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Reliability-based selective maintenance for redundant systems with dependent performance characteristics of components

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Abstract: The reliability-based selective maintenance (RSM) decision problem of systems with components that have multiple dependent performance characteristics (PCs) reflecting degradation states is addressed in this paper. A vine-Copula-based reliability evaluation method is proposed to estimate the reliability of system components with multiple PCs. Specifically, the marginal degradation reliability of each PC is built by using the Wiener stochastic process based on the PC’s degradation mechanism. The joint degradation reliability of the component with multiple PCs is established by connecting the marginal reliability of PCs using D-vine. In addition, two RSM decision models are developed to ensure the system accomplishes the next mission. The genetic algorithm (GA) is used to solve the constraint optimization problem of the models. A numerical example illustrates the application of the proposed RSM method.

Keywords: D-vine, genetic algorithm (GA), reliability-based selective maintenance (RSM), redundant system, Wiener stochastic process.

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1. Introduction

Many complex systems in the industry are required to execute continuous missions within a predetermined time. After finishing the current mission, some necessary maintenance actions must be performed on the system components during downtime to meet the reliability requirement for executing the next mission. However, within limited maintenance resources, not all system components can be restored to optimal conditions. Therefore, an appropriate maintenance policy should be developed to select the optimal maintenance components and the maintenance actions assigned to them. The selective maintenance proposed by Rice et al. [1] is specially developed for this kind of maintenance problem.

In classical reliability-based selective maintenance proposed by Cassady et al. [2], a system and its components are assumed to be in two possible states: functioning or failed. On the basis of that, a generalized modeling framework is further proposed by Cassady et al. [3] to solve the selective maintenance decision problem. From that, the selective maintenance is later extended by considering the varying state of components from perfect functioning to complete failure. For example, Pandey et al. [4] studied the imperfect selective maintenance problem and used the hybrid hazard rate approach to reflect whether a component was relatively young or old. Jiang et al. [5] proposed a reliability-centered predictive maintenance scheme for a complex structure system with several redundant components, where the reliability of system components is described by the Markov chain. Khatab et al. [6,7] proposed a condition-based selective maintenance model in a continuously monitored multi-component system, the reliability model of system components was established by using the Gamma stochastic process. Most research on reliability-based selective maintenance ignored the complex dependency of the system and its components exhibited in practical applications. More research on selective maintenance for multi-unit systems can be seen in [8].

To make the maintenance problem more practical, the reliability model, considering the dependency of systems components, has been developed in recent studies. Hong et al. [9] investigated the influence of dependent degradation components on the optimal maintenance decisions, and the dependency among degradation components was modeled by the Copula. Guo et al. [10] developed a joint reliability model via Copula for the non-repairable systems, which had dependent competition risks caused by component degradation and random shocks. Their study illustrated making maintenance decisions via the proposed reliability models could obtain a more precise result than that without considering the dependency. Ruiz et al. [11] presented a statistical and optimization frame-

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work for selective maintenance of a complex system. In their study, each system component had multiple hard sudden failure modes and a single soft degradation failure mode, and the reliability model was built on this basis.

However, most of the reliability-based models assumed system components have only one degraded performance characteristic (PC). According to existing related research, a component could exhibit multiple dependent PCs, especially for some mechanical components, such as pneumatic valves [12], smart electricity meter [13], and electro-hydraulic servo valve [14]. Accordingly, the dependency among multiple degraded PCs of system components should be considered when developing a reliability model or reliability-based maintenance. Moreover, a more accurate reliability modeling method should also be proposed to describe the actual influence of maintenance in the presence of multiple dependent PCs of components. The Copula function is widely used to solve such a reliability modeling problem since it can conveniently obtain the dependent relationship of component's PCs. Xu et al. [15] presented a multivariate failure behavior modeling and reliability assessment method based on vine-Copula. Fang et al. [16] investigated a coherent system, which had positively correlated degradation processes of its PCs, and the system reliability with multiple PCs was modeled by a Copula function. Pan et al. [17] proposed a reliability evaluation method for products with multi-performance degradation based on the Wiener process and Copula function. Sun et al. [18] developed a multivariate dependent accelerated degradation test model based on the random-effect Wiener process and drawable (D-) vine Copula to capture the dependence structure among the multiple PCs in modern products.

To obtain more credible system reliability, system components with multiple PCs are fully considered. Specifically, the reliability of those components is established by using the vine-Copula, and the marginal reliability of each PC is modeled by the general Wiener process. Based on that, the reliability-based selective maintenance (RSM) policy is developed. Within the limited maintenance resources, the proposed RSM can select and maintain some important components to achieve the reliability required of executing the next mission. Also, the proposed RSM considers the following problems: (i) dynamic evaluation of system reliability for executing the next mission; (ii) appropriate allocation of maintenance actions (including perfect and imperfect maintenance) for the selected components; (iii) cost-effectiveness of jointly maintaining multiple components.

The remainder of this study is organized as follows: Section 2 develops the reliability model for the system component with multiple dependent PCs. In Section 3, the maintenance decision-making and optimization process of the RSM policy are discussed. In Section 4, the implementation of the proposed RSM policy is illustrated through a numerical example. Conclusions are presented in Section 5.

