# Remaining useful life prediction based on nonlinear random coefficient regression model with fusing failure time data

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Abstract: Remaining useful life (RUL) prediction is one of the most crucial elements in prognostics and health management (PHM). Aiming at the imperfect prior information, this paper proposes an RUL prediction method based on a nonlinear random coefficient regression (RCR) model with fusing failure time data. Firstly, some interesting natures of parameters estimation based on the nonlinear RCR model are given. Based on these natures, the failure time data can be fused as the prior information reasonably. Specifically, the fixed parameters are calculated by the field degradation data of the evaluated equipment and the prior information of random coefficient is estimated with fusing the failure time data of congeneric equipment. Then, the prior information of the random coefficient is updated online under the Bayesian framework, the probability density function (PDF) of the RUL with considering the limitation of the failure threshold is performed. Finally, two case studies are used for experimental verification. Compared with the traditional Bayesian method, the proposed method can effectively reduce the influence of imperfect prior information and improve the accuracy of RUL prediction.

Keywords: remaining useful life (RUL) prediction, imperfect prior information, failure time data, nonlinear, random coefficient regression (RCR) model.

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### 1. Introduction

Engineering practice shows that prognostic and health management (PHM) can reduce the risk of failure events and improve the reliability and safety through both prognostic and health management [1-3]. It is widely used in mechanical, electronic, medical, and other high reliability fields [4-6]. Remaining useful life (RUL) prediction is one of the most crucial parts in PHM [7,8]. In recent years, with the improvement of reliability for complex

equipment, RUL prediction has attracted great attention by the scholars [9,10].

There are two common RUL prediction methods in PHM, namely physics-of-failure (PoF) and data-driven methods [10,11]. PoF approaches can provide more accurate RUL estimation based on the physics of underlying failure mechanisms [12]. However, it is typically difficult to obtain the physical failure mechanisms in advance for complex or large-scale engineering systems [13]. In contrast, the data-driven approaches can obtain the RUL of equipment through the condition monitoring (CM) data [14], such as vibration [15] and battery capacity [16], which are simpler than PoF approaches. The data-driven approaches include machine learning and statistical datadriven approaches [17]. Compared with machine learning approaches, the statistical data-driven approaches can derive an analytical expression of the probability density function (PDF) of RUL more easily. This is significant for determining the optimal maintenance time. In statistidata-driven approaches, the random coefficient cal regression (RCR) model is one of the earliest stochastic mathematical models for degradation modeling [14], which has been widely used in RUL prediction [18-21].

The CM data used by the data-driven approaches are mainly categorized into two classes, i.e., the historical degradation data of congeneric equipment and the field CM data of the evaluated equipment [2]. In general, the historical degradation data are used to estimate the fixed parameters [22] which describe the constant degradation features among all units of the population and the prior information of the random coefficient [23,24] that characterizes the unit-to-unit variability for a population of equipment. Then, under the Bayesian framework, the random coefficient is updated online by the field CM data. In this classical parameter estimation method, Bayesian theory is of paramount importance, which establishes a linkage between the historical degradation data and the field

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CM data [2,10]. For the convenience of comparison, this classical parameter estimation method is called the traditional Bayesian method in this paper. More details of this traditional Bayesian method can be found in [10,18,25-27].

However, the traditional Bayesian methods in [10,18,25–27] mentioned above all rely on the prior information of fixed parameters and random coefficient in the model, which requires the existence of historical degradation data of congeneric equipment. Unfortunately, imperfect prior information often occurs, i.e., the historical degradation data of congeneric equipment are often inaccurate, incomplete [7,28] or even non-existent [13] in practical application. The main reason for this phenomenon is that the cost of building a complete historical degradation database is too high. In the existing literature research, there are two approaches to solve such problems. The first approach is the combination of the Kalman filter and the expectation maximization (EM) algorithm, which was first proposed by Wang et al. [29] and has been widely applied to RUL prediction based on the Wiener process [7,13,30-34]. Its main principle is that the posterior distribution of the hidden random coefficient is calculated by Kalman filtering based on the field CM data of the evaluated equipment and then the fixed parameters and random coefficient in the model are solved by the EM algorithm. This approach can overcome the impact of improper prior information on the RUL prediction. However, it also completely gets rid of the impact of prior information and only depends on the field CM data [10]. The second approach is to fuse failure time data of congeneric equipment to reduce the impact of incomplete historical degradation data. Compared with building a complete degradation database, the failure time data is easier to be obtained from historical maintenance (or repair) records [28]. The related research of fusing failure time data can be referred to Lehmann [35], Zhang et al. [36], Zhao et al. [37] and Sun et al. [38]. However, the random effects, i.e., the unit-to-unit variability, are not considered in these literature [28,35-38].

Through the relevant research of the above literature regarding fusing failure time data, we can find that there are still some problems to address for the degradation process based on the RCR model, which mainly include the following points.

(i) Gebraeel et al. [28] did not give the reason why the method of fusing failure time data works well and the estimation method of fixed parameters. After that, Tang et al. [2] gave a method to estimate the fixed parameters. In addition, Tang et al. [2] also explained why the method of fusing failure time data works well based on the natures of parameters estimation presented in the paper.

However, [2] only aimed at the degradation process based on the Wiener process. Whether it is applicable to the degradation process based on the RCR model, including whether it has the same natures and whether the work with fusing failure time data is well, still remains to be solved.

(ii) The existing researches of fusing failure time data are mainly for the linear degradation process and the research on the RUL prediction based on the nonlinear RCR model with fusing failure time data still has not been presented. In actual conditions, many degradation trends are nonlinear.

