Journal of Systems Engineering and Electronics
Vol. 34, No. 3, June 2023, pp.674 – 681

Parameter estimation of LFM signals based on time reversal

MA Xinjie[1], QI Wei[2,*], CHE Kaijun[1], and WU Gang[3]

1. School of Electronic Science and Engineering, Xiamen University, Xiamen 361005, China; 2. Beijing Institute of Tracking and Telecommunications Technology, Beijing 100094, China; 3. Jiuquan Satellite Launch Center, Jiuquan 736200, China

Abstract: In this paper, parameter estimation of linear frequency modulation (LFM) signals containing additive white Gaussian noise is studied. Because the center frequency estimation of an LFM signal is affected by the error propagation effect, resulting in a higher signal to noise ratio (SNR) threshold, a parameter estimation method for LFM signals based on time reversal is proposed. The proposed method avoids SNR loss in the process of estimating the frequency, thus reducing the SNR threshold. The simulation results show that the threshold is reduced by 5 dB compared with the discrete polynomial transform (DPT) method, and the root-mean-square error (RMSE) of the proposed estimator is close to the Cramer-Rao lower bound (CRLB).

Keywords: linear frequency modulation (LFM) signal, time reversal, Cramer-Rao lower bound (CRLB), parameter estimation.

DOI: 10.23919/JSEE.2023.000014

1. Introduction

Linear frequency modulation (LFM) signals are widely used in the fields of radar, sonar, geological detection, and assisted driving [1,2]. The associated parameter estimation problem has been a hot spot in the field of signal processing. For example, in radar imaging, the motion parameters of the target must be estimated and then compensated to prevent defocusing and improve image quality. With the development of digital signal processing technology, how to estimate signal parameters quickly and accurately at low signal to noise ratios (SNRs) is of great significance [3,4]. The radar echo, which contains the second-order motion parameters of the target, can be modeled as an LFM signal. Doppler frequency estimation of radar echo essentially estimates the parameters of LFM signals corrupted by additive white Gaussian noise.

Regarding the parameter estimation of LFM signals, there are many methods. The maximum likelihood estimator (MLE) is a traditional method to estimate the parameters of LFM signals, but it is difficult to apply to engineering practice because of its complex calculation [5,6]. Most of the commonly used LFM signal parameter estimation methods are based on time-frequency characteristics, which transform one-dimensional (1D) signals into two-dimensional (2D) time-frequency planes and search in 1D or even 2D space, therefore they require a large amount of calculation [7−13]. Abatzoglou used 2D Newton iteration to calculate the MLE [14], which reduces the amount of calculation, but it needs to have an iterative initial value close to the true value. The discrete chirp Fourier transform (DCFT) [15] and fractional Fourier transform (FRFT) [16] use kernel functions matching LFM signals and exhibit good performance when used for parameter estimation of LFM signals. The performance of the parameter estimation method based on FRFT has been improved, which has certain engineering application value [17−19]. Computationally efficient algorithms for estimating the parameters of a complex LFM signal in white Gaussian noise were proposed in [20]. The discrete polynomial transform (DPT) method proposed in [21] and [22] needs only two Fourier transforms and two 1D searches to obtain the estimates of the starting frequency and the frequency modulation (FM) slope. The accuracy and scope of application of the DPT method was expanded in [23]. The phase method [24] requires a small amount of calculation, and the accuracy is close to the Cramer-Rao lower bound (CRLB) when it is higher than the SNR threshold. A novel method for estimating the parameters of the LFM signal was provided in [25] based on a modification to the filter bank approach for maximum likelihood estimation. This method was found to be especially effective in the case of low-SNR conditions. Chen et al. proposed a method to analyze multicomponent LFM signals, which eliminated cross terms in the conventional Wigner-Ville distribution (WVD) [26]. A parameter estimation algo-

Manuscript received April 09, 2021.
*Corresponding author.
This work was supported by the Regional Joint Fund for Basic and Applied Basic Research of Guangdong Province (2019B1515120009) and the Defense Basic Scientific Research Program (61424132005).

