maintenance service contract, so as to maximize the revenue for both the service agent and customers. The major contributions include: (i) three types of maintenance service contracts are designed by combining CM and PM, listed as follows: CM provided by the service agent, PM implemented by the customer, CM and PM provided by the service agent, respectively; (ii) a Nash non-cooperative game model is developed to optimize the price structure for both the service agent and customers; (iii) incentive and penalty terms are taken into account on the basis of multi-threshold failure time and in the third type of contract.

The remainder of this paper is organized as follows. Section 2 presents the details of the model formulation. Section 3 provides the procedure to analyze the optimal price structure of each contract. Section 4 conducts case study and sensitivity analysis for the parameters. Conclusions and future research directions are presented in Section 5.

2. Model formulation

2.1 Contract options

Previous studies indicated that the design and pricing mechanism of maintenance service contract has obvious impact on the availability of product, thus it will influence the benefits for both the customers and service agent. As the game player, customer usually prefers to gain higher availability of product and expected revenue with reasonable maintenance cost. On the contrary, the service agent aims to determine reasonable contract pricing and obtain higher expected profit with as little service cost as possible. In a sense, their interests are often conflicting. In this study, customer can choose an option for purchased product from one of the following three types of maintenance service contracts, listed as follows:

Option A_0: service agent performs CM at failures until the end of service contract period, and PM will not be adopted. Moreover, a fixed cost will be charged for each time of CM.

Option A_1: customer carries out PM at a regular interval, and the cost for each time of PM is C_{ps}. Moreover, the service agent performs CM when failures occur over the contract period, and the cost for each time of CM is C_c. In this case, when a failure occurs, customer calls service agent to repair the failed product.

Option A_2: in the fixed price of maintenance service contract, the service agent agrees to carry out PM and CM for the failures of the product until the end of service contract. Under this option, the service agent is rewarded or penalized according to the time spent for CM. If a failure is restored within the specified time τ_0, the service agent will be rewarded, and the reward coefficient is δ. Otherwise, if failure is not rectified within the specified time limit (i.e. τ_1), the service agent will incur a penalty, and the penalty coefficient is α. In addition, the reward and penalty are proportional to the difference of corresponding threshold of time.

2.2 Failure rate with imperfect PM

The product is subjected to CM and PM. For CM, failed unit will be restored to working state with minimal repair. Moreover, it does not change the product's failure rate. Imperfect maintenance is adopted for PM. It is supposed that there are N cycles of PMs, and the interval between PMs is T. Thus, the contract's duration (i.e. L) can be expressed as L=NT. Additionally, imperfect maintenance can reduce the failure rate, and the change formula of failure rate [8] can be defined as

$$\lambda_n(t) = r\lambda_{n-1}(t-T) + (1-r)\lambda_{n-1}(t) \tag{1}$$

where $\lambda_n(t)$ is the product's failure rate after the nth of PM, n=1, 2, $\cdots$, N−1; r is the improvement factor of PM, and $0 \leqslant r \leqslant 1$.

If r=0, then $\lambda_n(t) = \lambda_{n-1}(t)$. In this case, the failure rate is not disturbed. If r=1, then $\lambda_n(t) = \lambda_{n-1}(t-T)$. In this case, each time of overhaul restores the system to the condition of previous overhaul period. Therefore, Theorem 1 can be obtained.

Theorem 1 Let $\lambda'(t)$ denote failure rate of the product with periodic PM, and $\lambda(t)$ denote the failure rate without PM. For each integer $n \geqslant 1$, and $t \in [0, T)$, we have

$$\lambda'(n\cdot T+t) = \sum_{i=0}^{n} r^{n-i}(1-r)^i \lambda(i\cdot T+t). \tag{2}$$

Proof Let $\lambda_n(nT+t)$ denote failure rate of the pro duct after the nth PM, then $\lambda'(n\cdot T+t) = \lambda_n(n\cdot T+t)$. Note that $\lambda_0(t) = \lambda(t)$.

(i) For n=1, $\lambda'(T+t)$=$\lambda_1(T+t)$= $r\lambda(t)$+ (1-r) $\lambda_n(T+t)$, Theorem 1 holds;