(iii) The researches of fusing failure time data often ignore the existence of random effects. In 2009, Peng et al. [39] proved that the penalty of mis-fitting a randomeffect model by a fixed-effect model was more serious than that of mis-fitting a fixed-effect model by a randomeffect model.

(iv) The limitation of the failure threshold is not considered in most literature. For the equipment that does not fail at time  $t_k$ , it should be satisfied that  $\omega - x_k > 0$  [1,10], where  $\omega$  denotes the failure threshold and  $x_k$  denotes the actual degradation state of the equipment at time  $t_k$ .

To address the above problems, we first use the RCR model to model the nonlinear equipment degradation process. Then, the corresponding natures for parameters estimation of the RCR model are derived. This leads to the first contribution of this paper. Based on these natures of parameters estimation, we propose a parameters estimation method with fusing failure time data for the nonlinear RCR model. With the help of failure time data, this method uses the maximum likelihood estimation (MLE) to obtain the prior information of the random coefficient of the RCR model, which is the second contribution of this paper. Then, unlike the truncated cumulative distribution function (CDF) presented in Gebraeel et al. [18,19]. we use the truncated normal distribution (TND) to model the failure threshold of equipment and the PDF of the RUL based on the nonlinear RCR model is derived, which is the third contribution of this paper. Finally, we use a numerical example and a case studies to verify the effectiveness of the RUL prediction method proposed in this paper, which can not only reduce the influence of imperfect prior information, but also effectively improve the accuracy of RUL prediction compared with the traditional Bayesian method.

The remaining parts of this paper are organized as follows: Section 2 and Section 3 give some natures of parameters estimation. In Section 4, a parameter estimation method with fusing failure time data and the PDF of the RUL based on the nonlinear RCR model with considering the limitation of failure threshold are given. A numerical example and a case study are provided in SecWANG Fengfei et al.: Remaining useful life prediction based on nonlinear random coefficient regression...

tion 5. Section 6 draws the main conclusions.

# 2. Natures of parameters estimation for the evaluated equipment

First, the RCR model is used to model the nonlinear degradation process. The degradation process based on the nonlinear RCR model can be expressed as follows:

$$X(t) = x_0 + \lambda \Lambda(t;\theta) \tag{1}$$

where X(t) denotes the actual degradation state at time t;  $x_0$  is the initial degradation state;  $\lambda$  is the drift coefficient, which characterizes the rate of degradation;  $\Lambda(t;\theta)$  is a monotone continuous nonlinear function with t, characterizing a nonlinear degradation process, in which  $\theta$  is the fixed parameter, describing the nonlinear relationship between degradation state and time. The typical nonlinear functions are  $\Lambda(t;\theta) = t^{\theta}$  and  $\Lambda(t;\theta) = e^{\theta t} - 1$  [40]. Without loss of generality, we set  $x_0 = 0$ .

Due to measurement error between the actual degradation state X(t) and the observed degradation state Y(t), the observed degradation process can be expressed as

$$Y(t) = X(t) + \varepsilon \tag{2}$$

where  $\varepsilon$  denotes the measurement error and is normally distributed with zero mean and standard deviation  $\sigma_{\varepsilon}$ . In addition,  $\varepsilon$  is assumed to be identically distributed and s-independent with  $\lambda$ .

Before studying the natures of parameters estimation for the evaluated equipment, we first give the parameters estimation results based on the field CM data of the evaluated equipment.

#### 2.1 MLE of parameters for evaluated equipment

Suppose that at time  $t_k$ ,  $Y_{1:k} = \{y_1, y_2, \dots, y_k\}$  are the field CM data at times  $t_1, t_2, \dots, t_k$ , then based on (1) and (2), we can obtain that  $y_j \sim N(\lambda A(t_j; \theta), \sigma_{\varepsilon}^2)$ . Note that in this case, the drift coefficient  $\lambda$  is a constant for specific evaluated equipment. Therefore, the log-likelihood function can be written as

$$\ln L(\lambda, \sigma_{\varepsilon}^{2}|Y_{1:k}) = -\frac{k}{2}(\ln(2\pi) + \ln \sigma_{\varepsilon}^{2}) - \frac{1}{2\sigma_{\varepsilon}^{2}} \sum_{j=1}^{k} (y_{j} - \lambda \Lambda(t_{j}; \theta))^{2}.$$
(3)

Then, the parameters estimation of  $\lambda$  and  $\sigma_{\varepsilon}^2$  can be obtained [41] as follows:

$$\hat{\lambda} = \frac{\sum_{j=1}^{k} y_j \Lambda(t_j; \theta)}{\sum_{j=1}^{k} \left( \Lambda(t_j; \theta)^2 \right)},$$
(4)

$$\hat{\sigma}_{\varepsilon}^{2} = \frac{1}{k} \sum_{j=1}^{k} \left( y_{j} - \hat{\lambda} \Lambda(t_{j}; \theta) \right)^{2}.$$
(5)

**Remark 1** The existing methods cannot obtain the analytical expression of  $\hat{\theta}$ . In general, the estimated value of  $\theta$  is obtained through the Matlab function "FMIN-SEARCH".

#### 2.2 Natures of parameters estimation

For the parameters estimation shown in (4) and (5), we get some interesting natures, which are summed up in Theorem 1.