rithm of a multicomponent chirp signal based on an improved cubic phase function was proposed in [27]. The algorithm presented in [28] was found to have good noise immunity, good universality, high precision of parameter estimation, and better performance under low-SNR conditions. Phase-domain estimation has also been presented [29], and the SNR threshold was found to be lower than that of conventional phase-domain methods. A parameter estimation method based on morphology operations was proposed to estimate the parameters of LFM signals to resolve the contradiction between the estimation accuracy and the computation [30]. Song et al. proposed an algorithm to determine the optimal delay time and delay length in the autocorrelation sequence based on the sequence convolution method, and the accuracy of LFM signal estimation was still close to the CRLB at a lower SNR [31]. A novel and efficient parameter estimate technique for LFM signals was given in [32]. A new chirp rate estimation algorithm by multiple DPT and weighted combination was proposed to reduce the complexity in high-SNR scenarios [33]. The improved fast algorithm given in [34] can accurately estimate the initial frequency and the chirp rate of a wideband LFM signal. A novel parameter estimation method based on a modified convolution kernel function (MCKF) was proposed for a multicomponent LFM signal in [35]. This method has fewer external crossterms and a lower computational burden because of nonsearching operations. The problem of fast and accurate chirp signal parameter estimation in fractional Fourier domains was addressed in [36]. A fast and robust parameter estimation method for multicomponent LFM signals was proposed in [37], which has low computational complexity and favorable performance under a low SNR due to the low-order nonlinearity of generalized adjustable parameter correlation kernel (GAPCK).

Among the above methods, the DPT method involves quasi-maximum likelihood estimation, and its performance is close to that of the CRLB. It has many advantages, such as high accuracy and low calculation, and it is the most widely used method. On this basis, researchers have proposed many improved algorithms; however, these methods share the common problems of DPT: the estimation of chirp rate and carrier frequency are coupled, the estimation error of chirp rate spreads to the carrier frequency, and the delay conjugate multiplication in the calculation process leads to a greater decrease in the SNR. Therefore, a novel parameter estimation method for LFM signals is proposed here, which can solve the problem of the high SNR required by previous methods. First, we denoise the original signal and estimate its bandwidth. We use operations such as constructing reference signals and time reversal (TR) to obtain approximate estimates of the central frequency f and the chirp rate k. After low-pass filtering, the DPT method is used to obtain a fine estimate of k. We reincorporate k into the original signal to obtain a precise estimate of f. The simulation shows that the required SNR of the method proposed in this paper is lower than that of the DPT method, and the former has better accuracy, offering application value.

This paper is divided into five parts. After a brief introduction into the content of the paper, the signal model and the specific methods of this article are given. Then, the simulation results are shown. Finally, the content of this paper is summarized.

2. LFM signal model and problems of time-delay estimators

The mathematical expression of a noise-contaminated LFM signal is

$$\begin{aligned} r(n) &= s(n)+w(n) = \\ &\mathrm{rect}\left(\frac{t}{T}\right)\mathrm{e}^{\mathrm{j}(2\pi f n+\pi k n^2+\theta)}+w(n), \\ &n=-N,-N+1,\cdots,N \end{aligned} \tag{1}$$

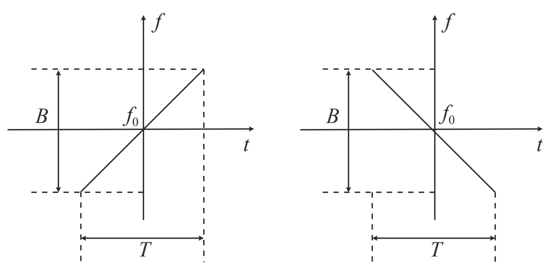
where f is the central frequency of the LFM signal, k is the chirp rate, T is the period of the signal, θ is the initial phase and $w(n)$ is additive Gaussian white noise with zero mean and variance σ^2, and $\mathrm{rect}\left(\frac{t}{T}\right)$ represents a rectangular signal:

$$\mathrm{rect}\left(\frac{t}{T}\right)=\begin{cases}1, & \left|\frac{t}{T}\right|<1 \\ 0, & \text{otherwise}\end{cases}. \tag{2}$$

The instantaneous frequency of the signal can then be written as

$$f(t)=f+kt,\ -\frac{T}{2}\leqslant t\leqslant\frac{T}{2}. \tag{3}$$

The sampling number is $2N+1$. Fig. 1 shows the relationship between f and k.



