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# Sequential quadratic programming-based non-cooperative target distributed hybrid processing optimization method

SONG Xiaocheng[1,*], WANG Jiangtao[2,3], WANG Jun[2,3], SUN Liang[2,3], FENG Yanghe[4], and LI Zhi[1]

1. Beijing Institute of Electronic Engineering, China Aerospace Science and Industry Corporation Limited, Beijing 100039, China;
2. School of Intelligence Science and Technology, University of Science and Technology Beijing, Beijing 100083, China;
3. Institute of Artificial Intelligence, University of Science and Technology Beijing, Beijing 100083, China;
4. School of Systems Engineering, National University of Defense Technology, Changsha 410073, China

**Abstract:** The distributed hybrid processing optimization problem of non-cooperative targets is an important research direction for future networked air-defense and anti-missile firepower systems. In this paper, the air-defense anti-missile targets defense problem is abstracted as a nonconvex constrained combinatorial optimization problem with the optimization objective of maximizing the degree of contribution of the processing scheme to non-cooperative targets, and the constraints mainly consider geographical conditions and anti-missile equipment resources. The grid discretization concept is used to partition the defense area into network nodes, and the overall defense strategy scheme is described as a nonlinear programming problem to solve the minimum defense cost within the maximum defense capability of the defense system network. In the solution of the minimum defense cost problem, the processing scheme, equipment coverage capability, constraints and node cost requirements are characterized, then a nonlinear mathematical model of the non-cooperative target distributed hybrid processing optimization problem is established, and a local optimal solution based on the sequential quadratic programming algorithm is constructed, and the optimal firepower processing scheme is given by using the sequential quadratic programming method containing non-convex quadratic equations and inequality constraints. Finally, the effectiveness of the proposed method is verified by simulation examples.

**Keywords:** non-cooperative target, distributed hybrid processing, multiple constraint, minimum defense cost, sequential quadratic programming.

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## 1. Introduction

Since defense resources are scarcely compared to the complex types and huge number of defense tasks, it has become an urgent problem to coordinate and schedule the limited defense resources to achieve the optimal utilization of resources and maximize the satisfaction of defense tasks. The defense system is a typical nonlinear time-varying system with numerous parameters affecting the final effectiveness of the system, which cannot be mathematically given as a closed-form analytical solution [1]. In practical engineering applications, the optimization of the non-cooperative target distributed hybrid processing method is often carried out by means of Brute Force, which requires a large number of solutions during the operation, especially when the number of intercepted attacking pipelines is large, and the computation is huge and slow when the range of processing points is selected, which adversely affects the time-sensitivity of the defense system [2].

Network flow theory was discovered and applied to the description of the combat defense system, which was established in the 1950s and has been widely applied in many fields [3], such as transportation network optimization, oil and gas transportation pipeline network construction, communication system information flow, and financial system cash flow. Network flow mainly contains start, middle, and end nodes, and the link path between nodes is called "edge". Because the carrying capacity of each link path is limited, the essence of solving the maximum flow problem from the starting point to the endpoint is how to find the optimal solution to meet the performance target based on the maximum carrying capacity of network nodes and link paths. In network flow theory, "feasible flow" refers to a path from the source node to the end node through the appropriate link path, which have two characteristics. The first is capacity limi-

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*Corresponding author.
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tation, which is the upper limit of the capacity of each edge in the network; the second is the equilibrium condition, that is, the total input of the intermediate node is equal to the total output. Under these two constraints, the total output of the source node should be equal to the total input of the end node, and the value is defined as the total flow of the network. For a network, a feasible flow always exists. The maximum flow problem is to find the feasible flow with the maximum total flow under the restriction condition [4]. The essence of the simple maximum flow problem is actually a linear programming problem.

For solving the problem of optimal processing of incoming noncooperative targets by a distributed defense system, the "threat streams" of one or more potential threat adversaries against multiple defended targets are mixedly processed after the processing of the equipment's mating system is given, which is similar to but different from the traditional network flow problem, which is reflected in the fact that the intermediate nodes of the network have the characteristics of capacity limitation, and the characteristics of the capacity limitation of each edge and the equilibrium condition of the intermediate nodes remain unchanged, hereafter referred to as the "defense flow" problem. The problem of defense flow mainly focuses on the process of deploying firepower to handle target groups by defense equipment clusters with different capabilities distributed in different areas [5,6]. Usually, different defense equipment clusters (intermediate node) lead to different defense cost for different incoming target groups, and the cost function usually considers various complex factors related to comprehensive cost. Because different defense systems often consists of different kinds of weapons and equipment, through the launch interceptor to intercept the target, the cost difference between different incoming noncooperative targets is very distinct. Even for the same target, the number of interceptors required in different jamming penetration scenarios will be different [7]. In addition, different interceptors have different capabilities; for example, weapons intercepted outside the atmosphere have a greater chance of interception and may achieve better results with a smaller number of interceptors compared to those intercepted inside the atmosphere, but the cost per shot will be higher, so optimization cost associated with the defense network flow problem needs to be evaluated in a comprehensive manner considering multiple constraints [8].

Based on the above observations, it is noted that the main theoretical challenge is the modeling and solving methods of the optimization problem for the complicated processing scheme of defensive equipment. Specifically, finding an effective and simplified optimization decision algorithm is also a difficult task. The non-cooperative target distributed hybrid processing optimization problem studied in this paper refers to finding a simple and efficient optimization scheme of interception strategy under the condition that the processing scheme of defensive equipment is determined so that the defense cost of the main incoming non-cooperative target group of the key protection target and the region is minimized. The problem can be considered as an optimization decision problem under constraints. In this paper, the overall defense strategy scheme is described as a nonlinear programming (NLP) problem to solve the minimum defense cost within the maximum defense capability of the defense system network, a nonlinear mathematical model of the incoming non-cooperative target optimization processing problem is established, an optimization solution method based on the sequential quadratic programming (SQP) algorithm is constructed, and the optimal interception allocation scheme is obtained by applying the SQP algorithm with nonlinear inequality constraints to solve the problem. A local optimal interception allocation scheme is obtained finally by using the SQP algorithm with nonlinear constraints and applied to the problem solution with good results.

## 2. Problem description

In fact, the most important defect in the traditional modeling and decision optimization algorithm for the threat target interception missions is that the multiple constraints on the optimization objective are not considered clearly for the mathematical description and solving procedures. Considering the deficiency of the traditional threat target interception strategy optimization method, the description method of the defense flow network in the defense system is illustrated firstly [9], then the mathematical description of the firepower equipment capability and the constraints on the incoming non-cooperative target processing are given, and the mathematical model of the non-cooperative target distributed hybrid processing system is established finally.