2. Reliability modeling

In this section, a system composed of multiple degradation components is considered. The system consists of M subsystems in parallel, and each subsystem i (i=1, 2, $\cdots$, M) is composed of m_i components C_{ij} (j=1, 2, $\cdots$, m_i). The components exhibit one or more kinds of degenerated PCs. To estimate the reliability of those components, the marginal reliability functions are established firstly for each of the PCs, and then the marginal functions are connected to model the component reliability by using vine-Copula. The system reliability can be calculated as a structure-function of the complex structures and the reliability of the components. The structure-function is directly created through the reliability block diagram method.

2.1 Marginal reliability modeling of PCs

Suppose that the degradation process of PCs can be observed, and the observed degradation can be sampled along with the PCs' degradation over time. In general, the degradations are assumed to have common sample time points. The degradations are stochastic for the randomness in the degradation process of components. Therefore, the general approach is to depict the degradation process as a stochastic process [19]. In this study, the Wiener process is introduced to model such a degradation process. Let $\{Y(t), t \geqslant 0\}$ denote the cumulative degradation of a PC at the simple time, then $\{Y(t), t \geqslant 0\}$ can be described as

$$Y(t) = \mu t + \sigma B(t) \tag{1}$$

where μ is a drift rate reflecting the mean rate of the degradation process of the PC. $\sigma > 0$ is a diffusion parameter quantifying the magnitude of the process. $B(t)$ is a standard Brownian motion presenting the randomness of the process.

For each PC, the failure time can be regarded as when its cumulative degradation first hits a pre-specified threshold [20]. Such first-passage time is defined as

$$T_f = \inf\{t | Y(t) \geqslant D_f | Y(0) = 0, t \geqslant 0\} \tag{2}$$

where D_f is the pre-specified failure threshold.

The Wiener process $Y(t)$ has the following properties:

$Y(0)=0$; $Y(t)$ has stationary independent increments that follow a normal distribution. According to the properties, the first-passage time T_f under knowing degradation $Y(\tau)=d$ can be derived as

$$T_f = \inf\{ Y(t+\tau) - Y(\tau) \geqslant D_f - Y(\tau) | t \geqslant 0\} = \inf\{ Y(t) \geqslant D_f - d | t \geqslant 0\} . \tag{3}$$

Correspondingly, the reliability of the PC at time τ can be proven following an inverse Gaussian distribution as follows:

$$r(t|d) = \Phi\left(\frac{D_f - d - \hat{\mu}t}{\hat{\sigma}\sqrt{t}}\right) - \exp\left(\frac{2\hat{\mu}(D_f - d)}{\hat{\sigma}^2}\right) \Phi\left(\frac{-(D_f - d) - \hat{\mu}t}{\hat{\sigma}\sqrt{t}}\right) \tag{4}$$

where Φ is the standard normal distribution function. $\boldsymbol{\Theta} = (\mu, \sigma^2)$ are the unknown parameters.

According to the property of the Wiener process, let Δy denote the degradation increments within the sample time increment interval Δt, then $\Delta y \sim N(\mu\Delta t, \sigma^2\Delta t)$. Estimation of the parameters $\boldsymbol{\Theta}=(\mu, \sigma^2)$ can be determined by maximizing the log-likelihood function of Δy.

If a component has only a single PC, (4) is used to directly depict its reliability, but if the component has multiple PCs simultaneously, (4) is used to depict the marginal reliability for each PC of the component. Then the component reliability can be modeled by using the marginal reliability of PCs. The remainder of this section focuses on the reliability modeling process of a component with multiple PCs.

2.2 Component reliability modeling by vine-Copula

In practical applications, if all PCs of a component are independent, the joint reliability distribution of the component can be calculated as follows:

$$R_{ij}(t) = P\{Y_{ij1}(t) < D_{f,ij1}, \cdots, Y_{ijN}(t) < D_{f,ijN}\} = P\{Y_{ij1}(t) < D_{f,ij1}\} \cdots P\{Y_{ijN}(t) < D_{f,ijN}\} = \prod_{k=1}^{N} r_{ijk}(t) \tag{5}$$

where $R_{ij}(t)$ is the reliability of component C_{ij} with N kinds of PCs and each of the PCs is expressed by $r_{ijk}(t)$; $Y_{ijk}(t)$ ($k=1, 2, \cdots, N$) denotes the cumulative degradation of the kth PC of the component; and $D_{f,ijk}$ is the failure threshold of the kth PC.

However, if the dependency exists among the multiple PCs, the interactions between PCs may impact the reliability evaluation results of the component. To describe this dependence, the Copula is introduced because it can provide a valid way to describe the joint reliability distribution for multiple dependence PCs. According to Sklar's theorem [21], the joint reliability of a component with multiple PCs can be defined as

$$R_{ij}(t) = C(r_{ij1}(t), \cdots, r_{ijN}(t); \theta_{ij}) \tag{6}$$

where $R_{ij}(t)$ denotes the reliability of component C_{ij}; $r_{ijk}(t)$ ($k = 1, 2, \cdots, N$) is the marginal reliability of the kth PC. For the sake of simplification, $r_{ijk}(t)$ is detonated as $r_k(t)$ ($k = 1, 2, \cdots, N$) in (7)−(8), and (21)−(23).

