

A New Filter Collaborative State Transition Algorithm for Two-Objective Dynamic Reactive Power Optimization

Hongli Zhang, Cong Wang*, and Wenhui Fan

Abstract: Dynamic Reactive Power Optimization (DRPO) is a large-scale, multi-period, and strongly coupled nonlinear mixed-integer programming problem that is difficult to solve directly. First, to handle discrete variables and switching operation constraints, DRPO is formulated as a nonlinear constrained two-objective optimization problem in this paper. The first objective is to minimize the real power loss and the Total Voltage Deviations (TVDs), and the second objective is to minimize incremental system loss. Then a Filter Collaborative State Transition Algorithm (FCSTA) is presented for solving DRPO problems. Two populations corresponding to two different objectives are employed. Moreover, the filter technique is utilized to deal with constraints. Finally, the effectiveness of the proposed method is demonstrated through the results obtained for a 24-hour test on Ward & Hale 6 bus, IEEE 14 bus, and IEEE 30 bus test power systems. To substantiate the effectiveness of the proposed algorithms, the obtained results are compared with different approaches in the literature.

Key words: dynamic reactive power optimization; filter collaborative state transition algorithm; Ward & Hale 6 bus; IEEE 14 bus; IEEE 30 bus

1 Introduction

Dynamic Reactive Power Optimization (DRPO) is very important for secure and economic operation of power systems. It is used to minimize the real power transmission loss and keep all the bus voltages within limits, while satisfying some equality and inequality constraints including the power flow equations, upper and lower voltage limits, and reactive power capacity restrictions in various reactive power sources such as generators, shunt capacitor banks, and transformer

taps^[1]. Because of the action number constraints of control equipment as well as the mutual influence during the movement of control equipment, DRPO is a large-scale multi-period mix-integer nonlinear programming problem. The key of solving DRPO problem is to efficiently deal with discrete variables and the action number constraints of reactive power control equipment.

Various optimization methods have been used to solve Reactive Power Optimization (RPO) problems, including classical methods such as linear programming^[1], nonlinear programming^[2], quadratic programming^[3], mixed integer programming^[4,5], and Newton method^[6]. These classical methods offer much faster convergence and noticeable convenience in handling inequality constraints compared to other methods. Nevertheless, these methods all require continuous and differentiable objective functions and have difficulties in handling very many variables; therefore, they are not suitable for solving DRPO.

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To overcome these limitations, many new models and intelligence algorithms have been applied in the literature for solving DRPO problem of power systems. Park et al.^[7] proposed a coordinated control method for an under-load tap changer with switching capacitors in distribution systems to reduce the operation numbers of both devices. The proposed method added the best optimization goal of the voltage quality. Liu et al.^[8] considered the action number constraints of transformers with on-load tap changers, and switchable shunt capacitor banks were described by the mathematical expressions of their control variables. Park et al.^[9] proposed the planning method for capacitor installation in a distribution system to reduce the installation costs and minimize the loss of electrical energy. Liu et al.^[10] proposed a DRPO method in the distribution network with distributed generators. References [7–10] are all single objective models, which cannot conform to the multi-objective requirements for the DRPO. Bhattacharyya and Raj^[11] considered loss minimization and cost economic operation of a system and used Particle Swarm Optimization (PSO), bacterial foraging algorithm, and bio-inspired PSO algorithms such as evolutionary PSO, adaptive PSO, and hybrid PSO to optimize a reactive power planning problem. Bhattacharyya and Badu^[12] used a Teaching-Learning-Based Optimization (TLBO) algorithm to reactive power planning problem. Ayan and Kürs^[13] optimized Reactive Power Flow (RPF) based on the artificial bee colony algorithm to minimize active power loss in power systems. Singh et al.^[14] presented PSO with an aging leader and challengers algorithm for the solution of optimal reactive power dispatch problem. Li et al.^[15] proposed a method based on the improved particle swarm-tabu search algorithm with three objectives: active power loss, voltage deviation, and static voltage stability margin. Xie et al.^[16] adopted a method of building multiple target reactive power optimization mathematical models and proposed a multi-objective firefly algorithm with elitism strategy, Pareto-dominated sort, and crowding distance sorting to optimize a reactive power optimization mathematical model. References [11–16] have shown successes in solving optimal reactive power dispatch problems. But the research object of these all methods is static reactive power optimization.

However, in the actual environment, load changes with time, which requires adjusting the real-time control equipment to meet the needs of the running

system. Considering the limited number of control equipment, action time strengthens the correlation between them, which makes the DRPO problem more complicated. Considerable work has been done for DRPO problems involving multiple criteria. Sun et al.^[17] proposed a multi-stage solution approach for DRPO, in which a complicated mixed-integer nonlinear programming problem is divided into a nonlinear programming and a mixed-integer programming problem. The results verified the effectiveness and applicability of the approach. However, this method increased the solution phase. Zhou et al.^[18] used the maximum network loss in a day, minimum tap changing times of on-load tap changer and minimum times of switching on/off capacitor banks as objective functions, and built a new multi-objective DRPO model. An improved multi-population ant colony algorithm was used to optimize solutions. This guaranteed finding the global optimal solution in the different states, but increased the algorithm complexity. Ding et al.^[19] proposed a mixed integer nonlinear programming model, considering the discrete reactive power equipment operation limit and power grid security constraints, to minimize the transmission losses. The optimal solutions could be quickly obtained by the proposed method. This method reduced the economic efficiency of DRPO^[10,19].