(ii) Assuming that Theorem 1 holds for n=m,

$$\lambda'(m\cdot T+t) = \sum_{i=0}^{m} r^{m-i}(1-r)^i \lambda(i\cdot T+t);$$

(iii) For n=m+1,

$$\lambda'((m+1)\cdot T+t)=\lambda_{m+1}((m+1)\cdot T+t)=r\lambda_m(m\cdot T+t)+(1-r)\lambda_m((m+1)\cdot T+t)=$$
$$r\sum_{i=0}^{m} r^{m-i}(1-r)^i\lambda(i\cdot T+t)+(1-r)\sum_{i=0}^{m} r^{m-i}(1-r)^i\lambda((i+1)\cdot T+t)=\sum_{i=0}^{m} r^{m+1-i}(1-r)^i\lambda(i\cdot T+t)+$$
$$\sum_{i=1}^{m+1} r^{m+1-i}(1-r)^i\lambda(i\cdot T+t)+\sum_{i=1}^{m+1} r^{m+1-i}(1-r)^i\lambda(i\cdot T+t)=r^{m+1}\lambda(t)+$$
$$\sum_{i=1}^{m} r^{m+1-i}(1-r)^i\lambda(i\cdot T+t)+(1-r)^{m+1}\lambda((m+1)\cdot T+t)=\sum_{i=0}^{m+1} r^{m+1-i}(1-r)^i\lambda(i\cdot T+t).$$

Therefore, Theorem 1 holds for each integer $n\geqslant 1$. □

2.3 Expected number of failures

Theorem 2 Let $H'(t)$ denote expected failures of the product during $[0, t]$ with periodic PM, and $H(t)$ denote expected failures of the product during $[0, t]$ without PM. For each integer $n\geqslant 1$,

$$H'(n\cdot T)=\sum_{i=0}^{n} r^{n-i}(1-r)^{i-1}H(i\cdot T). \tag{3}$$

Proof It can be proved by mathematical induction and Pascals identity [31,32].

(i) For n=1, $H'(T')=\int_0^T\lambda'(x)\mathrm{d}x=\int_0^T\lambda(x)\mathrm{d}x$, Theorem 2 holds;

(ii) Assuming that Theorem 2 holds for n=m,

$$H'(m\cdot T)=\sum_{i=0}^{m} r^{m-i}(1-r)^{i-1}H(i-T);$$

(iii) For n=m+1,

$$H'((m+1)\cdot T)=\int_0^{(m+1)\cdot T}\lambda'(x)\mathrm{d}x=\int_0^{m\cdot T}\lambda'(x)\mathrm{d}x+$$
$$\int_{m\cdot T}^{(m+1)\cdot T}\lambda'(x)\mathrm{d}x=\sum_{i=0}^{m} r^{m-i}(1-r)^{i-1}H(i\cdot T)+$$
$$\int_0^T\lambda'(m\cdot T+z)\mathrm{d}z=\sum_{i=0}^{m} r^{m-i}(1-r)^{i-1}H(i\cdot T)+$$
$$\sum_{i=0}^{m} r^{m-i}(1-r)^iH(i\cdot T+T)-$$
$$\sum_{i=0}^{m} r^{m-i}(1-r)^iH(i\cdot T)=$$
$$\sum_{i=1}^{m} r^{m+1-i}(1-r)^{i-1}H(i\cdot T)+$$
$$(1-r)^mH(m\cdot T+T)=$$
$$\sum_{i=0}^{m} r^{m+1-i}(1-r)^{i-1}H(i\cdot T).$$

Therefore, Theorem 2 holds for each integer $n\geqslant 1$. □

For Option A_0, the expected number of product's failures during the contract period is

$$H_0(N\cdot T)=\int_0^{N\cdot T}\lambda(t)\mathrm{d}t. \tag{4}$$

For Option A_1 and Option A_2, the expected number of the product's failures obtained from Theorem 2 are as follows:

$$H_1'(N\cdot T)=\sum_{i=0}^{N} {r_1}^{N-i}(1-r_1)^{i-1}H(i\cdot T), \tag{5}$$

$$H_2'(N\cdot T)=\sum_{i=0}^{N} {r_2}^{N-i}(1-r_2)^{i-1}H(i\cdot T), \tag{6}$$

where r_1 and r_2 are PM improvement factors in Option A_1 and Option A_2 respectively; and $H(i\cdot T)=\int_0^{i\cdot T}\lambda(t)\mathrm{d}t$.

The failure rate is assumed to obey a two-parameter Weibull distribution with the shape parameter (i.e. β) and scale parameter (i.e. η) respectively [33]. Then the failure rate (i.e. $\lambda(t)$) can be expressed as

$$\lambda(t)=\frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}. \tag{7}$$

Thus, during the contract period, the expected number of product's failures under Option A_0, Option A_1, and Option A_2 can be obtained as follows respectively:

$$H_0(N\cdot T)=\int_0^{N\cdot T}\lambda(t)\mathrm{d}t=\left(\frac{N\cdot T}{\eta}\right)^{\beta}, \tag{8}$$

$$H_1{}'(N\cdot T)=\sum_{i=0}^{N} {r_1}^{N-i}(1-r_1)^{i-1}\int_0^{i\cdot T}\lambda(t)\mathrm{d}t=\left(\frac{T}{\eta}\right)^{\beta}\sum_{i=0}^{N} {r_1}^{N-i}(1-r_1)^{i-1}i^{\beta}, \tag{9}$$

$$H_2'(N\cdot T)=\sum_{i=0}^{N} {r_2}^{N-i}(1-r_2)^{i-1}\int_0^{i\cdot T}\lambda(t)\mathrm{d}t=\left(\frac{T}{\eta}\right)^{\beta}\sum_{i=0}^{N} {r_2}^{N-i}(1-r_2)^{i-1}i^{\beta}. \tag{10}$$

