TSINGHUA SCIENCE AND TECHNOLOGY
ISSN 1007-0214 10/13 pp659–667
DOI: 10.26599/TST.2019.9010053
Volume 25, Number 5, October 2020

Analysis of a Firm's Optimal Size Based on Accessible Market Capacity

Xiao Sun, Jun Qian, Yueting Chai*, and Yi Liu

Abstract: E-commerce has dramatically reduced the limitation of space and time on economic activities, resulting in individuals having access to a huge number of consumers. In this paper, we propose a company's optimal size decision model containing management costs as a means of investigating the evolution of the company size in e-commerce. Given that production decisions are made based on accessible market capacity, we explain how a company enters the market, and we draw an evolutionary path of the optimal company size. The results show that in the early expansion stage of accessible market capacity, a firm's optimal size keeps increasing; after reaching a peak, the change in a firm's optimal size depends on its cost management. When the accessible market capacity reaches a threshold, the firm will no longer be in the market, and may no longer exist. Finally, we construct a simulation framework based on complex adaptive systems to validate our proposed model. A simulation experiment confirms our model and reveals the dynamic co-evolution process of individual producers and firms.

Key words: accessible market capacity; management cost; firm size; evolution

1 Introduction

As a transaction mode driven by computer and internet technology, e-commerce breaks restrictions of space and time on economic activities. The emergence of e-commerce fundamentally changes business rules and significantly enhances productivity. The publicity and sharing of production tools enable ordinary people to access a large number of consumers. This type of access has spawned some new business models, such as trading platforms like Taobao and JD, mobile applications like Didi and Eleme, and social network applications like Weibo and WeChat. Additionally, the convenience of online payment and express services greatly reduces the cost of transactions, and it can be observed that the number of contracted laborers and freelancers is gradually rising and becoming a new trend. We want to explore the changes that will happen to companies when the market capacity that individuals can access is unprecedentedly enlarged due to e-commerce and infrastructure.

The origin and nature of a company is a subject that has been extensively studied by economists, like Adam Smith, Karl Marx, and Alfred Marshall. Among them, the originator of new institutional economics is Ronald Coase, whose theory has greatly influenced modern thinking. He believes that companies are an expression of market inefficiency. The reason why firms appear is that the cost of some transactions carried out within firms is lower than that in the market. The boundary of company size is reached when the cost of the internal organizational transactions equals the cost of market organizational transactions[1]. Coase's theory organizes, defines, and deduces concepts. Currently, the formalization of his main ideas needs to be examined further. This paper uses the optimal decision model for

• Xiao Sun, Jun Qian, Yueting Chai, and Yi Liu are with the National Engineering Laboratory for E-Commerce Technologies, Tsinghua University, Beijing 100084, China. Yueting Chai and Yi Liu are also with Beijing National Research Center for Information Science and Technology, Beijing 100084, China. E-mail: sunxiao@mail.tsinghua.edu.cn; qianj18@mails.tsinghua.edu.cn; chaiyt@tsinghua.edu.cn; yiliu@mail.tsinghua.edu.cn.
∗ To whom correspondence should be addressed.
Manuscript received: 2019-07-15; revised: 2019-08-22; accepted: 2019-09-09

producers to model the business decision of business owners optimal business size. We parameterize a firm's internal management cost and formalize some of Coase's main ideas on the nature of companies.

Research on company size mainly focuses on two aspects. One is the study of the general rules of company size, such as the internal causes and mechanisms[2, 3] of the differences in the size of firms in various industries; the influence of the founders[4] on the scale of the companies, and the evolutionary characteristics of statistical distribution law on a company's size[5]. Although this kind of research considers the general rule of enterprise size (as does this article), it also focuses on the inside of the enterprise or industry. However, the influence of the social environment on the scale of the enterprise has not been explored. The other aspect analyzes the correlation and causality between the firm's size and other business management indicators. The relationship between the company's size and other factors, such as business life[6], the growth rate of firm revenue[7], firm social performance and financial performance[8], firm innovation and research and development investment[9–12], firm profitability and market[13, 14], and industrial agglomeration[15], is explored. This kind of research uses empirical methods to study the influencing factors on the company size from the micro-perspective of the enterprise. However, there are still some shortcomings in how the size of enterprises is affected by external macro-conditions, especially from the perspective of company development. Consequently, we study company law and explore how company size evolves when there is a change in external macro-conditions.

In this paper, we propose a decision model of the company's optimal size involving the firm's management cost coefficient. Accessible market capacity affects decision-making on production. Accordingly, we delve into how the firm as a form of production organization evolves from scratch to extinction, with the expansion of accessible market capacity from local consumers to all consumers. We constructed a simulation framework of collaborative evolution with both individual producers and firms that validated the proposed optimal business decision model. Simulation results are consistent with the model analysis results and reveal the dynamic characteristics of the collaborative evolution of an individual producer and a company.