In this paper, a two-objective DRPO model is built and optimized by Filter Collaborative State Transition Algorithm (FCSTA). Considering discrete variables and switching operation constraints, we build the first objective to minimize the actual power loss and Total Voltage Deviations (TVDs), and the second objective to minimize incremental system loss. The solutions of two objective functions were exchanged and used during the solving process. The DRPO problem is a multi-objective constraint problem. For constrained optimization problems, the commonly used method is the penalty function method, in which a penalty function is taken as a fitness function. The selection of penalty function parameters has a very important influence on the solution. The Filter Technique (FT)^[20] is presented to solve nonlinear planning problems. The fitness functions and the constraint violation degree functions make up filter pairs. To obtain the search direction, the new filter is detected if governed by the original filter subset; this avoids selecting the penalty factor of the penalty function. The State Transition Algorithm (STA),

proposed by Yang et al.^[21–23], is easy to understand, because of the fewer parameters and simple algorithm structure. Therefore, we present an FCSTA to solve the DRPO problem. Two populations corresponding to two different objectives are employed. FCSTA adopts double population parallel search method. In the iteration process, the solutions communicate with each other between populations and periodically share the optimal solution. The optimal solution of one objective function will be used by the other, which means the optimization solution of the first objective will be used as the initial value of the second one, and the optimization solution of the second objective will be added to the first objective function. In this way, the model can obtain the whole optimal solution. Finally, the proposed method is tested on Ward & Hale 6 bus, IEEE 14 bus, and IEEE 30 bus test power systems. To show the effectiveness of the proposed algorithms, the obtained results are compared with different approaches in the literature.

Details regarding the novelty of the proposed method are as follows:

(1) A two-objective DRPO model is formulated. Compared to other models, the objectives of the proposed model are to minimize the real power loss, TVDs, and incremental system loss. The first objective is to minimize the real power loss and the TVDs, and the second objective is to minimize incremental system loss. In this method, the minimum power loss and TVDs can be maintained, and also the minimum incremental system loss.

(2) Considering the security constraints of the system and unit operation and the discrete variable and action number constraints of relaxation reactive power control device on the whole day, the DRPO model is closer to the actual operation. It means the applicability of the proposed model is more extensive. The proposed two-objective DRPO model considers more actual factors.

(3) An FCSTA is first presented for solving DRPO problem. Comparing to other algorithms with other constraint processing technologies, the FCSTA has three advantages: First, the state transition algorithm has fewer parameters and a simple algorithm structure, which is a new intelligence algorithm and be easily understood. Second, FCSTA adopts a collaborative strategy, which is a double population parallel search method. Two populations corresponding to two different objectives are employed. In the iteration process, the solutions communicate with each other

between populations and periodically share the optimal solution. The collaborative strategy can guarantee the speed and accuracy of DRPO problem. Third, the FT is presented to solve the constrained DRPO problem. In dealing with the problem of constraint optimization with the line search method or trust region method, the penalty function is always used to ensure the convergence of the algorithm. These methods have different requirements for the penalty function, and the iterative process needs to consider the fall of the penalty function. In the proposed algorithm with filter technique, the fitness functions and the constraint violation degree functions make up filter pairs. To obtain the search direction, the new filter is detected if governed by the original filter subset; this avoids the selection of the penalty factor of penalty function.

The rest of the paper is organized as follows: In Section 2, the mathematical problem of DRPO is presented. In Section 3, basic STA and FT are presented in detail. In Section 4, FCSTA and its application to DRPO problems are discussed. Computational results and comparisons are given in Section 5. Finally, the paper ends with the conclusion and future work in Section 6.

2 Mathematical Problem Formulation of DRPO

The DRPO is a large-scale, multi-period, and mix-integer nonlinear programming problem. Considering security constraints of the system, unit operation, relaxing discrete variables and switching operation constraints, and action number constraints of reactive power control device on the whole day, the mathematical model is given.

2.1 First objective function and constraints

Considering discrete variables and switching operation constraints, we build the first objective optimization function and constraints as follows:

(1) Objective function

The first objective function is to minimize the real power loss and the TVDs.

$$f_1 = W_1 \sum_{t=1}^T \left\{ \sum_{i=1}^{N_E} \sum_{j=1}^{N_E} Y_{ij} [V_i(t)^2 + V_j(t)^2 - 2V_i(t)V_j(t) \cos \theta_{ij}(t)] \right\} + W_2 \sum_{t=1}^T \sum_{k=1}^{N_L} |V_k(t) - V_k(t)^{\text{ref}}| \quad (1)$$

Here, f_1 is the objective function, W_1 and W_2 are weight coefficients, T is the time of the whole day, $T = 1, 2, \dots, 24$, N_E is the number of network branches, N_L is the number of load buses, and Y_{ij} is the i -th row and j -th column element of bus admittance matrix. $V_i(t)$ and $V_j(t)$ are voltages at buses i and j , respectively, $\theta_{ij}(t)$ is voltage angle difference between buses i and j , and $V_k(t)^{\text{ref}}$ is the reference value of the voltage magnitude of the i -th bus which is equal to 1.0 p.u.

(2) Equality constraints

$$\begin{cases} P_{Gi}(t) - P_{Di}(t) = V_i(t) \sum_{j=1}^{N_i} V_j(t) Y_{ij} \cos \theta_{ij}(t); \\ Q_{Gi}(t) - Q_{Di}(t) = V_i(t) \sum_{j=1}^{N_i} V_j(t) Y_{ij} \sin \theta_{ij}(t) - Q_{Ci}(t) \end{cases} \quad (2)$$

Here t is the time. $P_{Gi}(t)$ and $Q_{Gi}(t)$ are the injected active power at bus i and injected reactive power at bus j , respectively. $P_{Di}(t)$ and $Q_{Di}(t)$ are demanded active power at bus i and demanded reactive power at bus j , respectively. N_i is the number of buses adjacent to bus i (including bus i), and $Q_{Ci}(t) = k_{Ci}(t) Q_{cN}$, where Q_{cN} is the capacitor reactive power injection for bus i on time t , Q_{cN} is the capacity of a single set capacitor, and $k_{Ci}(t)$ is the number of input capacitor on time t .