### 2.1 Mathematical description of defense system

#### 2.1.1 Mathematical description of defense flow network

The main components in the defense system, usually including radar, launcher, command vehicle, etc., are limited by the technical indicators and equipment characteristics, such as the power of radar, wave band, anti-interference ability, advanced command and control algorithms, computer board processing capabilities and other factors, the defense cluster composed of different equipment with multi-target defense capabilities also vary greatly, which means that the capacity of each directed edge of the defense network flow is different [10]. For any intermediate node in the defense network (i.e., a group of devices distributed in a certain geographic area), the defense equipment that enters the scope of its strike (interception) capability must be "responsive". A successful interception is not necessary, but not missing the processing is required, either the interception can be successful, or the former intermediate node misses it and hands it to the subsequent intermediate nodes for capability assessment [11]. In this paper, we improve the classical network flow model by defining that the target flow input to the defense network has a "threat component", such as a group of targets (e.g., ballistic missiles) carrying light and heavy decoys and jammers against a defended area, while the real threat component is the warhead. Therefore, the threat component (TC) constraint is considered in the improved model, and it is because of this constraint that the defense flow problem becomes a non-convex non-deterministic polynomail (NP) problem [12].

As shown in Fig. 1, the directed defense flow graph $G=(P,V)$ is defined, where $P$ and $V$ are all finite sets; $P$ denotes each node in the graph; $V$ represents directed connected edges; $T$ denotes the attacking target pool, and all types of attack weapon clusters distributed in different regions, such nodes flow out one-way, i.e., $T_t^-=\phi$, $\forall t\in T$; $Q$ denotes the defense weapons processing pool, which can be divided into two levels: exo-atmospheric interceptor weapons group and intra-atmospheric interceptor weapons group according to the processing level, i.e., $Q=P\backslash(T\cup E)$, where $E$ denotes the observation pool, which somehow corresponds to different defensive strongholds, with a one-way inflow of nodes of this type, i.e.,$E_e^+=\phi,\forall e\in E$; $P_i^+$ denotes the set of all neighboring nodes output from node $i$ in finite set $P$, and $P_i^-$ denotes the set of all neighboring nodes input to node $i$ in finite set $P$; $K$ denotes the set of threat components with combatant parts, such as warheads; the variable $f_{ij}$ denotes the flow from node $i$ to $j$ and $(i,j)\in V$; the variable $w_i^k$ denotes the content of the threat component $k$ in the defense weapon processing pool $i$, $i\in Q$ and $k\in K$; the constant $c_{ij}$ denotes the unit cost of the edge $(i,j)$ and $(i,j)\in V$; the constant $d_{ij}$ denotes the processing capacity limit of the edge $(i,j)$ and $(i,j)\in V$; the constant $b_i$ denotes the capacity limit of the point $i$ and $(i,j)\in V$; the constant $q_t^k$ denotes the content of the threat component $k$ in the threat set $t$ and $t\in T,k\in K$; the constant $q_e^k$ denotes the content of the threat component $k$ in the observation set $e$ and $e\in E,k\in K$.

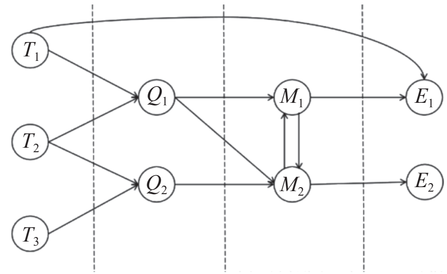


**Fig. 1 Distributed air-defense and anti-missile firepower systems**

### 2.1.2 Mathematical description of firepower equipment capability

In order to facilitate the description of the non-cooperative target distributed hybrid processing mathematical model, and to more intuitively portray the nodes of the overall defense system, the relationships among the nodes are characterized in the following in the form of a matrix. The number of units of the attacking target pool, the exo-atmospheric interceptor weapon cluster, the intra-atmospheric interceptor weapon cluster, and the observation pool in Fig. 1 are defined as $I$, $J$, $M$, $N$, and for the distributed defense system shown in Fig. 1, there is $I=3$, $J=2$, $M=2$, $N=2$, and the corresponding parameters of $I$, $J$, $M$, $N$ are denoted by $i$, $j$, $m$, $n$, respectively. Then, a $(I+J+M)\times(J+M+N)$ dimensional matrix $\boldsymbol{H}$ can be defined to describe the relationship between nodes in the network, where the rows of the matrix $\boldsymbol{H}$ denote nodes with output capability and the columns are nodes with input capability, if there is a pathway between two nodes, the corresponding element in the matrix $\boldsymbol{H}$ is 1, otherwise it is 0. From Fig. 1, the dimension of $\boldsymbol{H}$ is $(3+2+2)\times(2+2+2)=7\times 6$, and the corresponding matrix is

$$\boldsymbol{H}=\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

where the nodes corresponding to the rows are $[T_1,T_2,T_3,Q_1,Q_2,M_1,M_2]$ from top to bottom, and the nodes corresponding to the columns are $[Q_1,Q_2,M_1,M_2,E_1,E_2]$ from left to right.

In the non-cooperative target distributed hybrid defense process, achieving interdiction success against an attacking target requires an assessment of the deployed fire equipment capabilities. To achieve the requirement of describing the capabilities of firepower units as ideally as possible, the physical description of the capabilities of the defense equipment is shown mathematically. The capability of the exo-atmospheric interceptor weapon group $Q$ and the in-atmospheric interceptor weapon group $M$ as deployed fire equipment can be expressed as follows. The number of missiles at the launch node of the attacking target pool should be equal to the output number of fire equipment nodes, which is used to ensure the success of missile interception and to facilitate the description of the required cost. The mathematical description of the firepower equipment capability is

$$\sum_{i=1}^{3} h_{ij} f_{ij} = \sum_{m=2+1}^{2+2} h_{jm} f_{jm}, \quad j = 1,2 \tag{1}$$

where $h_{ij}$ is the element of the matrix $\boldsymbol{H}$, and the variable $f_{ij}$ represents the flow from point $i$ to $j$ with $(i, j) \in V$.

#### 2.1.3 Mathematical description of non-cooperative target distributed hybrid processing system constraint

In practice, incoming ballistic missiles usually carry multiple realistic warheads and decoys to increase penetration and difficulty in interception [13]. Therefore, this paper considers the "threat component" in the target stream input to the defense network (i.e., a group of targets carrying heavy or light decoys and jammers to attack a secure location, whose real threat component is warhead) as a class of constraints on the model in the optimization problem. In order to intercept all incoming non-cooperative targets, the number of missiles arriving in the observation pool from the incoming non-cooperative target pool and the missile defense pool cannot be greater than the number of defensive warheads deployed in the observation pool, so the constraint of threat component can be expressed as

$$q^k_{T_1} f_{T_1 E_1} + w^k_{M_1} f_{M_1 E_1} + w^k_{M_2} f_{M_2 E_2} \leqslant \\ q^k_{E_1} (f_{T_1 E_1} + f_{M_1 E_1}) + q^k_{E_2} f_{M_2 E_2} \tag{2}$$

where the constant $q^k_t$ denotes the content of threat components $k$ in threat concentration $t$ and $t \in T$ (the attacking target pool), $k \in K$; the constant $q^k_e$ denotes the content of threat component $k$ in the observation set $e \in E$ (the observation pool), $k \in K$; $w^k_{M_1}$ and $w^k_{M_2}$ respectively denote the content of threat components $k$ in the defense weapons disposal pool $M_1$ and $M_2$.

