Relay Selection Scheme for AF System with Partial CSI and Optimal Stopping Theory

Rui Zhu, Tao Li*, Jianxin Guo, and Yangchao Huang

Abstract: Relay selection for Relay Assisted (RA) networks is an economical and effective method to improve the spectrum efficiency. Relay selection performs especially well when the source node has accurate and timely Channel State Information (CSI). However, since perfect CSI knowledge is rarely available, research of relay selection with partial (statistical) CSI is of paramount importance. In this paper, relay selection for RA networks with statistical CSI is formulated as a Multiple-Decision (MD) problem. And, the cost of obtaining the CSI is also considered in the formulated problem. Two relay selection schemes, Maximal Selection Probability (MSP) and Maximal Spectrum Efficiency Expectation (MSEE), are proposed to solve the formulated MD problem under different optimal criteria assumptions based on the optimal stopping theory. The MSP scheme maximizes the probability that the Best Assisted Relay Candidate (BARC) can be selected, whereas the MSEE scheme provides the maximal expectation of the spectrum efficiency. Experimental results show that the proposed schemes effectively improve the spectrum efficiency, and the MSEE scheme is more suitable for stable communication cases. Meanwhile, the MSP scheme is more suitable for burst communication cases.

Key words: Relay-Assisted (RA) network; relay selection; optimal stopping

1 Introduction

With the rapid growth of information technology, wireless communication requires an exponential increase of spectrum efficiency. To meet this requirement, Relay Assisted (RA) communication with relay selection, a typical cooperative communication technique, has gained a lot of interest due to its ability to mitigate fading in wireless channels by introducing

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spatial diversity gain. To fully exploit the advantages of RA communication techniques, various relaying strategies and selection schemes have been proposed to achieve significant performance gains under various performance criteria and constraints[1–13].

Among the proposed relay strategies, Amplify-and-Forward (AF)[1] has been intensively studied because of its low complexity. In the decode-and-forward and compress-and-forward protocols, the received signal must be decoded and re-encoded before relay. Under the AF strategy, the assisted relay simply amplifies and forwards the received signal to the destination node under the relay power constraint without decoding and re-encoding. This makes the AF strategy easier and more convenient to employ in practice.

In regards to relay selection schemes, Bletsas et al.[2] and Jedrzejczak et al.[3] studied the spectrum efficient relay selection scheme based on the harmonic mean of Source-Relay (SR) and Relay-Destination (RD) channel gains, and various relay selection schemes

were proposed in Refs. [6–8] to optimize the outage of wireless powered RA networks. In order to study the coverage performance of the RA network, the corresponding relay selection scheme based on the outage probability criterion was design in Refs. [9–11]. Moreover, a relay selection scheme which can improve the bit error rate performance was considered[12, 13]. However, it is worthy of note that the achievements of the above mentioned work have been based on assumption of perfect Channel State Information (CSI), and it is known to be almost impossible to obtain the perfect CSI, due to the inevitability of noise and/or the time-varying nature of wireless channels.

Therefore, a more practical line of research looks at relay selection under the assumption of partial CSI[14–20]. Tabataba et al.[14] derived an approximate closed-form expression for the outage probability, and proposed the corresponding power allocation scheme of the source and the relay node for the AF-RA network under the assumption of a high Signal-to-Noise Ratio (SNR). Meanwhile, Seyfi et al.[17] and Amin et al.[18] presented relay selection schemes with the maximal effective SNR and equal power allocation. Wang and Chen[19] further analyzed the performance of relay selection schemes with limited feedback. Moreover, Koyuncu and Jafarkhani[20] proposed a relay selection scheme to maximize the capacity lower bound under the assumption of imperfect channel estimation. Nevertheless, compared to the perfect CSI case, the performance of relay selection with partial CSI is limited because of the challenging problem of channel estimation for the AF-RA network.

Fortunately, the development of the physical channel model[21–25] and optimal stopping theory[26–30] provides new methods to overcome this performance limitation. Unlike the perfect CSI, the statistical CSI can be obtained through long-term observation and the results of these studies show that the performance of an AF-RA network benefits from the use of the statistical CSI. A Multi-rate Cooperative Medium Access Control (MC-MAC) protocol, which selects the relay candidate and transmission rate based on the statistical CSI, was designed for networks with slow-changing channels[21]. Similarly, Zhan et al.[22] presented an AF network coded protocol that achieves excellent network performance, and Guo et al.[23] analyzed the ergodic capacity Upper Bound (UB) of an AF relay selection network with statistical CSI. Nevertheless, although the statistical CSI is beneficial

for the performance of RA networks, most of the existing research has not accounted for the cost of obtaining the statistical CSI. This oversight limits the application of the results.

In this paper, we formulate relay selection with statistical CSI as a Multistage Decision (MD) problem. Two relay selection schemes, Maximal Selection Probability (MSP) and Maximal Spectrum Efficiency Expectation (MSEE), are proposed to solve the formulated MD problem based on different optimal criteria. The MSP scheme maximizes the probability that the Best Assisted Relay Candidate (BARC) is selected, while the MSEE scheme maximizes the expectations of spectrum efficiency. Based on the optimal stopping theory, the optimal algorithms are designed for these two proposed relay selection schemes.

The reminder of this paper is organized as follows. In Section 2, the relay selection model and frame structure are presented. In Section 3, the MSP scheme and the optimal algorithm of the observation window size are proposed, followed by the MSEE scheme and the threshold optimal algorithm in Section 4. Section 5 shows the experimental results of the proposed schemes. Finally, conclusions are drawn in Section 6.

2 System and Signal Model

In this section, the AF-RA network model is proposed. The corresponding signal model and frame structure are also introduced.

2.1 System model

This paper studies a typical RA network with multiple users and Assisted Relay Candidates (ARCs) which is shown in Fig. 1. Each of the nodes in the network equips a single antenna, and the 2-hop AF protocol is taken
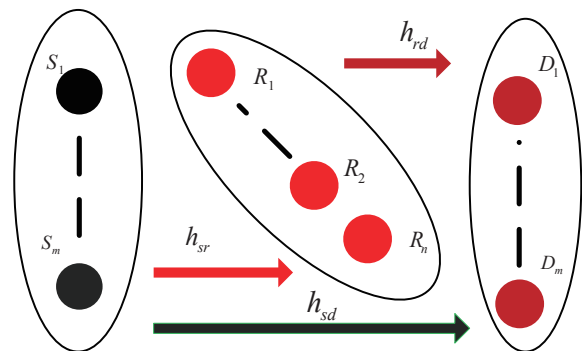

Fig. 1 Illustration of RA network with multiple users and assisted relay candidates.

by the relay node. For convenience, we assume any relay node can only cooperate with two users at once, and that any two users can adopt a single relay node to assist communication. Since there are multiple users in the RA network, we denote $P_{\text{free},i}$ as the available probability of the i-th ARC. Thus, a higher $P_{\text{free},i}$ equals more ARCs and/or fewer user nodes in the RA network, while a lower $P_{\text{free},i}$ equals fewer ARCs and/or more user nodes in the RA network.

