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Revised barrier function-based adaptive finite- and fixed-time convergence super-twisting control

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Abstract: This paper presents an adaptive gain, finite- and fixed-time convergence super-twisting-like algorithm based on a revised barrier function, which is robust to perturbations with unknown bounds. It is shown that this algorithm can ensure a finite- and fixed-time convergence of the sliding variable to the equilibrium, no matter what the initial conditions of the system states are, and maintain it there in a predefined vicinity of the origin without violation. Also, the proposed method avoids the problem of overestimation of the control gain that exists in the current fixed-time adaptive control. Moreover, it shows that the revised barrier function can effectively reduce the computation load by obviating the need of increasing the magnitude of sampling step compared with the conventional barrier function. This feature will be beneficial when the algorithm is implemented in practice. After that, the estimation of the fixed convergence time of the proposed method is derived and the impractical requirement of the preceding fixed-time adaptive control that the adaptive gains must be large enough to engender the sliding mode at time $t=0$ is discarded. Finally, the outperformance of the proposed method over the existing counterpart method is demonstrated with a numerical simulation.

Keywords: super-twisting algorithm, barrier function, fixed-time sliding mode control, adaptive control.

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1. Introduction

For controlling perturbed systems with matching disturbances, sliding mode control (SMC) has been one of the best choices [1,2]. In the past two decades, significant progress has been made with the higher-order SMC (HOSM), which was proven to be an effective way of reducing chattering [3–6]. Among the HOSMs, the continuous second order SMC (2-SMC), especially the super-twisting (STW) SMC, has been rigorously studied. In many practical cases, the perturbation bounds cannot be known, and the control gains are overestimated while implementing the STW control law, which further leads to the increase of the chattering [7]. Also, the control gains should increase and decrease according to the variation of the disturbances. In order to solve this problem, researchers have developed adaptive HOSM algorithms of that dynamically adjust the control gains to be as small as possible while staying large enough to counteract the disturbances. Fundamentally, there are two classes of approaches for creating adaptive SMC.

The first method is an equivalent control-based strategy [3,4,8–10]. It uses a low-pass filter to approximate the equivalent control which acts as an estimation of the disturbance. The equivalent control signal was first applied to super-twisting control in [4]. The aim was to avoid the gain overestimation of the discontinuous term in the integral part of STW control. As a result, the chattering of the system was reduced. However, the gain of the square-root term in the STW control was still overestimated [9]. Then the equivalent control method based STW control was extended to a so-called adaptive dual layer super-twisting algorithm (ADLSTA) by adapting both the two gains to further ameliorate the overestimation [3,8]. Tian et al. [11] further expanded the conventional single-input-and-single-output ADLSTA into a multivariable version and applied it to the attitude tracking control of quadrotors. However, the shortcoming of the equivalent control signal-based methods is that its filter constant must be chosen much smaller than the inverse of the upper bound of the derivative of the disturbance. Thus, the order of magnitude of the disturbance derivative upper bound is required in the control design.

The second method increases the adaptive gain until the sliding mode is achieved, and then the gain remains unchanged until the sliding mode is lost due to the growing disturbances. Then, the control gain grows again to reach the sliding mode once more [7,12]. The main disadvantage of this approach is that the adaptive gains are unable to decrease once the magnitude of the disturbances gets small. In order to deal with this problem, an

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increasing and decreasing adaptive gain method was proposed [13–15]. This method guarantees the convergence of the sliding variable to a vicinity of the origin while the overestimation of the adaptive gain is mitigated. Compared with the equivalent control-based strategy, the advantage of this method is that even the order of magnitude of the disturbance derivative upper bound is no longer needed to be known. However, this method still exhibits the problem that the sliding variable cannot be confined strictly in a predefined vicinity of the origin, and it is hard to estimate the final neighborhood in which the sliding variable is confined. Hence, the control precision with this method is not as high as that of the equivalent control-based method. Neither of these two methods can both guarantee the control precision and waive the knowledge of the disturbance derivative upper bound simultaneously.

Faced with this dilemma, recently, increasing and decreasing adaptive gain approaches based on a barrier function (BF) have been applied to the SMC [16–20]. With this method, the sliding variable can be confined strictly in the predefined vicinity of the origin and the overestimation of the adaptive gain is further reduced. However, when the method is implemented, the sampling step must be set small enough so that the variable cannot go beyond the predefined vicinity of the origin and the advantages of the BF can be maintained.