2 Model

The production decision-making model proposed in this paper is based on the "producer-consumer" environment setting. The world is a fixed two-dimensional plane, on which producers and consumers are randomly distributed. The numbers of producers and consumers are equal, and their locations are matched one-to-one. In this environment, we assume that consumers have the same preference and demand for different commodities. The only input by the producers in production activities is their labor force; that is, producers have their labor time endowment. In a production cycle, the labor time endowment of a producer is fixed and cannot be retained for the next production cycle. Notably, the demand of one consumer for each commodity is exactly equal to the output of individual producers, who evenly distribute their labor time endowment in all kinds of commodities. In the initial conditions, with external environment limitations, individual producers can only contact one local consumer who is provided with several necessary commodities. In this circumstance, the optimal decision of different producers is to distribute their labor force evenly on all products, so that the output can precisely meet the needs of the corresponding local consumer. Therefore, transactions between individual producers and consumers are concluded, and the market is cleared.

With the development of technology and the improvement of external market conditions, individual producers can access more consumers. So, different producers can choose whether and how to adjust their production mode to obtain stronger market competitiveness. Specifically, individual producers can choose to concentrate on fewer commodity categories, achieve a higher input-output ratio through specialized economic effects, and then trade with more consumers. Moreover, they can also cooperate with several other individual producers within the scope of accessible transactions to form a team. This mode of joint production can improve overall competitiveness through cooperation and the internal division of labor. The former is the mode of specialized individual producers in the market, and the latter is the company as the mode of production.

This paper focuses on how the company moves from the beginning to evolution with the evolvement of external conditions. Specifically, this research considers the changes that will take place in the firms' optimal size in the environment of e-commerce, when personal accessible market capacity has had unforeseen growth.

2.1 Basic form of model

For a market with n kinds of necessities, the company owner decides the size of the firm in an attempt to maximize profits. According to Coase's discussion on the nature of firms, companies use the internal division of labor to replace the inefficient market division of labor. This paper regards the company as an organization where the owner employs people to carry out production. The owner is responsible for production decision-making, including the number of employees and the mode of operation. In contrast, employees are subordinate to the management of the owner and specialize in one specific production activity. Another hypothesis in this paper is that the internal management behavior of a firm incurs costs: a single employee cannot put all his labor time endowment into production but always loses part of the time in communication and coordination with the company owner.

The primary decision-making model of the business scale is abstracted as Eq. (1),

$$\begin{aligned}
\max\quad & \sum_{i=1}^{n} s_i \times \text{price}(p_i),\\
\text{s.t.}\quad & s_i < N,\ \sum_{i=1}^{n} l_i < n,\\
& s_i \leqslant \frac{p_i}{p_{i0}},\ \sum_{i=1}^{n} l_i < N,\\
& p_i = (l_i')^b,\ b>1,\ l_i \in \{0,1\},\ p_{i0} = n^{-b},\\
& \eta = f\left(\sum_{i=1}^{n} l_i\right), l_i' = \eta l_i, s_i \in \mathbf{N}^+, \eta \in [0,1] \quad (1)
\end{aligned}$$

where n is the number of necessities, s_i is the sales of commodity i, and $\text{price}(p_i)$ is its market pricing. The objective function of the owner of the company's decision-making is to maximize the firm's profit $\sum_{i=1}^{n} s_i \times \text{price}(p_i)$. $s_i < N$ is a market constraint, N is the accessible market capacity of individual producers under the current level of technology. This market constraint means that the total number of commodities sold by a firm cannot be greater than the total number of consumers it can reach. l_i is a 0−1 value, it equals 0 when the company does not invest employees in commodity i and equals 1 when firm owners decide to allow one employee to produce a commodity i. $\sum_{i=1}^{n} l_i \leqslant n$ is the restriction of firm scale. It means firms invest at most one unit of labor in each direction of production. $\sum_{i=1}^{n} l_i \leqslant N$ is another restriction of firm scale, which means a firm can only hire the persons within the scope of current accessible market. $s_i \leqslant p_i / p_{i0}$ is the capacity constraint of a firm, which shows that the sales volume of commodity i should not exceed the upper limit of production capacity. p_i is the output of commodity i in the firm and p_{i0} is its minimum demand by a single consumer. $p_i = (l_i')^b$ is the individual producer's production function of commodity i. The parameter of b is the specialization economic coefficient. The equation $b>1$ implies that the production function is a strictly convex function, and production has specialized economic effects. The economic effect of specialization describes a phenomenon that the output value added by an individual's input in production increases. "Practice makes perfect" is a vivid description of the economic effect of specialization. η is the discount coefficient of firm's working time, which will be discussed below, and function f describes the relationship between firm size and η.