Without loss of generality, the flat fading channel with Gaussian noise is adopted in this paper, and the spectrum bandwidth is normalized for the sake of analysis. Therefore, there is no difference between channel capacity and spectrum efficiency in this paper.

2.2 Frame structure

As shown in Fig. 2, the communication process is spliced by a series of time slots, with each slot including a Negotiation Stage (NS) and a Transmission Stage (TS). During the NS, the source node negotiates with the available ARCs in sequence to determine the proper assisted relay node. If a negotiating ARC is adopted, the source node will turn to the TS. The TS is also divided into two steps due to the adoption of the 2-hop AF relay protocol. In the first step, the adopted relay node receives the signal from the source node; in the second step, this signal is relayed to the destination node. If we define the period of the communication slot as T, the duration time of the TS can be written as $T - k\tau$, where τ is the negotiation cost and k indicates the number of rejected ARCs.

2.3 Signal model

From the definition of the 2-hop AF protocol, the received signal of the relay node is easily obtained as

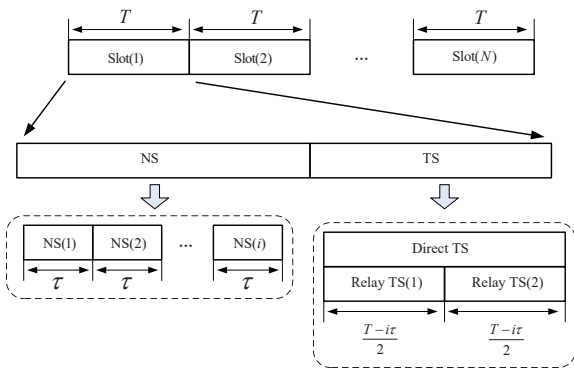$$y_{r_i} = h_{sr_i} \sqrt{P_s} x + n_{r_i} \quad (1)$$

Fig. 2 Illustration of the frame structure of the relay selection scheme.

where y_{r_i} represents the received signal of the i-th ARC. h_{sr_i} represents the gain of channel between the source node and the i-th ARC. P_s and x represent the transmit power and transmit signal of the source node, respectively, and n_{r_i} represents the Gaussian noise of the i-th ARC. Similarly, the signal in the second transmission step is

$$y_d = \beta h_{r_i d} y_{r_i} + n_d =$$
$$\beta h_{r_i d} h_{sr_i} \sqrt{P_s} x + \beta h_{r_i d} n_{r_i} + n_d \quad (2)$$

where y_d is the received signal of the destination node, $h_{r_i d}$ represents the gain of channel between the i-th ARC and the destination node, and n_d and β represent the Gaussian noise of the destination node and the magnification coefficient, respectively.

$$\beta = \sqrt{\frac{P_{r_i}}{|h_{sr_i}|^2 P_s + \sigma_{r_i}^2}} \quad (3)$$

where P_{r_i} is the relay power of the i-th ARC. If $\sigma_{r_i}^2$ and σ_d^2 are the variance of n_{r_i} and n_d, respectively, the variance of $\beta h_{r_i d} n_{r_i} + n_d$ can be denoted as $\beta^2 h_{r_i d}^2 \sigma_{r_i}^2 + \sigma_d^2$. Hence, when the maximal radio combining protocol is employed, the SNR of y_d can be formulated as

$$\gamma_{r_i} = \frac{P_s h_{sd}^2}{\delta_d^2} + \frac{P_s P_{r_i} h_{sr_i}^2 h_{r_i d}^2}{P_s h_{sr_i}^2 \delta_d^2 + P_{r_i} h_{r_i d}^2 \delta_{r_i}^2 + \delta_{r_i}^2 \delta_d^2} \quad (4)$$

Based on Eq. (4) and Fig. 2, the spectrum efficiency of the relay link with the i-th ARC can be represented as

$$C_{r,i} = \frac{T - i\tau}{2} \log_2(1 + \gamma_{r_i}), \quad i = 1, 2, \ldots, n \quad (5)$$

Similarly, the signal of the direct transmission mode is

$$y_d = h_{sd} \sqrt{P_s} x_s + n_d \quad (6)$$

and the spectrum efficiency of direct transmission in the i-th slot is

$$C_{d,i} = (T - i\tau) \log_2(1 + \gamma_d) =$$
$$(T - i\tau) \log_2 \left(1 + \frac{P_s h_{sd}^2}{\sigma_d^2}\right) \quad (7)$$

Therefore, the spectrum efficiency of AF-RA network by relay selection scheme can be formulated as

$$C_i = \max \left(C_{d,i}, C_{r,i} \left(\gamma_{r_i}\right)\right) \quad (8)$$

In order to achieve the best spectrum efficiency, the optimal relay selection scheme should guarantee that the BARC can be selected. So, the optimal problem of relay selection can be represented as

$$i^* = \arg\max \left\{\sum_i C_i I_i \mid i = 1, 2, \ldots, n\right\} \quad (9)$$

where I_i represents the indicator function, defined as

$$I_i = \begin{cases} 1, & i^* = i; \\ 0, & \text{else} \end{cases} \quad (10)$$

It is clear that the result of Eq. (9) provides the spectrum efficiency UB of the AF-RA network. If the perfect CSI is available, Eq. (9) can be solved by an existing optimal algorithm. However, as mentioned in the introduction above, it is almost impossible to solve Eq. (9) directly because in practice the perfect CSI is rarely available. We therefore propose two sub-optimal schemes in the following section.

3 MSP Scheme

Since the source node may fail to identify the BARC without perfect CSI, an alternative approach is to maximize the selection probability of the BARC. For the sake of convenience, we name it MSP scheme. The corresponding protocol and optimal algorithm of this scheme are given in this section.

3.1 Protocol of the MSP scheme

As mentioned above in the frame structure section, the communication slot is divided into an NS and a TS. For the MSP scheme, the NS is further split into a negotiation segment and a judgment segment. In the NS, the source node negotiates with the first k ARCs, and the maximal spectrum efficiency provided by the first k ARCs is taken as the threshold S. In the judgment segment, the source node negotiates with the other ARCs in sequence. If the spectrum efficiency provided by the negotiating ARC is larger than the threshold S, the negotiating ARC is adopted and the source node turns to the transmission stage, otherwise the negotiating ARC is rejected. The rejected ARCs will be reconsidered until the next communication slot. The protocol of the MSP scheme is shown in Fig. 3.