Although most of the HOSM methods could guarantee finite time convergence, their convergence time always depends on the initial conditions of the sliding variable. Thus, the convergence time may become infinite when the initial values of the variables grow unboundedly [21]. For this reason, the finite- and fixed-time convergence controller and observer has been extensively studied recently. The fixed-time controller can avoid the dependence of the convergence time on the initial conditions and provide the controller designer with an estimation of convergence time. Furthermore, the fixed-time observer enables us to safely employ the separation principle [22]. A recent overview of this field was given in [22]. In the field of fixed-time control, scalar systems, multivariable systems and multi-dimensional systems have all been studied [21,23,24]. The case when the initial conditions of the disturbance are unknown is also investigated [25]. However, the adaptive fixed-time convergence control method has not been studied in depth, except for the contribution by [13].

In this paper, the study of adaptive finite- and fixed-time convergence control is carried further. It can be seen as an improvement of the work in [13]. Inspired by [18], a revised-BF(RBF)-based strategy is proposed and applied in adapting the control gains of the STW. Moreover, it is the first time that BF is used to handle the case when the bounded disturbances (square-root growth disturbances) and disturbances with bounded rates (Lipschitz disturbances) are present together.

The contributions of this paper are as follows:

(i) The sliding variable converges in a fixed time to a predefined vicinity of the origin and then is strictly confined in it rather than to an unknown vicinity of the origin like the previous method. No knowledge of the upper bounds of the disturbances is required and the adaptive gains are no longer overestimated.

(ii) A revised version of the BF is proposed so that the computational burden caused by the necessity of setting sampling step quite small is mitigated effectively.

(iii) Different from [13], the impractical requirement that the initial value of the adaptive gain is large enough to engender the sliding mode is avoided. The adapting time of the control gain is included in the convergence time. The estimation of the fixed convergence time is still viable.

This paper is organized as follows. Section 2 presents the statement of the control problem under assumptions. Section 3 is dedicated to the proposed adaptive controller design. Section 4 offers simulation examples to verify the efficacy of the proposed method. The conclusions are given in Section 5.

2. Problem statement and definitions

2.1 Control objective

Consider a first-order system

$$\dot{x} = u + \zeta \tag{1}$$

where $x \in \mathbf{R}$ is the system state, $u \in \mathbf{R}$ is the control input, and $\zeta \in \mathbf{R}$ represents the disturbance which could be formulated as $\zeta = \zeta_1 + \zeta_2$. Assume that

$$|\zeta_1| \leqslant K|x|^{1/2} \tag{2}$$

and

$$|\zeta_2| \leqslant Lt \tag{3}$$

where K and L are unknown upper bounds of square-root growth disturbance ζ_1 and Lipschitz disturbance ζ_2 respectively. The design objective could be summarized as Problem 1.

Problem 1 Given the scalar control system in (1) subject to assumptions in (2) and (3), the objective is to design a continuous adaptive control law so that the resulting closed-loop system is fixed-time convergent and subsequently confined in a given vicinity of the origin. In addition, the convergence (settling) time is estimated.

2.2 Basic definitions

The definitions of finite- and fixed-time convergence to the origin and the vicinity of the origin are given as follows [13,22].

Definition 1 The control system in (1) is called fixed-time convergent to the origin, if there exists a time T such that the system state x is equal to zero for $t \geqslant T$, even if x starts from any initial condition $x(0) = x_0 \in \mathbf{R}$.

Definition 2 The control system in (1) is called fixed-time convergent to the vicinity $S \subset \mathbf{R}$ of the origin, if there is a time T such that the state x belongs to S for $t \geqslant T$, even if x starts from any initial condition $x_0 \in \mathbf{R}$.

3. Controller design and stability proof

3.1 RBF

Before the main result of the adaptive control law is introduced, the definition of the RBF is given as follows.

Definition 3 Let us suppose that some $\delta > 0$ is given and fixed, then the RBF can be defined as an even continuous function α_b: $x \in [-\delta, \delta] \rightarrow \alpha_b(x) \in [\bar{F}, \infty]$ that is strictly increasing on $[0, \delta]$. Then

$$\alpha_b = \frac{\bar{F}\delta^{1/m}}{\delta^{1/m} - |x|^{1/m}} \tag{4}$$

where m is a positive integer to be chosen and $\bar{F} > 0$ is the minimum value that α_b could attain; furthermore, $\lim\limits_{|x| \to \delta} \alpha_b(x) = \infty$. The benefit of the revised version is that it can alleviate the problem that the sampling step must be chosen very small when the BF-based method is applied in practice. This trait of the RBF will be illustrated later.