Considering the practical arrangement of air defense and anti-missile firepower in the real environment, the influence of many natural conditions including geographical conditions and environmental factors should be considered, and the most important factor should be the constraint of anti-missile equipment resources themselves [14].

The flow of each directed edge $f_{ij}$ in Fig. 1 is limited, which means the processing capability of each defense flow to the number of missiles is limited, and the constraint can be described as

$$0 \leqslant f_{ij} \leqslant d_{ij} \tag{3}$$

where $d_{ij}$ is the processing capability limitation of each directed edge in Fig. 1.

In addition, not only the constraint of the defense flow in the directed defense flow graph has to be considered, but also the constraint of the defense nodes has to be considered. According to the distributed air defense and anti-missile firepower system in Fig. 1, all nodes in the defense process are required to be within their capacity limits. The constraints on the capacity of the nodes are described in two parts based on the offensive-defensive dichotomy.

Based on Fig. 1 and the matrix $\boldsymbol{H}$, the constraint on the observation pool node capacity can be described as

$$\sum_{i=1}^{3} h_{in} f_{in} + \sum_{j=3+1}^{3+2} h_{jn} f_{jn} + \\ \sum_{m=3+2+1}^{3+2+2} h_{mn} f_{mn} \leqslant b_n, \quad n = 5,6 \tag{4}$$

where $b_n$ is the upper capacity constant in the observation pool node, i.e., the number of missiles arriving at that node must not exceed the upper limit in order to ensure the successful completion of the defense mission [15].

Similarly, the mathematical description of the node capacity of the attacking target and defense weapon pool is

$$\sum_{j=1}^{2} h_{ij} f_{ij} + \sum_{m=2+1}^{2+2} h_{im} f_{im} + \sum_{n=2+2+1}^{2+2+2} h_{in} f_{in} \leqslant b_i, \quad i \in [1,7] \tag{5}$$

where $b_i$ is the upper capacity limit constant in the remaining nodes after removing observations.

### 2.2 Mathematical model for distributed hybrid processing of non-cooperative targets

Based on the above mathematical description of the defense area with the quantitative description of the fire armament capabilities and threat components with constraints, the flow rate $f_{ij}$ for each directed edge in the directed defense flow diagram is viewed as the decision variable, then the threat component content $w^k_i$ in the exo-atmospheric interceptor weapons group is also treated as an another decision variable, and the intra-atmospheric interceptor weapons groups are used as decision variables. Furthermore, the cost minimization is used as the objective function, which is defined as

$$F=\sum_{i=1}^{3}\sum_{j=1}^{2}\omega_i h_{ij}c_{ij}f_{ij}+\sum_{j=3+1}^{3+2}\sum_{m=2+1}^{2+2}\omega_j h_{jm}c_{jm}f_{jm}+\sum_{m=3+2+1}^{3+2+2}\sum_{n=2+2+1}^{2+2+2}\omega_m h_{mn}c_{mn}f_{mn}+\sum_{i=1}^{3}\sum_{n=2+2+1}^{2+2+2}\omega_i h_{in}c_{in}f_{in}+\sum_{m_1=3+2+1}^{3+2+2}\sum_{m_2=2+1}^{2+2}\omega_{m_1}h_{m_1m_2}c_{m_1m_2}f_{m_1m_2} \quad (6)$$

where $h_{ij}$ is an element of matrix $\boldsymbol{H}$; the variable $f_{ij}$ denotes the flow from point $i$ to $j$; the constant $c_{ij}$ denotes the unit cost of edge $(i,j)$; $\omega_i$ is used to determine whether there are incoming missiles launched from the incoming target pool; $\omega_j$ is used to determine whether there is a threat component in the exo-atmospheric interceptor weapon population that is subject to a threat from the incoming target pool, and $\omega_j$ is zero if and only if $\omega_i$ is always zero; $\omega_m$ is used to determine whether there is a threat component in the in-atmospheric interceptor weapon population subject to an unresolved threat in the exo-atmospheric interceptor weapon population, and they can be expressed as

$$\omega_i=\begin{cases}0, & q_i^k=0\\ 1, & q_i^k\neq 0\end{cases};i=1,2,3,$$

$$\omega_j=\begin{cases}0, & \sum_{i=1}^{3}h_{ij}\omega_i=0;j=4,5\\ 1, & \text{otherwise}\end{cases},$$

$$\omega_m=\begin{cases}0, & \omega_j=0\\ 1, & \text{otherwise}\end{cases};m=6,7;\forall j=4,5.$$

The constraint on the objective function is

$$\sum_{i=1}^{3}\sum_{j=1}^{2}h_{ij}f_{ij}+\sum_{i=1}^{3}\sum_{m=2+1}^{2+2}h_{im}f_{im}+\sum_{i=1}^{3}\sum_{n=2+2+1}^{2+2+2}h_{in}f_{in}=C \quad (7)$$

where $C$ is a constant value, which is the sum of the traffic input from the incoming target pool to the whole network, and its value must be less than the maximum defense capacity of the defense system network.

## 3. SQP-based distributed hybrid processing optimization method

The solution of the non-cooperative target distributed hybrid processing problem is essentially an NLP problem for solving the minimum defense cost. To simplify this problem, in this paper, we consider that $C$ exceeds the maximum capacity that the network can handle at a given moment when there is no feasible solution for $C$ above a certain value. Through the established nonlinear mathematical model of the non-cooperative target distributed hybrid processing problem, an optimization method based on the SQP algorithm is designed, and the local optimal defense processing solution is obtained by applying the SQP method containing non-convex quadratic equations and inequality constraints.

### 3.1 SQP solution

It can be seen from the model that the minimum defense cost is a nonlinear optimization problem. To solve this problem, an SQP algorithm is used below. SQP is one of the most effective and recently developed techniques to solve NLP problems [16–22], which is currently the best algorithm for small-scale NLP problems. In this algorithm, the original problem is transformed into a series of quadratic programming (QP) subproblems to obtain the optimum solution of the original problem. By taking quadratic approximation of the Lagrange function, the degree of approximation of the quadratic programming subproblems is improved such that nonlinear optimization problems can also be calculated. The SQP algorithm is an algorithm that transforms complex nonlinear constrained optimization problems into relatively simple QP problems [23]. The so-called QP problem is the optimization problem with quadratic objective function and linear constraint function. The QP problem is the simplest nonlinear constrained optimization problem.

The basic idea of the SQP method is to simplify the original NLP problem into a QP problem about a certain approximate solution and then the local optimal solution is obtained [24]. If there is a locally optimal solution, it is considered to be the locally optimal solution of the original NLP problem; otherwise, a new QP problem is replaced by an approximate solution and computational iteration continues [25].