**Theorem 1** For the parameters estimation of  $\lambda$  and  $\sigma_{\varepsilon}^2$  given in (4) and (5), the expectations and variances of  $\lambda$  and  $\sigma_{\varepsilon}^2$  can be calculated respectively as follows:

 $\sum_{i=1}$ 

$$\mathbf{E}(\hat{\lambda}) = \lambda, \tag{6}$$

$$D(\hat{\lambda}) = \sigma_{\varepsilon}^{2} \frac{1}{\sum_{k}^{k} \Lambda(t:\theta)^{2}},$$
(7)

$$\mathbf{E}(\hat{\sigma}_{\varepsilon}^{2}) = \left(1 - \frac{1}{k}\right)\sigma_{\varepsilon}^{2},\tag{8}$$

$$\mathbf{D}(\hat{\sigma}_{\varepsilon}^{2}) = 2\left(\frac{k-1}{k^{2}}\right)\sigma_{\varepsilon}^{4}.$$
(9)

**Proof** From (1) and (2), we can obtain

$$y_j \sim N(\lambda \Lambda(t_j; \theta), \sigma_{\varepsilon}^2).$$
 (10)

According to the natures of the normal distribution, we have

$$\sum_{j=1}^{k} y_j \Lambda(t_j; \theta) \sim N\left(\lambda \sum_{j=1}^{k} \Lambda(t_j; \theta)^2, \sigma_{\varepsilon}^2 \sum_{j=1}^{k} \Lambda(t_j; \theta)^2\right).$$
(11)

Then,

$$\hat{\lambda} = \frac{\sum_{j=1}^{k} y_j \Lambda(t_j; \theta)}{\sum_{j=1}^{k} \Lambda(t_j; \theta)^2} \sim \mathbf{N} \left( \lambda, \frac{\sigma_{\varepsilon}^2}{\sum_{j=1}^{k} \Lambda(t_j; \theta)^2} \right).$$
(12)

Thus,

$$E(\hat{\lambda}) = \lambda, \tag{13}$$

$$D(\hat{\lambda}) = \sigma_{\varepsilon}^{2} \frac{1}{\sum_{j=1}^{k} \Lambda(t_{j}; \theta)^{2}}.$$
(14)

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For (8), we have

$$\mathbf{E}\left(\hat{\sigma}_{\varepsilon}^{2}\right) = \mathbf{E}\left(\frac{1}{k}\sum_{j=1}^{k}\left(y_{j}-\hat{\lambda}\Lambda(t_{j};\theta)\right)^{2}\right) = \frac{1}{k}\mathbf{E}\left(\sum_{j=1}^{k}\left(\left(y_{j}-\lambda\Lambda(t_{j};\theta)\right)-\left(\hat{\lambda}\Lambda(t_{j};\theta)-\lambda\Lambda(t_{j};\theta)\right)\right)^{2}\right) = \frac{1}{k}\sum_{j=1}^{k}\mathbf{E}\left(y_{j}-\lambda\Lambda(t_{j};\theta)\right)^{2}-\sum_{j=1}^{k}\Lambda(t_{j};\theta)^{2}\mathbf{E}\left(\hat{\lambda}-\lambda\right)^{2} = \frac{1}{k}\left(\sum_{j=1}^{k}\mathbf{D}\left(y_{j}\right)-\mathbf{D}\left(\hat{\lambda}\right)\sum_{j=1}^{k}\Lambda(t_{j};\theta)^{2}\right) = \left(1-\frac{1}{k}\right)\sigma_{\varepsilon}^{2}.$$
 (15)

In addition, it is easy to know that  $y_j \sim N(\lambda \Lambda(t_j; \theta), \sigma_{\varepsilon}^2)$ and

$$\hat{\lambda} = \sum_{j=1}^{k} y_j \Lambda(t_j; \theta) / \left( \sum_{j=1}^{k} \Lambda(t_j; \theta)^2 \right) \sim N\left( \lambda, \sigma_{\varepsilon}^2 / \left( \sum_{j=1}^{k} \Lambda(t_j; \theta)^2 \right) \right).$$

Then,

$$\frac{1}{\sigma_{\varepsilon}^{2}} \sum_{j=1}^{k} \left( y_{j} - \hat{\lambda} \Lambda(t_{j}; \theta) \right)^{2} \sim \chi^{2}(k-1), \qquad (16)$$

$$\mathbf{D}\left(\hat{\sigma}_{\varepsilon}^{2}\right) = \mathbf{D}\left(\frac{1}{k}\sum_{j=1}^{k}\left(y_{j}-\hat{\lambda}\Lambda(t_{j};\theta)\right)^{2}\right) = \frac{1}{k^{2}}\mathbf{D}\left(\sum_{j=1}^{k}\left(y_{j}-\hat{\lambda}\Lambda(t_{j};\theta)\right)^{2}\right) = 2\left(\frac{k-1}{k^{2}}\right)\sigma_{\varepsilon}^{4}.$$
 (17)

**Remark 2** Since the estimation of  $\theta$  cannot get the analytical expression, to further analyze the natures of parameters estimation, it is assumed that the estimation of  $\theta$  is equal to the actual  $\theta$  here. The corresponding nature of  $\hat{\theta}$  is proved by simulation data in Subsection 5.1.

From (7) and (9) in Theorem 1, the accuracy of parameters estimation of  $\lambda$  and  $\sigma_{\varepsilon}^2$  is mainly effected by the number *k* of the detection time. The variances of parameters estimation of  $\lambda$  and  $\sigma_{\varepsilon}^2$  become small as the number *k* increases.