3.2 Control structure

The structure of adaptive super-twisting-like controller is

$$u = -\alpha|x|^{1/2}\mathrm{sign}(x) - \alpha_1|x|^p\mathrm{sign}(x) - \frac{\beta}{2}\int_0^t \mathrm{sign}(x(s))\,\mathrm{d}s \tag{5}$$

where α, α_1, β are positive and $p > 1$. α and β are RBF-based adaptive parameters and parameter α_1 is fixed. It can be shown that this super-twisting-like controller reduces to the conventional super-twisting algorithm if α_1 is chosen to be zero. $|x|^p\mathrm{sign}(x)$ acts as a higher-order term to speed up the convergence. Accordingly, the dynamic equation of the closed-loop system could be derived as

$$\begin{cases} \dot{x} = -\alpha|x|^{1/2}\mathrm{sign}(x) - \alpha_1|x|^p\mathrm{sign}(x) + y + \zeta_1 \\ \dot{y} = -\dfrac{\beta}{2}\mathrm{sign}(x) + \dot{\zeta}_2 \\ x(0) = x_0 \\ y(0) = 0 \end{cases} \tag{6}$$

where $\dot{\zeta}_2$ exists almost everywhere. The solution of (6) is understood in the sense of Filippov [26]. The main result of this proposed method is given as follws.

3.3 Main results

Lemma 1 Consider a closed-loop control system (6) with disturbance $\zeta = \zeta_1 + \zeta_2$ subject to assumptions (2) and (3) for some unknown constants K and L. Suppose that, at time t_1, $|x(t_1)| < \delta$ and the adaptive control parameters α and β are chosen as

$$\begin{cases} \alpha(x,t) = \alpha_b \\ \beta(x,t) = 2\varepsilon\alpha + 2\left(\lambda + 4\varepsilon^2\right)\alpha_1\varepsilon_1^{p-1/2} \end{cases} \tag{7}$$

where α_b is the RBF as in (4), λ, ε and $\varepsilon_1 > \delta$ are positive constants selected by the designer. Then, for all $t \geqslant t_1$, $|x|$ is strictly confined in the domain $|x| < \delta$.

Proof Define a new state vector $\boldsymbol{z} = [z_1, z_2]^{\mathrm{T}}$ as

$$\begin{cases} z_1 = |x|^{1/2}\mathrm{sign}(x) \\ z_2 = y \end{cases}.$$

Based on the assumptions on the disturbances ζ_1 and ζ_2 in (2) and (3), define

$$\zeta_1 = \rho_1 z_1, \tag{8}$$

$$\dot{\zeta}_2 = \frac{\rho_2 z_1}{2|z_1|}, \tag{9}$$

where coefficients ρ_1 and ρ_2 are uncertain and meet the conditions:

$$\begin{cases} |\rho_1| \leqslant K \\ |\rho_2| \leqslant 2L \end{cases}. \tag{10}$$

Invoking dynamic (6), the derivative of $\boldsymbol{z}$ can be written as

$$\dot{\boldsymbol{z}} = \frac{1}{2|z_1|}\left(\boldsymbol{A}\boldsymbol{z} + \begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix} z_1\right) \tag{11}$$

where

$$\boldsymbol{A} = \begin{bmatrix} -\alpha - \alpha_1|z_1|^{2p-1} & 1 \\ -\beta & 0 \end{bmatrix}. \tag{12}$$

Note that $\mathrm{sign}(x) = \mathrm{sign}(z_1)$ and x, y will converge to the origin in a finite and fixed time T if z_1, z_2 converge to the origin in a finite and fixed time T.

Consider a Lyapunov function candidate

$$V(\boldsymbol{z}) = \boldsymbol{z}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{z} \tag{13}$$

where

$$\boldsymbol{P} = \begin{bmatrix} \lambda + 4\varepsilon^2 & -2\varepsilon \\ -2\varepsilon & 1 \end{bmatrix},\ \lambda > 0; \varepsilon > 0. \tag{14}$$

The derivative of the Lyapunov function candidate in (14) is

$$\dot{V}(z) = \dot{z}^{\mathrm{T}} \boldsymbol{P} z + z^{\mathrm{T}} \boldsymbol{P} \dot{z} = \frac{1}{2|z_1|}\left[z^{\mathrm{T}}\left(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}\right) z + 2 z^{\mathrm{T}} \boldsymbol{P} \left[\begin{array}{c} \rho_1 \\ \rho_2 \end{array} \right] z_1 \right] = -\frac{1}{2|z_1|} z^{\mathrm{T}} \boldsymbol{Q}(z) z$$

where

$$\boldsymbol{Q}(z) = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{12} & Q_{22} \end{pmatrix} \tag{15}$$

with

$$\begin{cases} Q_{11} = 2\lambda\alpha + 4\varepsilon(2\varepsilon\alpha - \beta) + 4\left(\lambda + 4\varepsilon^2\right)\alpha_1 |z_1|^{2p-1} - \\ \quad 2\left(\lambda + 4\varepsilon^2\right)\rho_1 + 4\varepsilon\rho_2 \\ Q_{12} = \beta - 2\varepsilon\alpha - 2\left(\lambda + 4\varepsilon^2\right)\alpha_1 |z_1|^{2p-1} - \lambda - \\ \quad 4\varepsilon^2 + 2\varepsilon\rho_1 - \rho_2 \\ Q_{22} = 4\varepsilon \end{cases}.$$