(a) Up-chirp (k>0) (b) Down-chirp (k<0)

Fig. 1 Typical LFM signal

Due to the error propagation effect of the DPT method, the estimation error of k affects the estimation accuracy of the frequency f. In addition, chirp rate estimation requires a higher SNR threshold, which indirectly raises the SNR threshold of frequency estimation. It is found that the odd and even modulation terms in the signal can be separated by appropriate operation between the TR signal and the original signal. According to this property, we propose a parameter estimation method that can avoid the effect of error propagation and reduce the SNR threshold of LFM parameter estimation. Next, we briefly introduce the error propagation effect of the DPT method.

We choose a fixed delay amount τ ($1 \leqslant \tau < 2N+1$); the estimation of the central frequency can be obtained by finding the $\hat{\omega}$ that maximizes $\mathrm{DAF}(\boldsymbol{r},\omega,\tau)$ as

$$\hat{\omega} = \arg\max_{\omega}\left\{|\mathrm{DAF}(\boldsymbol{r},\omega,\tau)|^2\right\}, \tag{4}$$

Referring to (8) in [38] for $\mathrm{DAF}(\boldsymbol{r},\omega,\tau)$. Then, the estimated value of the frequency modulation slope is given by

$$\hat{k} = \frac{\hat{\omega}}{2\pi\tau}. \tag{5}$$

Removing the FM term from the received signal, we have

$$r^{(1)}(n) = r(n)\mathrm{e}^{-\mathrm{j}\pi\hat{k}n^2}, \quad n = -N, -N+1, \cdots, N \tag{6}$$

and we maximize the new $\mathrm{DAF}(\boldsymbol{r},\omega,\tau)$, i.e., $\mathrm{DFT}(\boldsymbol{r}^{(1)}, f)$ with respect to f, i.e.,

$$\hat{f} = \arg\max_{f}\left\{\left|\mathrm{DFT}(\boldsymbol{r}^{(1)}, f)\right|^2\right\}. \tag{7}$$

As seen from the introduction of the DPT method, to estimate the center frequency f of the LFM signal, it is necessary to estimate the chirp rate k first. After the chirp slope component in the received signal is removed by (6), the estimate of the center frequency can be obtained from the residual signal $r^{(1)}(n)$. The estimation error of the chirp rate k is brought in (6), which is the error propagation effect of LFM signal parameter estimation. In addition, when the discrete ambiguity function is formed, the delay correlation operation causes the length of the signal to change from $2N+1$ to $2N+1-\tau$, which leads to an increase in the SNR threshold required for the center frequency estimation.

3. TR LFM parameter estimator

The TR operation can divide the phase term of a complex signal in the form of a higher-order polynomial into two parts, i.e., odd terms and even terms [39]. Based on this property, the chirp rate and the center frequency of the LFM signal can be decoupled, and the problem of error propagation can be solved. At the same time, the problem of signal length shortening can be avoided, which makes the SNR threshold of LFM parameter estimation lower. Next, the TR algorithm is introduced, and then the parameter estimation method based on TR for LFM signals is given.

Performing a TR operation on the noise-free LFM signal $s(n)$, we have

$$s(-n) = A\mathrm{e}^{\mathrm{j}(-2\pi fn+\pi kn^2+\theta)}. \tag{8}$$

Conducting vector dot multiplication to $s(n)$ with $s(-n)$ and $s^*(-n)$, we obtain C_o and C_e, respectively, as

$$C_o = r(n)\cdot s(-n) = A^2\mathrm{e}^{\mathrm{j}2(\pi kn^2+\theta)} \tag{9}$$

and

$$C_e = r(n)\cdot s^*(-n) = A^2\mathrm{e}^{\mathrm{j}4\pi fn}. \tag{10}$$

The even-order and odd-order terms in the phase modulation of the LFM signal are contained in C_o and C_e respectively, which indicates that the even-order and odd-order terms of the phase modulation are decoupled.