(3) Inequality constraints

State variable constraint. All bus voltages (including slack bus) and reactive power outputs of slack bus must be restricted within their respective lower and upper limits as stated in Eqs. (3) and (4).

$$\begin{cases} x_{\min}(t) \leq x(t) \leq x_{\max}(t); \\ x(t) = [V_1(t), V_2(t), \dots, V_{N_E}(t), P_{\text{Gslack}}(t)]^T \end{cases} \quad (3)$$

Equation (3) can also be

$$\begin{cases} V_{\min}(t) \leq V_i(t) \leq V_{\max}(t), i = 1, 2, \dots, N_E; \\ P_{\min}(t) \leq P_{\text{Gslack}}(t) \leq P_{\max}(t) \end{cases} \quad (4)$$

Here P_{Gslack} is reactive power output of slack bus, and min and max represent the minimum and maximum values of the variables, respectively.

Control variable constraint. The reactive power of generator and the number of reactive power compensation capacitor switching and transformation ratios of on-load voltage-regulating transformer must be restricted within their respective lower and upper limits as stated in the following.

$$\begin{cases} u_{\min}(t) \leq u(t) \leq u_{\max}(t); \\ u(t) = [Q_{Gi}(t), k_{Ci}(t), k_{Ti}(t)]^T \end{cases} \quad (5)$$

Equation (5) can also be

$$\begin{cases} Q_{Gi\min}(t) \leq Q_{Gi}(t) \leq Q_{Gi\max}(t); \\ k_{Ci\min}(t) \leq k_{Ci}(t) \leq k_{Ci\max}(t); \\ k_{Ti\min}(t) \leq k_{Ti}(t) \leq k_{Ti\max}(t) \end{cases} \quad (6)$$

Here, $k_{Ci}(t)$ is compensation capacitor constraint, and $k_{Ti}(t)$ is the on-load voltage regulating transformer constraint, and it is the alteration ratio of on-load voltage-regulating transformer of bus i in time t .

2.2 Second objective function and constraints

Based on the sensitivities of control variables corresponding to the objective function, and considering the discrete variables subject to switching operation constraints, the second objective function and constraint are given as follows.

(1) Objective function

The second objective function f_2 is to minimize incremental system loss.

$$f_2 = \min \sum_{t=1}^T S_{Pu}(t) [u'(t) - u^0(t)] T_{\text{step}} \quad (7)$$

Here $u'(t) = [k'_{Ci}(t), k'_{Ti}(t)]$ is preparative optimization value of discrete control variable, and $u^0(t) = [k^0_{Ci}(t), k^0_{Ti}(t)]$ is the optimized result from first objective function. $S_{Pu}(t) = [S_{PQc}(t), S_{PT}(t)]$, where $S_{Pu}(t)$ is the sensitivity matrix of system network loss with control variables in time t , S_{PQc} is the system network loss with capacitor variables, and S_{PT} is the system network loss with transformer, and $T_{\text{step}} = [T_{C\text{step}}, T_{T\text{step}}]$, which is the action step of reactive power control device.

(2) Constraints

The state variables must be adjusted within their respective lower and upper limits, as stated in Eq. (8).

$$V_{\min}(t) \leq S_{Vu}(t) [u'(t) - u^0(t)] T_{\text{step}} + V_i(t) \leq V_{\max}(t) \quad (8)$$

Here $S_{Vu}(t) = [S_{VQc}(t), S_{VT}(t)]$, $S_{Vu}(t)$ is the sensitivity matrix of bus voltage with control variables in time t , S_{VQc} is the bus voltage with capacitor variables, and $S_{VT}(t)$ is the bus voltage with transformer variable.

The constraint of state variables (number of reactive power compensation capacitor switching and

transformation ratios of on-load voltage-regulating transformer) is shown as follows:

$$\text{int}(u'^0(t)) \leq u'(t) \leq \text{int}(u'^0(t) + 1) \quad (9)$$

Here, int means using only integer.

The action number of control equipment constraints for the whole day is given as Eq. (10).

$$\sum_{t=1}^{T-1} |u'(t) - u'(t+1)| \leq K \quad (10)$$

Here, $K = [K_{C_{\max}}, K_{T_{\max}}]^T$, $K_{C_{\max}}$ is the maximum value of $K_C(t)$ and $K_{T_{\max}}$ is the maximum value of $K_T(t)$.

3 State Transition Algorithm and Filter Technique

3.1 State transition algorithm

The state transition algorithm was proposed in 2011 by Zhou et al.^[21] A solution of specific optimization problem is regarded as a state, and the optimization algorithm is regarded as a state transition. Therefore, the process of solving the optimization problem can be regarded as a state transition process.

The state transition algorithm has fewer parameters and a simple, easily understandable algorithm structure. The state transition can be defined as function.

$$\begin{cases} x_{k+1} = A_k x_k + B_k u_k; \\ x_k = f(x_{k+1}) \end{cases} \quad (11)$$

Here, $x_k \in R^n$ is a state and corresponds to a solution to the optimization problem; $A_k, B_k \in R^{n \times n}$ are the operators of the optimization algorithm, which is also called state transition matrixes, and $R^{n \times n}$ are state transition matrixes, which are the operators of the optimization algorithm; $u_k \in R^n$ is the function of the state x_k and its history state; $f(x_k)$ is the objective function. This algorithm has four operators. The details of the four operators are shown as follows.

Rotation Transformation (RT) has the function of searching in a hypersphere.

$$x_{k+1} = x_k + \alpha \frac{1}{n \|x_k\|_2} R_r x_k \quad (12)$$

Translational operation (TT) has the function of searching along a line from x_{k-1} to x_k at the starting point x_k , with the maximum length of β .

$$x_{k+1} = x_k + \beta R_t \frac{x_k - x_{k-1}}{\|x_k - x_{k-1}\|_2} \quad (13)$$

Expansion Transformation (ET) has the function of expanding the components in x_k to the range of

$[-\infty, +\infty]$, searching in the whole space.

$$x_{k+1} = x_k + \gamma R_e x_k \quad (14)$$

Axesion transformation (AT) aims to search along the axes and strengthens single dimensional search.

$$x_{k+1} = x_k + \delta R_a x_k \quad (15)$$

In Eqs. (12)–(15), $x_k \in R^n$ is the state of STA, α, β, γ , and δ are rotation, translation, expansion, and axesion factors, respectively, and they are all positive constants. $R_r \in R^{n \times n}$ is a random matrix, whose elements belonging to the range of $[-1, 1]$. $\|x_k\|_2$ is the 2-norm of x_k . $R_t \in R$ is a random variable, which has elements belonging to the range of $[0, 1]$. $R_e \in R^{n \times n}$ is a random diagonal matrix, and its elements obey the Gaussian distribution. $R_a \in R^{n \times n}$ is a random diagonal matrix, whose elements obeying the Gaussian distribution and only one random index has value.