# 3. Natures of drift coefficient with considering random effects

With considering the existence of the random effects, the drift coefficient  $\lambda$  is assumed to be a random variable that follows normal distribution, i.e.,  $\lambda \sim N(\mu_{\lambda}, \sigma_{\lambda}^2)$ , which describes the unit-to-unit variability among equipment. Theorem 1 only analyzes the natures of parameters estimation based on the evaluated equipment. Thus, in this section, we further analyze the natures of drift coefficient with considering the random effects.

# 3.1 Parameters estimation of drift coefficient with considering random effects

In order to address the problem of obtaining a negative variance of the drift parameter, Tang et al. [10] simplified the two-step MLE method proposed by Lu and Meeker [42] and applied it to the Wiener process. In addition, the analytical expressions of  $\mu_{\lambda}$  and  $\sigma_{\lambda}^2$  can be obtained by the two-step MLE method, which is more convenient to analyze the natures of drift coefficient than the traditional MLE method. Then, we apply it to the RCR model.

Without loss of generality, it is assumed that there are *n* items with the same type and the degradation data at time  $t_1, t_2, \dots, t_m$  of the *i*th item is  $y_i = \{y_{i,1}, y_{i,2}, \dots, y_{i,m}\}$ .

Then, the parameters estimation of  $\mu_{\lambda}$  and  $\sigma_{\lambda}^2$  can be calculated by the two-step MLE as follows:

$$\hat{\mu}_{\lambda} = \frac{1}{n} \sum_{i=1}^{n} \frac{\sum_{j=1}^{m} y_{i,j} \Lambda(t_j; \theta)}{\sum_{i=1}^{m} \Lambda(t_j; \theta)^2},$$
(18)

$$\hat{\sigma}_{\lambda}^{2} = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{\sum_{j=1}^{m} y_{i,j} \Lambda(t_{j}; \theta)}{\sum_{j=1}^{m} \Lambda(t_{j}; \theta)^{2}} - \hat{\mu}_{\lambda} \right)^{2}.$$
 (19)

#### 3.2 Natures of drift coefficient

Inspired by Tang et al. [2], we obtain some natures of drift coefficient based on the nonlinear RCR model with considering the random effects, which are summarized in Theorem 2.

**Theorem 2** For the nonlinear RCR model described in (2), the expectations and variances of  $\mu_{\lambda}$  and  $\sigma_{\lambda}^2$  can be expressed respectively as follows:

$$\mathbf{E}(\hat{\mu}_{\lambda}) = \mu_{\lambda},\tag{20}$$

$$\mathbf{D}(\hat{\mu}_{\lambda}) = \frac{1}{n} \cdot \frac{\sigma_{\lambda}^{2} \sum_{j=1}^{m} \Lambda(t_{j}; \theta)^{4} + \sigma_{\varepsilon}^{2} \sum_{j=1}^{m} \Lambda(t_{j}; \theta)^{2}}{\left(\sum_{j=1}^{m} \Lambda(t_{j}; \theta)^{2}\right)^{2}}, \quad (21)$$

$$\mathbf{E}\left(\hat{\sigma}_{\lambda}^{2}\right) = \left(1 - \frac{1}{n}\right) \cdot \frac{\left(\sigma_{\lambda}^{2} \sum_{j=1}^{m} \Lambda(t_{j};\theta)^{4} + \sigma_{\varepsilon}^{2} \sum_{j=1}^{m} \Lambda(t_{j};\theta)^{2}\right)}{\left(\sum_{j=1}^{m} \Lambda(t_{j};\theta)^{2}\right)^{2}},$$
(22)

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$$D(\hat{\sigma}_{\lambda}^{2}) = \frac{(n-1)}{n^{2}} \cdot \frac{2\left(\sigma_{\lambda}^{2} \sum_{j=1}^{m} \Lambda(t_{j}; \theta)^{4} + \sigma_{\varepsilon}^{2} \sum_{j=1}^{m} \Lambda(t_{j}; \theta)^{2}\right)^{2}}{\left(\sum_{j=1}^{m} \Lambda(t_{j}; \theta)^{2}\right)^{4}}.$$
(23)

**Proof** From (1) and (2), we can obtain  $y_{i,j} = \lambda \Lambda(t_j; \theta) + \varepsilon$ .

In addition, since  $\lambda \sim N(\mu_{\lambda}, \sigma_{\lambda}^2)$ ,  $\varepsilon \sim N(0, \sigma_{\varepsilon}^2)$ , and  $\varepsilon$  is *s*-independent with  $\lambda$ ,  $y_{i,j} \sim N(\mu_{\lambda}\Lambda(t_j;\theta), \sigma_{\lambda}^2\Lambda(t_j;\theta)^2 + \sigma_{\varepsilon}^2)$  can be obtained.

Thus,  $y_{i,i}\Lambda(t_i;\theta)$  also obeys a normal distribution:

$$y_{i,j}\Lambda(t_j;\theta) \sim N\Big(\mu_{\lambda}\Lambda(t_j;\theta)^2, \sigma_{\lambda}^2\Lambda(t_j;\theta)^4 + \sigma_{\varepsilon}^2\Lambda(t_j;\theta)^2\Big).$$
(24)

According to the natures of the normal distribution, we have

$$\frac{\sum_{j=1}^{m} y_{i,j} \Lambda(t_j; \theta)}{\sum_{j=1}^{m} \Lambda(t_j; \theta)^2} \sim N\left(\mu_{\lambda}, \frac{\sigma_{\lambda}^2 \sum_{j=1}^{m} \Lambda(t_j; \theta)^4 + \sigma_{\varepsilon}^2 \sum_{j=1}^{m} \Lambda(t_j; \theta)^2}{\left(\sum_{j=1}^{m} \Lambda(t_j; \theta)^2\right)^2}\right).$$
(25)