Since the parameters of the received signal are unknown, the noise-free signal $s(n)$ in (8) should be replaced by the received signal $r(n)$. However, if $r(n)$ is used to generate the TR signal, i.e., C_o and C_e, there is a large loss of SNR since $r(n)$ contains noise. To solve the SNR loss problem, it is necessary to construct a noise-free reference signal, so the parameters of the LFM signal must be estimated coarsely. However, the classic LFM signal parameter estimation methods, namely, DPT estimators, inevitably suffer from SNR loss when performing the discrete ambiguity operation, i.e., $\mathrm{DAF}(\boldsymbol{r},\omega,\tau)$ in [38]. Therefore, if we can approximately estimate the chirp rate of the LFM signal, the squared term $\mathrm{e}^{\mathrm{j}\pi kn^2}$ of the LFM signal can be removed by using this coarse estimate so that most of the energy of the signal is focused on one to several spectral lines. Therefore, the center frequency can be estimated under the condition of a lower SNR. Next, we discuss how to estimate the chirp rate under low-SNR conditions.

According to the relationship between the chirp rate k and the bandwidth B, i.e., $k = B/T$ (where T is the pulse width), as long as the bandwidth B is estimated, the estimate of the chirp rate can be obtained. Because the energy of the LFM signal is dispersed in its bandwidth B, the spectral peak of the signal may be lower than the noise when the SNR is low. However, even though the spectral peaks are submerged in noise, all the spectral peaks of the LFM signal are distributed continuously in its bandwidth B and can be viewed as a block. A denoising method based on the block threshold in [40] is proposed and can be used for denoising the received signal $r(n)$. Block thresholding can be seen as an automatic hypothesis test. Select a set of important variables (wavelet coefficients) by ignoring unimportant variables and adapt the data to a model that contains only important variables. After denoising, the noise term is adaptively smoothed, and the signal term is basically retained. A comparison between the original signal spectrum and the smoothed signal spectrum is shown in Fig. 2.

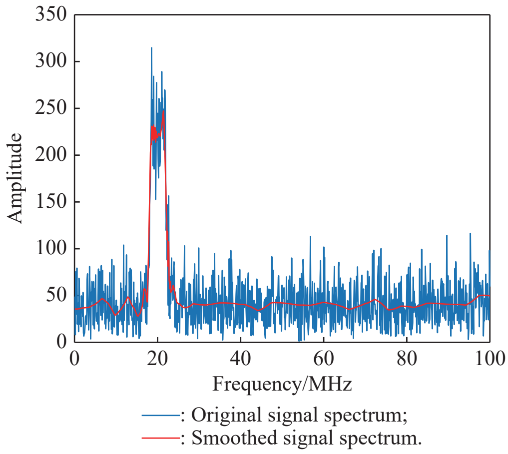


Fig. 2 Signal spectrum comparison before and after smoothing

The estimate of the signal bandwidth can be easily obtained from the smoothed signal spectrum. Assume that the sampling frequency is f_s, and the number of samples is N. Then, the frequency quantization interval corresponding to each sample is f_s/N. Taking the maximum spectral peak of the smoothed signal spectrum as a reference, the 3 dB bandwidth can be obtained by calculating the number of spectral peaks around the largest peak that are larger than 0.5 times the largest peak. Let the number of peaks calculated be m; then, the bandwidth estimate is obtained by

$$\tilde{B} = m \cdot f_s/N. \tag{11}$$

Then, the estimate of the chirp rate is $\tilde{k} = \tilde{B}/T$.

It is now possible to construct a noise-free reference signal

$$s_1(n) = \mathrm{e}^{\mathrm{j}\pi\tilde{k}n^2}. \tag{12}$$

The TR signal is

$$s_1(-n) = \mathrm{e}^{\mathrm{j}\pi\tilde{k}(-n)^2} = \mathrm{e}^{\mathrm{j}\pi\tilde{k}n^2}. \tag{13}$$

Multiplying $s_1^*(-n)$ with $r(n)$, we have

$$C_e{}' = r(n)\cdot s_1^*(-n) = A\mathrm{e}^{\mathrm{j}(2\pi fn+\pi kn^2+\theta)}\cdot \mathrm{e}^{-\mathrm{j}\pi\tilde{k}n^2} = A\mathrm{e}^{\mathrm{j}(2\pi fn+\theta)}\mathrm{e}^{\mathrm{j}\pi(k-\tilde{k})n^2}. \tag{14}$$

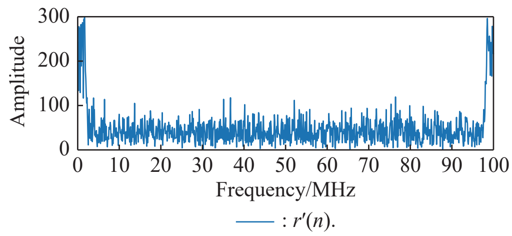
By estimating the frequency of $C_e{}'$, a coarse estimate of the center frequency can be approximately estimated and denoted as $\tilde{f}$.