The nonlinear constrained optimization problem is modeled as

$$\begin{aligned}&\min f(\boldsymbol{X})\\ &\text{s.t.}\ \begin{cases}g_u(\boldsymbol{X})\leqslant 0, & u=1,2,\cdots,p\\ h_v(\boldsymbol{X})=0, & v=1,2,\cdots,m\end{cases}\end{aligned} \quad (8)$$

where $\boldsymbol{X}\in\mathbf{R}$ and $f(\boldsymbol{X})$ is the objective function; $g_u(\boldsymbol{X})$ is the equality constraint function; $h_v(\boldsymbol{X})$ is the inequality constraint function; $p$ and $m$ are positive integers.

Firstly, the objective function of nonlinear constraint optimization problem (8) is simplified to a quadratic function at the iterative point $\boldsymbol{X}^k$ by using Taylor expansion, and the QP problem can be described after the constraint function is simplified to the following linear function:

$$\min f(\boldsymbol{X}) = \frac{1}{2}[\boldsymbol{X}-\boldsymbol{X}^k]^{\mathrm{T}}\nabla^2 f(\boldsymbol{X}^k)[\boldsymbol{X}-\boldsymbol{X}^k]+ \nabla f(\boldsymbol{X}^k)^{\mathrm{T}}[\boldsymbol{X}-\boldsymbol{X}^k]$$

$$\text{s.t.} \begin{cases} \nabla g_u(\boldsymbol{X}^k)^{\mathrm{T}}[\boldsymbol{X}-\boldsymbol{X}^k]+g_u(\boldsymbol{X}^k) \leqslant 0, & u=1,2,\cdots,p \\ \nabla h_v(\boldsymbol{X}^k)^{\mathrm{T}}[\boldsymbol{X}-\boldsymbol{X}^k]+h_v(\boldsymbol{X}^k) = 0, & v=1,2,\cdots,m \end{cases} \quad (9)$$

where $\nabla f(\boldsymbol{X}^k), \nabla g_u(\boldsymbol{X}^k), \nabla h_v(\boldsymbol{X}^k)$ are the gradients of the functions $f, g_u, h_v$ at the point $\boldsymbol{X}^k$, and $\nabla^2 f(\boldsymbol{X}^k)$ is the second derivative of the function $f$. This problem is approximate to the original constrained optimization problem, but its solution may not be feasible to the original problem. Then,

$$\boldsymbol{S} = \boldsymbol{X}-\boldsymbol{X}^k. \quad (10)$$

The QP problem described in (9) is transformed into an optimization problem about variable $\boldsymbol{S}$, so

$$\min f(\boldsymbol{X}) = \frac{1}{2}\boldsymbol{S}^{\mathrm{T}}\nabla^2 f(\boldsymbol{X}^k)\boldsymbol{S}+\nabla f(\boldsymbol{X}^k)^{\mathrm{T}}\boldsymbol{S}$$

$$\text{s.t.} \begin{cases} \nabla g_u(\boldsymbol{X}^k)^{\mathrm{T}}\boldsymbol{S}+g_u(\boldsymbol{X}^k) \leqslant 0, & u=1,2,\cdots,p \\ \nabla h_v(\boldsymbol{X}^k)^{\mathrm{T}}\boldsymbol{S}+h_v(\boldsymbol{X}^k) = 0, & v=1,2,\cdots,m \end{cases}. \quad (11)$$

Define the symbols as

$$\begin{cases} \boldsymbol{H} = \nabla^2 f(\boldsymbol{X}^k) \\ \boldsymbol{C} = \nabla f(\boldsymbol{X}^k) \\ \boldsymbol{A}_{\mathrm{eq}} = [\nabla h_1(\boldsymbol{X}^k), \nabla h_2(\boldsymbol{X}^k), \cdots, \nabla h_m(\boldsymbol{X}^k)]^{\mathrm{T}} \\ \boldsymbol{A} = [\nabla g_1(\boldsymbol{X}^k), \nabla g_2(\boldsymbol{X}^k), \cdots, \nabla g_p(\boldsymbol{X}^k)]^{\mathrm{T}} \\ \boldsymbol{B}_{\mathrm{eq}} = [h_1(\boldsymbol{X}^k), h_2(\boldsymbol{X}^k), \cdots, h_m(\boldsymbol{X}^k)]^{\mathrm{T}} \\ \boldsymbol{B} = [g_1(\boldsymbol{X}^k), g_2(\boldsymbol{X}^k), \cdots, g_p(\boldsymbol{X}^k)]^{\mathrm{T}} \end{cases}. \quad (12)$$

By substituting (12) into (11), the general form of QP problem can be obtained as

$$\min \frac{1}{2}\boldsymbol{S}^{\mathrm{T}}\boldsymbol{H}\boldsymbol{S}+\boldsymbol{C}^{\mathrm{T}}\boldsymbol{S}$$

$$\text{s.t.} \begin{cases} \boldsymbol{A}\boldsymbol{S} \leqslant \boldsymbol{B} \\ \boldsymbol{A}_{\mathrm{eq}}\boldsymbol{S} = \boldsymbol{B}_{\mathrm{eq}} \end{cases}. \quad (13)$$

This QP problem is further solved by taking the optimal solution $\boldsymbol{S}^*$ as the next search direction $\boldsymbol{S}^k$ for the original problem and performing a constrained one-dimensional search of the objective function for the original constrained problem to obtain an approximate solution $\boldsymbol{X}^{k+1}$ of the original constrained problem. By repeating this process, the local optimal solution of the original problem can be obtained [26].

The solution of QP problem can be divided into the following two situations.

(i) QP with equality constraints

$$\min f(\boldsymbol{X}) = \frac{1}{2}\boldsymbol{S}^{\mathrm{T}}\boldsymbol{H}\boldsymbol{S}+\boldsymbol{C}^{\mathrm{T}}\boldsymbol{S}$$

$$\text{s.t.} \ \boldsymbol{A}_{\mathrm{eq}}\boldsymbol{S} = \boldsymbol{B}_{\mathrm{eq}} \quad (14)$$

If the Lagrangian is zero, then

$$\min L(\boldsymbol{S},\boldsymbol{\lambda}) = \frac{1}{2}\boldsymbol{S}^{\mathrm{T}}\boldsymbol{H}\boldsymbol{S}+\boldsymbol{C}^{\mathrm{T}}\boldsymbol{S}+\boldsymbol{\lambda}^{\mathrm{T}}(\boldsymbol{A}_{\mathrm{eq}}\boldsymbol{S}-\boldsymbol{B}_{\mathrm{eq}}) \quad (15)$$

where $\boldsymbol{S} = \boldsymbol{X}-\boldsymbol{X}^k$, $\boldsymbol{H} = \nabla^2 f(\boldsymbol{X}^k)$ is the positive definite matrix, and $\boldsymbol{\lambda}$ is the Lagrange multiplier. Then from the extremum condition $\nabla L(\boldsymbol{S},\boldsymbol{\lambda}) = 0$ of multivariate function, we can obtain

$$\begin{cases} \boldsymbol{H}\boldsymbol{S}+\boldsymbol{C}+\boldsymbol{A}_{\mathrm{eq}}^{\mathrm{T}}\boldsymbol{\lambda} = 0 \\ \boldsymbol{A}_{\mathrm{eq}}\boldsymbol{S}-\boldsymbol{B}_{\mathrm{eq}} = 0 \end{cases}. \quad (16)$$