In the initial state, there are no firms in the market, and each producer provides n essential commodities to a local consumer. As the assumption mentioned in the first paragraph of this section, the demand for n kinds of necessities by one consumer is identical and could be just met if he distributes his working time on n production directions. Therefore, the consumer's demand for each commodity is $n-b$, and we have $p_{i0} = n^{-b}$, $l_i' = \eta l_i$, $\eta \in [0,1]$, which reveals the characteristics of the company's internal management cost. The expression $\eta \in [0,1]$ signifies that employees should bear certain management costs when they engage in production activities; it is reflected in the discount of working time for employees. So η is the coefficient of firm management cost, and it is a function of the total number of employees.

It is necessary to emphasize here that our proposed model concentrates on the function of a firm as a production unit: say human capital is seen as a form of input from a macro perspective. It ignores its other characteristics, including external effects, social responsibility, supply chain, management mode, and so on. In our point of view, production is the proximate driving force of a company's evolution.

2.2 Pricing strategy

In the initial state, the market capacity is up to 1 and no firm exists. Individual producers provide n kinds of commodities to a local consumer. Assuming that the price of each unit of a commodity is c, the revenue of a single producer $n \times c$ is the survival standard for an individual producer. A single producer is chosen to become a firm owner and employs $\sum_{i=1}^{n} l_i$ employees to produce, needs to make at least a profit $\left(1+\sum_{i=1}^{n} l_i\right)\times n\times c$ in each production cycle to survive. Without considering the cost of internal management (the owner does not know about this cost before initiating the company), the total output of this firm is $\sum_{i=1}^{n} l_i^b$, which can sell at most $\sum_{i=1}^{n} (nl_i)^b$. For the company to survive, the lowest price per unit of commodities, i.e., the base value of pricing, is shown in the following:

$$\text{basePrice}=\frac{\left(1+\sum\limits_{i=1}^{n} l_i\right)\times n\times c}{\sum\limits_{i=1}^{n}\frac{l_i^b}{n^{-b}}}=c\times n^{1-b}\times\left(\frac{1}{\sum\limits_{i=1}^{n} l_i^b}+1\right) \tag{2}$$

As a firm, the minimum value of $\sum_{i=1}^{n} l_i$ is 1, and the maximum value is n. So the basePrice needs to satisfy

$$c\times n^{2-b}\times(n+1)\leqslant \text{basePrice}\leqslant c\times 2n^{1-b} \tag{3}$$

For a company to evolve from an "individual producer-consumer" market structure, it must be more competitive than individual producers. Consequently, the basic value of firm pricing needs to be lower than other individual producers. From Eq. (3), Eq. (4) can be obtained,

$$\text{basicPrice}\begin{cases}\leqslant c, & n^{1-b}<0.5;\\ \geqslant c, & n^{2-b}(n+1)\geqslant 1\end{cases} \tag{4}$$

When $n^{2-b}(n+1)\geqslant 1$, the basicPrice is bigger than c, so the firm as a production organization does not have a competitive advantage, and there will never be a company in such a market. Whereas, when $n^{1-b}<0.5$, the basicPrice is smaller than c. Then, compared with individual producers, the firm has a competitive advantage, and when external conditions are appropriate, it will evolve endogenously. The emergence of a firm would reduce commodity prices in the market, resulting in an improvement in the level of consumer welfare. Considering the profitability and competitiveness of firms, this part of the increased welfare, which can neither be fully occupied by consumers nor by companies, should be distributed among consumers and firms proportionately. The proportion of this allocation is defined as the allocation coefficient ξ ($\xi\in(0,1)$), which is used to indicate the proportion of the firm in ($c-$basePrice). Therefore, when $n^{1-b}<0.5$, firms' pricing of commodities is lower than that of individual producers, this firm may become a form of production organization with competitive advantages. The firm's pricing of commodity i is expressed in the following:

$$\begin{aligned}&\text{price}(p_i)=\text{basePrice}+\xi\times(\text{initialPrice}-\text{basePrice})=\\&c\times n^{1-b}\times\left(\frac{1}{\sum\limits_{i=1}^{n} l_i^b}+1\right)+\\&\xi\times\left[c-c\times n^{1-b}\times\left(\frac{1}{\sum\limits_{i=1}^{n} l_i^b}+1\right)\right]=\\&\left(\xi+n^{1-b}-\xi n^{1-b}\right)\times c+\\&(1-\xi)\times c\times n^{1-b}\times\frac{1}{\sum\limits_{i=1}^{n} l_i^b}\end{aligned} \tag{5}$$