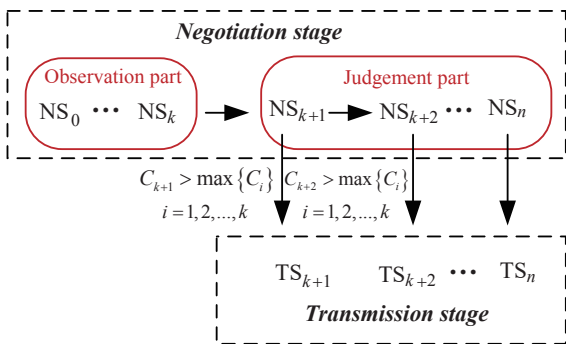Because of the specifics of the MSP protocol, the

Fig. 3   Illustration of the protocol of MSP scheme.

performance of the MSP scheme depends deeply on the value of the observation window size k. The next section describes an algorithm designed to optimize k based on the optimal stopping theory.

3.2 Optimal algorithm of MSP scheme

With the observation window size represented by k, the selection probability of BARC under the MSP scheme can be formulated as

$$\mathscr{P}(k) = \sum_{i=k+1}^{n} \mathscr{P}_{\max}(i,n) \mathscr{P}_{\text{sel}}(i | \mathscr{P}_{\max}(i,n)) \quad (11)$$

where $\mathscr{P}_{\max}(i,n)$ defines the probability that the i-th ARC is the BARC among n candidates, and $\mathscr{P}_{\text{sel}}(i | \mathscr{P}_{\max}(i,n))$ defines the conditional probability that the i-th ARC is selected when it is the BARC among n candidates. Hence, when the i-th ARC is the BARC among n candidates, the Cumulative Distribution Function (CDF) of the spectrum efficiency is formulated as

$$\mathscr{P}_j(C_i) = \mathscr{P}\left\{C_j < C_i\right\} = \mathscr{P}\left\{\gamma_j < 2^{\frac{2C_i}{T-i\tau}} - 1\right\} =$$

$$\int_{-\infty}^{2^{\frac{2C_j}{T-i\tau}} - 1} f(\gamma_i) \mathrm{d}\gamma_i, \ j = 1,\ldots,n, j \neq i \quad (12)$$

where $f(\gamma_i)$ is the Probability Density Function (PDF) of $\gamma_i, i = 1, 2, \ldots, n$. If we then denote the spectrum efficiency of the direct transmission mode as $C_{d,0}$, the probability $\mathscr{P}_{\max}(i,n)$ can be written as

$$\mathscr{P}_{\max}(i,n) = \mathscr{P}_{\text{others}} \mathscr{P}_{\text{free},i} \int_{2^{\frac{2C_{d,0}}{T-i\tau}} - 1}^{\infty} f(\gamma_i) \mathrm{d}\gamma_i \quad (13)$$

where

$$\mathscr{P}_{\text{others}} = \mathscr{P}_{\text{free},j} \prod_{\substack{j=1, \\ j \neq i}}^{n} \mathscr{P}_j(C_i) + \mathscr{P}_{\text{busy},j} \quad (14)$$

and

$$\mathscr{P}_{\text{free},i} + \mathscr{P}_{\text{busy},i} = 1, \quad i = 1, 2, \ldots, n \quad (15)$$

Similarly, if we take $\mathscr{P}_{\max}(0,n)$ to denote the probability that the direct transmission mode can provide the maximal spectrum efficiency, then it can be calculated as

$$\mathscr{P}_{\max}(0,n) = \prod_{j=1}^{n} \left(\mathscr{P}_{\text{free},j} \mathscr{P}_j(C_{d,0}) + \mathscr{P}_{\text{busy},j}\right) \quad (16)$$

Moreover, based on the protocol of the MSP scheme, when the i-th candidate is the BARC among n candidates, it will be selected if the BARC among previous $i-1$ candidates located in the observe window. Therefore, $\mathscr{P}_{\text{sel}}(i | \mathscr{P}_{\max}(i,n))$ can be calculated as

$$\mathscr{P}_{\text{sel}}(i | \mathscr{P}_{\max}(i,n)) = \sum_{j=1}^{k} \mathscr{P}_{\max}(j, i-1) \quad (17)$$

From Eqs. (11), (13), (16), and (17), we know that the probability equation depends on the PDF of $\mathscr{P}(k)$ and observation window size k. In general, γ_i emerges from the statistical CSI, so k is the optimal variable.

$$k^* = \arg \max \{\mathscr{P}(k) \mid k = 1, 2, \ldots, n - 1\} \quad (18)$$

It is hard to design a general algorithm to solve Eq. (18) for the various $f(\gamma)$ values, except via enumeration. Fortunately, enumeration is an acceptable method for the MSP scheme because there are few ARCs in the RA network. In particular, when γ_i is independent and identically distributed (iid) and

$$\tau = \mathscr{P}_{\text{busy},i} = 0, \quad i = 1, \ldots, n \quad (19)$$

then Eq. (13) can be simplified as

$$\mathscr{P}_{\max}(i, n) = \int_{-\infty}^{\infty} f(\gamma_i) \mathrm{d}\gamma_i \prod_{\substack{j=1, \\ j \neq i}}^{n} \int_{-\infty}^{\gamma_i} f(\gamma_j) \mathrm{d}\gamma_j = \frac{1}{n} \quad (20)$$

Equation (17) can be rewritten as

$$\mathscr{P}_{\text{sel}}(i \mid \mathscr{P}_{\max}(i, n)) = \frac{k}{i - 1} \quad (21)$$

and Eq. (11) can be calculated by

$$\mathscr{P}(k) = \frac{1}{n} \sum_{i=k+1}^{n} \frac{k}{i - 1} \quad (22)$$

Equation (22) is the solution of the classical secretary problem[26], so the MSP scheme is in fact an extended example of the classical secretary problem. The MSP process is summarized in Table 1.

4 MSEE Scheme

An alternative to the MSP scheme is to maximize the

Table 1   Process of MSP scheme.

(1) Determine the observation window size k based on Eq. (18).

(2) Negotiate with the first k assisted relay candidates and calculate the relay selection threshold $S = \max(C_{r,i}, i = 1, 2, \ldots, k)$.

(3) Negotiate with the i-th ($i = k+1, k+2, \ldots, n-1$) assisted relay candidate.

   (a) Calculate the spectrum efficiency of relay link C_i.

   (b) If the spectrum efficiency is greater than the threshold, $C_i > S$, the i-th assisted relay candidate is adopted and the source node turns to the TS.