Next, we obtain the fine estimate of the chirp rate. Although the coarse estimation of k can be obtained through denoising, the block denoising operation leads to distortion of the signal spectrum, resulting in loss of estimation accuracy. For this reason, we need to consider other methods of chirp rate estimation under low SNR conditions.

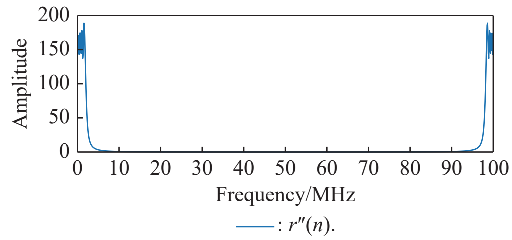
Converting $r(n)$ to baseband using $\tilde{f}$,

$$r'(n) = r(n)\cdot \mathrm{e}^{-\mathrm{j}2\pi\tilde{f}n}. \tag{15}$$

Performing low-pass filtering on $r'(n)$ yields $r''(n)$. The low-pass filter can be designed with reference to $\tilde{B}$. The principle is that the bandwidth of the filter is slightly larger than $\tilde{B}$ to ensure that the signal energy is not filtered out by the low-pass filter. The spectrogram before and after low-pass filtering is shown in Fig. 3.



(a) Spectrogram before low-pass filtering



(b) Spectrogram after low-pass filtering

Fig. 3 Signal spectrum comparison before and after low-pass filtering

Note that $r''(n)$ has a higher SNR than $r'(n)$ after low-pass filtering. Assuming that the bandwidth of the low-pass filter is B_{LP}, the SNR improves by approximately $10\lg(f_s/B_{\mathrm{LP}})$ dB.

We use the DPT method to estimate $\widehat{k}$ for $r''(n)$. The noise-free reference signal is constructed again as

$$s_2(n) = \mathrm{e}^{\mathrm{j}(2\pi\tilde{f}n+\pi\hat{k}n^2)}. \tag{16}$$

Conducting TR on $s_2(n)$ yields

$$s_2(-n) = \mathrm{e}^{\mathrm{j}[-2\pi\tilde{f}n+\pi\hat{k}(-n)^2]} = \mathrm{e}^{\mathrm{j}(-2\pi\tilde{f}n+\pi\hat{k}n^2)}. \tag{17}$$

Multiplying $s_2^*(-n)$ with $r(n)$ yields

$$C_e{}'' = r(n)\cdot s_2^*(-n) = A\mathrm{e}^{\mathrm{j}(2\pi fn+\pi kn^2+\theta)}\cdot \mathrm{e}^{\mathrm{j}(-2\pi\tilde{f}n+\pi\hat{k}n^2)} \approx A\mathrm{e}^{\mathrm{j}(2\pi\Delta fn+\theta)} \tag{18}$$

where $\Delta f = f - \tilde{f}$. Using the sinusoidal frequency estimator to find the estimate $\Delta\hat{f}$ [14], the final estimate of the center frequency can be obtained by $\hat{f} = \tilde{f} + \Delta\hat{f}$. The specific algorithm flow is shown in Fig. 4.