2.3 Model solution

Optimizing Eq. (1) is a discrete non-convex optimization problem, which is difficult to solve analytically. We deconstructed and simplified it to investigate the nature of its solution. Based on Eqs. (1) and (5), we have price $(p_i)>0$ and $s_i<0$. Therefore, to maximize the objective function, s_i should be maximized in the feasible region, and then we can relax the positive integer constraints on s_i. When $s_i=\min\left(N, l_i^b n^b\eta^b\right)$, the optimal solution can be obtained. Let $A=\left(1+\xi-\xi n^{1-b}\right)\times c$ and $B=\xi\times c\times n^{1-b}$, the model is simplified in the following:

$$\begin{aligned}\max\quad&\frac{1}{1+\sum\limits_{i=1}^{n} l_i}\times\sum_{i=1}^{n}\min(N,l_i^b n^b\eta^b)\times\\&\left[A+B\times\frac{1}{\sum_{i=1}^{n} l_i^b}\right],\\ \text{s.t.}\quad&\sum_{i=1}^{n} l_i\leqslant n,\ \sum_{i=1}^{n} l_i\leqslant N,\\&A=(1+\xi-\xi n^{1-b})\times c>0,\\&B=\xi n^{1-b}c>0,\\&l_i\in\{0,1\},\eta=f\left(\sum_{i=1}^{n} l_i\right)\end{aligned} \tag{6}$$

It is supposed that the firm's management cost coefficient $\eta=f\left(\sum_{i=1}^{n} l_i\right)$ is a decreasing function of $\sum_{i=1}^{n} l_i$. Thus, with the increase in the company's size, the management cost for each person increases. If $l_i\neq 0$, we have $\max\left(l_i^b n^b\eta^b\right)=n^b f(1)^b$, $\min(l_i^b n^b\eta^b)=n^b f(n)^b$. Considering that the accessible market capacity N increases from 1 to M, where M is the total population and $M\gg n^b f(1)^b$, the optimal solution of the model varies in the following three cases.

(1) When $N > n^b f(1)^b$, the model is transformed into the following form:

$$\begin{aligned} \max \quad & n^b \eta^b \times \left(\frac{A \sum_{i=1}^{n} l_i + B}{1 + \sum_{i=1}^{n} l_i} \right), \\ \text{s.t.} \quad & \sum_{i=1}^{n} l_i \leqslant n, \\ & A = (1 + \xi - \xi n^{1-b}) \times c > 0, \\ & B = \xi n^{1-b} c > 0, \\ & l_i \in \{0, 1\}, \eta = f\left(\sum_{i=1}^{n} l_i \right) \end{aligned} \tag{7}$$

The optimal solution is the value of $\sum_{i=1}^{n} l_i$ at the maximum value of sequence $f(n)^b(An + B)/(n + 1)$, $f(n-1)^b(An+B-A)/n$, $f(n-2)^b(An+B-2A)/(n-1), \ldots, f(1)^b(A+B)/2$. $f\left(\sum_{i=1}^{n} l_i\right)$ is an increasing function of $\sum_{i=1}^{n} l_i$ and $\left(A\sum_{i=1}^{n} l_i + B\right) \Big/ \left(1+\sum_{i=1}^{n} l_i\right)$ is a decreasing function of $\sum_{i=1}^{n} l_i$, so the optimal solution depends on the comprehensive effect of these two functions. Specifically, if $f\left(\sum_{i=1}^{n} l_i\right)$ decreases slightly with the increase of $\sum_{i=1}^{n} l_i$ and the decreasing extent of $f\left(\sum_{i=1}^{n}\right) l_i$ can be ignored compared with the increase of $A\sum_{i=1}^{n} l_i - B$, then when $N > n^b f(1)^b$, the optimal operational size of the firm is n. This circumstance implies that the optimal decision of the owner is to employ one employee for each production direction. If $f\left(\sum_{i=1}^{n} l_i\right)$ decreases greatly with the increase of $\sum_{i=1}^{n} l_i$ and the decreasing extent of $f\left(\sum_{i=1}^{n}\right) l_i$ is much larger than the effect of $\left(A\sum_{i=1}^{n} l_i + B\right) \Big/ \left(1 + \sum_{i=1}^{n} l_i\right)$, then when $N > n^b f(1)^b$, the company's optimal size is 1, the optimal decision of a firm owner is to employ only one employee for only one production direction. Otherwise, the optimal decision of the entrepreneur depends on the trade-off between $f\left(\sum_{i=1}^{n} l_i\right)$ and $\left(A\sum_{i=1}^{n} l_i + B\right) \Big/ \left(1 + \sum_{i=1}^{n} l_i\right)$, and the optimal operation scale of the firm is an intermediate value between 1 and n.

(2) When $1 \leqslant N \leqslant n^b f(n)$, the original optimal decision model can be simplified to the following:

$$\begin{aligned} \max \quad & N \times \left(\frac{A \sum_{I=1}^{n} l_i + B}{1 + \sum_{I=1}^{n} l_i} \right), \\ \text{s.t.} \quad & \sum_{i=1}^{n} l_i \leqslant N, \ \sum_{i=1}^{n} l_i \leqslant n, \\ & A = \left(1 + \xi - \xi n^{1-b}\right) \times c > 0, \\ & B = \xi n^{1-b} c > 0, \\ & l_i \in \{0, 1\} \end{aligned} \tag{8}$$

Optimal solution is $\sum_{i=1}^{n} l_i = \min(N, n)$ because $A > 0$, which corresponds to two situations: when $0 < N \leqslant n$, with the increase of N, the firm grows from start and increases continuously until it reaches the maximum scale of n; when $n \leqslant N \leqslant n^b f(n)$, firm's optimal scale remains n.