   (c) If the channel capacity is smaller than the threshold, $C_i \leqslant S$, the i-th assisted relay candidate is rejected and the source node will negotiate with the $(i + 1)$-th assisted relay candidate.

   (d) If the first $(n - 1)$-th assisted relay candidates are rejected, the source node will adopt the n-th assisted relay candidate and turn to the TS.

(4) Turn to transmission stage.

expectation of the spectrum efficiency, that is MSEE. The corresponding protocol and optimal algorithm are introduced in this section.

4.1   Protocol of the MSEE scheme

Like the MSP scheme, the MESS scheme is also divided into an NS and a TS. Unlike the MSP scheme, the MESS scheme will decide the polling order and threshold set S_i ($i = 0, 1, \ldots, n$) based on the statistical CSI before the communication begins. Then, during the i-th NS, the source node will estimate the spectrum efficiency of relay transmission $C_{r,i}$ and direct transmission $C_{d,i}$. If the $C_{r,i}$ or $C_{d,i}$ is larger than the threshold S_i, the NS will end and the source node will turn to the corresponding transmission mode, otherwise, the negotiating ARC will be rejected and the source node will negotiate with the next ARC. The threshold optimal algorithm is given in the next section, and the process of the MESS protocol is shown in Fig. 4, where DTS is Direct TS, and RTS is Relay TS.

4.2   Optimal algorithm of the MSEE algorithm

In order to overcome the shortage caused by the time variation γ_i, a classical optimal stopping algorithm, Stochastic Dynamic Progress (SDP), is adopted to cooperate with the MSEE scheme. From the definition of the MSEE scheme, the optimal problem of relay selection given in Eq. (9) can be rewritten as

$$i^* = \arg \max \left\{ \sum_i E_{\gamma_{r_i}} \left[C_i \left(\gamma_{r_i} \right) I_i \right] \right\} \quad (23)$$

where $i = 1, 2, \ldots, n$. We denote π_i as the state set and

$$\pi_i = \begin{cases} \{W_i, T_i, O_i\}, & i = 1, 2, \ldots, n - 1; \\ \{T_n, O_n\}, & i = n \end{cases} \quad (24)$$

where T denotes the transmission state of ARC, W and O represent the negotiation state and busy state of ARC, respectively. From the definition of the MSEE scheme, the threshold S_i can then be written as
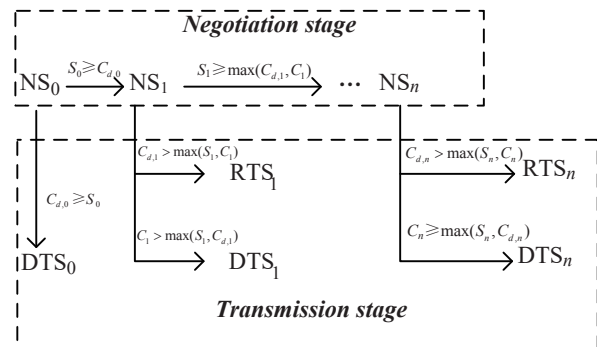

Fig. 4   Illustration of the protocol of MSEE scheme.

$$S_i = \begin{cases} E_{\gamma_i} \left[C_i \left(\gamma_{r_i} | W_i, \pi_{i+1} \right) \right], & i = 1, \ldots, n-1; \\ 0, & i = n \end{cases}$$
$$(25)$$

where $E_{\gamma_{r_i}} \left[C_i \left(\gamma_{r_i} | W_i, \pi_{i+1} \right) \right]$ is the conditional expectation of spectrum efficiency. Based on the idea of backtracking, Eq. (25) can be further represented as

$$S_i = E_{\gamma_{r_i}} \left[C_i \left(\gamma_{r_i} \right) | W_i, \pi_{i+1} \right] =$$
$$\sum_{\pi_{i+1}} q_{W_i, \pi_{i+1}} E_{\gamma_{r_{i+1}}} \left[C_{i+1} \left(\gamma_{r_{i+1}} \right) | W_i, \pi_{i+1} \right] \quad (26)$$

where $q_{W_i, \pi_{i+1}}$ is the one step transition probability, which can be represented as

$$q_{W_i, \pi_{i+1}} = \left(q_{W_i, W_{i+1}}, \quad q_{W_i, T_{i+1}}, \quad q_{W_i, O_{i+1}} \right),$$
$$i = 1, \ldots, n-1 \quad (27)$$

Based on Eqs. (26) and (27), the one step transition probability between the last two ARCs is

$$q_{W_{n-1}, \pi_n} = \left(q_{W_{n-1}, W_n}, \quad q_{W_{n-1}, T_n}, \quad q_{W_i, O_{i+1}} \right) =$$
$$\left(0, \quad \mathscr{P}_{\text{free}, n}, \quad \mathscr{P}_{\text{busy}, n} \right) \quad (28)$$

where, as defined above, $\mathscr{P}_{\text{free}, n}$ and $\mathscr{P}_{\text{busy}, n}$ represent the available probability and unavailable probability of the n-th ARC, respectively. From Eqs. (26) and (28), the threshold of the penultimate stage S_{n-1} can be calculated by

$$S_{n-1} = \mathscr{P}_{\text{free}, n} E_{\gamma_{r_n}} \left[C_n \left(\gamma_{r_n} \right) \right] + \mathscr{P}_{\text{busy}, n} C_{d, n} \quad (29)$$

where

$$E_{\gamma_{r_n}} \left[C_n \left(\gamma_{r_n} \right) \right] = \int_{-\infty}^{\infty} C_n \left(\gamma_{r_n} \right) f(\gamma_{r_n}) \mathrm{d}\gamma_{r_n} \quad (30)$$

Similarly, the one step transition probability between the i-th and $(i+1)$-th ARCs is

$$q_{W_i, \pi_{i+1}} = \left(q_{W_i, W_{i+1}}, q_{W_i, T_{i+1}}, q_{W_i, O_{i+1}} \right)^{\mathrm{T}} =$$
$$\begin{pmatrix} \mathscr{P}_{\text{free}, i} \int_{-\infty}^{2^{\frac{2S_{i+1}}{T-(i+1)\tau}} -1} f(\gamma_{r_{i+1}}) d\gamma_{r_{i+1}} \\ \mathscr{P}_{\text{free}, i} \int_{2^{\frac{2S_{i+1}}{T-(i+1)\tau}} -1}^{\infty} f(\gamma_{r_{i+1}}) d\gamma_{r_{i+1}} \\ \mathscr{P}_{\text{busy}, i} \end{pmatrix} \quad (31)$$

where the threshold $S_i \ (i = 1, 2, \ldots, n-2)$ is

$$S_i = \sum_{\pi_{i+1}} E_{\gamma_{r_{i+1}}} \left[C_{i+1} \left(\gamma_{r_{i+1}} \right) | \pi_{i+1} \right] =$$
$$q_{W_i, W_{i+1}} S_{i+1} + q_{W_i, T_{i+1}} V_{i+1} + q_{W_i, O_{i+1}} C_{d, i+1} \quad (32)$$

where V_{i+1} is the expectation of the spectrum efficiency for γ, calculated as

$$V_{i+1} = \int_{2^{\frac{2S_{i+1}}{T-(i+1)\tau}} -1}^{\infty} C_{i+1} \left(\gamma_{r_{i+1}} \right) f(\gamma_{r_{i+1}}) \mathrm{d}\gamma_{r_{i+1}}.$$

From Eqs. (28) to (32), each of the threshold $S_i \ (i = 1, 2, \ldots, n-1)$ is determined. The process of the MSEE scheme is summarized in Table 2.