The framework of the basic STA is illustrated in Fig. 1. RT denotes rotation transformation, TT denotes translational operation, ET denotes expansion transformation, and AT denotes axesion transformation.

3.2 Filter technique

Fletcher et al.^[24] proposed a filter technique for solving constraint optimization problems in 2002. The constraint optimization problem can be shown as

$$\begin{cases} \min f(x); \\ \text{s.t. } g_i(x) \leq 0, \quad i \in I = \{1, 2, \dots, p\} \end{cases} \quad (16)$$

Here, $f(x) : R^n \rightarrow R$ is the objective function. $g_i(x) : R^n \rightarrow R$ is the constraint function, and I indicates a set of constraints. All points which satisfy the constraint conditions are called the feasible points, and the set of all feasible points is a feasible region denoted as X .

$$X = \{x | g_i(x) \leq 0, \quad i = 1, 2, \dots, p\} \quad (17)$$

The conception and properties of a filter are given in the following.

Definition 1. The function value f and constraint violation degree h can constitute a filter (f, h) , where

$$f = f(x), h = \sum_{i=1}^p \max(0, g_i(x)).$$

Definition 2. The filter (f_i, h_i) corresponds to the point i , and the filter (f_j, h_j) corresponds to the point j . When $f_i \leq f_j$ and $h_i \leq h_j$, then point i dominates point j .

Definition 3. F is a filter set, which is defined to be a set of filter. All filters cannot dominate each other.

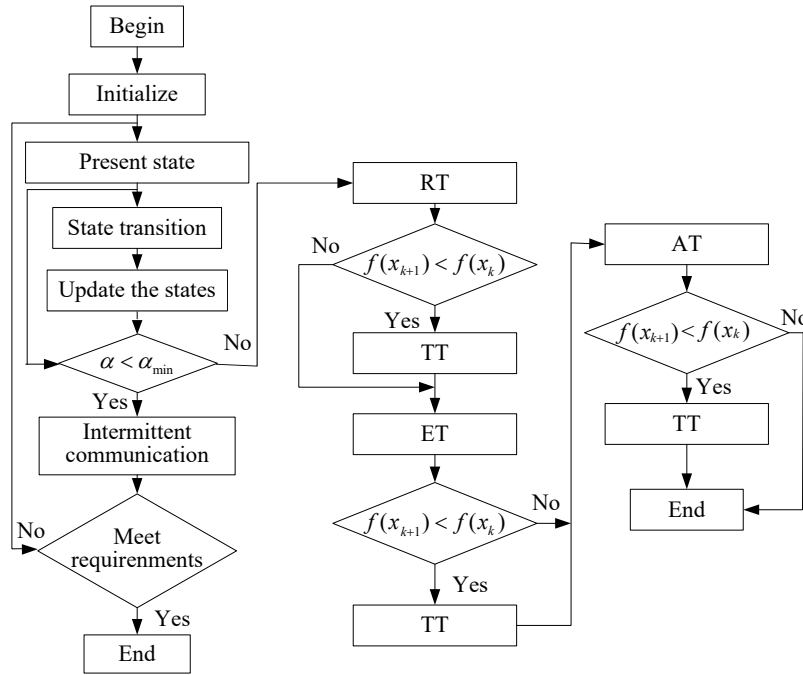


Fig. 1 Framework of the STA.

Definition 4. If x_k is the newly generated points or individuals, then a point x_k is to be added to the filter, which means adding its filter (f_k, h_k) to the list of pairs in the filter, and pairs in the filter that are dominated by the new pair are removed. The filter is used as an alternative to a penalty function to decide whether or not to accept a new point (or a new solution) in constraint optimization problems.

4 FCSTA and FCSTA’s Application to DRPO Problem

4.1 Filter collaborative state transition algorithm

To solve DRPO problems, we propose the FCSTA. In FCSTA, filter technique is used to solve objective function and constraint functions, which can be seen in Section 3.2. Collaborative state transition algorithm means that two state populations corresponding to two different objectives are employed. FCSTA adopts double population parallel search method. In the iteration process, the solutions communicate with each other between populations and share periodically the optimal solution. In this search method, the algorithm can have a rapid convergence.

Supposing there are two populations S_1 and S_2 , the number of S_1 and S_2 is N , the number of iteration is T , the optimal solution is $x = [x_1, x_2]$, and the filter sets are F_1 and F_2 , the number of F_1 and F_2 are N_1 and N_2 ,

and the filters are (f_1, h_1) and (f_2, h_2) . The flowchart of FCSTA is illustrated in Fig. 2, and the steps are given below.

Step 1 Initialization. All parameters are sets of S_1 and S_2 . The number of S_1 and S_2 is N , and the number of iteration is T . The filter sets are also initialized.

Step 2 Generation of initial states. The initial states of all the individuals in populations S_1 and S_2 are randomly generated within their respective minimum and maximum values.

Step 3 Expansion of the filter sets. According to (f_1, h_1) , the filter set F_1 is expanded, that is, all filters that cannot dominate the new pair is added to F_1 . Likewise, according to (f_2, h_2) , the filter set F_2 is expanded, that is, all filters that cannot dominate the new pair is added to F_2 .

Step 4 Implementation of the four operations. All individuals in populations S_1 and S_2 are operated by RT, TT, ET, and AT.

Step 5 Updating. After the four operations are implemented, the objective function values are calculated. Supposing the new filter (f_i, h_i) and the filter (f_j, h_j) is any one in filter set, when $f_i \leq f_j$ and $h_i \leq h_j$, the new filter (f_i, h_i) can be accepted by the filter set. We get the new filter sets F'_1 and F'_2 .