Thus,

$$\mathbf{E}(\hat{\mu}_{\lambda}) = \mathbf{E}\left(\frac{1}{n}\sum_{i=1}^{n} \left(\frac{\sum_{j=1}^{k} y_{i,j}\Lambda(t_{j};\theta)}{\sum_{i=1}^{k} \Lambda(t_{j};\theta)^{2}}\right)\right) = \mu_{\lambda}, \quad (26)$$

$$D(\hat{\mu}_{\lambda}) = \frac{1}{n} \frac{\sigma_{\lambda}^{2} \sum_{j=1}^{m} \Lambda(t_{j}; \theta)^{4} + \sigma_{\varepsilon}^{2} \sum_{j=1}^{m} \Lambda(t_{j}; \theta)^{2}}{\left(\sum_{j=1}^{m} \Lambda(t_{j}; \theta)^{2}\right)^{2}}.$$
 (27)

In addition, according to the natures of the Chi-squared distribution, we have

$$\frac{\left(\sum_{j=1}^{m} \Lambda(t_j; \theta)^2\right)^2}{\sigma_{\lambda}^2 \sum_{j=1}^{m} \Lambda(t_j; \theta)^4 + \sigma_{\varepsilon}^2 \sum_{j=1}^{m} \Lambda(t_j; \theta)^2} \cdot \sum_{i=1}^{n} \left(\frac{\sum_{j=1}^{m} y_{i,j} \Lambda(t_j; \theta)}{\sum_{j=1}^{m} \Lambda(t_j; \theta)^2} - \mu_{\lambda}\right)^2 \sim \chi^2(n-1).$$
(28)

 $\mathbf{E}\left(\hat{\sigma}_{\lambda}^{2}\right) = \left(1 - \frac{1}{n}\right) \cdot \frac{\left(\sigma_{\lambda}^{2} \sum_{j=1}^{m} \Lambda(t_{j}; \theta)^{4} + \sigma_{\varepsilon}^{2} \sum_{j=1}^{m} \Lambda(t_{j}; \theta)^{2}\right)}{\left(\sum_{j=1}^{m} \Lambda(t_{j}; \theta)^{2}\right)^{2}},$  (29)  $\mathbf{D}\left(\hat{\sigma}_{\lambda}^{2}\right) = \frac{(n-1)}{n^{2}} \cdot \frac{2\left(\sigma_{\lambda}^{2} \sum_{j=1}^{m} \Lambda(t_{j}; \theta)^{4} + \sigma_{\varepsilon}^{2} \sum_{j=1}^{m} \Lambda(t_{j}; \theta)^{2}\right)^{2}}{\left(\sum_{j=1}^{m} \Lambda(t_{j}; \theta)^{2}\right)^{4}}.$  (30)

**Remark 3** From (21) and (23), the accuracy of parameters estimation of  $\mu_{\lambda}$  and  $\sigma_{\lambda}^2$  is mainly effected by number *N* of the congeneric equipment sample and number *k* of the detection time. When the detection time is fixed for equipment, increasing number *N* of sample can improve the accuracy of parameters estimation of  $\mu_{\lambda}$  and  $\sigma_{\lambda}^2$ .

# 4. RUL prediction with fusing failure time data

In practical application, the traditional Bayesian method may be unable to obtain the accurate prior information of the unknown parameters in the model due to the imperfect prior information, which could decrease the RUL accuracy. In order to solve this problem, according to the natures of parameters estimation derived in Section 2 and Section 3, this section proposes an RUL prediction method based on the nonlinear RCR model that reasonably fuses the failure time data of congeneric equipment and the field CM data of the evaluated equipment. The flow chart of this method is shown in Fig. 1.



Fig. 1 Flow chart of RUL prediction with fusing failure time data

Then, we can obtain

The main steps are as follows:

**Step 1** Based on the field CM data of the evaluated equipment, the fixed parameters that describe constant degradation features among all units of the population can be obtained. Then, the prior information of the random coefficient that characterizes the unit-to-unit variability for a population of equipment can be calculated by fusing failure time data of congeneric equipment.

**Step 2** The prior information of the random coefficient is updated online based on the field CM data of the evaluated equipment under Bayesian framework. Then, RUL of the evaluated equipment with considering the limitation of the failure threshold can be obtained.

Note that the method with fusing failure time data proposed in this paper only utilizes the failure time data and the field CM data, and avoids using the historical degradation data, which is the most obvious difference from the traditional Bayesian method. Next, according to the flow chart, calculate the RUL of the evaluated equipment.

#### 4.1 Parameters estimation

The prior parameters in the degradation model based on the nonlinear RCR model are  $\{\mu_{\lambda}, \sigma_{\lambda}^2, \sigma_{\varepsilon}^2, \theta\}$ . In the following, the prior parameters are solved according to the flow chart, as shown in Fig. 1.

(i) Estimating the fixed parameters based on the field CM data of the evaluated equipment.

From (5), we have

$$\hat{\sigma}_{\varepsilon}^{2}(\hat{\theta}) = \frac{1}{k} \sum_{j=1}^{k} \left( y_{j} - \hat{\lambda} \Lambda(t_{j}; \theta) \right)^{2}.$$
 (31)

Then, by substituting (4) and (31) into (3), after simplification, we obtain the profile log-likelihood function of  $\theta$  as follows:

$$\ln L(\theta|Y_{1:k}) = -\frac{k}{2}\ln(2\pi) - \frac{k}{2}\ln(\hat{\sigma}_{\varepsilon}^{2}) - \frac{k}{2}.$$
 (32)

The estimation of  $\theta$  can be obtained by maximizing the profile log-likelihood function in (32) through Matlab function "FMINSEARCH". Then, the estimations of  $\hat{\sigma}_{\varepsilon}^2$  can be obtained by bring  $\hat{\theta}$  into (31).