Table 2   Process of MSEE scheme.

(1) Calculate the spectrum efficiency threshold $S_i, i = 1, 2, \ldots, n$.
   (a) Set the threshold $S_n = 0$.
   (b) Calculate the threshold S_{n-1} based on Eq. (29).
   (c) Calculate the one step transition probability between the i-th ARC to the $(i+1)$-th ARC based on Eq. (31).
   (d) Calculate the threshold S_i based on Eq. (32).
(2) Negotiation stage.
   (a) Calculate the spectrum efficiency $C_{d,i}$ of user without RA before the source node negotiates with the i-th ARC,
   (b) Confirm whether the i-th assisted relay candidate is available.
   (c) If the i-th ARC is busy and $C_{d,i} > S_i$, the source node turns to the DTS.
   (d) If the i-th ARC is busy and $C_{d,i} \leqslant S_i$, the source node will negotiate with the next AR candidate and go back to Step (a).
   (e) If the i-th ARC is free and $\max(C_{d,i}, C_{r,i}) > S_i$, the source node turns to the DTS when $C_{d,i} > C_{r,i}$, or RTS when $C_{d,i} \leqslant C_{r,i}$.
   (f) If the i-th ARC is free and $\max(C_{d,i}, C_{r,i}) \leqslant S_i$, the source node will negotiate with the next ARC and go back to Step (a).
(3) Transmission stage.

5   Numerical Results

In this section, we present the numerical results of the MSP and MSEE schemes. We assume the SNR provided by the ARCs follows the uniform distribution $U(\gamma_d, \gamma_{\max})$, and the SNR of the direct transmission mode γ_d is -10 dB. The Monte Carlo simulation is repeated 100 000 times.

5.1   Numerical results of MSP scheme

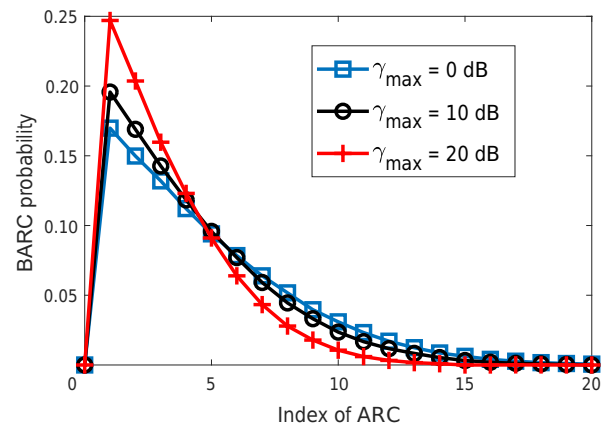The y-coordinate of Fig. 5 represents the probability that the i-th ARC is BARC. Moreover, the red line with

Fig. 5   Probability as the BARC under different SNR conditions.

plus signs shows the probability when γ_{max} is 20 dB, the black line with circles shows the probability when the γ_{max} is 10 dB, and the blue line with squares shows the probability when the γ_{max} is 0 dB. The 0 point of the ARC index denotes the direction communication mode. The results show that the probability decreases with an increase in the ARC index i, because the total negotiation cost $i\tau$ increases. Figure 5 also shows that the probability approaches 0 at a sufficiently high index i, indicating that a very large number of ARCs is unnecessary for the MSP scheme. Furthermore, we also note that the γ_{max} value has an insignificant effect on probability, indicating that the MSP scheme is robust under different SNR conditions.

Figure 6 shows the relationship between the probability of selecting the BARC and the observation window size k under different γ_{max} conditions when the negotiation cost τ is 0.001. The lines have the same meanings as in Fig. 5. The results show that a smaller observation window size k expects a higher γ_{max}, and that the optimal observation window size k is not sensitive to the SNR. Consistent with the results shown in Fig. 5, this also affirms the robustness of the MSP scheme is under different SNR conditions.

Figure 7 shows the relationship between the selective probability of BARC and the observation window size k under different negotiation cost conditions when the γ_{max} is 20 dB. Similar to the results shown in Fig. 6, the observation window size k increases slowly as the negotiation cost τ decreases.

In Fig. 8, the spectrum efficiency performance under different γ_{max} conditions is shown when the negotiation cost τ is 0.001. The black line shows the spectrum efficiency performance without ARC. The solid blue line with squares shows the spectrum efficiency UB with 10 ARCs in the AF-RA network, the dashed blue
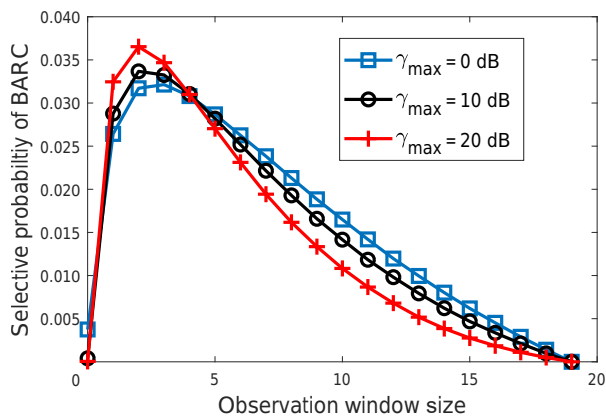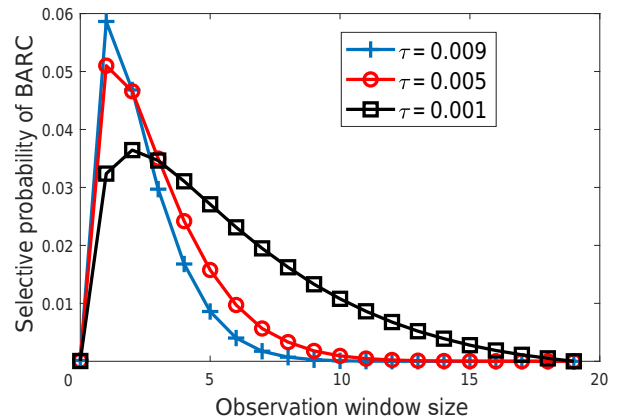