And it can be further written in a matrix form as

$$\begin{bmatrix} \boldsymbol{H} & \boldsymbol{A}_{\mathrm{eq}}^{\mathrm{T}} \\ \boldsymbol{A}_{\mathrm{eq}} & \boldsymbol{0} \end{bmatrix}\begin{bmatrix} \boldsymbol{S} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} -\boldsymbol{C} \\ \boldsymbol{B}_{\mathrm{eq}} \end{bmatrix}. \quad (17)$$

Equation (17) is a system of linear equations with $[\boldsymbol{S},\boldsymbol{\lambda}]^{\mathrm{T}}$ as the variables. The number of variables and equations is $n+m$. From the knowledge of linear algebra, it is clear that this equation either has no solution or a unique solution. If there is a solution, then the unique solution of this equation can be easily found by using the elimination transformation, denoted as $[\boldsymbol{S}^{k+1},\boldsymbol{\lambda}^{k+1}]^{\mathrm{T}}$. If the multiplicative vector $\boldsymbol{\lambda}^{k+1}$ in this solution is not zero, then $\boldsymbol{S}^{k+1}$ is the optimal solution $\boldsymbol{S}^*$ of the equation constrained QP problem (14), namely $\boldsymbol{S}^* = \boldsymbol{S}^{k+1}$, according to the condition $k-t$.

(ii) QP with general constraints

For QP problem (13) with general constraints, we should find out the constraints acting on iteration points $\boldsymbol{X}^k$ in inequality constraints, and form new constraints by combining equality constraints and working constraints

$$\min f(\boldsymbol{X}) = \frac{1}{2}\boldsymbol{S}^{\mathrm{T}}\boldsymbol{H}\boldsymbol{S}+\boldsymbol{C}^{\mathrm{T}}$$

$$\text{s.t.} \sum_{i\in E\bigcup I_k}\sum_{j=1}^{n} a_{ij}s_j = b_j \quad (18)$$

where $E$ represents the set of equality constraints, and $I_k$ represents the set of subscripts acting constraints in inequality constraints [27].

Referring to the solution process of (14), the solution of (18) can be found [28]. After obtaining the solution $[\boldsymbol{S}^{k+1},\boldsymbol{\lambda}^{k+1}]^{\mathrm{T}}$ of (18), if the multipliers corresponding to the constraints of the original equation are not zero and the multipliers corresponding to the parts of the functioning constraints are not less than zero, then $\boldsymbol{S}^{k+1}$ is the optimal solution $\boldsymbol{S}^*$ of (13) for the general constraint QP problem according to condition $k-t$ [29].

### 3.2 Optimization solution algorithm of non-cooperative target distributed hybrid processing systems

The solution steps of non-cooperative target distributed hybrid processing optimization based on SQP are as follows [30].

**Step 1** Given the initial point $\boldsymbol{X}^0$ and convergence precision $\varepsilon$, let $\boldsymbol{H}^0=\boldsymbol{I}$ ( $\boldsymbol{I}$ is the identity matrix), set $k=0$.

**Step 2** Simplify the original problem as the QP problem at point $\boldsymbol{X}^k$, as shown in (18).

**Step 3** Solve the QP problem and let $\boldsymbol{S}^k=\boldsymbol{S}^*$.

**Step 4** In the direction $\boldsymbol{S}^k$ of the original problem, perform the objective function-based constraint one-dimensional search to get the solution $\boldsymbol{X}^{k+1}$.

**Step 5** If $\boldsymbol{X}^{k+1}$ satisfies the given convergence accuracy $\varepsilon$, then let $\boldsymbol{X}^*=\boldsymbol{X}^{k+1}$, $f^*=f(\boldsymbol{X}^{k+1})$, and output the local optimal solution, then terminate the calculation; otherwise, go to Step 6.

**Step 6** Modify $\boldsymbol{H}^{k+1}$ according to the variable metric method or quasi-Newton method in the proposed Newton formula, make $k=k+1$, and go to Step 2 to continue iterations.

## 4. Simulation verifications

Given the values of the parameters of the missile defense stream. The unit costs for different interceptor missile defenses are given as follows. $c_{TQ}=2\,000$ for the unit cost in the exo-atmospheric interceptor weapon group, which is $c_{11}=c_{21}=c_{22}=c_{32}=2\,000$; $c_{QM}=800$ for the unit cost of interception in the intra-atmospheric interceptor weapon group, which is $c_{43}=c_{44}=c_{54}=800$; in addition there is an internal mutual aid interception unit cost of $c_{M_1M_2}=800$, which is $c_{64}=c_{73}=800$; the unit interception cost in the observation pool includes the interception cost from the self-defense pool, $c_{ME}=1$, that is $c_{65}=c_{76}=1$, and the interception cost to the incoming target pool, $c_{TE}=1$, that is $c_{15}=1$; the different processing capability. The incoming target pool to the exo-atmospheric interceptor group is limited to $d_{TQ}=10$, namely $d_{11}=d_{21}=d_{22}=d_{32}=10$; the exo-atmospheric interceptor group to the in-atmospheric interceptor group is limited to $d_{QM}=15$, namely $d_{43}=d_{44}=d_{54}=15$; the interceptor group within the in-atmospheric interceptor group is limited to $d_{M_1M_2}=15$, namely $d_{64}=d_{73}=15$; the in-atmospheric interceptor group to the observation pool is limited to $d_{ME}=25$, namely $d_{65}=d_{76}=25$, and the incoming target pool to the observation pool is limited to $d_{TE}=10$, namely $d_{15}=10$. The capacity limit of the nodes in the incoming target pool is $b_T=30$, that is, $b_1=b_2=b_3=30$; the capacity limit of the nodes in the exo-atmospheric interceptor weapon group is $b_Q=8$, that is, $b_4=b_5=8$; the capacity limit of the nodes in the intra-atmospheric interceptor weapon group is $b_M=8$, that is, $b_6=b_7=8$; the capacity limit of the nodes in the observation pool is $b_E=15$, that is, $b_{e1}=b_{e2}=15$; the threat component $k$ content $q_t^k$ in the threat set $t$ is 60%; the threat component $k$ content $q_e^k$ in the observation set $e$ is 10%.