(ii) Calculating the prior information of the random coefficient with fusing failure time data of congeneric equipment.

According to the natures of the nonlinear RCR model, the PDF of failure lifetime  $T_{\nu}$  of equipment can be written as

$$f_{T_{v}|\omega}(t_{v}|\omega) = \frac{\omega\Lambda'(t_{v};\theta)}{\sqrt{2\pi\sigma_{\lambda}^{2}\Lambda(t_{v};\theta)^{4}}} \cdot \exp\left(-\frac{(\omega-\mu_{\lambda}\Lambda(t_{v};\theta))^{2}}{2\sigma_{\lambda}^{2}\Lambda(t_{v};\theta)^{2}}\right).$$
(33)

Suppose that there are *M* items and their failure time data are  $T_{1:M} = \{t_1, t_2, \dots, t_m\}$ , then, the log-likelihood function can be expressed as follows:

$$\ln L\left(\mu_{\lambda}, \sigma_{\lambda}^{2} | T_{1:M}\right) = m \ln \omega - m \ln \Lambda'(t_{\nu}; \theta) - \frac{m}{2} \ln (2\pi) - \frac{m}{2} \ln \sigma_{\lambda}^{2} - 2m \ln \Lambda(t_{\nu}; \theta) - \frac{1}{2\sigma_{\lambda}^{2}} \sum_{\nu=1}^{m} \frac{(\omega - \mu_{\lambda} \Lambda(t_{\nu}; \theta))^{2}}{(\Lambda(t_{\nu}; \theta))^{2}}.$$
(34)

Taking the first partial derivatives of  $\ln L(\mu_{\lambda}, \sigma_{\lambda}^2 | T_{1:M})$ with respect to  $\mu_{\lambda}$  and  $\sigma_{\lambda}^2$  gives

$$\frac{\partial \ln L(\mu_{\lambda}, \sigma_{\lambda}^{2} | T_{1:M})}{\partial \mu_{\lambda}} = \frac{1}{\sigma_{\lambda}^{2}} \sum_{\nu=1}^{m} \frac{(\omega - \mu_{\lambda} \Lambda(t_{\nu}; \theta)) \Lambda(t_{\nu}; \theta)}{(\Lambda(t_{\nu}; \theta))^{2}}$$
(35)

and

$$\frac{\partial \ln L(\mu_{\lambda}, \sigma_{\lambda}^{2}|T_{1:M})}{\partial \sigma_{\lambda}^{2}} = -\frac{m}{2} \frac{1}{\sigma_{\lambda}^{2}} + \frac{1}{2(\sigma_{\lambda}^{2})^{2}} \cdot \sum_{\nu=1}^{m} \frac{(\omega - \mu_{\lambda}\Lambda(t_{\nu};\theta))^{2}}{(\Lambda(t_{\nu};\theta))^{2}}.$$
(36)

Then, by setting these derivatives to zeros, the parameters estimation of  $\mu_{\lambda}$  and  $\sigma_{\lambda}^2$  can be written as

$$\hat{\mu}_{\lambda} = \frac{\omega}{m} \sum_{\nu=1}^{m} \frac{1}{\Lambda(t_{\nu};\theta)},$$
(37)

$$\hat{\sigma}_{\lambda}^{2} = \frac{1}{m} \sum_{\nu=1}^{m} \frac{(\omega - \hat{\mu}_{\lambda} \Lambda(t_{\nu}; \theta))^{2}}{(\Lambda(t_{\nu}; \theta))^{2}}.$$
(38)

#### 4.2 Online parameter updating

Let the parameters estimation of  $\mu_{\lambda}$  and  $\sigma_{\lambda}^2$  in the previous subsection be the prior information  $\mu_{\lambda 0}$  and  $\sigma_{\lambda 0}^2$  of the random coefficient, i.e.,  $\lambda \sim N(\mu_{\lambda 0}, \sigma_{\lambda 0}^2)$ . Given the field CM data  $y_{1:k}$ , the posterior distribution can be calculated by Bayesian theory [43] as follows:

$$\lambda | y_{1:k} \sim \mathcal{N}(\mu_{\lambda,k}, \sigma_{\lambda,k}^2) \tag{39}$$

where

$$\mu_{\lambda,k} = \frac{\sigma_{\lambda 0}^2 \sum_{i=1}^k y_i \Lambda(t_i; \theta) + \sigma_{\varepsilon}^2 \mu_{\lambda 0}}{\sigma_{\lambda 0}^2 \sum_{i=1}^k \Lambda(t_i; \theta)^2 + \sigma_{\varepsilon}^2},$$

$$\sigma_{\lambda,k}^2 = \frac{\sigma_{\lambda 0}^2 \sigma_{\varepsilon}^2}{\sigma_{\lambda 0}^2 \sum_{i=1}^k \Lambda(t_i; \theta)^2 + \sigma_{\varepsilon}^2}.$$
(40)

#### 4.3 RUL prediction

For the degradation process based on the RCR model, lifetime *T* can be defined as the time that the degradation process exceeds a pre-set failure threshold. Let  $T = t_k + l_k$ 

denote the lifetime of equipment, where  $t_k$  denotes the current time and  $l_k$  denotes the RUL at time  $t_k$ . Then, the degradation process in (1) can be transformed into

$$Z(l_k) = X(l_k + t_k) - X(t_k) = \lambda \Delta \Lambda(l_k; \theta)$$
(41)

where  $\Delta \Lambda(l_k; \theta) = \Lambda(l_k + t_k; \theta) - \Lambda(t_k; \theta)$ . Without loss of generality, we set Z(0) = 0.