To evaluate the multivariate distribution $R_{ij}(t)$ in (6), the vine-Copula is introduced. Vine-Copula can decompose a multivariate Copula into multiple bivariate pair-Copulas and connect the pair-Copulas through a vine graphical representation model [22−24]. Thanks to that, multivariate distribution can be vividly expressed as a joint of bivariate distributions. The specific forms of the bivariate distribution used in the study are given in the following.

The specific forms of the bivariate Copula used in the study are as follows:

Gaussian:

$$C(u,v;\theta) = \int_{-\infty}^{\phi^{-1}(u)} \int_{-\infty}^{\phi^{-1}(v)} \frac{1}{2\pi\sqrt{1-\theta^2}} \cdot \exp\left\{-\frac{p^2 - 2\theta pq + q^2}{2(1-\theta^2)}\right\} \mathrm{d}p\mathrm{d}q$$

where $\theta \in (-1, 1)$.

Frank:

$$C(u,v;\theta) = -\frac{1}{\theta}\ln\left(1 + \frac{(\exp(-\theta u) - 1)(\exp(-\theta v) - 1)}{\exp(-\theta) - 1}\right)$$

where $\theta \in (-\infty, +\infty) \backslash \{0\}$.

Clayton:

$$C(u,v;\theta) = (u^{-\theta} + v^{-\theta} - 1)^{-1/\theta}$$

where $\theta \in (0, +\infty)$.

Gumbel:

$$C(u,v;\theta) = \exp(-((-\ln u)^{\theta} + (-\ln v)^{\theta})^{1/\theta})$$

where $\theta \in [1, +\infty)$.

The corresponding probability density functions and partial derivative functions for those Copulas are listed in Table 1.

Table 1 Probability density functions and partial derivative functions

Copula	$c(u, v; \theta)$	$h(u, v; \theta)$
Gaussian	$\frac{1}{\sqrt{1-\theta^2}}\exp(-\frac{\theta^2(\phi^{-2}(u)+\phi^{-2}(v)) - 2\theta\phi^{-1}(u)\phi^{-1}(v)}{2(1-\theta^2)})$	$\Phi(\frac{\Phi^{-1}(u) - \theta\Phi^{-1}(v)}{\sqrt{1-\theta^2}})$

Continued

Copula	$c(u, v; \theta)$	$h(u, v; \theta)$
Frank	$\dfrac{-\theta\exp(-\theta u)\exp(-\theta v)(\exp(-\theta)-1)}{((\exp(-\theta)-1)+(\exp(-\theta u)-1)(\exp(-\theta v)-1))^2}$	$\dfrac{\exp(-\theta v)(\exp(-\theta u)-1)}{(\exp(-\theta)-1)+(\exp(-\theta u)-1)(\exp(-\theta v)-1)}$
Clayton	$u^{-\theta-1}v^{-\theta-1}(1+\theta)(u^{-\theta}+v^{-\theta}-1)^{-1/\theta-2}$	$v^{-\theta-1}(u^{-\theta}+v^{-\theta}-1)^{-1-1/\theta}$
Gumbel	$C(u,v)(uv)^{-1}((-\ln u)^{\theta}+(-\ln v)^{\theta})^{-2+2/\theta}(\ln u\ln v)^{\theta-1}\cdot(1+(\theta-1)((-\ln u)^{\theta}+(-\ln v)^{\theta})^{-1/\theta})$	$C(u,v)v^{-1}(-\ln v)^{\theta-1}((-\ln u)^{\theta}+(-\ln v)^{\theta})^{1/\theta-1}$

However, there exists a significant number of selectable pair-Copulas with the increase of the dimension of vector $\boldsymbol{x}$. Thus, a D-vine [25] is adopted for it is free to arrange dependent pairs among multiple PCs. According to the D-vine, the component reliability $R_{ij}(t)$ can be written as

$$R_{ij}(t)=r_N(t)h_{N-1|N}(r_{N-1}(t),r_N(t);\theta_{N-1|N})\cdots h_{1N|2\cdots(N-1)}(r_{1|2\cdots(N-1)}(t),r_{N|2\cdots(N-1)}(t);\theta_{1N|2\cdots(N-1)}) \tag{7}$$

where θ is the parameter of the conditional density function of the pair-Copulas. $h(\cdot)$ denotes the conditional density, which can be expressed [16] as

$$h_{k,v_s|v_{-s}}(r_{k|v_{-s}}(t),r_{v_s|v_{-s}}(t);\theta_{k,v_s|v_{-s}})=\frac{\partial C_{k,v_s|v_{-s}}(r_{k|v_{-s}}(t),r_{v_s|v_{-s}}(t);\theta_{k,v_s|v_{-s}})}{\partial r_{v_s|v_{-s}}(t)} \tag{8}$$

where $\boldsymbol{v}$ is the conditioning vector, v_s is an arbitrarily chosen variable of $\boldsymbol{v}$, v_{-s} is the vector $\boldsymbol{v}$ excluding v_s. $C(\cdot)$ is a bivariate Copula function. $h(\cdot)$ denotes the partial derivative concerning the second parameter of $C(\cdot)$.