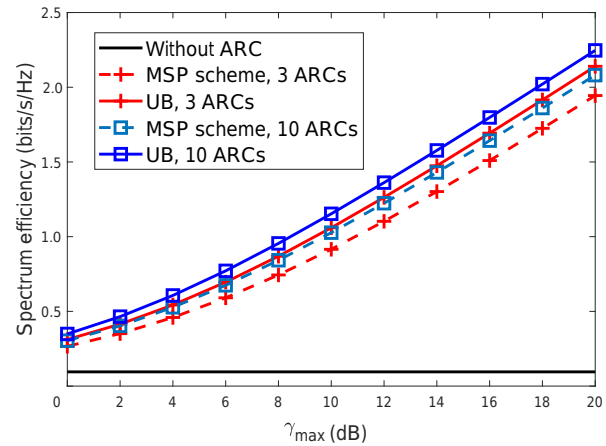**Fig. 7   Selective probability of the BARC under different negotiation cost conditions.**

Fig. 8   Spectrum efficiency of MSP scheme under different SNR conditions.

line with squares is the spectrum efficiency of the MSP scheme with 10 ARCs, and the red lines with plus signs are the spectrum efficiency UB (solid) and the spectrum efficiency of the MSP scheme (dashed) with 3 ARCs. From the results shown in Fig. 8, it is clear that the performance of the MSP scheme effectively approaches the UB. Furthermore, the gap between the spectrum efficiency of the MSP scheme and performance UB increase with an increase in γ_{max} indicating that the MSP scheme is more suitable for the lower γ_{max} case.

The results shown in Figs. 5 – 8 reveal that the performance of the MSP scheme effectively approaches the performance UB, and that the observation window size of the MSP scheme is insensitive to the SNR and negotiation cost. These results mark the MSP scheme as simple, stable, and easily applicable to a real-world environment.

5.2   Numerical results of MSEE scheme

Figure 9 shows the relationship between the spectrum

Fig. 6   Selective probability of the BARC under different SNR conditions.

efficiency and the number of ARCs under different γ_{max} conditions. The solid lines with squares and plus signs show the spectrum efficiency UB when the γ_{max} is 3 dB and 10 dB, respectively, while the dashed lines with squares and plus signs show the same results for the MSEE scheme. The black line denotes the spectrum efficiency without ARC. The results shown in Fig. 9 reveal the spectrum efficiency UB and the spectrum efficiency of MSEE scheme increases as the number of ARCs increases. The performance of the MSEE scheme effectively approaches the spectrum efficiency UB, and with more than 8 ARCs, the two almost converge. This shows that increasing the number of ARCs is not always beneficial for the spectrum efficiency of an AF-RA network.

Figure 10 shows the relationship between the spectrum efficiency and γ_{max}. The dashed lines with squares and plus signs represent the spectrum efficiency of MSEE scheme when there are 8 and 4 ARCs in
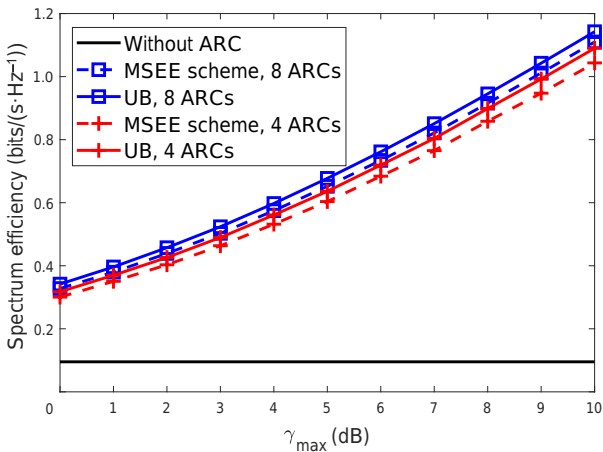
the AF-RA network, respectively, while the solid lines show the corresponding spectrum efficiency UB. The results show that increasing γ_{max} is beneficial for the spectrum efficiency of an AF-RA network, and that the gap in spectrum efficiency between the MSEE scheme and UB increases with an increase in γ_{max}, these findings are similar to those of the MSP scheme.

Figure 11 shows the spectrum efficiency performance under different probability of ARC availability assumptions. A higher probability of ARC availability means that there are fewer users in the RA network. The dashed lines with squares and plus signs represent the performance of the MSEE scheme with 8 and 4 ARCs, respectively, and the solid lines with squares and plus signs show the spectrum efficiency UB when there are 8 and 4 ARCs in the RA network, respectively. The results show that the spectrum efficiency increases with an increase in the probability of ARC availability, and that the performance of the MSEE scheme is closer to the spectrum efficiency UB when the probability of ARC availability is lower. This points to that the MSEE scheme shows a higher performance under conditions of low probability of ARC availability. Moreover, the spectrum efficiencies almost converge when the probability of a free ARC is higher than 0.6, indicating that the presence of a large number ARCs gives only a limited advantage for the spectrum efficiency of an AF-RA network.

Figure 12 shows the relationship between the spectrum efficiency and the negotiation cost τ. The meaning of the lines is the same as in Fig. 11. From the results shown in Fig. 12, it is clear that the spectrum efficiency decreases sharply with an increase in the negotiation cost τ, and that such an increase
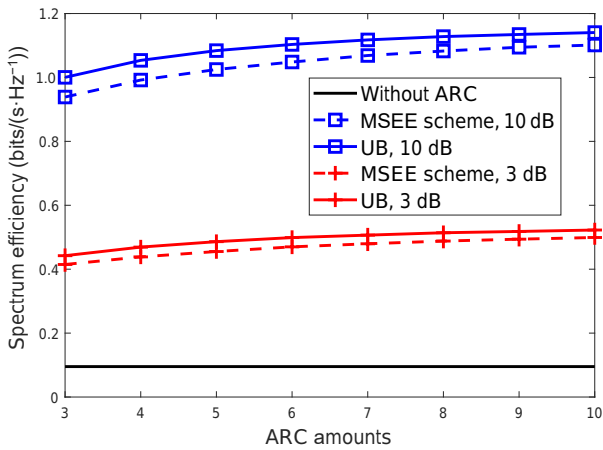

Fig. 9 Spectrum efficiency of MSEE scheme under different ARC amounts conditions.