2.4 Expected time of repairs

In this paper, the repair time (i.e. y) is assumed to be exponential distribution [34], and the repair rate is μ, then the probability density function (PDF) of repair time can be obtained as

$$f(y) = \mu e^{-\mu y}. \tag{11}$$

Therefore, during the contract period, the expected repair time is expressed as

$$E[Y_i] = \int_0^{+\infty} y f(y) dy = \int_0^{+\infty} \mu y e^{-\mu y} dy = \frac{1}{\mu} \tag{12}$$

where Y_i is the repair time of the ith failure.

In addition, the expected penalty time and expected reward time for Option A_2 are expressed as follows respectively:

$$E[\max\{0, Y_i - \tau_1\}] = \int_{\tau_1}^{+\infty} (y - \tau_1) f(y) dy, \tag{13}$$

$$E[\max\{0, \tau_0 - Y_i\}] = \int_0^{\tau_0} (\tau_0 - y) f(y) dy. \tag{14}$$

2.5 Decision problem for the customers

Option A_0: for the customer, its revenue majorly consists of revenue generated from the product's operation, charge for CM and product's purchase price. Hence, the expected profit (i.e. $W(A_0)$) is given by

$$W(A_0) = R\left(N \cdot T - \sum_{i=1}^{H_0(N \cdot T)} Y_i\right) - C_c H(N \cdot T) - C_e \tag{15}$$

where R is revenue of the product per unit time; C_c is the cost for each CM; and C_e is the purchase price of the product.

Option A_1: the customer's revenue consists of the revenue generated from the product's operation, the charge of CM, the charge of PM and the product's purchase price. Hence, the expected profit (i.e. $W(A_1)$) is given by

$$W(A_1) = R(N \cdot T - \sum_{i=1}^{H_1'(N \cdot T)} Y_i - (N-1)T_p) - C_c H_1'(N \cdot T) - C_{ps}(N-1) - C_e \tag{16}$$

where T_p is the time for PM of the product.

Option A_2: the customer's revenue consists of the revenue generated from the operation of the product, penalty fee, incentive fee, fixed price of maintenance service contract and product's purchase price. Hence, the expected profit (i.e. $W(A_2)$) is given by

$$W(A_2) = R\left(N \cdot T - \sum_{i=1}^{H_2'(N \cdot T)} Y_i - (N-1)T_p\right) + C_P - C_I - P_M - C_e \tag{17}$$

where C_P, C_I denote the penalty cost and incentive cost for service agent respectively.

C_P and C_I can be obtained as follows:

$$C_P = \alpha\left(\sum_{i=1}^{H_2'(N \cdot T)} \max\{0, Y_i - \tau_1\}\right) = \alpha H_2'(NT) E[\max\{0, Y_i - \tau_1\}], \tag{18}$$

$$C_I = \delta\left(\sum_{i=1}^{H_2'(N \cdot T)} \max\{0, \tau_0 - Y_i\}\right) = \delta H_2'(NT) E[\max\{0, \tau_0 - Y_i\}]. \tag{19}$$

2.6 Decision problem of the service agent

Option A_0: the service agent is responsible only for CM, so the revenue is the difference value between CM cost and CM charge paid by the customer. Hence, the expected profit (i.e. $U(A_0)$) of service agent is given by

$$U(A_0) = (C_c - C_{cm}) H_0(N \cdot T) \tag{20}$$

where C_{cm} is the CM cost for service agent.

Option A_1: as with Option A_0, the revenue of service agent comes from CM. Hence, the expected profit (i.e. $U(A_1)$) for service agent is given by

$$U(A_1) = (C_a - C_{cm}) H_1'(N \cdot T). \tag{21}$$

Option A_2: for service agent, the revenue consists of fixed price of maintenance service contract, penalty fee, incentive fee, CM cost and PM cost, respectively. Hence, the expected profit (i.e. $U(A_2)$) of service agent is given by

$$U(A_2) = P_M + C_I - C_P - C_{cm} H_2'(N \cdot T) - C_{pm}(N-1) \tag{22}$$

where C_{pm} is the PM cost of service agent.

3. Model analysis with Nash game

To achieve long-term cooperation, it is crucial to reach a win-win situation between customers and service agent. As the players of the game, customers and service agent need to negotiate the service contract options and pricing mechanism together, with the goal to maximize their respective expected revenue. Therefore, it is appropriate to apply Nash non-cooperative game theory to determine the price of service contract. The optimal pricing of service contract is that can meet the conditions of Nash equilibrium. Under such a situation, the revenues for both parties are the same, and it is half of the total profit respectively [35]. By solving the Nash equilibrium of non-cooperative game, the optimal decision of service agent is obtained.