Step 6 Exchange of optimal solutions. The two populations will exchange their optimal solutions in this iteration. Then the two filter sets will be updated.

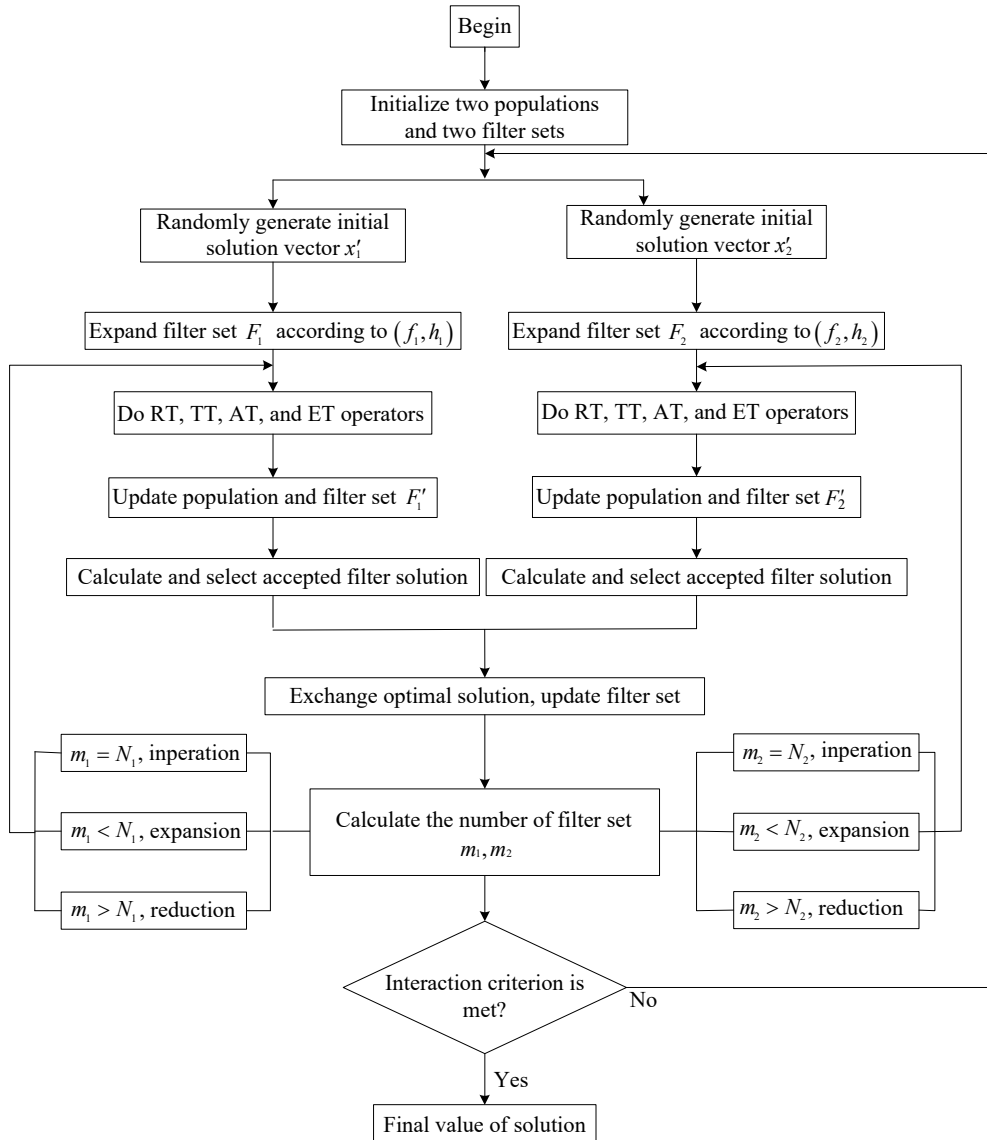


Fig. 2 Flowchart of the FCSTA.

Step 7 Calculation of the number of new filter sets. The number of new filter sets is calculated as m_1, m_2 . Throughout the processing, the number of the two filter sets must be remained unchanged. Taking S_1 as an example,

If $m_1 = N_1$, do the next iteration;

If $m_1 < N_1$, we randomly add $(N_1 - m_1)$ filters in the new filter set into the filter to keep the number unchanged;

If $m_1 > N_1$, according to the objective function f , we sort all filters. From the sequence of all filters, we choose N_1 optimal filters to compose the new filter set.

Step 8 Termination condition check. If the number of iteration meets the number of criterion, the algorithm terminates. Otherwise, it returns to Step 2 for a new round of iteration.

4.2 FCSTA as applied to DRPO problems

The FCSTA is used to solve the DRPO problems of different test cases. We used population S_1 to optimize the first objective function and constraints. The state vector is $x(t) = [V_1(t), V_2(t), \dots, V_{N_E}(t), P_{Gslack}(t)]^T$ and the control variable is $u(t) = [Q_{Gi}(t), k_{Ci}(t), k_{Ti}(t)]^T$. Similarly, population S_2 is to optimize the second objective function and constraints. Essentially, the algorithm deals with a population (chosen as 40) of similar such vectors and optimize their variables of the vector during the iteration. While implementing the FCSTA in the DRPO problem, the objective functions stated in Eqs. (1) and (7) are considered subject to the first constraints mentioned in Eqs. (2) and (6) and second constraints

presented in Eqs. (8)–(10). Finally, the values of control variables, the actual power loss, the TVDs, and the number of capacitor and transformer action times are taken as output from the program.

5 Simulation Results and Discussions

In this study, FCSTA was applied to Ward & Hale 6 bus, IEEE 14 bus, and IEEE 30 bus test power systems for the solution of a DRPO problem. For solving different test power systems problem, FCSTA algorithm and the two populations of FCSTA set the same parameters, which are shown in Table 1.