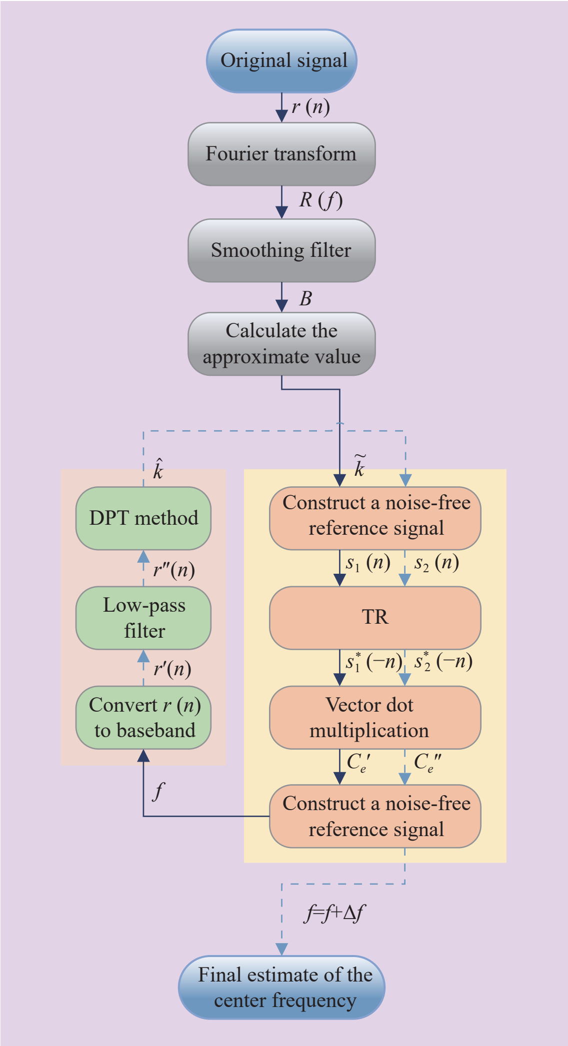


Fig. 4 Algorithm flow chart of the TR method

The purpose of the operation shown in Fig. 2 is to obtain an estimate of the signal bandwidth. The denoising process has little effect on the final fine estimation, and the algorithm performance can be guaranteed as long as the signal bandwidth estimation result can make the baseband signal, i.e., (16), be within the bandwidth of the low-pass filter.

4. Simulation

In this section, simulations are conducted to demonstrate the effectiveness of the proposed method. In each simulation, a Monte Carlo simulation (with 1 000 iterations) is carried out to compute the root-mean-square errors (RMSEs).

In our simulations, the parameters are defined as follows: sampling frequency $f_s = 100$ MHz, center frequency $f = 20$ MHz, and chirp rate $k = 4\times10^{11}$. The values of N are 255 and 511, and the corresponding sample numbers are 511 and 1 023. The CRLBs of the LFM signal center frequency and chirp rate estimation [14] are defined as

$$\mathrm{CRLB}(f) \approx \frac{6}{(2\pi)^2 \cdot \mathrm{SNR} \cdot \Delta^2 N^3} \tag{19}$$

and

$$\mathrm{CRLB}(k) \approx \frac{90}{\pi^2 \cdot \mathrm{SNR} \cdot \Delta^4 N^5} \tag{20}$$

where the SNR is defined as $\mathrm{SNR} = \dfrac{A^2}{2\sigma^2}$.

The performance of the proposed method (for convenience, abbreviated as TR) is compared with that of DPT and the method proposed in [36] (for convenience, abbreviated as FrFT-GSS). The results of center frequency estimation are shown in Fig. 5 and Fig. 6. The chirp rate estimation results are shown in Fig. 7 and Fig. 8. Note that since the simulation conditions in this paper are the same as those in [36], the simulation results do not show the performance curve of FrFT-GSS.

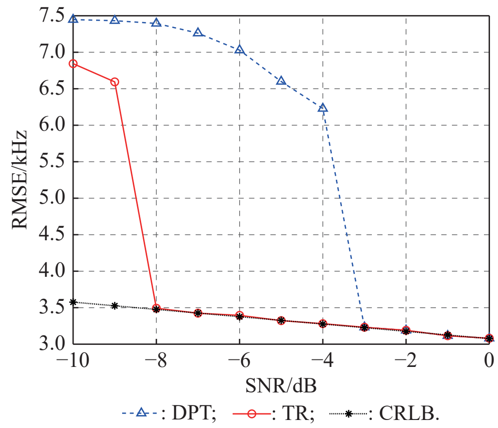


Fig. 5 Comparing the frequency estimation of the DPT and TR methods ($N = 511$)