Option A_0: according to the Nash non-cooperative game theoretic formulation, with (15) and (20), the bargaining price can be obtained as $W(A_0)^*=U(A_0)^*$, that is

$$R(N\cdot T-\sum_{i=1}^{H_0(N\cdot T)}Y_i)-C_c^*H_0(N\cdot T)-C_e=(C_c^*-C_{\rm cm})H_0(N\cdot T). \quad (23)$$

Then the optimal price of CM is given by

$$C_c^*=\frac{1}{2H_0(N\cdot T)}\left(R\left(N\cdot T-\sum_{i=1}^{H_0(N\cdot T)}Y_i\right)+C_{\rm cm}H_0(N\cdot T)-C_e\right). \quad (24)$$

Consequently, the expected profit of service agents is

$$U({\rm A}_0;C_c^*)=\frac{R}{2}\left(N\cdot T-\sum_{i=1}^{H_0(N\cdot T)}Y_i\right)-\frac{C_{\rm cm}}{2}H_0(N\cdot T)-\frac{C_e}{2}. \quad (25)$$

Option A_1: according to (16) and (21), and by using the similar approach as in Option A_0, the bargaining price can be obtained as $W(A_1)^*=U(A_1)^*$:

$$R(N\cdot T-\sum_{i=1}^{H_1'(N\cdot T)}Y_i-(N-1)T_p)-C_c^*H_1'(N\cdot T)-C_{ps}(N-1)-C_e=(C_c^*-C_{\rm cm})H_1'(N\cdot T). \quad (26)$$

Then the optimal price of CM is given by

$$C_c^*=\frac{1}{2H_1'(N\cdot T)}(R(N\cdot T-\sum_{i=1}^{H_1'(N\cdot T)}Y_i-(N-1)T_p)+C_{\rm cm}H_1'(N\cdot T)-C_{\rm ps}(N-1)-C_e). \quad (27)$$

Consequently, the expected profit of service agent is

$$U({\rm A}_1;C_c^*)=\frac{R}{2}(N\cdot T-\sum_{i=1}^{H_1'(N\cdot T)}Y_i-(N-1)T_p)-\frac{C_{\rm cm}}{2}H_1'(N\cdot T)-\frac{C_{\rm ps}}{2}(N-1)-\frac{C_e}{2}. \quad (28)$$

Option A_2: according to (17) and (22), and by using similar approach as in Option A_0, the bargaining price can be obtained as $W(A_2)^*=U(A_2)^*$, that is

$$R\left(N\cdot T-\sum_{i=1}^{H_2'(N\cdot T)}Y_i-(N-1)T_p\right)+C_{\rm P}-C_{\rm I}-P_M^*-C_e=P_M^*+C_{\rm I}-C_{\rm P}-C_{\rm cm}H_2'(N\cdot T)-C_{\rm pm}(N-1). \quad (29)$$

Then, the optimal price of service contract is given by

$$P_M^*=\frac{R}{2}(N\cdot T-\sum_{i=1}^{H_2'(N\cdot T)}Y_i-(N-1)T_p)+C_{\rm P}-C_{\rm I}+\frac{C_{\rm cm}}{2}H_2'(N\cdot T)+\frac{C_{\rm pm}}{2}(N-1)-\frac{C_e}{2}. \quad (30)$$

Thus, we obtain the expected profit of service agent as follows:

$$U({\rm A}_2;P_M^*)=\frac{R}{2}(N\cdot T-\sum_{i=1}^{H_2'(N\cdot T)}Y_i-(N-1)T_p)-\frac{C_{\rm cm}}{2}H_2'(N\cdot T)-\frac{C_{\rm pm}}{2}(N-1)-\frac{C_e}{2}. \quad (31)$$

4. Numerical experiments

To verify the effectiveness of Nash non-cooperative game model, two numerical experiments are conducted in this section. The first experiment is to analyze the benefit of different contract options under fixed contract period. The second experiment is to calculate the optimal pricing and optimal number of PM cycles under a flexible contract period. Due to the inconsistent length of contract period in the experiments, the daily-expected revenue is used as the index for analysis, and its formula is as follows:

$$f(L)=\frac{U}{L} \quad (32)$$

where U denotes the expected profit of service agent, L is the duration of contract.

4.1 Experiment 1

We consider a product, and its failure rate follows Weibull distribution with the shape parameter of β=2 and scale parameter of η=200. The other parameters are given in Table 1.