(3) When $n^b f(n)^b < N \leqslant n^b f(1)^b$, the optimal company size is discussed in three cases. (a) If the firm management cost coefficient $f\left(\sum_{i=1}^{n} l_i\right)$ decreases slightly with the increase of operation scale $\sum_{i=1}^{n} l_i$, the optimal scale remains n with the increase of N, the owner of the firm employs a labor force for each direction of production. (b) If the firm management cost coefficient $f\left(\sum_{i=1}^{n} l_i\right)$ decreases significantly with the expansion of the firm operation scale $\sum_{i=1}^{n} l_i$, the optimal scale decreases continuously with the increase of N and finally decreases to 1, the owner of the firm employs a labor force in one specific production direction. (c) If the firm management cost coefficient $f\left(\sum_{i=1}^{n} l_i\right)$ has neither a too large nor a too small decrease with the expansion of the firm's size, then the optimal scale decreases continuously with the increase of N and remains unchanged after reaching an optimal median value.

2.4 Cooperative evolution of individual producers and firms

According to the analysis results in Section 2.3, as an organizational form for production, a company can be formed spontaneously in the market due to the improvement in technology. However, can the company die? To answer this question, we need to compare the competitiveness of individual producers and firms. As described in Section 2.1, in the primary market, an individual producer provides commodities to a local consumer, and no firm exists. With the progress of technology, accessible market capacity has been continuously expanded. When it reaches the critical threshold, firms can achieve a higher input-output ratio through the internal division of labor and then sell more units of commodities. They become more competitive than individual producers; it is the beginning of a firm's evolution.

After that, as described in Section 2.3, when accessible market capacity N is greater than $n^b f(1)^b$, the firm evolves to its final form, and maintains its optimal operating scale. However, there is always a loss of management costs within a firm, even though its owner does not directly engage in production activities, but shares the labor income of employees. Therefore, although employees in the firm are fully specialized in producing a particular commodity, compared with individual producers, they always have losses of labor endowment and are unable to form per capita profits. Thus, we can imagine that when accessible market capacity expands to such a degree that a fully specialized individual producer can sell all the products they produce, companies will lose their competitive advantage. Correspondingly, firms will disappear, and many fully specialized individual producers will occupy the market.

Apart from the two circumstances mentioned earlier, individual producers and firms evolve together when $1 \leqslant N \leqslant n^b f(1)^b$. The mode of production that is more competitive depends on accessible market capacity. The evolutionary path of firms is summarized in Table 1.

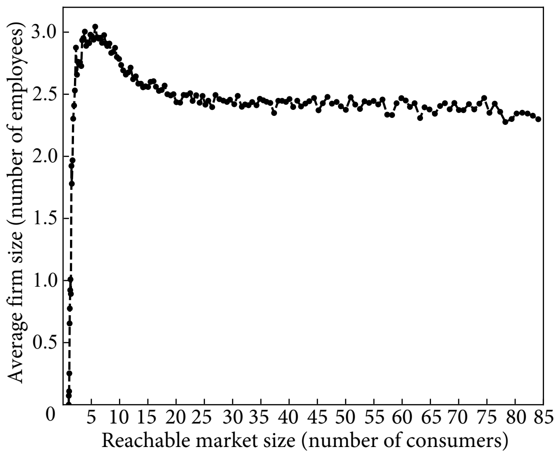
3 Simulation Results

To validate the results of the decision-making model of a company's optimal size, based on the method of complex adaptive systems and multi-agent-based modeling, a simulation framework of "individual producer-firm" collaborative evolution is constructed. Parameters of the simulation experiment are as follows. A total of 400 individual producers and consumers are randomly generated on the two-dimensional plane of 5×5. Consumers in the market need three kinds of essential commodities. The specialized economic effect parameters of these three commodities are all three. That is to say, the production function of individual producers for each commodity is $p_i = l_i^3, l_i \in \{0, 1\}$. Accessible market capacity ranges from 1 to 85. Two sets of firm management cost coefficients [0.95, 0.85, 0.7] and [0.95, 0.9, 0.85] are set. They correspond to the situation that $f(n)$ decreases rapidly with the increase of n and decreases linearly with the increase of n. Simulation results are taken as an average of 10 times.