The corresponding RUL at time  $t_k$  can be written as

$$L_{k} = \{l_{k} : X(x_{k} + l_{k}) > \omega | x_{0} < \omega\} = \{l_{k} : Z(l_{k}) \ge \omega - x_{k} | Z(0) < \omega - x_{k}\}.$$
(42)

Then, if  $\omega - x_k$  is given, the PDF for the RUL of the equipment can be obtained as follows:

$$f_{L_{k}|\omega_{k}}(l_{k}|\omega_{k}) = \frac{\omega_{k}\Delta\Lambda'(l_{k};\theta)}{\sqrt{2\pi\sigma_{\lambda,k}^{2}\Delta\Lambda(l_{k};\theta)^{4}}} \cdot \exp\left(-\frac{(\omega_{k}-\mu_{\lambda,k}\Delta\Lambda(l_{k};\theta))^{2}}{2\sigma_{\lambda,k}^{2}\Delta\Lambda(l_{k};\theta)^{2}}\right)$$
(43)

where  $\omega_k = \omega - x_k$  and  $x_k$  denotes the actual degradation state at time  $t_k$ .

From (2), we have  $x_k \sim N(y_k, \sigma_{\varepsilon}^2)$ . Therefore,  $(\omega - x_k) \sim N(\omega - y_k, \sigma_{\varepsilon}^2)$ . Then, in order to satisfy the condition that  $\omega - x_k > 0$ , we use the TND to model  $\omega - x_k$ , that is,  $(\omega - x_k) \sim TN(\omega - y_k, \sigma_{\varepsilon}^2)$ .

Therefore, given the field CM data  $y_{1:k}$ , based on Lemma 1 proposed by Tang et al. [10], the PDF of the RUL based on the nonlinear RCR model with considering  $\omega - x_k > 0$  can be derived by using the law of total probability as follows:

$$f_{L_{k}|y_{1:k}}(l_{k}|y_{1:k}) = \frac{\sqrt{G}\Delta\Lambda'(t;\theta)}{2\pi\Delta\Lambda(t;\theta)D\Phi(\mu \cdot \sigma_{\varepsilon}^{-1})} \cdot \exp\left(-\frac{\mu_{\lambda,k}^{2}\Delta\Lambda(t;\theta)^{2}\sigma_{\epsilon}^{2} + \mu^{2}\sigma_{\lambda,k}^{2}\Delta\Lambda(t;\theta)^{2}}{2G}\right) + \frac{E\Delta\Lambda'(t;\theta)\Phi\left(\frac{E}{\sqrt{DG}}\right)}{\sqrt{2\pi\Delta\Lambda(t;\theta)^{2}D^{3}}\Phi(\mu \cdot \sigma_{\varepsilon}^{-1})} \cdot \exp\left(-\frac{(\mu - \mu_{\lambda,k}\Delta\Lambda(t;\theta))^{2}}{2D}\right)$$
(44)

where

$$\mu = \omega - y_k, \tag{45}$$

$$D = \mu_{\lambda,k}^2 \Delta \Lambda(t;\theta)^2 + \sigma_{\varepsilon}^2, \qquad (46)$$

$$E = \mu_{\lambda,k} \Delta \Lambda(t;\theta) \sigma_{\varepsilon}^{2} + \mu \sigma_{\lambda,k}^{2} \Delta \Lambda(t;\theta)^{2}, \qquad (47)$$

$$G = \mu_{\lambda,k}^2 \Delta \Lambda(t;\theta)^2 \sigma_{\varepsilon}^2.$$
(48)

## 5. Experiment study

#### 5.1 Simulation experiments

First, we use a numerical example to show the effective-

ness of Theorem 1. Let  $\Lambda(t;\theta) = t^b$ . The parameters in the degradation process are assumed as  $\lambda = 1$ ,  $\sigma_{\varepsilon}^2 = 0.04$  and b = 1.5. Let  $t_k = 1\ 000$  and  $\Delta t = 1$ . The corresponding degradation path is shown in Fig. 2.



Based on the simulated degradation data, the estimations of  $\lambda$  and  $\sigma_{\epsilon}^2$  with the change of t can be calculated respectively by (4) and (5) as shown in Fig. 3 and Fig. 4.



Fig. 8.

From Fig. 3 and Fig. 4, we can find that the estimations of  $\lambda$  and  $\sigma_s^2$  tend to the actual value as detection time t increases. In addition, the fluctuations of the estimations for  $\lambda$  and  $\sigma_{\varepsilon}^2$  tend to be stable as detection time t gets larger, which also proves that increasing the number of on-site monitored degradation data can improve the accuracy of parameters estimation for fixed parameters. The estimation of  $\theta$  with the change of t obtained through the Matlab function "FMINSEARCH" is shown in Fig. 5, which also shows that the estimation of b converges rapidly to the actual value as t increases. Although the nature of the fixed parameter  $\theta$  cannot be analyzed by the analytical expression of  $\hat{\theta}$  in theory, the simulation results indicate that the fixed parameter  $\theta$  can be calculated based on the field CM data of the evaluated equipment.