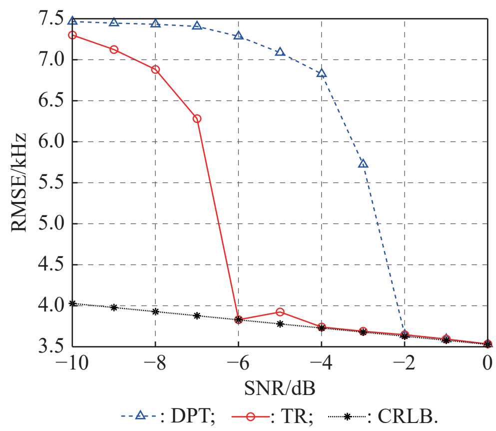


Fig. 6 Comparing the frequency estimation of the DPT and TR methods ($N = 255$)

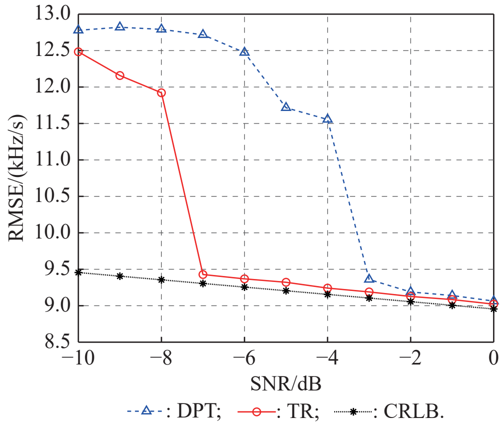


Fig. 7 Comparing the chirp rate of the DPT and TR methods ($N = 511$)

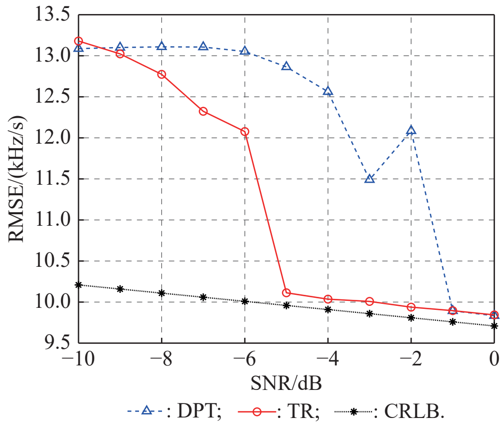


Fig. 8 Comparing the chirp rate of the DPT and TR methods ($N = 255$)

The simulation results show that when the number of samples is 511 (i.e., $N = 255$), the SNR thresholds of the TR estimator for frequency and chirp rate estimation are −6 dB and −5 dB, respectively. The SNR thresholds of the DPT method are −2 dB and −1 dB, respectively. From [36], when the number of samples is 512, the SNR threshold of the FrFT-GSS estimator for chirp rate estimation is approximately −3 dB. When the number of samples is 1 023 (i.e., $N = 511$), the SNR thresholds of the TR estimator for frequency and the chirp rate estimation are −8 dB and −7 dB, respectively. The SNR thresholds of the DPT method are both −3 dB. From [36], when the number of samples is 1 023, the SNR threshold of the FrFT-GSS estimator for chirp rate estimation is approximately −6 dB.

The simulation results show that the TR method is better than DPT and FrFT-GSS. In addition, the computational complexity of the TR estimator is only slightly larger than that of the DPT estimator but smaller than that of the FrFT-GSS estimator. Table 1 shows a comparison of the computational complexity of TR and FrFT-GSS.

Table 1 Comparison of computational costs of different algorithms

Algorithm	Computational cost
FrFT-GSS	$O(KN \lg^2 N)$
Proposed algorithm	$O(N \lg N)$

The algorithm is verified by the measured data, which was collected by Altium designer (AD) board based on TI company. The emitter may be a radar emitter with a center frequency of about 3 220 MHz. The collected radar pulse signal is shown in Fig. 9.