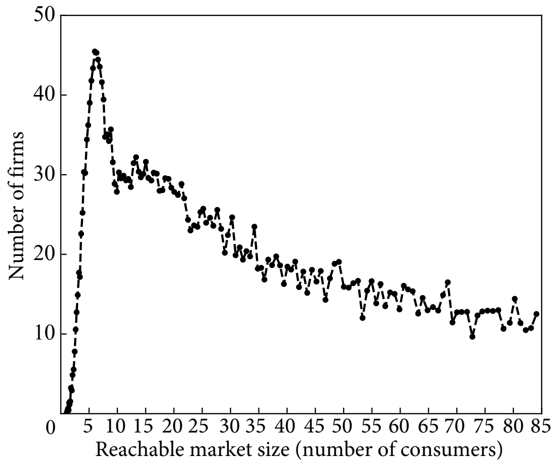
Figure 1 is the simulation results of $\eta_1 = [\eta_{11}, \eta_{12}, \eta_{13}] = [0.95, 0.85, 0.7]$ and Fig. 2 is that under $\eta_2 = [\eta_{21}, \eta_{22}, \eta_{23}] = [0.95, 0.9, 0.85]$. Compared Figs. 1a and 2a, with the expansion of the accessible market capacity, the average firm size increases rapidly at the beginning, which corresponds to the analysis results in Table 1 when $1 < N \leqslant n^b f(1)^b$. At this stage, with the increase of N, the optimal scale has increased from 1 to n. After the average firm size reaches its peak, with a further expansion of accessible market capacity, the average firm size in Figs. 1a and 2a show different trends. In Fig. 1a, with the expansion of accessible market capacity, the average firm size has a sharp reduction and finally stabilizes at about 2.5. This finding corresponds to the analysis in Table 1 where the attenuation of $f\left(\sum_{i=1}^n l_i\right)$ with the increase of $\sum_{i=1}^n l_i$ is moderate when $n^b f(1)^b < N < n^b$. The optimal company size gradually decreases from n and stabilizes at an intermediate value between 1 and n at this phase. In Fig. 2a, when $\eta_2 = [0.95, 0.9, 0.85]$, with the expansion of accessible market capacity, there is no obvious decline in the average firm size. It oscillates between 2.5 and 3.0 for a long time. This finding corresponds to the case in Table 1, where the attenuation

Table 1 Evolutionary path of company size with a change in accessible market capacity.

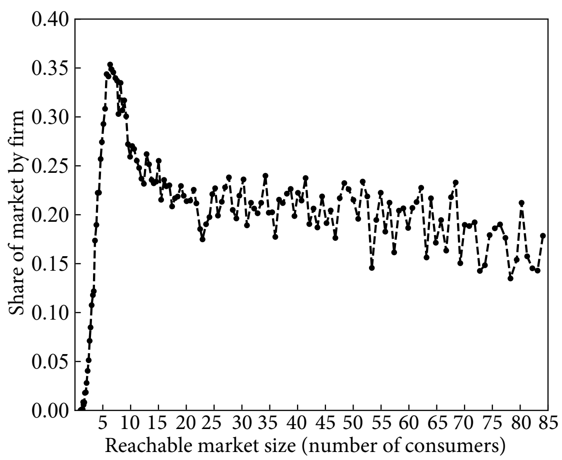
<table>
<tr><th>Accessible market capacity</th><th colspan="3">Phase of firm evolution</th></tr>
<tr><td>$N = 1$</td><td colspan="3">No firm. Individual producers distribute their labor time endowments evenly and provide all kinds of commodities to a local consumer.</td></tr>
<tr><td>$1 < N \leqslant n^b f(1)^b$</td><td colspan="3">Firms emerge. Optimal scale keeps rising, finally reaches n.</td></tr>
<tr><td rowspan="3">$n^b f(1)^b < N < n^b$</td><td rowspan="3">Attenuation of $f\left(\sum_{i=1}^n l_i\right)$ with the increase of $\sum_{i=1}^n l_i$</td><td>Small</td><td>Optimal scale stays to n.</td></tr>
<tr><td>Normal</td><td>Optimal scale decreases, finally stabilizes at a value between 1and n.</td></tr>
<tr><td>Large</td><td>Optimal scale decreases, finally reaches 1.</td></tr>
<tr><td>$n^b \leqslant N \leqslant M$</td><td colspan="3">Firms die. Individual producers provide consumers with a commodity.</td></tr>
</table>



(a)

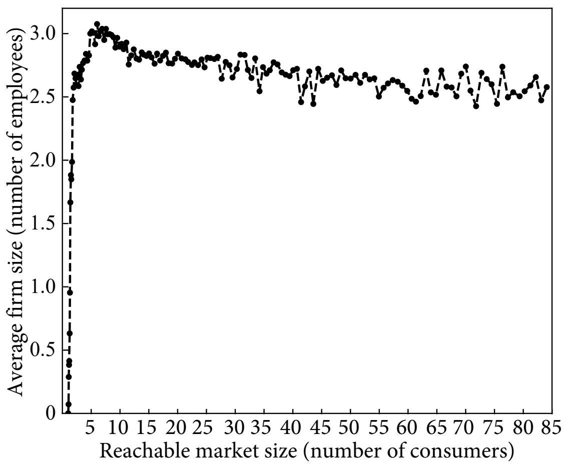


(b)