After obtaining the marginal reliability of PCs, only the form and parameter of the pair-Copulas should be determined when establishing a D-vine model of component reliability. To address this problem, the Akaike information criterion (AIC) principle [26] is first used to select the optimal form of the pairwise connection function in $R_{ij}(t)$. Then, estimate the global parameters of the D-vine Copula model based on the determined forms of pair-Copulas by using the maximum likelihood estimation. Finally, the global parameters of the pair-Copulas of $R(t)$ can be estimated together based on the forms. The value of AIC can be calculated as follows:

$$\text{AIC}=-2\ln L+2\lambda \tag{9}$$

where L is the maximum value of the likelihood function of the candidate-specific pair-Copulas, and λ is the number of parameters. The parameters estimation of the optimal form should be with the minimum value of AIC.

To sum up, based on the marginal reliability of the PCs, the component reliability with multiple PCs can be computed by the D-vine. Then, after obtaining the component reliability, system reliability can be easily evaluated according to the established reliability block diagram.

3. Reliability-based selective maintenance

The RSM mainly focuses on the reliability estimation method of components with multiple dependent degradation PCs. The specific decision process of RSM is shown in Fig. 1. When the current mission is completed, system reliability is compared with a pre-specified safety reliability threshold R_0 to decide if selective maintenance is needed. If so, the RSM decision could be made based on the component reliability and maintenance resources to select the optimal components. Specifically, if the component has only a single degraded PC, the reliability can be directly calculated by (4). While if a component contains multiple PCs, the marginal reliability of each PC should be recomputed, and the reliability of the component also needs to be re-evaluated according to (7).

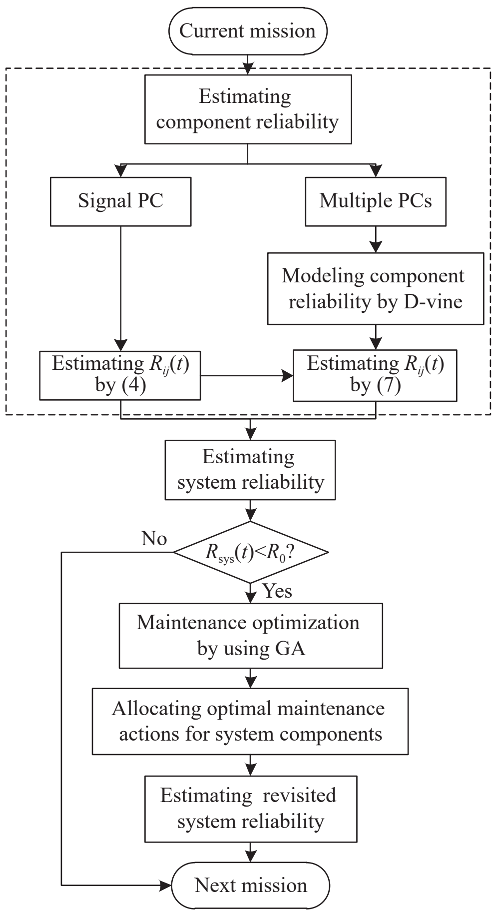


Fig. 1 Maintenance decision process of RSM

Throughout this study, it is assumed that: (i) components reliability as well as the system reliability can be monitored by sensors; (ii) no maintenance action is allowed during the mission; (iii) the system does not work during the maintenance break, and degradation of the components is ignored until the next mission starts.

3.1 Reliability revisited model

The revisited reliability of components is determined by the maintenance actions. Multiple maintenance actions that including do-nothing (l = 0), imperfect maintenance (l = 1, 2, $\cdots$, $n-1$), and replacement ($l = n$), are considered, where l is the level of maintenance action. Components can be restored "as good as new" after replacement, and the component reliability returns to 1. The imperfect maintenances are mainly employed to maintain components from different levels. Examples of this kind of maintenance action are disassembling and washing of machinery, repair, and maintenance of the moving parts and calibrations. These actions can restore components to a level between "as good as new" and "as bad as old". In the case where do-nothing is carried out, component reliability is invariant.

To evaluate the revisited reliability of components, the following decision variable is defined firstly:

$$z(l)=\begin{cases}1, & l=1,2,\cdots,n\\ 0, & l=0\end{cases}. \tag{10}$$

Assume that the system has just completed the current mission ω and is ready for maintenance. The degradation of the kth PC of a component C_{ij} is d_{ijk}, then the revisited reliability of the component's PC for executing the next mission ω+1 can be derived from (4) as

$$r_{ijk}^{\omega+1}(t|d_{ijk}^{*})=\varPhi\left(-\frac{\mu_{ijk}t-(D_{f,ijk}-d_{ijk}^{*})}{\sigma_{ijk}\sqrt{t}}\right)-\exp\left(\frac{2\mu_{ijk}(D_{f,ijk}-d_{ijk}^{*})}{\sigma_{ijk}^{2}}\right)\varPhi\left(\frac{-(D_{f,ijk}-d_{ijk}^{*})-\mu_{ijk}t}{\sigma_{ijk}\sqrt{t}}\right) \tag{11}$$

where d_{ijk}^{*} is the updated degradation level of the kth PC (k=1, 2, $\cdots$, N) of component C_{ij} at the beginning of the next mission, such that:

$$d_{ijk}^{*}=[\eta_l\cdot z_{ijk}(l)+(1-z_{ijk}(l))]\cdot d_{ijk} \tag{12}$$

where d_{ijk} is the degradation level of the kth PC of component C_{ij} when the current mission is completed, and η_l is the degradation reduction coefficient of maintenance level l. Equation (12) illustrates that if a maintenance action with suitable level l is performed on the kth PC of the component at the end of the current mission, the revisited degradation can be determined by the degradation level d_{ijk} and the degradation reduction coefficient η_l. Based on (7) and (11), the revisited component reliability with multiple PCs can be computed by D-vine.