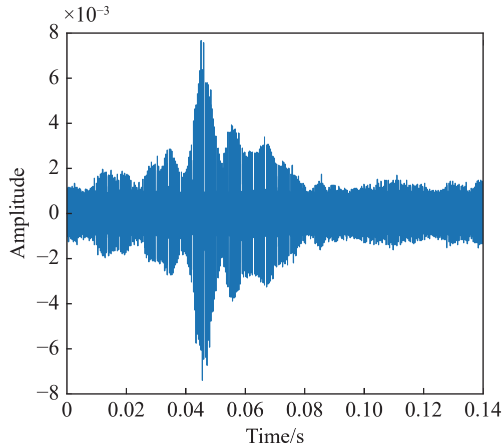


Fig. 9 Radar pulse train signal

The waveform of one of these pulses is shown in Fig. 10. I and Q are the real and imaginary parts of the signal, respectively. It shows that the signal is a typical LFM signal with a high SNR of about 23 dB. Add Gaussian white noise of different magnitudes to the signal, and we use the DPT and TR methods to estimate the parameters respectively.

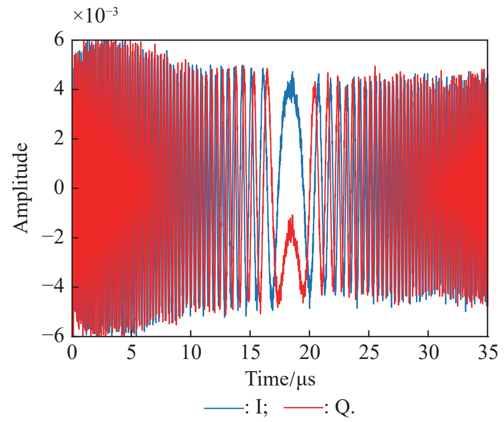


Fig. 10 Time domain waveform of a pulse

The performance of center frequency estimation is shown in Fig. 11. It shows that the simulation results of measured data are similar to the simulation data, and the

SNR threshold of the TR method is about 4 dB lower than that of the DPT method.

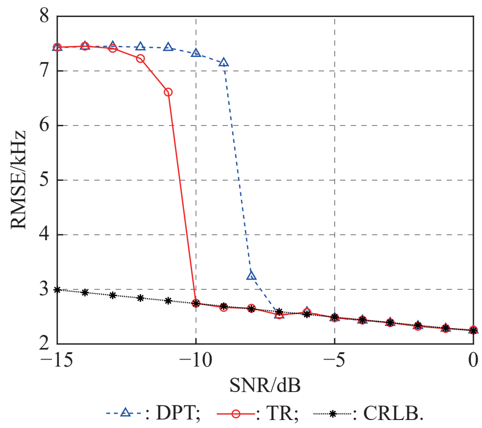


Fig. 11 Comparing the frequency estimation of the DPT and TR methods (N = 1 842)

5. Conclusions

The LFM parameter estimation algorithm based on TR proposed in this paper can estimate the center frequency and the chirp rate of the LFM signal under the condition of a lower SNR. The proposed method decouples the center frequency and the chirp rate of the LFM signal by a TR operation and avoids the effect of error propagation in LFM parameter estimation. The SNR threshold of the proposed method is decreased by a few decibels, while the accuracy is also improved. When the SNR is higher than the SNR threshold, the estimation accuracy reaches the Cramer-Rao bound. Simulation and measured data processing results show the effectiveness of the proposed method.

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Biographies

MA Xinjie was born in 1998. She received her B.S. degree in communication engineering in 2019 from Xiamen University of Technology, Xiamen, China. She is currently pursuing her M.E. degree in electronic and communication engineering at Xiamen University. Her current research interests include radar target detection and radar signal processing.
E-mail: 23120191150233@stu.xmu.edu.cn

QI Wei was born in 1982. He received his Ph.D. degree in engineering mechanics from Beihang University in 2011. He is currently a senior engineer in Beijing Institute of Tracking and Telecommunications Technology, Beijing, China. His research interests include space engineering, radar signal processing, target tracking recognition, and satellite navigation.
E-mail: robyche@163.com

CHE Kaijun was born in 1985. He is currently an associate professor with the Department of Electronics Engineering, Xiamen University, Xiamen, China. His research interests include semiconductor microlasers, plasmonic microdevices, optical nonlinearities, and optical devices' applications.
E-mail: chekaijun@xmu.edu.cn

WU Gang was born in 1980. He received his B.E. degree in measurement and control technology and instrument from Nanjing University of Aeronautics and Astronautics, Nanjing, China, in 2003, and M.S. degree in electronic and communication engineering from Xidian University, Xi'an, China, in 2012. His current research interests include target detection and recognition.
E-mail: 37192275@qq.com