In order to guarantee the positive definiteness of the matrix $\boldsymbol{Q}(z)$, we enforce

$$\beta = 2\varepsilon\alpha + 2\left(\lambda + 4\varepsilon^2\right)\alpha_1 \varepsilon_1^{p-1/2}. \tag{16}$$

Matrix $\boldsymbol{Q}(z)$ will be positive definite with a minimum eigenvalue $\lambda_{\min}(\boldsymbol{Q}(z)) > 2\varepsilon$, if $\alpha > \alpha^*$, where

$$\alpha^* = \frac{K(\lambda + 4\varepsilon^2) - \varepsilon(4L+1)}{\lambda(1-\gamma)} + \frac{(2\varepsilon K - 2L - \lambda - 4\varepsilon^2)^2}{12\varepsilon\lambda(1-\gamma)} \tag{17}$$

with positive constant $\gamma < 1$. Therefore

$$\dot{V}(z) \leqslant -\frac{\varepsilon}{|z_1|}\|z\|^2. \tag{18}$$

Since

$$\lambda_{\min}(\boldsymbol{P})\|z\|^2 \leqslant V(z) \leqslant \lambda_{\max}(\boldsymbol{P})\|z\|^2, \tag{19}$$

and

$$|z_1| \leqslant \sqrt{z_1^2 + z_2^2} = \|z\| \leqslant \frac{V^{1/2}(z)}{\sqrt{\lambda_{\min}(\boldsymbol{P})}}, \tag{20}$$

then

$$\dot{V}(z) \leqslant -\mu V^{1/2}(z) \tag{21}$$

where

$$\mu = \frac{\varepsilon\sqrt{\lambda_{\min}(\boldsymbol{P})}}{\lambda_{\max}(\boldsymbol{P})}. \tag{22}$$

Considering that $|x(t_1)| < \delta$ and $\alpha_b(|x|)$ is a monotonically increasing function with respect to $|x|$ when $|x| \in [0, \delta]$, there exists $x_1 < \delta$ as follows:

$$x_1 = \begin{cases} \delta\left(1 - \dfrac{\bar{F}}{\alpha^*}\right)^m, & \bar{F} < \alpha^* \\ 0, & \bar{F} \geqslant \alpha^* \end{cases} \tag{23}$$

such that if $|x| > x_1$, then $\alpha_b > \alpha^*$, and (21) is satisfied. Then, for $x_1 \leqslant |x| < \delta$, $\dot{V}(z)$ is negative. Therefore, $|x|$ will remain confined in $|x| \leqslant x_1 < \delta$ for $t > t_1$. □

Theorem 1 Consider a closed-loop control system in (6) with disturbance $\zeta = \zeta_1 + \zeta_2$ subject to assumptions in (2) and (3) for some unknown constants K and L. Suppose that after time t_0 conditions

$$\begin{cases} \alpha > \alpha^* \\ \alpha > K \\ \beta > 2L \end{cases} \tag{24}$$

hold and $|x(t_0)| > \delta/2$. Then, for any initial condition $x(0) = x_0$, there exists a finite and fixed time:

$$T_f \leqslant t_0 + \frac{1}{\alpha_1(p-1)\varepsilon_1^{p-1}} + \frac{\beta(t_0)/2 + L}{\beta(t_0)/2 - L} t_0 + \frac{2}{\mu}\left[\left(\lambda + 4\varepsilon^2\right)\varepsilon_1 + M^2\left(\frac{1}{\alpha_1(p-1)\varepsilon_1^{p-1}} + t_0\right)^2 - 4\varepsilon\varepsilon_1^{1/2} M\left(\frac{1}{\alpha_1(p-1)\varepsilon_1^{p-1}} + t_0\right)\right] \tag{25}$$

in which $x(t)$ and $y(t)$ converge and are then strictly confined in a vicinity of the origin for $t \geqslant T_f$, and P is a positive definite matrix defined as in (14), via the following adaptive control parameters α and β:

$$\alpha(x,t) = \begin{cases} \alpha_a, & 0 < t \leqslant \bar{t} \\ \alpha_b, & t > \bar{t} \end{cases}, \tag{26}$$

$$\dot{\alpha} = k, 0 < t \leqslant \bar{t}, \tag{27}$$

$$\beta(x,t) = 2\varepsilon\alpha + 2\left(\lambda + 4\varepsilon^2\right)\alpha_1\varepsilon_1^{p-1/2}, \tag{28}$$

$$\alpha(x(0), 0) = \alpha_0, \tag{29}$$

$$\beta(x(0), 0) = \beta_0, \tag{30}$$

where $\lambda, \varepsilon, \kappa, \gamma < 1$, α_0, $\varepsilon_1 > \delta/2$ are positive constants selected by the designer; $\bar{t}$ is the moment when $|x|$ becomes $|x| \leqslant \delta/2$ for the first time. The indicated vicinities for $|x|$ and $|y|$ are δ and η respectively.