3.2 Maintenance cost and time model

In RSM, the maintenance actions allocated to system components are related to the degradation states of the component's PCs. If a component is functioning at the end of the current mission, it can be allocated to imperfect maintenance and replacement. If it fails, the component must be replaced. In addition, a component is also considered replaced if it is functioning but its degradation state cannot meet the reliability requirement of executing the next mission. Allocating different maintenance actions to system components may consume different maintenance costs and time.

The maintenance cost can be computed as the sum of the maintenance cost of all selected components at the end of the current mission. Besides, considering that the total cost of joint maintaining multiple components can be saved by sharing the basic maintenance cost, which includes some necessary maintenance resources such as labor, materials, and tools [27,28]. Thus, the total maintenance cost of CMC should be minus the cost-saving. The CMC can be evaluated as

$$\mathrm{CMC}=\sum_{i=1}^{M}\sum_{j=1}^{m_i}\sum_{l=1}^{n}(c_{p,ij}(l)\cdot\varepsilon_{ij}+c_{r,ij}(l)\cdot(1-\varepsilon_{ij}))\cdot z_{ij}(l)-\left(\sum_{i=1}^{M}\sum_{j=1}^{m_i}\sum_{l=1}^{n}z_{ij}(l)-1\right)\cdot c_s \tag{13}$$

where $c_{r,ij}$ denotes the replacement cost of component j in subsystem i, $c_{p,ij}$ denotes the cost of imperfect maintenance, and $c_{r,ij} > c_{p,ij}$. ε_{ij} is an indicator variable, ε_{ij} =1 means imperfect maintenance is allocated to component C_{ij}, while ε_{ij} = 0 means replacement. c_s denotes the basic maintenance cost incurred by maintenance actions (except do-nothing).

Similarly, the same computation is adapted to the total maintenance time, which is the sum of the maintenance time of all selected components. The total maintenance time can be evaluated as

$$\mathrm{TMT}=\sum_{i=1}^{M}\sum_{j=1}^{m_i}\sum_{l=1}^{n}(t_{p,ij}(l)\cdot\varepsilon_{ij}+t_{r,ij}(l)\cdot(1-\varepsilon_{ij}))\cdot z_{ij}(l) \tag{14}$$

where $t_{r,ij}$ denotes the replacement time of component C_{ij}, and $t_{p,ij}$ denotes the imperfect maintenance time.

3.3 Optimization model of RSM

Let $R_{sys}(\omega+1)$ denote the system reliability for executing the next mission ω+1. The RSM schedule of completing the next mission can be attained by solving the following two optimization models.

Model 1: the reliability maximization model. This model focuses on maximizing the system's reliability after maintenance within limited maintenance cost and time. The optimization model can be defined as follows:

$$\text{Max}: R_{\text{sys}}(\omega+1), \tag{15}$$

subject to

$$\text{CMC} \leqslant c_0, \tag{16}$$

$$\text{TMT} \leqslant t_0, \tag{17}$$

$$\sum_{l_{ij}=1}^{n} z_{ij}(l) \leqslant 1, \ \ \forall z \in \{0,1\}, \tag{18}$$

where c_0 is the maximum maintenance cost, and (16) is the constraint of the total maintenance cost, t_0 is the maximum time required for maintenance, the constraint in (17) guarantees that the total maintenance time does not exceed the available time t_0. Equation (18) states that if a component C_{ij} is selected to maintain, only one maintenance level l (l =1, 2, $\cdots$, n) can be allocated to the component.

Model 2: the maintenance cost minimum model. This model devotes to minimizing maintenance costs while meeting the reliability requirements of executing the next mission within limited maintenance time. The optimization model can be defined as follows:

$$\text{Min}: \text{CMC} = \sum_{i=1}^{M}\sum_{j=1}^{m_i}\sum_{l=1}^{n}(c_{p,ij}(l)\cdot\varepsilon_{ij} + c_{r,ij}(l)\cdot (1-\varepsilon_{ij}))\cdot z_{ij}(l) - \left(\sum_{i=1}^{M}\sum_{j=1}^{m_i}\sum_{l=1}^{n} z_{ij}(l) - 1\right)\cdot c_s, \tag{19}$$

subject to (17), (18) and

$$0 \leqslant R_0 \leqslant R_{\text{sys}}(\omega+1) \leqslant 1, \tag{20}$$

where R_0 is the required reliability level for executing the next mission, and (20) is the constraint of the revisited reliability of the system.

In this study, the GA [29,30] is introduced to find the optimal maintenance components that can maximize the system's reliability and minimize the maintenance cost. Firstly, the maintenance vectors that contain all the system components coded by binary are randomly generated to establish the initial population. Secondly, the fitness and constraints of each vector are evaluated. Thirdly, the fitness-based vectors are selected from the population to be parents for crossover and mutation. After exiting the iteration (the iterations $g \geqslant 200$), the optimal maintenance components can be defined. The optimization process is shown in Fig. 2.