5.1 Test system 1: Ward & Hale 6 bus

Ward & Hale 6 bus is taken as the test system 1, which comprises six buses, two generators, two transformers, seven circuits, and two reactive power compensation capacitors^[17]. Five capacitor groups are set in buses 4 and 6, respectively, and the unitary capacity of each capacitor group is 0.01 and 0.011, respectively. The branches 3-4 and 5-6 connect the on-load voltage-regulating transformers (T4-3 and T6-5) with a range of 0.9 p.u to 1.1 p.u. The structure and parameters of system can be seen in Ref. [17].

To further and fairly comparing the results of this study to those of other researchers, we choose the interior-point method and Multi-Stage Solution Method (MSSM), which were used in Ref. [17], and the same parameters are used in the comparison methods.

To demonstrate the effectiveness of dealing with discrete variables, the lower and upper limits of control variables, system initial state, and the optimization results are shown in Table 2. It can be seen from Table 2 that Q_{G1} and Q_{G2} of three methods all conform to the constraints. The five capacitor groups are set in buses 4 and 6, which means when the reactive power is put into the system, the power loss can be reduced. The power loss of FCSTA is 0.0839 p.u, which is larger than the interior-point method, but less than the multi-stage solution method. The optimal power loss value of interior-point method is a theoretical value. When considering the discreteness of control variables, the

Table 2 Optimization results of the Ward & Hale 6 bus system. (p.u)

Control variable	Constraints		Initial value	Optimization results		
	Lower	Upper		TPM ^[17]	MSSM ^[17]	FCSTA
Q_{G1}	-0.2	1.0	1.050	0.4016	0.3775	0.3912
Q_{G2}	-0.2	1.0	1.097	0.1569	0.1836	0.1631
k_{C4}	0.0	5.0	0.000	5.0000	5.0000	5.0000
k_{C6}	0.0	5.0	0.000	5.0000	5.0000	5.0000
T4-3	0.9	1.1	1.025	0.9821	1.0000	0.9936
T6-5	0.9	1.1	1.100	0.9485	0.9750	0.9810
TVD	—	—	—	—	—	0.0498
Power loss	—	—	0.116	0.0888	0.0907	0.0839

result cannot be practically applied. Furthermore, the FCSTA method has a better accuracy than the multi-stage solution method in dealing with discrete variables. The FCSTA has a TVD of only 0.0498 p.u, which is lower than those of some other methods.

To verify the effectiveness of dealing with the action number constraints of control equipment, we also increase the capacitor groups to nine in buses 4 and 6 as in Ref. [17]. To verify the influence of reactive power control device action on load change, Ward & Hale case system load is divided into 24 h. Its change law is shown in Fig. 3.

The system power loss considering different switching operation limits is shown in Table 3. As the number of capacitor switching operations and adjustment space of reactive equipment increases, the optimization problem has a larger feasible region and the system power loss shows to gradually decline. It

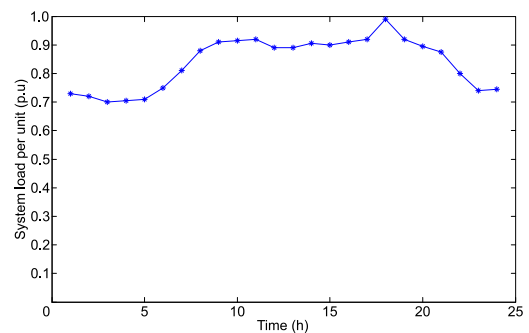


Fig. 3 Ward & Hale case system load.

Table 1 Parameters of FCSTA, STA, and PSO.

Parameter	Swarm size	Max number of iterations	Rotation factor α	Other factors $\beta, \gamma,$ and δ	Constant coefficient f_c	Communication Frequency (CF)	Primal dual parameter ϵ
FCSTA	40	100	1 to 10^{-4}	1	2	30	10^{-5}
STA	40	100	1 to 10^{-4}	1	2	30	10^{-5}
PSO	40	100	—	—	—	—	—

Table 3 System loss considering different switching operation limits.

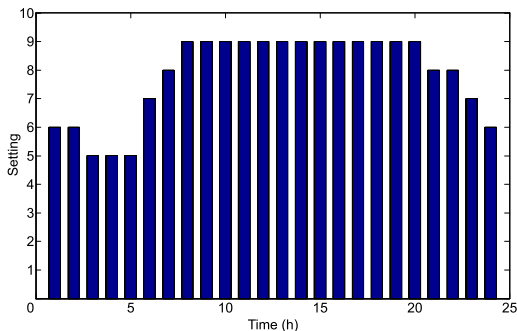
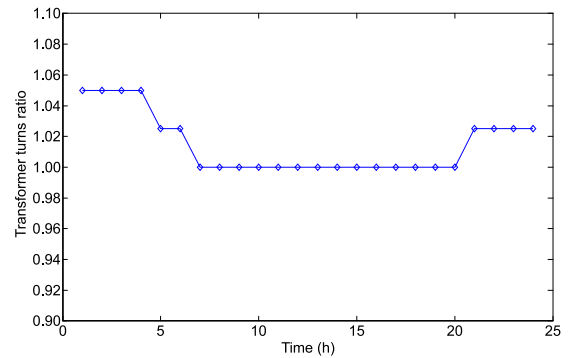
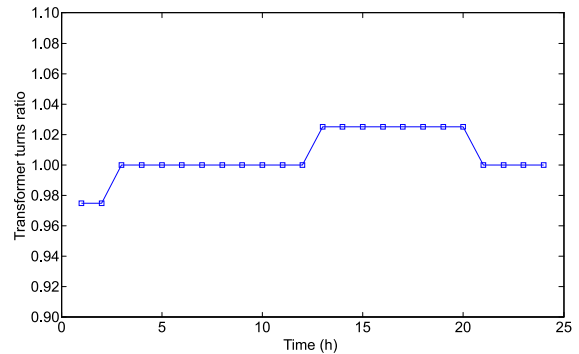
Number of switching operation limit $K_{C_{max}}$	Capacitor at bus 4		Capacitor at bus 6		System power loss (p.u)	
	MSSM	FCSTA	MSSM	FCSTA	MSSM	FCSTA
3	3	3	3	3	1.532 589 16	1.532 588 28
4	3	3	3	3	1.532 589 16	1.532 588 28
5	5	4	0	0	1.532 104 49	1.532 120 13
6	5	5	0	0	1.532 104 49	1.532 100 13
7	7	7	0	0	1.532 025 68	1.532 015 21
8	8	7	0	0	1.532 016 70	1.532 015 21

may be noted from the table that the proposed FCSTA and MSSM methods can effectively solve discrete variable problems and also meet the constraints of control equipment action number throughout the day. For any number of switching operation limits ($K_{C_{max}}$), the system power loss values of FCSTA are all less than those of MSSM. When $K_{C_{max}} \geq 5$, the capacitors of bus 6 run the whole day. This means that the power loss can be decreased if the reactive power is injected.