Proof First, prove that $|x|$ will reach $\delta/2$ in a finite and fixed time T_f.

Step 1 Consider first that $x(t_0) > \varepsilon_1 > \delta/2$ and $y(t_0)$ is zero or $\mathrm{sign}(y(t_0))$ is opposite to $\mathrm{sign}(x(t_0))$. With the condition $\alpha > K$, the first equation in (6) yields

$$\frac{\mathrm{d}|x|}{\mathrm{d}t} \leqslant -\alpha_1|x|^p, \tag{31}$$

taking into account that $\mathrm{sign}(y)$ will remain opposite to $\mathrm{sign}(x)$ for $t > t_0$, while x does not cross the axis $x = 0$. Equation (31) can be solved to yield

$$\frac{1}{1-p}\left(|x|^{1-p} - |x(t_0)|^{1-p}\right) \leqslant -\alpha_1(t - t_0),$$

which leads to

$$|x|^{p-1} \leqslant \frac{1}{\alpha_1(p-1)(t-t_0)}.$$

Thus, $|x|$ decreases and reaches the value ε_1 for time

$$T_1 - t_0 \leqslant \Delta T_1 = \frac{1}{\alpha_1(p-1)\varepsilon_1^{p-1}},$$

which corresponds to the second term in (25). Note that this term is independent on $x(t_0)$ and further independent on the unknown initial condition x_0.

Step 2 Consider the case when $\operatorname{sign}(y(t_0))$ is the same as $\operatorname{sign}(x(t_0))$. Then from t_0, as $\beta > 2L$, $|y|$ will converge to the origin. Also, as x cannot reach the origin before y does, $\operatorname{sign}(y)$ changes to the opposite of $\operatorname{sign}(x(t_0))$ while $\operatorname{sign}(x)$ remains unchanged. After $\operatorname{sign}(y)$ becomes the opposite of $\operatorname{sign}(x)$, suppose that condition $x(t_0+\Delta T_2) > \varepsilon_1$ remains, Step 1 can be executed. The time for $\operatorname{sign}(y)$ changing to its opposite can be estimated as

$$\Delta T_2 \leqslant \frac{(\beta(t_0)/2+L)t_0}{\beta(t_0)/2-L}.$$

Thus, the time in which $|x|$ decreases and reaches value ε_1 is added by ΔT_2. Then there exists

$$T_1 - t_0 \leqslant \frac{1}{\alpha_1(p-1)\varepsilon_1^{p-1}} + \frac{\beta(t_0)/2+L}{\beta(t_0)/2-L}t_0.$$

Step 1 ends with $|x| = \varepsilon_1$. Thus, if $x(t_0) \leqslant \varepsilon_1$ in the first place, Step 1 and 2 would not be executed; therefore the second and third term in (25) will be absent.

Step 3 Prove that, for $t > T_1$, $|x|$ will continue to decrease until it reaches $\delta/2$ in a certain time T_f. Consider the same Lyapunov function candidate as in (13) and its derivative is

$$\dot{V}(z) = -\frac{1}{2|z_1|}z^{\mathrm{T}}\boldsymbol{Q}(z)z$$

where $\boldsymbol{Q}(z)$ is the same as (15). With β taken as the same expression as in (16) and if $\alpha > \alpha^*$, by using (18)–(22), there exists

$$\dot{V}(z) \leqslant -\mu V^{1/2}(z). \tag{32}$$

By integrating both sides of (32), the time from T_1 to state z converging to the origin is bounded by

$$T_f - T_1 \leqslant \Delta T_3 = \frac{2}{\mu}V^{1/2}(z(T_1)).$$

To further calculate $V^{1/2}(z(T_1))$, knowledge of the bounds of $z_1(T_1)$ and $z_2(T_1)$ at T_1 is required. $|z_1(T_1)|$ is bounded by $\varepsilon_1^{1/2}$. With regard to the bound of $|z_2(T_1)|$, if $y(t_0)$ is zero or $\operatorname{sign}(y(t_0))$ is opposite to $\operatorname{sign}(x(t_0))$, then the bound is

$$|z_2(T_1)| \leqslant M\left(\frac{1}{\alpha_1(p-1)\varepsilon_1^{p-1}} + t_0\right) \tag{33}$$

where

$$M = \frac{\max\limits_{t\in(0,T_1)}\beta}{2} + L, \tag{34}$$

and $\max\limits_{t\in(0,T_1)}\beta$ can be calculated as

$$\max_{t\in(0,T_1)}\beta = 2\varepsilon(\alpha(t_0) + \kappa(T_1 - t_0)) + 2\left(\lambda + 4\varepsilon^2\right)\alpha_1\varepsilon_1^{p-1/2}.$$

If sign $(y(t_0))$ is the same as sign $(x(t_0))$, then y will first go through the process of reaching the origin from t_0 to $t_0 + \Delta T_2$, and there exists $|z_2(T_1)| \leqslant M(T_1 - (t_0 + \Delta T_2))$, which further leads to (33).