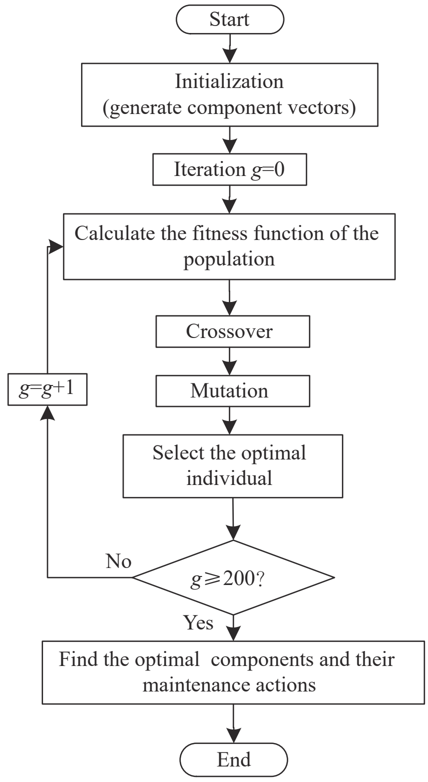


Fig. 2 Optimization process of GA

4. Simulation example

To illustrate the application of the RSM, the presented policy is applied to a series-parallel redundant system. The reliability block diagram of the system is shown in Fig. 3. The system is composed of two subsystems, and each subsystem is composed of five degradation components. The component C_{1j} and C_{5j} are particular kinds of components, and each of them contains multiple degradation PCs. The PCs can be wear, corrosion, crack propagation, oil-contamination accumulation, and so on. The components C_{2j}, C_{3j}, and C_{4j} are assumed to have only one PC.

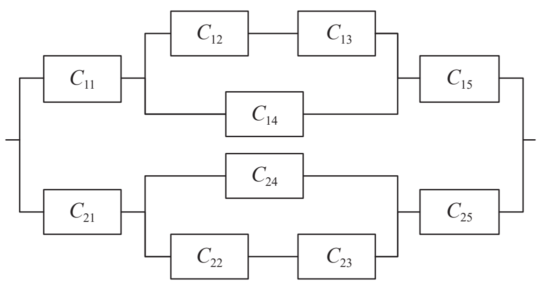


Fig. 3 Reliability block diagram of the series-parallel redundant system

4.1 System reliability assessment

The degradation process of each component of PCs is depicted by the Wiener process. The values of drift rate μ, diffusion parameter σ, and the threshold degradation D_f are given in Table 2, where j=1, 2; k=1, 2, $\cdots$, 3. Table 2 also shows the components' degradation level when the current mission is completed.

Table 2 Simulation parameters of the system components

Parameter	C_{1jk}	C_{2j}	C_{3j}	C_{4j}	C_{5jk}
μ	(0.3172, 0.1399 0.0965)	0.2401	0.4213	0.1165	(0.2066, 0.1164, 0.1414)
σ	(2.8908, 2.6244, 1.5906)	1.001	2.9238	2.5922	(1.9917, 1.6394, 1.4731)
D_f	(80, 130, 100)	80	102	68	(120, 98, 70)
d	(55, 112, 80)	68	52	44	(98, 82, 48)

According to the maintenance decision process of RSM in Fig.1. Firstly, the reliability distribution of C_{1j} and C_{5j} should be built. By using the simulation parameters in Table 1, the marginal reliability of component PCs can be computed by (4). Based on the D-vine model, the reliability of C_{1j} and C_{5j} is the joint reliability distribution of the PCs and can be decomposed as (8) shows. The D-vine connection model of the PCs can be constructed as Fig. 4.

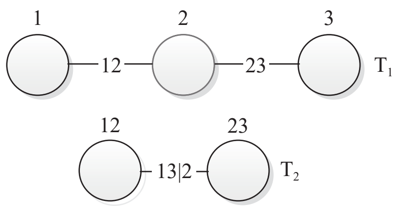


Fig. 4 D-vine model of the PCs of C_{1j} and C_{5j}

The corresponding joint reliability distribution can be defined as follows:

$$R_{ij}(t) = R_3(t)h_{23}(R_2(t), r_3(t); \theta_{23})h_{13|2}(h_{12}(r_1(t), r_2(t); \theta_{12}), h_{32}(r_3(t), r_2(t); \theta_{23}); \theta_{13|2}). \tag{21}$$

Secondly, the optimal Copula function of the pair-Copulas in (21) should be estimated. The candidate Copula function is Gaussian, Frank, Clayton, and Gumbel (See Table 1 for specific form). Each of them is respectively optimized by using the AIC principle. The most appropriate Copula function for the pair-Copulas of components C_{1j} and C_{5j} is shown in Table 3.