When $K_{C_{max}} = 8$ and $K_{T_{max}} = 6$, the settings of capacitors for a whole day at bus 4 are presented in Fig. 4, and the profiles of transformers for a day are presented in Figs. 5 and 6. It can be seen from Fig. 4 that the settings of capacitors all meet the nine groups constraints and the switching time does not exceed 8th-hour limit. From Figs. 5 and 6, the two transformers turning ratio are all in the range of 0.90 p.u to 1.10 p.u.

The results of this DRPO problem optimized by FCSTA, STA, and PSO are shown in Table 4.

The power loss and TVD curves for one day are given in Figs. 7 and 8, respectively. The obtained results of control variables by the three algorithms all meet the constraints. The Average TVD (ATVD) of FCSTA is 0.0498 p.u, which is lower than that of STA (0.0535 p.u) and PSO (0.0560 p.u). The Average Power Loss (APL) optimized by FCSTA is lower than that of STA and PSO. Moreover, the power losses and all the TVDs obtained by FCSTA for the whole day are the lowest (Figs. 7 and 8).

**Fig. 4** Whole day setting capacitor.**Fig. 5** Profile of transformer T4-3.**Fig. 6** Profile of transformer of T6-5.**Table 4** Optimization results of the Ward & Hale 6 bus system. (p.u)

Control variables	Constraints		Initial value	Optimization results		
	Lower	Upper		STA	PSO	FCSTA
Q_{G1}	-0.20	1.00	1.050	0.5123	0.4561	0.3912
Q_{G2}	-0.20	1.00	1.097	0.5678	0.5412	0.1631
k_{C4}	0.00	5.00	0.000	5.0000	5.0000	5.0000
k_{C6}	0.00	5.00	0.000	5.0000	5.0000	5.0000
T4-3	0.90	1.10	1.025	0.1031	1.1201	0.9936
T6-5	0.90	1.10	1.100	0.9982	0.1021	0.9810
Average TVD	—	—	—	0.0535	0.0560	0.0498
Average power loss	—	—	0.113	0.0963	0.0908	0.0839

5.2 Test system 2: IEEE 14-bus

The IEEE 14-bus system^[18] is used to test the performance of the proposed technique. It is assumed that each bus has the same load profile with a constant

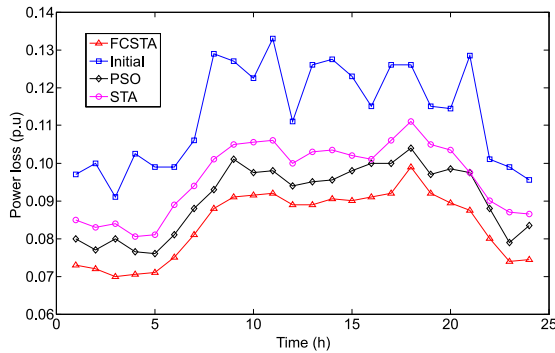


Fig. 7 Power loss curves for one day.

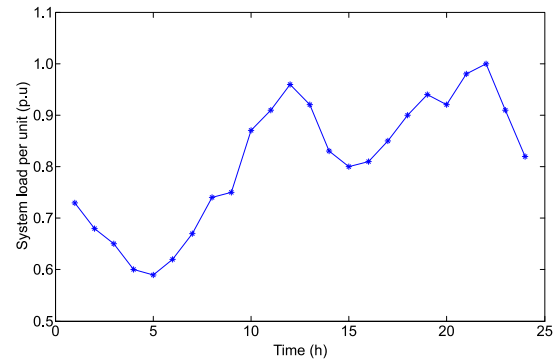


Fig. 9 IEEE 14 case system load per unit profile.

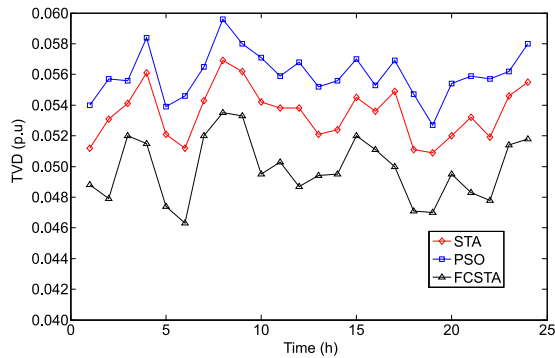


Fig. 8 TVD curves for one day.

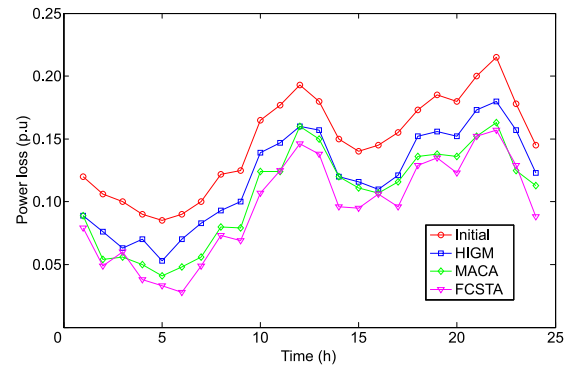


Fig. 10 Power loss curves for one day.

power factor. Shunt capacitor sets are allocated at buses 9, 10, 13, and 14. At bus 9, there are four shunt capacitors with a rating of 0.2 p.u. each. The lower and upper limits of all bus voltage are 0.90 p.u and 1.10 p.u, respectively. At buses 10, 13, and 14, four shunt capacitors are with a rating of 0.02 p.u each. There are three transformers in the system, with a range of 0.90 p.u to 1.10 p.u. The step size of transformer tap ratios is 0.01 p.u. And the reactive power control equipment are allowed to switch ten times in a day. The on-load tap changers are allowed to switch three times in adjacent time intervals.