Therefore, according to (13) and (14), there exists

$$V(z(T_1)) \leqslant \left(\lambda + 4\varepsilon^2\right)\varepsilon_1 + M^2\left(\frac{1}{\alpha_1(p-1)\varepsilon_1^{p-1}} + t_0\right)^2 - 4\varepsilon\varepsilon_1^{1/2}M\left(\frac{1}{\alpha_1(p-1)\varepsilon_1^{p-1}} + t_0\right).$$

Thus, ΔT_3 can be computed and $T_f - t_0$ can be calculated by adding $T_1 - t_0$ by ΔT_3.

Note that T_f is the time when $z(t)$ converges to the origin. Thus, before T_f, the states have already converged to the domain $|x| \leqslant \delta/2$; therefore, $T_f \geqslant \bar{t}$.

Second, prove that, after T_f, $|x|$ is strictly confined by $|x| \leqslant \delta$ and $|y| \leqslant \eta$. As there must exist $\bar{t}$ when $|x| \leqslant \delta/2$, the control gain α then performs as the strategy α_b in (4). Thus, according to Lemma 1, $|x|$ is strictly confined by $|x| \leqslant \delta$.

If $\delta/2 < x_1$, where x_1 is denoted as in (23), then it will take a certain period of time τ for $|x|$ to increase from $\delta/2$ to x_1. Thus, the bound η of $|y|$ can be estimated as

$$\eta = \int_0^{T_f+\tau}\left(\frac{\beta(s)}{2} + L\right)\mathrm{d}s. \tag{35}$$

□

Remark 1 Theoretically, the proposed algorithm is implemented in a continuous system. Thus, according to Lemma 1, as long as $|x|$ enters the domain $|x| < \delta$, no matter how the disturbance changes, $|x|$ will be confined strictly in it afterwards. However, in practice, when the algorithm is implemented in a discrete manner, the BF presents the following problem.

First, we present conventional BF [18] as follows :

$$\alpha = \frac{\bar{F}\delta}{\delta - |x|}.$$

Write $|x|$ as a function of α renders:

$$|x| = \delta\left(1 - \frac{\bar{F}}{\alpha}\right).$$

Note that if $\bar{F}$ is chosen small, when α becomes large, the margin between $|x|$ and the barrier δ will become so small that it is very easy for $|x|$ to go beyond the barrier. Thus, the sampling step must be chosen small enough so that the attractive features of the BF could be maintained [18]. However, a too small sampling step may cause great computational burden.

One way to avoid using too small step is to increase $\bar{F}$. However, $\bar{F}$ determines the minimum value that α could reach and it must be chosen small enough to guarantee the feature of adaptiveness.

The proposed RBF in (4) can mitigate this problem effectively without increasing the value of $\bar{F}$. By setting m in (4) large enough, the margin between $|x|$ and the barrier δ is increased greatly. Here we present an example to illustrate the benefit of the RBF by comparing it with BF. The barrier value δ and minimum reachable value $\bar{F}$ of the two functions are selected the same: $\delta = 0.02$, $\bar{F} = 0.01$. Parameter m in RBF is selected as 20. Fig. 1 shows the curves of $|x|$ with the BF and RBF when α varies from 0 to 200. From the figure, we can observe that the $|x|$ curve of the BF rises steeply during $\alpha = 0$ to $\alpha = 10$ and the gap between $|x|$ and the barrier value (0.02) becomes quite limited afterwards. This means that the sampling step must be set small enough so that the magnitude of $|x|$ will not transcend such a small gap in one sampling step. In contrast, the curve of the RBF renders a much broader margin which can grant a larger sampling step and reduce the computation burden. In the simulation section below, when m is set to 5, the order of magnitude of the step can increase by 1 compared with BF.