### 4.1 Simulation example 1

When the non-cooperative target defense situation is shown in Fig. 2, where one node of the incoming non-cooperative target pool launches missiles to the observation pool, then the mathematical description is as follows:

$$\begin{cases} q_1^k=q_3^k=0 \\ q_2^k\neq 0\Rightarrow \omega_1=\omega_3=0 \\ \omega_2=\omega_4=\omega_5=\omega_6=\omega_7=1 \end{cases}.$$

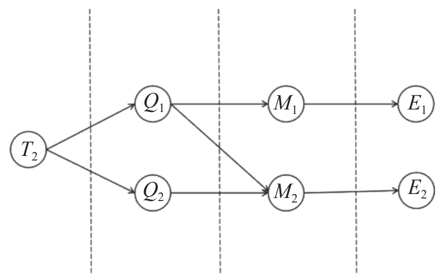


**Fig. 2 Schematic diagram of air defense and anti-missile fire system when the incoming target is 1**

The objective function is

$$F=c_{21}f_{21}+c_{22}f_{22}+c_{43}f_{43}+c_{44}f_{44}+ \\ c_{54}f_{54}+c_{65}f_{65}+c_{76}f_{76}.$$

Combined with the established model and simulation parameters, the corresponding constraint conditions can be obtained.

(i) Network flow constraint is

$$f_{21}+f_{22}=C.$$

(ii) Processing capacity constraints are

$$f_{21}+f_{22}\leqslant b_2;f_{43}+f_{44}\leqslant b_4;f_{54}\leqslant b_5; \\ f_{65}\leqslant b_6;f_{76}\leqslant b_7;f_{65}\leqslant b_{e1};f_{76}\leqslant b_{e2}.$$

(iii) Ability balance constraints are

$$Q_1:f_{21}=f_{43}+f_{44}; \\ Q_2:f_{22}=f_{54}; \\ M_1:f_{43}=f_{65}; \\ M_2:f_{54}+f_{44}=f_{76}.$$

(iv) Constraint of threat component is

$$q_6^kf_{65}+q_7^kf_{76}\leqslant q_{e_1}^kf_{65}+q_{e_2}^kf_{76}.$$

To facilitate the solution, the ability balance constraint is substituted into the objective function $F$ and the constraint conditions (1), (2) and (4) are further simplified. The objective function is

$$F = (c_{22}+c_{54}+c_{76})f_{22}+(c_{43}+c_{21}+c_{65})f_{43}+(c_{44}+c_{21}+c_{76})f_{44}.$$

Network flow constraint is

$$f_{22}+f_{43}+f_{44}=C.$$

Processing capacity constraints are

$$f_{43}+f_{44}\leqslant b_4;f_{22}\leqslant b_5;f_{43}\leqslant b_6;f_{22}+f_{44}\leqslant b_7.$$

Threat component constraint is

$$q_6^k f_{43}+q_7^k(f_{22}+f_{44})\leqslant q_{e_1}^k f_{43}+q_{e_2}^k(f_{22}+f_{44}).$$

In the simulation example 1, the value of $C$ is set as 8. According to the proposed SQP-based optimization method, simulation verification is carried out. By calling the fmincon function in Matlab toolbox and setting the corresponding simulation parameters to solve the problem, the local optimal solution of simulation example 1 can be derived. As the total traffic input to the defense system network increases, the minimum defense cost increases, as shown in Fig. 3, where the abscissa $C$ and the ordinate $F$ denotes the value of the total flow input to the defense system network and the minimum defense cost respectively. Furthermore, to verify the effectiveness of the algorithm, a defense scenario is considered in this paper, assuming that the variation of the flow $C$ sent to the defense system network satisfies a quadratic function as time $t$ varies, the equation is

$$C=-0.08t^2+1.6t,\quad t\in[0,10].$$

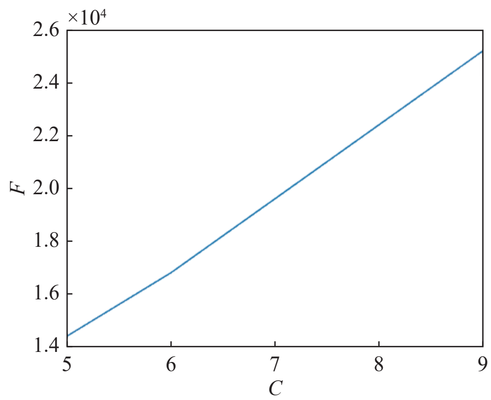


**Fig. 3 Minimum defense cost varying with the value of total flow $C$ input to the defense system network (example 1)**

Using the algorithm and optimization process proposed in this paper, the variation curve of the minimum defense cost is given, as shown in Fig. 4, where $t$, $C$, $F$ coordinate denotes the time, the value of total flow input to the defense system network and the minimum defense cost respectively, and it can be seen that the algorithm is able to give all optimal solutions within the defense capability.

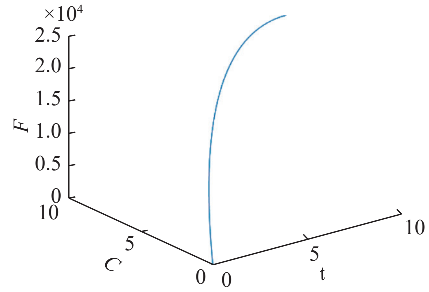


**Fig. 4 Curve of minimum defense cost after time and traffic changes (example 1)**

### 4.2 Simulation example 2

The missile defense situation is shown in Fig. 5, where two nodes of the incoming target pool launch missiles to the observation pool, then the mathematical description is

$$\begin{cases} q_1^k \neq 0 \\ q_2^k \neq 0 \\ q_3^k = 0 \Rightarrow \omega_3 = 0 \\ \omega_1=\omega_2=\omega_4=\omega_5=\omega_6=\omega_7=1 \end{cases}.$$

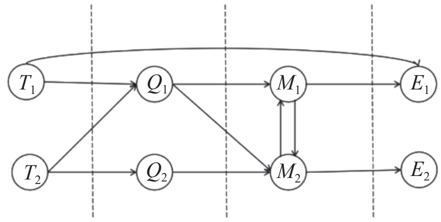


**Fig. 5 Schematic diagram of air defense and anti-missile fire system when the incoming target is 2**

The objective function is

$$F = c_{11}f_{11}+c_{15}f_{15}+c_{21}f_{21}+c_{22}f_{22}+c_{43}f_{43}+c_{44}f_{44}+c_{54}f_{54}+c_{65}f_{65}+c_{76}f_{76}.$$

Combined with the established model and simulation parameters, the corresponding constraint conditions can be obtained.