Table 3 Forms and parameters for the pair-Copulas

h-function	Candidate Copulas	Parameter of the pair-Copulas (θ)		AIC	
		C_{1j}	C_{5j}	C_{1j}	C_{5j}
h_{12}	Gaussian	0.0645	−0.0091	0.3321	1.9669
	Clayton	0.0270	**0.0144**	1.8199	1.9407
	Frank	**0.3992**	−0.0070	0.0384	1.9994
	Gumbel	1.0351	1.0000	0.6659	2.0001
h_{23}	Gaussian	0.1251	0.1164	−4.3102	−3.4530
	Clayton	0.1376	0.1510	−2.7710	−3.3602
	Frank	**0.8052**	**0.6710**	−5.7434	−3.5202
	Gumbel	1.0758	1.0698	−3.0865	−2.9090
$h_{13\|2}$	Gaussian	0.0292	0.0080	1.6586	1.9743
	Clayton	**0.0135**	**0.0567**	1.3759	0.9508
	Frank	0.1576	0.0196	1.7052	1.9954
	Gumbel	1.0227	1.0000	1.9449	2.0000

According to the optimal value of AIC in Table 3, Frank, Frank, Clayton are proposed for h_{12}, h_{23}, and $h_{13|2}$ of component C_{1j}, and Clayton, Frank, Clayton are proposed for h_{12}, h$_{23}$, and $h_{13|2}$ of component C_{5j}. After obtaining the optimal forms of the pair-Copulas, the reliability distribution of C_{1j} and C_{5j} can be defined as follows:

$$R_{C_{1j}}(t) = r_3(t)h_{23}(r_2(t), r_3(t); \theta_{\text{Frank}})\cdot h_{13|2}(h_{12}(r_1(t), r_2(t); \theta_{\text{Frank}})h_{32}(r_3(t), r_2(t); \theta_{\text{Frank}}); \theta_{\text{Clayton}}), \tag{22}$$

$$R_{C_{5j}}(t) = r_3(t)h_{23}(r_2(t), r_3(t); \theta_{\text{Frank}})\cdot h_{13|2}(h_{12}(r_1(t), r_2(t); \theta_{\text{Clayton}})h_{32}(r_3(t), r_2(t); \theta_{\text{Frank}}); \theta_{\text{Clayton}}). \tag{23}$$

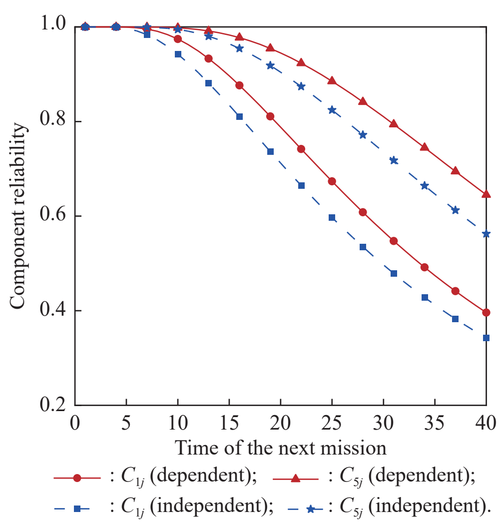
The parameters of pair-Copulas(θ) in (22) and (23) are re-estimated together by the maximum likelihood estimation method to determine the reliability distribution of C_{1j} and C_{5j}. Here, the fminsearch function is used to address the optimization problem of the likelihood function. The estimations of the global parameters for the pair-Copulas are shown in Table 4.

Table 4 Re-estimation results of the parameters of pair-Copulas(θ)

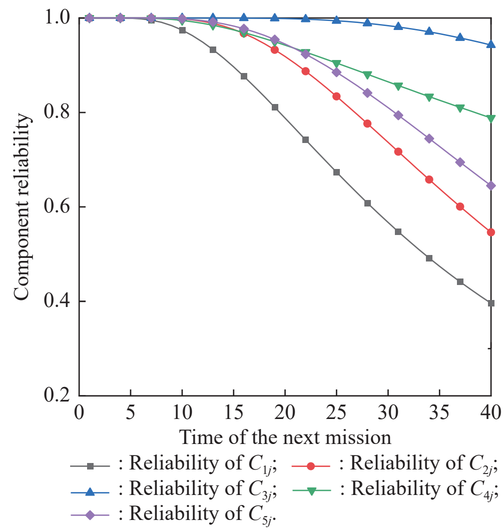
Component	C_{1j}			C_{5j}		
	h_{12}	h_{23}	$h_{13\mid 2}$	h_{12}	h_{23}	$h_{13\mid 2}$
Copula	Frank	Frank	Clayton	Clayton	Frank	Clayton
Estimation	0.1133	0.1155	0.5730	0.1014	0.1029	0.1476

Finally, with the marginal reliability of component PCs, the joint reliability of C_{1j} and C_{5j} can be easily computed according to (22) and (23).

According to the current degradation level, the component reliability of the series-parallel redundant system for performing the next mission can be computed by (4) and (7). The reliability of system components is shown in Fig. 5. Besides, the reliability of particular components, C_{1j} and C_{5j}, when their PCs are considered independent, is also shown in Fig. 5.



(a) Reliability of components C_{1j} and C_{5j}



(b) Reliability of system components

Fig. 5 Component reliability for executing the next mission

As shown in Fig. 5(a), the full lines in the figure denote the joint reliability of C_{1j} and C_{5j} based on the D-vine model, and dotted lines denote the reliability irrespective of the correlation among their degradation PCs. From Fig. 5, it can be concluded that the component reliability seems to be conservative when the dependency among multivariate performance parameters is ignored. Moreover, if there are components with multiple PCs existing in the system, the effectiveness of RSM will be affected by the conservative reliability of the system since the system reliability is an important basis when formulating a reasonable RSM policy.