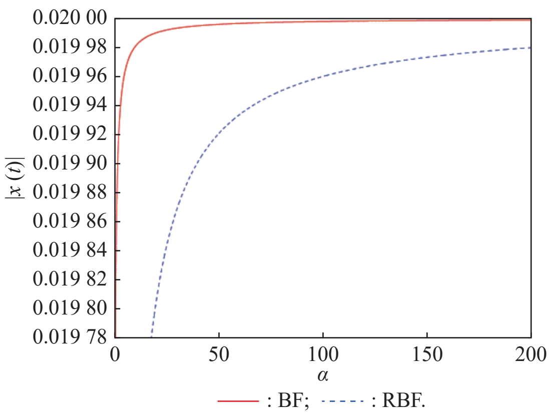

Fig. 1 Comparison of the change of $|x|$ between BF and RBF (δ=0.02, $\bar{F}$=0.01, m=20)

Remark 2 Note that Theorem 1 is subject to the assumption that the condition of (24) is satisfied. If (24) is not valid at time $t = 0$, it will take time t_0 for α and β to grow until (24) is met. Thus, if the whole fixed time T_f of convergence is to be estimated, t_0 must be included. To avoid the requirement of knowing t_0, in [13], α_0 and β_0 are set to meet (24) at time $t = 0$, which is not practical when K and L are unknown. In this paper, α_0 and β_0 can be set arbitrarily and do not have to meet (24). As $y(0) = 0$, y changes randomly in t_0 and two cases for $y(t_0)$ can be distinguished:

(i) $\operatorname{sign}(y(t_0))$ is opposite to $\operatorname{sign}(x(t_0))$ or $y(t_0) = 0$;

(ii) $\operatorname{sign}(y(t_0))$ is the same as $\operatorname{sign}(x(t_0))$, for the sake of discussion in the proof of Theorem 1. t_0 can be estimated as

$$t_0 \leqslant (\alpha^* - \alpha_0)/\kappa.$$

As α^* contains unknown parameters K and L, in order to estimate t_0, the order of magnitude of K and L should be known. Alternatively, t_0 could also be estimated via simulation.

4. Simulation and discussion

In the simulation, we illustrate the effectiveness of the proposed finite- and fixed-time convergence law and its advantages over the adaptive algorithm presented in [13].

First, simulations are conducted for system in (6) based on the proposed control law with the disturbance $\zeta = 3|x| + 5\sin(2t)$. The parameters are assigned as $\alpha_0 = 0$, $\alpha_1 = 1$, $p = 3/2$, $m = 5$, $\kappa = 15$, $\lambda = 1$, $\varepsilon = 1$ and $\varepsilon_1 = 1$. The initial conditions are set as $x_0 = 10$, $x_0 = 10^3$ and $x_0 = 10^6$, respectively; the predefined vicinity of the origin is set as $\delta = 0.02$. The results show that the convergence time of $|x|$ to $|x| < \delta$ corresponding to the three initial conditions are 0.905, 1.001 and 1.026, respectively. This demonstrates that with the same disturbance and control parameters, there exists a uniformity of the convergence time independent of the initial conditions. Note that although the initial value of the adaptive gain α is 0 instead of having to be set larger than the predefined value α^* as in reference [13], the uniformity of the convergence time is still guaranteed, which renders the proposed method more practical.

The simulation results corresponding to the initial values $x_0 = 10^3$ and $x_0 = 10^6$ are shown in Fig. 2–Fig. 6 and in Fig. 7–Fig. 10, respectively. It can be seen that not only the convergence time shows uniformity, but the plots of x, y and the adaptive gain α perform uniformly under different initial conditions as well. Note that although there exists a sudden change of α due to the change of adaptive strategy at time $\bar{t}$ in the BF-based method as shown in Fig. 5, its influence is trivial, for $|x|$ at this time is quite small, making the change of u negligible. As shown in Fig. 6, the corresponding change of u is rather smooth.