(i) Network flow constraint is

$$f_{11}+f_{15}+f_{21}+f_{22}=C.$$

(ii) Processing capacity constraints are

$$f_{12}+f_{15}\leqslant b_1;f_{21}+f_{22}\leqslant b_2;f_{43}+f_{44}\leqslant b_4;$$
$$f_{54}\leqslant b_5;f_{65}\leqslant b_6;f_{76}\leqslant b_7;f_{15}+f_{65}\leqslant b_{e1};f_{76}\leqslant b_{e2}.$$

(iii) Ability balance constraints are

$$\begin{aligned} Q_1 &: f_{11} + f_{21} = f_{43} + f_{44}; \\ Q_2 &: f_{22} = f_{54}; \\ M_1 &: f_{43} = f_{65}; \\ M_2 &: f_{54} + f_{44} = f_{76}. \end{aligned}$$

(iv) Constraint of threat component is

$$q_1^k f_{15} + q_6^k f_{65} + q_7^k f_{76} q_{e_1}^k (f_{15} + f_{65}) + q_{e_1}^k f_{76}.$$

To facilitate the solution, the ability balance constraint is substituted into the objective function and the constraint conditions (i), (ii) and (iv) are further simplified. The objective function is

$$F = (c_{11} - c_{21}) f_{11} + c_{15} f_{15} + (c_{22} + c_{54} + c_{76}) f_{22} + (c_{43} + c_{21} + c_{65}) f_{43} + (c_{44} + c_{21} + c_{76}) f_{44}.$$

The network flow constraint is

$$f_{15} + f_{22} + f_{43} + f_{44} = C.$$

Processing capacity constraints are

$$\begin{aligned} & f_{11} + f_{15} \leqslant b_1; f_{43} + f_{44} + f_{22} - f_{11} \leqslant b_2; \\ & f_{43} + f_{44} \leqslant b_4; f_{22} \leqslant b_5; f_{43} \leqslant b_6; \\ & f_{22} + f_{44} \leqslant b_7; f_{15} + f_{43} \leqslant b_{e1}. \end{aligned}$$

The threat component constraint is

$$\begin{aligned} & q_1^k f_{15} + q_6^k f_{43} + q_7^k (f_{22} + f_{44}) \leqslant \\ & q_{e_1}^k (f_{15} + f_{43}) + q_{e_2}^k (f_{22} + f_{44}). \end{aligned}$$

In the simulation example 2, the value of $C$ is set as 16. According to the proposed SQP-based optimization method, simulation verification is carried out. By calling the fmincon function in Matlab toolbox and setting the corresponding simulation parameters to solve the problem, the local optimal solution of simulation example 2 can be derived. As the total traffic input to the defense system network increases, the minimum defense cost increases, as shown in Fig. 6, where the abscissa $C$ and the ordinate $F$ denote the value of total flow input to the defense system network and the minimum defense cost respectively. To verify the effectiveness of the algorithm, a defense scenario is considered in this paper, assuming that the variation of the flow $C$ sent to the defense system network satisfies a quadratic function as time $t$ varies, the equation is as follows:

$$C = -0.16t^2 + 3.2t, \quad t \in [0, 10].$$

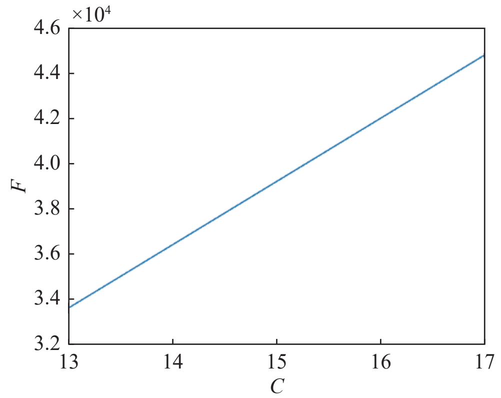


**Fig. 6 Minimum defense cost varying with the value of total flow $C$ input to the defense system network (example 2)**

Using the algorithm and optimization process proposed in this paper, the variation curve of the minimum defense cost is given, as shown in Fig. 7, where $t$, $C$, $F$ coordinates denote the time, the value of total flow input to the defense system network and the minimum defense cost respectively, and it can be seen that the algorithm is able to give all optimal solutions within the defense capability.

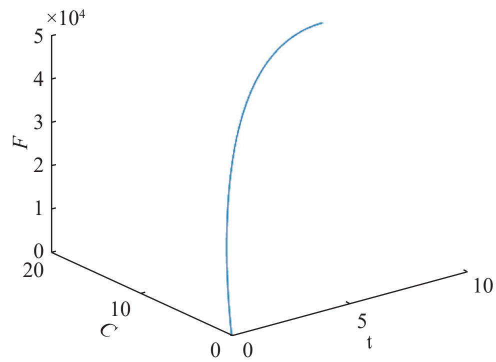


**Fig. 7 Curve of minimum defense cost after time and traffic changes (example 2)**

### 4.3 Simulation example 3

The missile defense situation is shown in Fig. 1, where three nodes of the incoming target pool launch missiles to the observation pool, then the mathematical description is

$$\begin{cases} q_1^k \neq 0 \\ q_2^k \neq 0 \\ q_3^k = 0 \Rightarrow \omega_3 = 0 \\ \omega_1 = \omega_2 = \omega_4 = \omega_5 = \omega_6 = \omega_7 = 1 \end{cases}.$$

The objective function is

$$F = c_{11} f_{11} + c_{15} f_{15} + c_{21} f_{21} + c_{22} f_{22} + c_{32} f_{32} + c_{43} f_{43} + c_{44} f_{44} + c_{54} f_{54} + c_{64} f_{64} + c_{65} f_{65} + c_{73} f_{73} + c_{76} f_{76}.$$

Combined with the established model and simulation parameters, the corresponding constraint conditions can be obtained.

(i) The network flow constraint is

$$f_{11}+f_{15}+f_{21}+f_{22}+f_{32}=C.$$

(ii) Processing capacity constraints are

$$lf_{11}+f_{15}\leqslant b_1; f_{21}+f_{22}\leqslant b_2; f_{32}\leqslant b_3;$$
$$f_{43}+f_{44}\leqslant b_4; f_{54}\leqslant b_5; f_{64}+f_{65}\leqslant b_6;$$
$$f_{73}+f_{76}\leqslant b_7; f_{15}+f_{65}\leqslant b_{e1}; f_{76}\leqslant b_{e2}.$$

(iii) Ability balance constraints are

$$Q_1: f_{11}+f_{21}=f_{43}+f_{44};$$
$$Q_2: f_{22}+f_{32}=f_{54};$$
$$M_1: f_{43}+f_{73}=f_{64}+f_{65};$$
$$M_2: f_{44}+f_{54}+f_{64}=f_{73}+f_{76}.$$

(iv) The constraint of threat component

$$q_1^k f_{15}+q_6^k f_{65}+q_7^k f_{76}\leqslant q_{c_1}^k(f_{15}+f_{65})+q_{e_1}^k f_{76}.$$

To facilitate the solution, the ability balance constraint is substituted into the objective function $F$ and the constraint conditions (i), (ii) and (iv) are further simplified. The objective function is

$$F=c_{15}f_{15}+(c_{21}-c_{11})f_{21}+(c_{22}+c_{54}+c_{76})f_{22}+$$
$$(c_{32}+c_{54}+c_{76})f_{32}+(c_{43}+c_{11}+c_{65})f_{43}+$$
$$(c_{41}+c_{11}+c_{76})f_{44}+(c_{64}+c_{76}-c_{65})f_{64}+$$
$$(c_{73}+c_{65}-c_{76})f_{73}.$$