Fig. 5 Estimation of b with the change of t

Then, the parameters in the degradation process are assumed as  $\mu_{\lambda} = 0.8$ ,  $\sigma_{\lambda}^2 = 0.0225$ ,  $\sigma_{\varepsilon}^2 = 1.44$ , and b = 1.5. Let  $t_k = 50$  and N = 5, 10, and 20. The corresponding partial degradation path is shown in Fig. 6.



Based on simulated degradation data, the estimations of  $\mu_{\lambda}$  and  $\sigma_{\lambda}^2$  with the change of t can be calculated

1.4 1.3 1.2 1.1 Ę 1.0 0.9 0.8 0.7 10 20 30 50 40 0 Time/h : N=5: : N=10; : N=20; \* : Actual value.

respectively by (18) and (19) as shown in Fig. 7 and

Fig. 7 Estimation of  $\mu_{\lambda}$  with the change of N



From Fig. 7 and Fig. 8, we can find that the estimations of  $\mu_{\lambda}$  and  $\sigma_{\lambda}^2$  tend to the actual value as the number of equipment sample *N* increases. In addition, the fluctuations of the estimations for  $\mu_{\lambda}$  and  $\sigma_{\lambda}^2$  tend to be stable as the number of equipment sample *N* gets larger, which also proves that increasing the number *N* of sample can

improve the accuracy of parameters estimation for ran-

# dom coefficients.**5.2** Case study

In this subsection, we use the degradation data collected from the National Aeronautics and Space Administration (NASA) Ames Prognostics Center of Excellence to illustrate the effectiveness of the RUL prediction presented in this paper. The degradation data of lithium-ion batteries are shown in Fig. 9. The relaxation effect (RE) of battery capacity during the rest time [44] will affect the accuracy of RUL prediction. Thus, we use degradation data of lithium-ion batteries after eliminating the RE as shown in Fig. 10. The method of eliminating the RE can refer to Jin et al. [44] and Tang et al. [10].





The failure threshold of lithium-ion battery is defined as 70%—80% of its rated capacity. In this paper, the failure threshold is set as 70% capacity. Without loss of generality, let  $\Lambda(t;\theta) = t^b$ . In order to compare the method

with fusing failure time data presented in this paper with the traditional Bayesian method, we select No.5 lithiumion battery as the evaluated equipment and the degradation data of the other lithium-ion batteries are treated as historical degraded data of congeneric equipment to calculate the correct prior information of parameters in the traditional Bayesian method.

For simplicity, the method of RUL prediction based on the RCR model with fusing the failure time data is referred to  $M_0$ , the traditional Bayesian method based on the RCR model is referred to  $M_1$  and the traditional

Bayesian method based on the Wiener process is referred to  $M_2$  [2]. First, based on the degradation data of No.6, No.7, and No.18 lithium-ion batteries, the prior information of parameters based on the RCR model can be calculated as follows:  $\mu_{\lambda} = 0.0041$ ,  $\sigma_{\lambda}^2 = 1.85 \times 10^{-6}$ ,  $\sigma_{\varepsilon}^2 =$  $4.97 \times 10^{-4}$ , and b = 1.1565. And, the failure lifetimes of No.6, No.7, and No.18 batteries are 69.5, 110.3, and 51 respectively.

Then, the RUL distributions calculated by  $M_0$ ,  $M_1$  and  $M_2$  at some points are shown in Fig. 11 where it can be found that the RUL distributions calculated by three methods can cover the actual RUL of the battery. However, the RULs predicted by  $M_0$  are closer to the actual RUL and focused, which shows that the method proposed in this paper is more accurate.



In order to show the effectiveness of these methods more intuitively, we further calculate the mean squared errors (MSEs) and REs at some points as shown in Fig. 12 and Fig. 13.



Fig. 12 MSEs at some CM points by  $M_0$ ,  $M_1$ , and  $M_2$ 

The MSEs and RE at each observation point are calculated as follows:

$$MSE_{k} = \int_{0}^{+\infty} (l_{k} + t_{k} - T)^{2} f_{L_{k}|y_{1:k}} (l_{k}|y_{1:k}) dl_{k}, \qquad (49)$$

$$\operatorname{RE}_{k} = |l_{k} + t_{k} - T|.$$
(50)

The results show that the MSEs and REs of RUL predicted by our method are better than those predicted by the traditional Bayesian method at all CM points, which reflects the superiority of our method.



Fig. 13 REs at some CM points by  $M_0$ ,  $M_1$ , and  $M_2$ 

### 6. Conclusions

Improving the accuracy of RUL prediction is the core of RUL prediction. This paper proposes an RUL prediction method with fusing the failure time data to reduce the influence of imperfect prior information. First, some natures of parameters estimation based on the nonlinear RCR model are given. Second, based on the natures of parameters estimation, the fixed parameters and random coefficient are obtained. Then, an RUL prediction method with fusing failure time data is proposed. From above works, the main contributions can be summarized as follows:

(i) Based on the nonlinear RCR model, this paper gives the natures of parameters estimation. This gives the theoretical basis of the parameters estimation method with fusing failure time data.

(ii) Based on the natures of parameters estimation, we propose a parameter estimation method with fusing failure time data and field CM data for the degradation process based on the nonlinear RCR model with considering the random effects. This method utilizes the failure time data instead of the historical degradation data. Thus, it can reduce the impact of imperfect prior information.

(iii) TND is used to model the failure threshold in predicting the RUL to satisfy the limitation. The PDF of the RUL based on the nonlinear RCR model is derived.

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