Fig. 10 Spectrum efficiency of MSEE scheme under different SNR conditions.
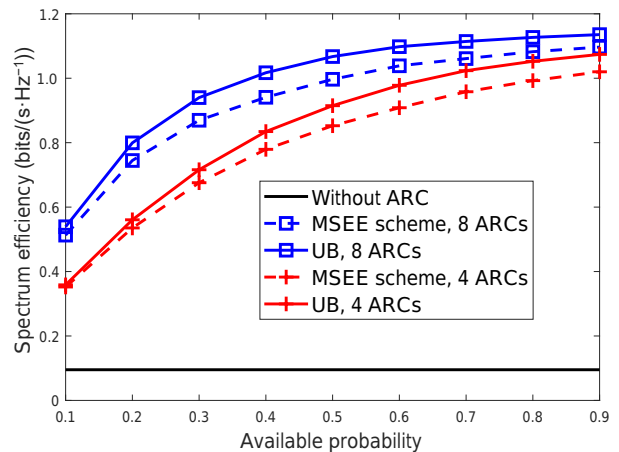

Fig. 11 Spectrum efficiency of MSEE scheme under different available probability conditions.
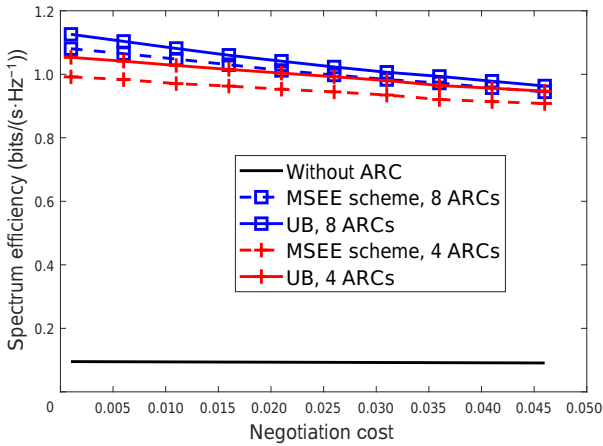
Fig. 12 Spectrum efficiency of MSEE scheme under different negotiation cost conditions.

also decreases the performance gap between a 8-ARC MSEE and a 4-ARC MSEE scheme. From the results shown in Fig. 12, we can conclude that the MSEE scheme is more suitable for low negotiation cost cases.

5.3 Comparing the MSP and MSEE schemes

Figure 13 shows the results of a performance comparison between the MSP and MSEE schemes with different numbers of ARCs. The dotted and dashed lines with squares are the spectrum efficiency performance of the MSP and MSEE schemes, respectively, with the negotiation cost $\tau = 0.01$, while the dotted and dashed lines with plus signs represent the spectrum efficiency performance of the MSP and MSEE schemes, respectively, with the negotiation cost $\tau = 0.001$. The solid lines with squares and plus signs show the spectrum efficiency UB under different negotiation cost conditions. From the results shown in Fig. 13, it is clear that the spectrum efficiency performance of the MSEE
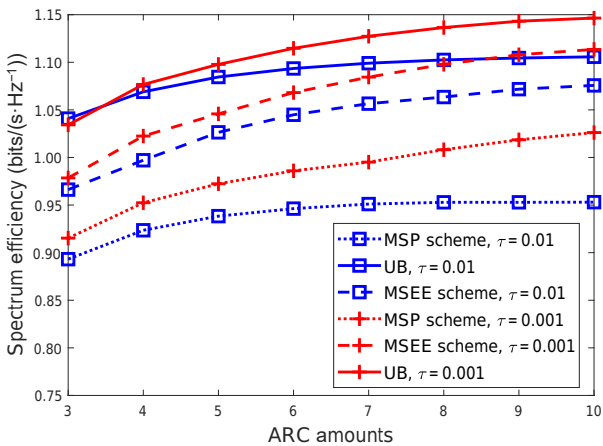
scheme is superior to that of the MSP scheme, and that the superiority of the MSEE scheme increases with an increasing number of ARCs.

We also compare the performance of the MSP and MSEE schemes under different γ_{max} conditions, with the results shown in Fig. 14. The meaning of the lines is same as in Fig. 13. Figure 14 also shows that the performance of the MSEE scheme to be superior to that of the MSP scheme, and that superiority of the MSEE scheme increases with an increase in γ_{max}.

6 Conclusion

In this paper, two relay selection schemes, MSP and MSEE, are proposed for AF-RA networks, with the optimal stopping theory employed to cooperate with the schemes. From the simulation results, it is clear that the spectrum efficiency performance of the MSEE scheme is superior to that of the MSP scheme, even though both of them effectively approach the performance UB. Nonetheless, the performance advantage of the MSEE scheme comes at the cost of complexity, and the performance gap is negligible with a sufficiently low maximum SNR (γ_{max}) value and/or a sufficiently high negotiation cost (τ) value. These findings indicate that MSP is more suitable for lower SNR and/or higher negotiation cost cases, since it can provide the same performance with low complexity. However, the MSEE scheme should be adopted in higher SNR and/or lower negotiation cost cases because of its superior performance.

Acknowledgment

Fig. 13 Compare of spectrum efficiency under different ARC amounts conditions.
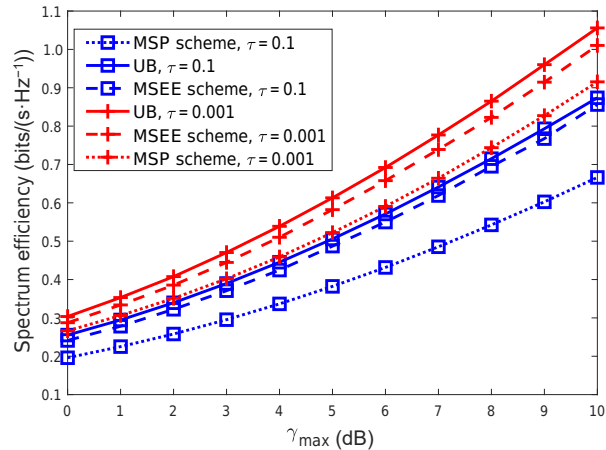
Fig. 14 Compare of spectrum efficiency under different SNR conditions.

References

[1] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, Cooperative diversity in wireless networks: Efficient protocols and outage behavior, *IEEE Transactions on Information Theory,* vol. 50, no. 12, pp. 3062–3080, 2004.

[2] A. Bletsas, A. Khisti, D. P. Reed, and A. Lippman, A simple cooperative diversity method based on network path selection, *IEEE Journal on Selected Areas in Communications,* vol. 24, no. 3, pp. 659–672, 2006.

[3] J. Jedrzejczak, G. J. Anders, M. Fotuhi-Firuzabad, H. Farzin, and F. Aminifar, Reliability assessment of protective relays in harmonic-polluted power systems, *IEEE Transactions on Power Delivery,* vol. 32, no. 1, pp. 556–564, 2017.