Table 1 Parameters setting of Experiment 1

Notation	Description	Value
C_e/\$	Purchase price of the product	150 000
R/\$	Daily revenue of the product	400
μ	Repair rate	0.4
$C_{\rm ps}$/\$	Charge of PM carried out by the customer	500
C_p/\$	Charge of PM carried out by the agent	1 000
r_1	Improvement factor of PM carried out by the customer	0.5
r_2	Improvement factor of PM carried out by the agent	0.8
$C_{\rm cm}$/\$	Cost of CM carried out by the agent	1 100
$C_{\rm pm}$/\$	Cost of PM carried out by the agent	700
δ/(\$/d)	Reward coefficient	400
α/(\$/d)	Penalty coefficient	300
τ_0/d	Time limit for rewards	2
τ_1/d	Time limit for punishment	3.5
T_p/d	Time for PM	1

Assuming that the duration of contract is L=2 000 d, the cycle for PM (i.e. N) is an integer and $N\in[2,20]$. The experiment is solved by applying Nash non-cooperative game theory. Fig. 1 shows the variation rule of daily-expected revenue with the PM cycle under different contract options.

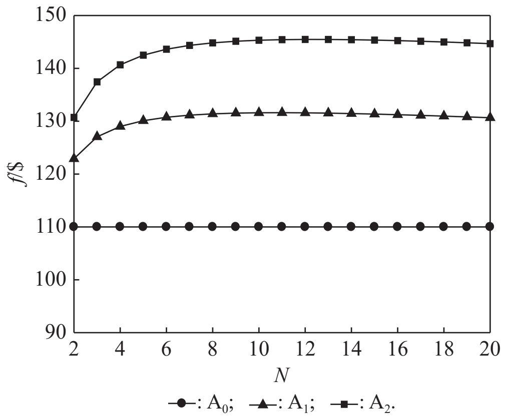


Fig. 1 Changes of daily-expected revenue

The results show that:

(i) Since there is no PM in Option A_0, the daily-expected revenue is fixed. For Option A_1 and Option A_2, with the increase of PM cycle, the daily-expected revenue increases rapidly in the initial stage and then it decreases slowly.

(ii) The daily-expected revenues of Option A_1 and Option A_2 are higher than that of Option A_0. The reason is that the existence of PM can promote the availability of product, and operating revenue of the product is increased.

(iii) Under the same cycle of PM, the daily-expected revenue of Option A_2 is higher than that of Option A_1. The major reason is that the maintenance quality of service agent is higher than that of customer, and it brings higher revenue.

Table 2 presents the optimal maintenance pricing and daily-expected revenue for different contract options. For Option A_0, the optimal CM pricing of service agent is C_c= 3 300 $, and the optimal daily-expected revenue is f(A_0)=110 $. For Option A_1, the optimal CM pricing of service agent is C_c=5 926 $, the optimal daily-expected revenue is f (A_1)=132 $, and the preventive maintenance cycle is N=11. For Option A_2, the optimal service contract price for service agent is P_M=326 270 $, the optimal daily-expected revenue is f (A_2)=145 $, and the PM cycle is N=12. By comparison, it can be found that the optimal daily-expected revenue of Option A_0 is far lower than that of Option A_1 and Option A_2. The reason is that in Option A_0 only CM is adopted and PM is not be adopted. It will lead to more failures and longer time of maintenance for the product, and finally result in lower revenue of the product.

Table 2 Optimal pricing and revenue of maintenance service contract

Contract options		A_0	A_1	A_2
Optimal pricing	C_c /$	3 300	5 926	–
	P_M /$	–	–	326 270
Optimal daily expected revenue f /$		110	132	145
Optimal PM cycle N		–	11	12

Considering that the parameters in the model have great influence on the price and revenue for the maintenance service contract. Here, sensitivity analysis is carried out for three major parameters in the model, including contract period, repair rate and improvement factor, respectively.

Table 3 shows the optimal decision results of each contract option under different periods of the contract. Fig. 2 shows the variation rule of optimal daily-expected revenue for each contract option change within the contract period. For a given contract period, the optimal revenue of service agent is $f(A_2)> f(A_1)> f(A_0)$. For a given option, with the increase of contract period, the optimal price for CM continues to decrease, while the optimal price of the service contract continues to rise. The optimal daily-expected revenue shows a tendency of increasing firstly and then decreasing. Furthermore, they will achieve their maximum daily-expected revenue in 1 000—2 000, 2 000—3 000 and 3 000—4 000 days for Options A_0, A_1 and A_2, respectively.

Table 3 Sensitivity analysis of contract period.