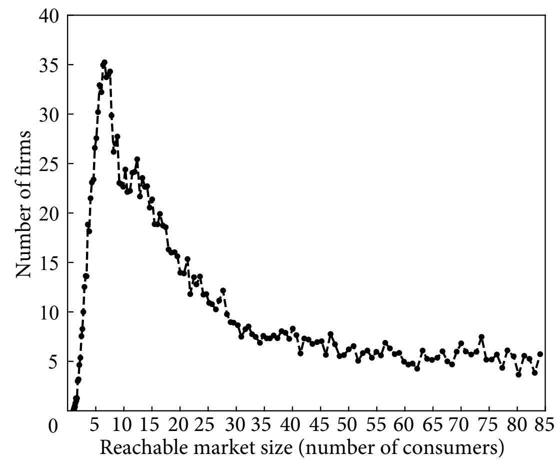


(c)

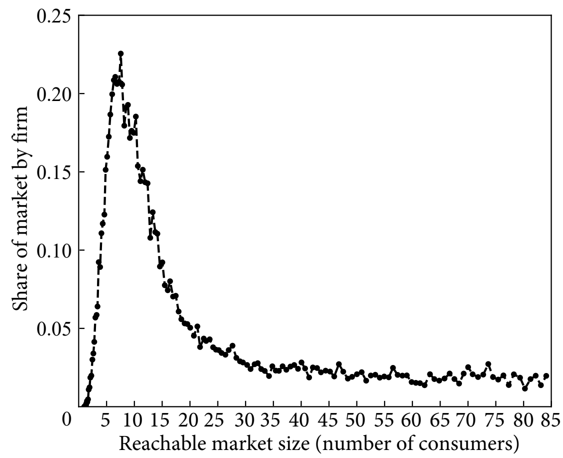
Fig. 1 Firm size evolution when management cost coefficient decreases exponentially with the increase of firm size. (a) Average size of firms varies with accessible market capacity. (b) Number of surviving firms varies with accessible market capacity. (c) Proportion of a firm's sales varies with accessible market capacity.



(a)



(b)



(c)

Fig. 2 Firm size evolution when management cost coefficient decreases linearly with the increase of firm size. (a) Average size of firms varies with accessible market capacity. (b) Number of surviving firms varies with accessible market capacity. (c) Proportion of a firm's sales varies with accessible market capacity.

of $f\left(\sum_{i=1}^{n} l_i\right)$ with the increase of $\sum_{i=1}^{n} l_i$ is small when $n^b f(1)^b < N < n^b$. At this time, the optimal operation scale of the firm remains n.

Simulation results also reveal the characteristics of individual producer-firm collaborative evolution. As can be seen in Figs. 1b and 1c and Figs. 2b and 2c, with the expansion of accessible market capacity, the number of firms in the market increases first. It then decreases, so does the proportion of total social demand satisfied by the firms. This result shows that in the early stage of expansion of accessible market capacity, the production system of firms has competitive advantages over individual producers. However, when the accessible market capacity exceeds a certain threshold, the competitive advantages of firms reduce, and many specialized individual producers gradually occupy the market.

Also, the downward trend of the curves in Figs. 1b and 1c is slower than that in Figs. 2b and 2c. Figure 1c shows that even at the end of the evolution, firms still account for 15% to 20% of total social consumption, while in Fig. 2c, this figure is less than 5%. We can deduce that firms cannot survive in the market when accessible market capacity exceeds the critical threshold. This phenomenon can be attributed to the difference in the companies' management cost coefficients. The setting of management cost coefficients in Fig. 1 allows firms to improve their market competitiveness by reducing the company's size. Therefore, firms in Fig. 1 are more likely to survive competition from individual producers. The interval in which a firm can survive is minimal, so a company would soon disappear due to the competitive pressure of individual producers.

4 Discussion

In this paper, we propose a decision-making model of company size by including management costs. By considering the influence of accessible market capacity on production decision-making, we study the process of a company's evolution from start to extinction. The results of the model analysis show that after the expansion of accessible market capacity to a specific value, the firm as a production organization has a competitive advantage over individual producers because of the complete internal division of labor and begins to appear in the market. With the continuous expansion of accessible market capacity, the optimal firm size has gradually increased from 1 to n. The largest firm size is n. Then, because of the difference in the management cost coefficient, a company's optimal size has three different paths with the increase of accessible market capacity. When the management cost coefficient decreases very little with the expansion of the firm scale, the optimal scale keeps n; when the management cost coefficient decreases sharply with the increase of the firm size, the optimal size decreases gradually to 1. When the management cost coefficient decreases moderately with the increase of business size, the optimal business scale gradually decreases, and finally stabilizes in the middle of 1 and n. After that, when the accessible market capacity expands further until it can carry the function of all fully specialized individual producers, individual producers have a competitive advantage over firms, and the company gradually disappears.