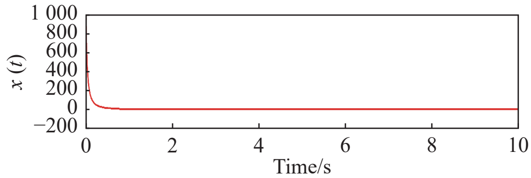

Fig. 2 Plot of x with proposed control corresponding to $x(0) = 10^3$

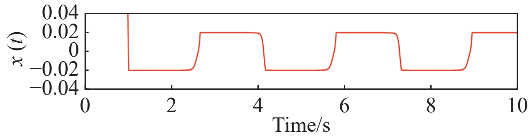

Fig. 3 Zoomed-in plot of x with proposed control corresponding to $x(0) = 10^3$

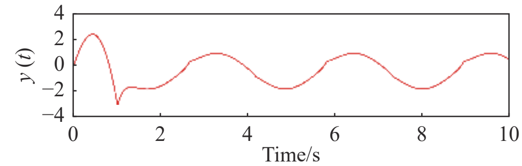

Fig. 4 Plot of y with proposed control corresponding to $x(0) = 10^3$

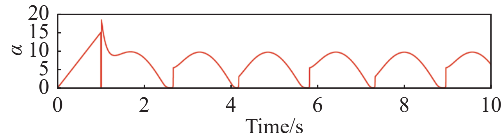

Fig. 5 Plot of α with proposed control corresponding to $x(0) = 10^3$

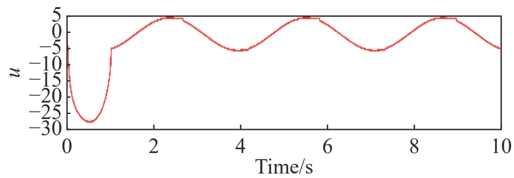

Fig. 6 Plot of u with proposed control corresponding to $x(0) = 10^3$

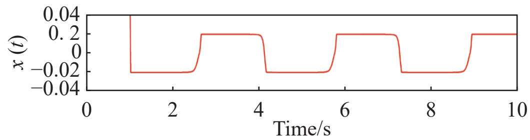

Fig. 7 Zoomed-in plot of x with proposed control corresponding to $x(0) = 10^6$

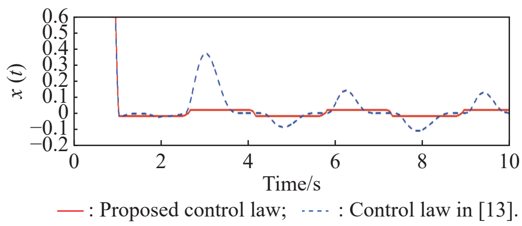

Fig. 8 Comparison of plots of x under the two control laws corresponding to $x(0) = 10^6$

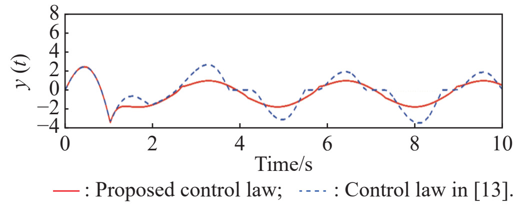

Fig. 9 Comparison of plots of y under the two control laws corresponding to $x(0) = 10^6$

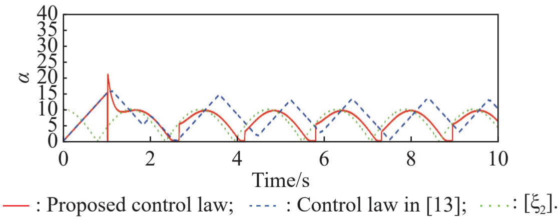

Fig. 10 Comparison of plots of gain α under the two control laws corresponding to $x(0) = 10^6$

For comparison, the same simulations are also conducted using the control law provided in [13] where the adaptive law is

$$\dot{\alpha} = \begin{cases} \kappa \operatorname{sign}(|x| - \delta), & \alpha - \alpha_{\min} > 0 \text{ or } |x| > \delta \\ 0, & \alpha - \alpha_{\min} \leqslant 0 \text{ and} |x| \leqslant \delta \end{cases},$$

$$\alpha(0) = \alpha_0,$$

$$\beta = 2\varepsilon\alpha + 2\left(\lambda + 4\varepsilon^2\right)\alpha_1 \varepsilon_1^{p-1/2},$$

with $\alpha_{\min} = 0.02$. The other conditions are identical. The results under the two control laws with the initial value, $x_0 = 10^6$, are compared in Fig. 8−Fig. 10. In Fig. 7 and Fig. 8, it is illustrated that for the proposed control law, $|x|$ is strictly confined in the predefined domain, $|x| < \delta$. In contrast, the control strategy in [13] has a much inferior control precision and there is no guarantee that the state can be confined in a certain domain. Fig. 10 compares the plots of the adaptive gains of the two control laws and the absolute value of the derivative of disturbance ζ_2, i.e., $5\sin(2t)$. We can observe that the proposed control law outperforms the other one by solving the problem of overestimation of adaptive gain as it follows the change of disturbance much more closely. That is due to the fact that the adaptive gains in the proposed method respond to the change of disturbance instantly while there is a lag of time in the change of the adaptive gains of the method in [13]. With the proposed method, the vicinity of the origin that $|y|$ converges to is also smaller than the one using the other control law as shown in Fig. 9.

5. Conclusions

This paper presents a RBF based adaptive finite- and fixed-time STW control law for the case of first-order disturbed systems under perturbations with unknown bounds. The finite- and fixed-time convergence of the states is established while the state of the system is guaranteed to be confined in a predefined vicinity of the origin. An estimate for the convergence time is derived based on the proposed control law. The proposed RBF is shown to be capable of reducing the computational burden compared with the traditional BF. Several simulation examples are carried out to verify the efficiency of the proposed method. The results show that the proposed method combines both the advantages of the uniform control law and the BF adaptive strategy and it outperforms the non-BF-based finite- and fixed-time control in several aspects, which is shown in the simulation tests.

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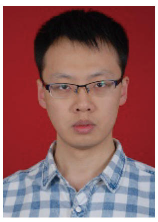
Biographies



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