L/d	Option A_0		Option A_1			Option A_2		
	C_c^*/$	f^*/$	C_c^*/$	f^*/$	N^*	P_M^*/$	f^*/$	N^*
1 000	5 050	99	8 263	107	5	125 630	114	6
1 500	4 050	111	7 062	126	8	225 966	136	9
2 000	3 300	110	5 926	132	11	326 267	145	12
2 500	2 770	104	5 057	133	14	426 531	150	15
3 000	2 383	96	4 386	131	16	526 917	153	19
3 500	2 091	87	3 877	128	19	626 793	154	20
4 000	1 863	76	3 462	124	20	726 469	153	20

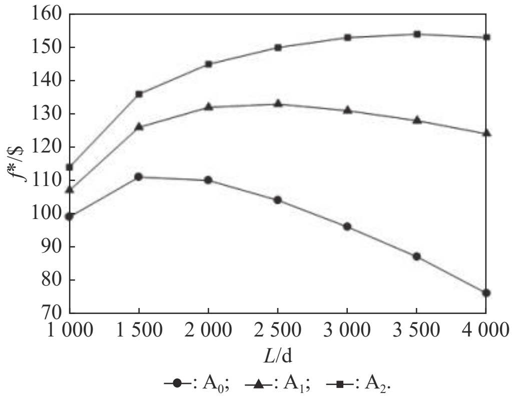


Fig. 2 Sensitivity analysis of the contract period

Table 4 shows the optimal decision of each contract option under different repair rates. Fig. 3 shows the variation rule of optimal daily-expected revenue with repair rate under different contract options. The larger the repair rate is, the shorter the expected repair time will be. Obviously, it can help to obtain higher availability of the product and increase the expected revenue. The results of Table 4 and Fig. 3 indicate that: (i) in Option A_0 and Option A_1, with the increase of repair rate, the optimal CM price and daily-expected revenue show a tendency of increase. (ii) In Option A_2, with the increase of repair rate, the optimal service contract price shows a tendency of decrease, while the optimal daily-expected revenue tends to increase. (iii) The impact of repair rate on Option A_0 is greater than the other two contract options. Therefore, by improving maintenance policy and ensuring maintenance resources, the service agent can improve repair rate and acquire more revenue.

Table 4 Sensitivity analysis of repair rate

μ	Option A_0		Option A_1			Option A_2		
	C_c^*/\$	f^*/\$	C_c^*/\$	f^*/\$	N^*	P_M^*/\$	f^*/\$	N^*
0.30	3 133	102	5 792	127	12	327 712	143	13
0.35	3 229	106	5 854	130	11	326 949	144	13
0.40	3 300	110	5 926	132	11	326 267	145	12
0.45	3 356	113	5 981	133	11	325 878	146	12
0.50	3 400	115	5 986	134	10	325 582	147	12

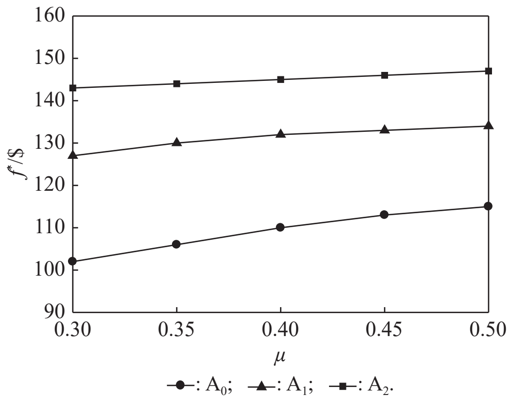


Fig. 3 Sensitivity analysis of repair rate

Table 5 and Table 6 show optimal solutions for various improvement factors in Option A_1 and Option A_2 respectively. The results indicate that with the increase of improvement factors r_1 and r_2, the optimal pricing and optimal daily-expected revenue for Options A_1 and A_2 tend to increase. It demonstrates that with the increase of improvement factor, the product's reliability and expected revenue tend to increase for both the parties. Therefore, to maximize daily-expected revenue, the quality of PM and improvement factors should be increased as much as possible.

Table 5 Sensitivity analysis for improvement factor r_1 in Experiment 1

r_1	N^*	C_c^*/\$	f^*/\$
0.40	10	5 065	127
0.45	10	5 444	129
0.50	11	5 926	132
0.55	11	6 460	134
0.60	12	7 162	136

Table 6 Sensitivity analysis for improvement factor r_2 in Experiment 1

r_2	N^*	P_M^*/\$	f^*/\$
0.70	12	326 135	141
0.75	12	326 201	143
0.80	12	326 267	145
0.85	13	326 490	148
0.90	13	326 557	150

4.2 Experiment 2

This section analyzes optimal decision of service agent under various contract options in case of flexible contract periods. In Option A_1 and Option A_2, $N\in[2,20]$ and $T\in[100, 500]$ d; and in Option A_0, $L\in[200, 10\,000]$ d. Other parameters in Experiment 2 are consistent with

Experiment 1.

(i) Option A_0

Nash non-cooperative game is adopted to solve the variation rule of daily-expected revenue under Option A_0, as shown in Fig. 4. With the increase of contract period, the daily-expected revenue will first increase rapidly and then decrease gradually. When the contract period is 1 690 days, the maximum value of daily-expected revenue(i.e. $f(A_0)^*$) is 111 \$, and the optimal CM price is C_c=3 733 \$.