[4] L. Peter, Large-scale cooperative relaying network with optimal coherent combining under aggregate relay power constraints, https://wireless.kth.se/wp-content/uploads/2010/09/Peter1.1.pdf, 2003.

[5] J. N. Laneman and G. W. Wornell, Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks, *IEEE Transactions on Information Theory,* vol. 49, no. 10, pp. 2415–2425, 2003.

[6] M. Butt, A. Salama, A. Mohamed, and M. Guizani, Relay selection schemes to minimize outage in wireless powered communication networks, *IET Signal Processing letter,* vol. 10, no. 3, pp. 203–209, 2016.

[7] H. Eghbali, S. Muhaidat, S. A. Hejazi, and Y. Ding, Relay selection strategies for single-carrier frequency-domain equalization multi-relay cooperative networks, *IEEE Transactions on Wireless Communications,* vol. 12, no. 5, pp. 2034–2045, 2013.

[8] P. Das and N. B. Mehta, Direct link-aware optimal relay selection and a low feedback variant for underlay CR, *IEEE Transactions on Communications,* vol. 63, no. 6, pp. 2044–2055, 2015.

[9] B. Choi, S. J. Bae, K. Cheon, A. Park, and M. Y. Chung, Relay selection and resource allocation schemes for effective utilization of relay zones in relay-based cellular networks, *IEEE Communications Letters,* vol. 15, no. 4, pp. 407–409, 2011.

[10] S. N. Islam, M. A. Mahmud, and A. M. T. Oo, Relay aided smart meter to smart meter communication in a microgrid, in *2016 IEEE International Conference on Smart Grid Communications (SmartGridComm),* Sydney, Australia, 2016, pp. 128–133.

[11] C. Zhao, V. Angelakis, D. Yuan, and B. Timus, Evaluation of cell selection algorithms in LTE-advanced relay networks, in *2013 IEEE 18th International Workshop on Computer Aided Modeling and Design of Communication Links and Networks (CAMAD),* Berlin, Germany, 2013, pp. 114–118.

[12] Y. Jing and H. Jaffarkhani, Single and multiple relay selection schemes and their achievable diversity orders, *IEEE Transactions on Wireless Communications,* vol. 8, no. 3, pp. 1414–1423, 2009.

[13] Y. W. Hong, W. J. Huang, F. H. Chiu, and C. C. J. Kuo, Cooperative communications in resource-constrained wireless networks, *IEEE Signal Process Magzine,* vol. 24, no. 3, pp. 45–57, 2007.

[14] F. S. Tabataba, P. Sadeghi, and M. R. Pakravan, Outage probability and power allocation of amplify and forward relaying with channel estimation errors, *IEEE Transactions on Wireless Communication,* vol. 10, no. 1, pp. 124–134, 2011.

[15] O. Amin, R. Mesleh, S. S. Ikki, M. H. Ahmed, and O. A. Dobre, Performance analysis of multiple-relay cooperative systems with signal space diversity, *IEEE Transactions on Vehicular Technology,* vol. 64, no. 8, pp. 3414–3425, 2015.

[16] R. Madan, N. Mehta, A. Molisch, and J. Zhang, Energy-efficient cooperative relaying over fading channels with simple relay selection, *IEEE Transactions on Wireless Communications,* vol. 7, no. 8, pp. 3013–3025, 2008.

[17] M. Seyfi, S. Muhaidat, and J. Liang, Amplify-and-forward selection cooperation over Rayleigh fading channels with imperfect CSI, *IEEE Transactions on Wireless Communications,* vol. 11, no. 1, pp. 199–209, 2012.

[18] O. Amin, S. Ikki, and M. Uysal, On the performance analysis of multirelay cooperative diversity systems with channel estimation errors, *IEEE Transactions on Vehicular Technology,* vol. 60, no. 5, pp. 2050–2059, 2011.

[19] C. Wang and J. Chen, Power allocation and relay selection for AF cooperative relay systems with imperfect channel estimation, *IEEE Transactions on Vehicular Technology,* vol. 65, no. 9, pp. 7809–7813, 2016.

[20] E. Koyuncu and H. Jafarkhani, Distributed beamforming in wireless multiuser relay-interference networks with quantized feedback, *IEEE Transactions on Information Theory,* vol. 58, no. 7, pp. 4538–4576, 2012.

[21] X. Pi, Y. Cai, and W. Yang, A multirate cooperative MAC protocol employing statistic channel state information, in *2013 IEEE Third International Conference on Information Science and Technology (ICIST),* Yangzhou, China, 2013, pp. 1131–1135.

[22] A. Zhan, C. He, and L. Jiang, A channel statistic based power allocation in a butterfly wireless network with network coding, in *2010 IEEE International Conference on Communications Workshops,* Capetown, South Africa, 2010, pp. 1–5.

[23] S. Guo, J. Tang, Y. Wei, W. Li, and C. Zhang, Research on relay selection based on statistical channel state information, in *2012 International Symposium on Instrumentation and Measurement, Sensor Network and Automation (IMSNA),* Sanya, China, 2012, pp. 176–178.

[24] J. P. Coon and R. Cepeda, Power loading in parallel diversity channels based on statistical channel information, *IEEE Communications Letters,* vol. 16, no. 7, pp. 1060–1063, 2012.

[25] H. Jiang, S. Zhang, and W. Zhou, Energy-efficient distributed relay selection based on statistical channel state

information, *Journal of Applied Sciences*, vol. 98, pp. 388–399, 2012.

[26] D. Seale and A Rapoport, Optimal stopping behavior with relative ranks: The secretary problem with unknown population size, *Journal of Behavioral Decision Making,* vol. 13, no. 4, pp. 391–411, 2000.

[27] C. Delakouridis and L. Kazatzopoulos, On the use of optimal stopping theory for secret sharing scheme update, *Intelligent Technologies and Techniques for Pervasive Computing,* doi: 10.4018/978-1-4666-4038-2.ch015.

[28] P. Fatouros, I. Konstantelos, D. Papadaskalopoulos, and G. Strbac, Stochastic dual dynamic programming for operation of DER aggregators under multidimensional uncertainty, *IEEE Transactions on Sustainable Energy,* vol. 10, no. 1, pp. 459–469, 2019.

[29] A. Papavasiliou, Y. Mou, L. Cambier, and D. Scieur, Application of stochastic dual dynamic programming to the real-time dispatch of storage under renewable supply uncertainty, in *2018 IEEE Power and Energy Society General Meeting (PESGM),* Portland, OR, USA, 2018, p. 1.

[30] A. Shapiro, Analysis of stochastic dual dynamic programming method, *European Journal of Operational Research*, vol. 209, no. 1, pp. 63–72, 2011.

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