Further, a simulation framework based on complex adaptive systems is constructed to verify the analysis results. We set up two sets of simulation parameters to validate the proposed model. The simulation results are consistent with the model analysis results, and reveal the dynamic process of collaborative evolution among individual producers and firms. Given the discussion on accessible market capacity in this paper, we can refine the critical effects of accessible market capacity on geography and time.

Notwithstanding the conclusion that the optimal firm size decreases with the improvement of transaction efficiency, business observations seem to be different. Giant and transnational companies still dominate the economic system. Two reasons may help to understand this perception. First, small size refers to the number of people hired by one firm; it does not mean that a small company cannot have "large" sales volumes, revenue, or economic influence. Second, the evolutionary path of companies is illustrated as an evolutionary stable state. Still, the dynamically changing process is absent in this approach, which may lead to the gap between theory and reality. Moreover, the proposed model and theoretical analysis in this paper still need to be further improved and investigated, more roles and their characteristics should be considered. An empirical study may be planned, in which the proposed model could be validated and adjusted.

Acknowledgment

This work was supported by the National Key Research & Development Program of China (No. 2017YFB1400104).

References

[1] R. H. Coase, The nature of the firm, *Economica*, vol. 4, no. 16, pp. 386–405, 1937.

[2] D. Besanko and U. Doraszelski, Capacity dynamics and endogenous asymmetries in firm size, *Computing in Economics & Finance*, vol. 35, no. 1, pp. 23–49, 2002.

[3] P. Pagano and F. Schivardi, Firm size distribution and EPL in Italy, *The Scandinavian Journal of Economics*, vol. 105, no. 2, pp. 255–274, 2003.

[4] P. W. Roberts, S. Klepper, and S. Hayward, Founder backgrounds and the evolution of firm size, *Industrial and Corporate Change*, vol. 20, no. 6, pp. 1515–1538, 2011.

[5] L. M. B. Cabral and J. Mata, On the evolution of the firm size distribution: Facts and theory, *American Economic Review*, vol. 93, no. 4, pp. 1075–1090, 2003.

[6] T. C. Fort, J. Haltiwanger, R. S. Jarmin, and J. Miranda, How firms respond to business cycles: The role of firm age and firm size, *IMF Economic Review*, vol. 61, no. 3, pp. 520–559, 2013.

[7] T. Yasuda, Firm growth, size, age and behavior in Japanese manufacturing, *Small Business Economics*, vol. 24, no. 1, pp. 1–15, 2005.

[8] M. Orlitzky, Does firm size comfound the relationship between corporate social performance and firm financial performance? *Journal of Business Ethics*, vol. 33, no. 2, pp. 167–180, 2001.

[9] Z. Zhou and K. Luo, Firm size and innovation: Evidence from China's province-level data, (in Chinese), *China Economic Quarterly*, vol. 4, no. 3, pp. 623–638, 2005.

[10] F. Malerba, Innovation and the evolution of industries, *Journal of Evolutionary Economics*, vol. 16, no. 1, pp. 3–23, 2006.

[11] J. A. DiMasi, Pharmaceutical R&D performance by firm size: Approval success rates and economic returns, *American Journal of Therapeutics*, vol. 21, no. 1, pp. 26–34, 2014.

[12] J. Bai, Firm size, market structure and innovation efficiency: Evidence from high-tech industry, (in Chinese), *Economic Issues in China*, no. 5, pp. 65–78, 2011.

[13] R. Israel and T. J. Moskowitz, The role of shorting, firm size, and time on market anomalies, *Journal of Financial Economics*, vol. 108, no. 2, pp. 275–301, 2013.

[14] N. E. Coviello, R. J. Brodie, and H. J. Munro, An investigation of marketing practice by firm size, *Journal of Business Venturing*, vol. 15, no. 5, pp. 523–545, 2000.

[15] Y. Lu, D. Li, Q. Fang, and X. Chen, Industrial agglomeration and firm size: Evidence from China, (in Chinese), *Management World*, no. 8, pp. 84–89, 2010.

Xiao Sun received the PhD and BEng degrees from Tsinghua University in 2016 and 2011, respectively. He is now an assistant researcher of National Engineering Laboratory for E-Commerce Technology (NELECT). His research interests include human behavior model and simulation approach, evolutionary game theory, and dynamics of collective intelligence.

Jun Qian received the BS degree from the University of Science and Technology Beijing in 2018. She is currently a PhD candidate in the Department of Automation at Tsinghua University. Her research interests include human behavior and dynamics of collective intelligence.

Yueting Chai received the PhD degree from Tsinghua University in 2004 and served as deputy chief engineer of National CIMS Engineering Research Center since 1996. He is now the director of National Engineering Laboratory for Ecommerce Technologies (NELECT) with a research focus on crowd science and technology.

Yi Liu received the MEng degree from Tsinghua University in 1991. She became an associate professor in the Department of Automation at Tsinghua University from 1994. She is now focusing on e-commerce and supply chain management.