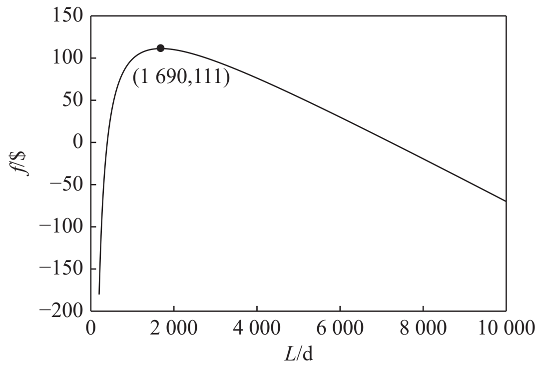
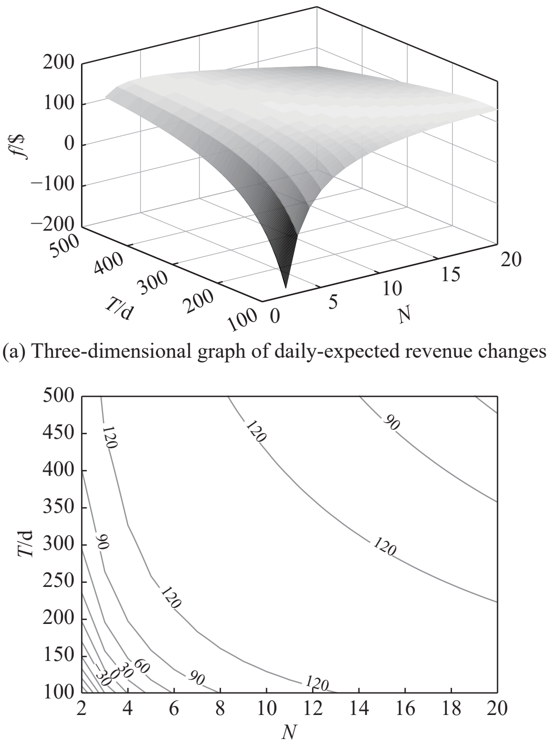


Fig. 4 Changes of daily-expected revenue under Option A_0

(ii) Option A_1

Similarly, Nash non-cooperative game is used to solve variation rule of daily-expected revenue under Option A_1. Fig. 5 shows the three-dimensional and contour plot of daily-expected revenue function.



(a) Three-dimensional graph of daily-expected revenue changes

(b) Contour plot of daily-expected revenue changes

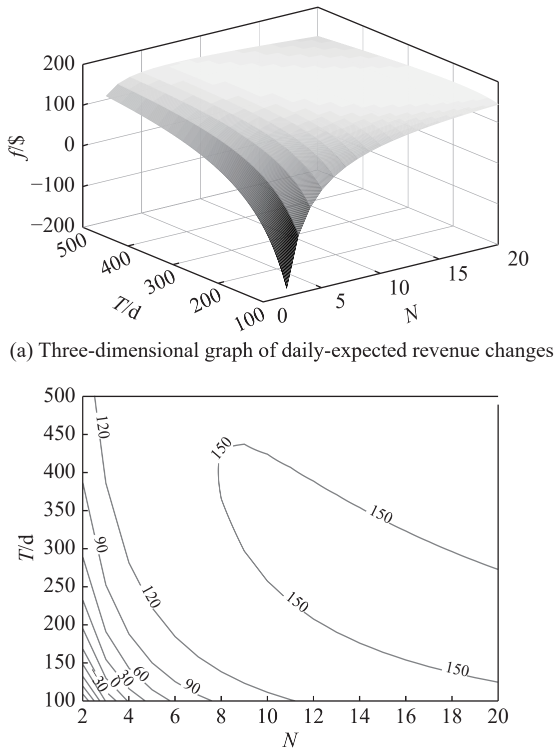
Fig. 5 Changes of daily-expected revenue under Option A_1

In Fig. 5(a), the daily-expected revenue curve is a convex function.

As shown in Fig. 5(b), with the change of N and T, daily-expected revenue keeps converging and reaches a maximum value. According to the calculation, when N=13 and T=183 d, the maximum value of expected daily revenue is 133 \$, that is, $f(A_1)^*$=133 \$, and the corresponding optimal CM price is C_c=3 329 \$.

(iii) Option A_2

By using similar approach as in Option A_1, the variation rule of daily-expected revenue under Option A_2 is obtained, as shown in Fig. 6. As can be seen from Fig. 6(a), the daily-expected revenue curve is a convex function with a maximum value. In addition, through analyzing the contour plot shown in Fig. 6(b), it can be found that: when N and T change, the value of daily-expected revenue continues to converge; and when N=20 and T=184 d, the maximum value of daily-expected revenue is 154 \$, that is, $f(A_2)^*$=154 \$, the corresponding optimal service contract price is P_M=925 152 \$.