4.2 Maintenance optimization

According to the reliability estimation results in the previous section, a reasonable RSM policy can be made for the system shown in Fig. 3. Assume that the RSM contains four maintenance actions. They are do-nothing, cleaning and adjusted, and preventative maintenance, and replacement, where cleaning and adjusted, preventive maintenance are two kinds of imperfect maintenance actions. Do-nothing means no component is selected for maintenance. It also consumes no maintenance resources, and the corresponding maintenance efficiency is 0. Except for do-nothing, Table 5 presents the maintenance cost and time allocated to the system components when operating different maintenance actions. The degradation reduction efficiency of maintenance actions is also shown in Table 5.

Table 5 Maintenance cost, time, and efficiency for each kind of maintenance action

Maintenance action	Efficiency (η)	Maintenance cost/time				
		C_{1j}	C_{2j}	C_{3j}	C_{4j}	C_{5j}
Replacement	1	49/3	47/3.2	48/3.2	50/3	53/3.4
Preventative maintenance	0.3	35/2.7	36/2.8	39/2.5	41/2.3	43/2.1
Cleaning and adjusted	0.1	32/1.4	24/1.5	27/1.5	33/1.6	35/1.4

According to the current degradation level of system components presented in Table 2, all of the system components in the system are operational at the end of the current mission. Assume that the next mission is expected to be completed in 40 days. Equations (4) and (7) can be used to estimate the reliability of the system components for executing the next mission. The estimation results are shown in Table 6. Assumed that if the component reliability is lower than 0.4, the component is considered to be failed for executing the next mission. In contrast, if the component reliability is higher than 0.9, that means the component needs no maintenance actions. Thus, the component C_{3j} should be allocated do-nothing action, C_{1j} should be replaced, and the other components have a chance to perform imperfect maintenance actions.

Table 6 Component reliability for executing the next mission

Component	C_{1j}	C_{2j}	C_{3j}	C_{4j}	C_{5j}	System reliability
Reliability (dependent)	0.3961	0.5463	0.9428	0.7885	0.6450	0.4060
Reliability (independent)	0.3423	0.5463	0.9428	0.7885	0.5626	0.3158

It is also assumed that, for Model 1 (maximizing system reliability), the time available for maintaining the system is 8 h, and the constrained total cost for maintenance actions is 120. For Model 2 (minimizing the cost), the maintenance time is invariable, and the constrained system reliability for executing the next mission is 0.9. The basic maintenance c_s is 5. Accordingly, GA is used to solve the constraint optimization problems of Model 1 and Model 2. Optimization results are shown in Table 7.

Table 7 Optimization results

Policy	Model	Cost	Time	System reliability	
				Dep. PCs	Ind. PCs
RSM	Model 1	118	7.9	0.9888	0.9884
	Model 2	87	5.1	0.9193	0.9138

As shown in Table 7, the system reliability of Model 1 for executing the next mission is elevated to 0.9888, and the maintenance cost and time are 118 h and 7.9 h, respectively. Under the same constraint of maintenance time, the minimum maintenance cost of Model 2 attained by GA is 87, the system reliability is 0.9193, and the maintenance time is 5.1. It can be concluded that system reliability for executing the next mission of Model 1 is 6.95% higher than that of Model 2 and it correspondingly consumes 31 more costs and 2.8 more hours.

To illustrate the effects of the proposed RSM, optimization results of the system when ignoring the dependence of component PCs are also given in Table 7. Under the same maintenance constraints, the reliability of the system with independent PCs of component for executing the next mission is 0.9884 for Model 1, and 0.9138 for Model 2, both of which are lower than that of considering dependence PCs. This means that ignoring the dependency among multiple PCs also results in a conservative estimation of the system reliability. The optimal maintenance actions of the selected system components when operating different maintenance policies are shown in Fig. 6.

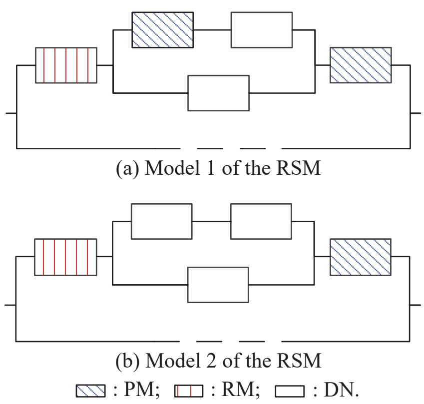


Fig. 6 Optimal maintenance actions

From Fig.6 (a), it can be seen that components C_{1j} should be replaced, and components C_{2j} and C_{5j} are required to perform preventive maintenance action. From Fig.6(b), for minimizing the total maintenance cost, the preventive maintenance action of C_{2j} is abandoned.

5. Conclusions

In this paper, the RSM of systems with dependent PCs of components is proposed. To obtain more accurate reliability estimations, a joint reliability model of the component with dependent PCs is developed by using D-vine. Based on that, two RSM policy models are formulated to ensure the system completes the next mission successfully, with the objectives of maximizing the system reliability and minimizing the maintenance cost, respectively. The usage and benefits of the proposed RSM policy are illustrated by applying that to a simulation example, the series-parallel redundant system with ten degradation components. The numerical results illustrate that the system reliability for executing the next mission is conservative when the dependency between PCs of components is ignored. This means that when a component contains multiple dependent degradation PCs, taking the dependency of PCs into account can obtain a more accurate system reliability estimation result. Furthermore, supported by the accurate system reliability, the proposed RSM can better assist the maintenance decision-makers in specifying a reasonable maintenance schedule.

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