The network flow constraint is

$$f_{15}+f_{22}+f_{32}+f_{43}+f_{44}=C.$$

Processing capacity constraints are

$$f_{43}+f_{44}+f_{15}-f_{21}\leqslant b_1; f_{21}+f_{22}\leqslant b_2; f_{32}\leqslant b_3;$$
$$f_{43}+f_{44}\leqslant b_4; f_{22}+f_{32}\leqslant b_5; f_{43}+f_{73}\leqslant b_6;$$
$$f_{22}+f_{32}+f_{44}+f_{64}\leqslant b_7; f_{15}+f_{43}+f_{73}-f_{64}\leqslant b_{e1};$$
$$f_{22}+f_{32}+f_{44}+f_{64}-f_{73}\leqslant b_{c2}.$$

The threat component constraint is

$$q_1^k f_{15}+q_6^k f_{43}+q_7^k(f_{22}+f_{44})\leqslant q_{e^k}^k(f_{15}+f_{43})+$$
$$q_{e_2}^k(f_{22}+f_{44})\left(q_1^k-q_e^k\right)f_{15}+\left(q_6^k-q_e^k\right)f_{43}+$$
$$\left(q_7^k-q_{e_2}^k\right)(f_{22}+f_{44})\leqslant 0.$$

In the simulation example 3, the value of $C$ is set as 15. According to the proposed SQP-based optimization method, simulation verification is carried out. By calling the fmincon function in Matlab toolbox and setting the corresponding simulation parameters to solve the problem, the local optimal solution of simulation example 3 can be derived. As the total traffic input to the defense system network increases, the minimum defense cost increases, as shown in Fig. 8, where the abscissa $C$ and the ordinate $F$ denote the value of total flow input to the defense system network and the minimum defense cost respectively. To verify the effectiveness of the algorithm, a defense scenario is considered in this paper, assuming that the variation of the flow $C$ sent to the defense system network satisfies a quadratic function as time $t$ varies, the equation is

$$C=-0.15t^2+3t,\quad t\in[0,10].$$

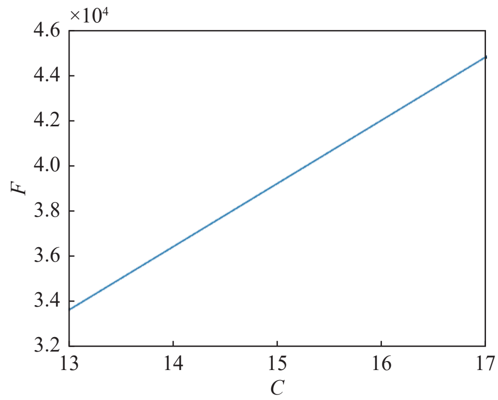


**Fig. 8 Minimum defense cost varing with the value of total flow $C$ input to the defense system network (example 3)**

Using the algorithm and optimization process proposed in this paper, the variation curve of the minimum defense cost is given, as shown in Fig. 9, where $t$, $C$, $F$ coordinates denote the time, the value of total flow input to the defense system network and the minimum defense cost respectively, and it is seen that the algorithm is able to give optimal solution within the defense capability.

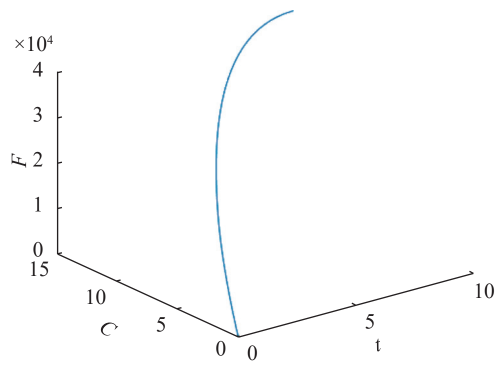


**Fig. 9 Curve of minimum defense cost after time and traffic changes (example 3)**

## 5. Conclusions

In this study, the problem of distributed hybrid processing of non-cooperative targets in networked defense systems is investigated, i.e., finding a reasonable and efficient fire interdiction strategy optimization scheme that allows for the least costly defense of key protection targets and the main group of incoming non-cooperative targets in the area. Compared with other types of missile defense optimization network mathematical models, the mathematical model established in this paper takes into account more comprehensive constraints and is more suitable for practical engineering applications. In addition, the method proposed in this paper is universal and can solve the same type of optimization problems. The main conclusions are as follows:

(i) The grid discretization concept is used to partition the defense zone into network nodes, abstract the distributed hybrid processing scheme for non-cooperative targets into a NP problem that solves for the minimum defense cost, and solve for the defense zone node defense capability of the defense system against incoming non-cooperative targets in different directions according to the basic constraint requirements for intercepting non-cooperative targets in networked defense systems.

(ii) A distributed hybrid processing optimization model for non-cooperative targets with minimization of defensive fire resources as the objective function is established, a SQP-based optimization solving method is constructed, and the optimal interception strategy scheme is obtained by applying the SQP method containing nonconvex quadratic equations and inequality constraints to solve the problem. Finally, the effectiveness of the proposed method is verified by simulation examples.

In this paper, under the complicated multiple constraints, the optimization result is a local optimal solution, but it can meet the non-cooperative target distributed hybrid processing optimization requirements in engineering. In future, the deep learning-based global optimization method [31] will be studied for solving the problem of distributed hybrid processing of non-cooperative targets in networked defense systems.

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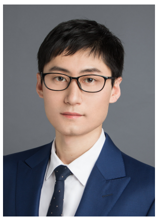
## Biographies

**SONG Xiaocheng** was born in 1989. He received his master degree from Harbin Institute of Technology in 2013. He is an engineer in Beijing Institute of Electronic Engineering, China Aerospace Science and Industry Corporation Limited. His research interests are air-defense and anti-missile technology, and artificial intelligence.
E-mail: sxchitman@126.com

**WANG Jiangtao** was born in 2000. He graduated from University of Science and Technology Beijing in 2022, majoring in intelligent science and technology. He is pursuing his master degree in University of Science and Technology Beijing. His research interest is reinforcement learning-based control and optimization.
E-mail: M202210564@xs.ustb.edu.cn

**WANG Jun** was born in 1997. He graduated from Shanxi University in 2019, majoring in automation. He received his master degree from University of Science and Technology Beijing. He is pursuing his Ph.D. degree in Beijing Institute of Technology. His research interest is spacecraft control and optimization.
E-mail: wangjun_ustb@163.com

**SUN Liang** was born in 1986. He received his master and Ph.D. degrees from Beihang University. He is an associate professor in University of Science and Technology Beijing. His research interests are nonlinear mechanical systems control, intelligent control, and aerospace control.
E-mail: liangsun@ustb.edu.cn



**FENG Yanghe** was born in 1985. He received his master and Ph.D. degrees from National University of Defense Technology. He is an associate professor in National University of Defense Technology. His research interests are system engineering and artificial intelligence.
E-mail: fengyanghe@nudt.edu.cn

**LI Zhi** was born in 1961. He received his mater degree from Beihang University and Ph.D. degree from University of Electronic Science and Technology of China. He is a professor in China Aerospace Science and Industry Corporation Limited. He is an academician of Chinese Academy of Sciences. His research interests are air-defense and anti-missile technology, detection guidance and control technology.
E-mail: lizhi1961@163.com