(a) Three-dimensional graph of daily-expected revenue changes

(b) Contour plot of daily-expected revenue changes

Fig. 6 Changes of daily-expected revenue under Option A_2

By changing repair rate and improvement factor, sensitivity analysis for the parameters is conducted. As indicated in Table 7 and Fig. 7, with the increase of repair rate, optimal CM price for both Option A_0 and Option A_1 decreases, while optimal length of contract period and daily-expected revenue tend to increase. The optimal contract period, both the service contract pricing and daily-expected revenue of Option A_2 show an increase tendency. In engineering application, the increase of repair rate usually means that the expected repair time will be shorter, the product availability and benefit will be increased. Thus, to maximize daily-expected revenue for the service agent, it should improve maintenance quality and increase repair rate as far as possible.

Table 7 Sensitivity analysis of repair rate

μ	Option A_0			Option A_1				Option A_2			
	L^*/d	C_c^*/$	f^*/$	N^*	T^*/d	C_c^*/$	f^*/$	N^*	T^*/d	P_M^*/$	f^*/$
0.30	1 570	3 762	104	13	170	3 406	127	20	171	892 424	150
0.35	1 636	3 748	108	13	178	3 353	130	20	178	911 693	152
0.40	1 690	3 733	111	13	183	3 329	133	20	184	929 152	154
0.45	1 737	3 717	114	13	189	3 288	134	20	189	943 482	155
0.50	1 777	3 702	116	13	193	3 266	136	20	194	960 513	156

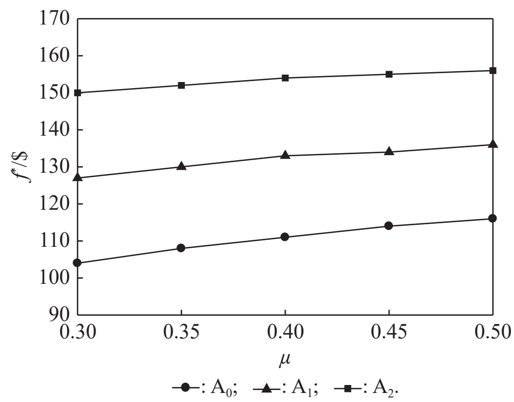


Fig. 7 Sensitivity analysis of repair rate

Table 8 and Table 9 show the sensitivity analysis results of improvement factors r_1 and r_2, respectively. It can be clearly observed that:

(i) With the increase of improvement factor of r_1, both the optimal contract period and optimal daily-expected revenue of Option A_1 show an increasing tendency. It indicates that the product availability is improved, and the production revenue will also increase.

(ii) With the increase of improvement factor of r_2, the optimal length of contract period for Option A_2 will increase. In this case, the operational benefit of customers will increase, and the price of service contract will also increase. Therefore, it can result in a win-win situation. Thus, to maximize the daily-expected revenue, the improvement factors of PM (i.e. r_1 and r_2) should be increased as much as possible.

Table 8 Sensitivity analysis of improvement factor r_1 in Experiment 2

r_1	N^*	T^*/d	L^*/d	C_c^*/$	f^*/$
0.40	11	198	2 178	3 435	127
0.45	12	190	2 280	3 377	130
0.50	13	183	2 379	3 329	133
0.55	14	179	2 506	3 264	136
0.60	16	167	2 672	3 177	139

Table 9 Sensitivity analysis of improvement factor r_2 in Experiment 2

r_2	N^*	T^*/d	L^*/d	P_M^*/$	f^*/$
0.70	18	180	3 240	729 869	148
0.75	20	177	3 540	835 364	152
0.80	20	184	3 680	929 152	154
0.85	20	216	4 320	1 131 079	160
0.90	20	249	4 980	1 402 628	166

5. Conclusions

To maximize the revenues of both the service agent and customers, Nash non-cooperative game is adopted to negotiate the pricing mechanism in the maintenance service contract, and manufacturer can share the expected profits with customers in a bargaining way. In the models, three types of maintenance service contracts are designed by combining different maintenance strategies. In addition, minimal repair and imperfect maintenance are implemented, and failure rate of the product is considered. Moreover, to reduce the maintenance service cost and improve the product performance, penalty and incentive mechanisms are also taken into account. It is found that by considering incentive and penalty mechanism, revenue can be improved for both the parties. Case study indicates that by optimizing the price of contract, improving the maintenance quality as well as increasing the repair rate, the benefits can be promoted for both parties.

To make the model more realistic, in the future multiple types of customers and multiple service agents can be integrated into the model. Another interesting topic is to apply different game models to analyze the service contracts. Further, more types of reward and punishment mechanisms can be taken into account.

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