To further compare the results of this study to those of other researchers, we choose the multi-population Hybrid Immune Genetic Method (HIGM)^[25] and Multiple Ant Colony Algorithm (MACA)^[18] as the comparison methods. The profile of load per unit is showed in Fig. 9. The initial APL for the whole day is 0.1466 p.u.

The obtained results are separately compared with those of 24 h MACA and HIGM reactive power optimization results. The power loss curves (Fig. 10) show that all these three methods can reduce the power loss. The power losses obtained by FCSTA for the whole day are the least, except for those obtained at the 3rd and 23rd hours (Fig. 10). The switching number of shunt capacitors at buses C9, C10, C13, and C14, settings of transformers T4-7, T4-9, and T5-6 for the whole day, and APL and the ATVD are all shown in Table 5. It can be observed that the MACA, HIGM, and FCSTA all meet the switching number constraints. The control variable switching number of FCSTA is close to that of HIGM. But the whole day APL of FCSTA is 0.0958 p.u, which is lower than that of HIGM (0.1192 p.u). Moreover, the whole day ATVD of FCSTA is 0.0512 p.u, which is the least. These results show that the FCSTA is efficient and has superior optimization ability.

Table 5 Results of switching number.

Method	Switching number of the whole day								APL (p.u)	ATVD (p.u)
	C9	C10	C13	C14	T4-7	T4-9	T5-6	Total		
MACA	6	5	6	8	6	6	9	46	0.1053	0.0587
HIGM	5	7	4	9	6	4	6	41	0.1192	0.0634
FCSTA	4	9	7	3	5	5	6	39	0.0958	0.0512

The profile of discrete control variables are shown in Fig. 11. The settings of shunt capacitors at buses 9, 10, 13, and 14 are showed in curves C9, C10, C13, and C14, respectively. The settings of transformers are T4-7, T4-9, and T5-6. Figure 11a shows the profile of C9, C10, C13, and C14. Figure 11b shows the profile of T4-7, T4-9, and T5-6.

The results of IEEE 14 with voltage are given in Table 6. All the large peak values after optimization exceed 0.9 p.u., and all the small peak values after optimization exceed 1.0 p.u., which are observed from Table 6. These optimized results by MACA, HIGM, and FCSTA are all

meet the requirements.

The obtained optimal values of control variables from the proposed FCSTA method are compared with those of PSO and STA, using the same DRPO model. The resulting APL, ATVD, and optimal control variables are presented in Table 7. The simulation results clearly show that the obtained power loss from the proposed FCSTA approach is 0.0958 p.u., which is less than that of the PSO average (0.1103 p.u) by 0.0145 p.u and that of STA by 0.0063 p.u. Thus, PSO yielded the highest result. The value of TVD improved for the FCSTA-based approach and is reported as 0.0512 in Table 7, which is lower than STA and PSO.

5.3 Test system 3: IEEE 30-bus

The IEEE 30-bus^[26] network consists of 41 branches, 6 generator-buses, 21 load-buses, and 9 shunt compensators (C12, C15, C18, C19, C21, C24, C26, C28, and C30). Four branches (6, 9), (6, 10), (4, 12), and (28, 27) are under load tap setting transformer branches, and are presented as T6-9, T6-10, T4-12, and T28-27, where each transformer has a tap-changer with

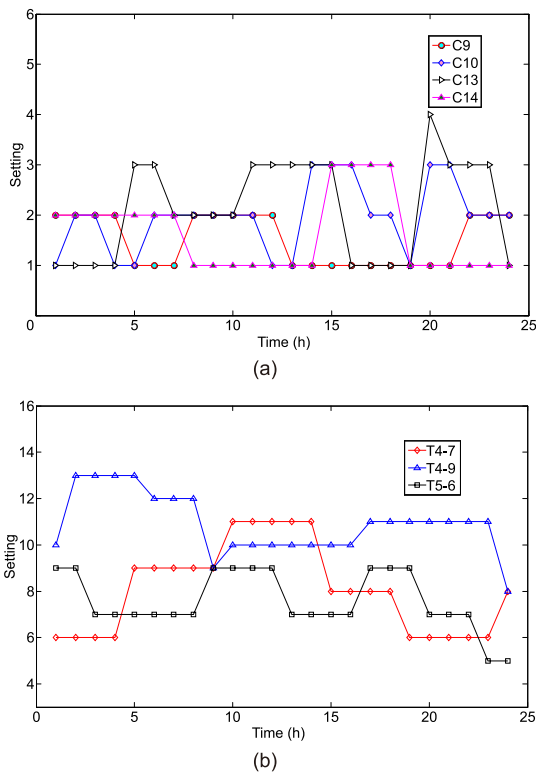


Fig. 11 Profile of discrete control variables by dynamic reactive power optimization.

Table 7 Comparison of simulation results for IEEE 14. (p.u)

Control variable	Limit		Algorithm		
	Lower	Upper	STA	PSO	FCSTA
V_{g1}	0.90	1.10	0.996	1.021	0.989
V_{g2}	0.95	1.05	0.972	1.015	0.986
V_{g3}	0.95	1.05	1.034	1.028	1.001
V_{g6}	0.95	1.05	0.965	0.981	1.010
V_{g8}	0.95	1.05	1.003	0.954	0.962
T4-7	0.90	1.10	1.082	1.035	1.041
T4-9	0.90	1.10	0.996	0.997	0.983
T5-6	0.90	1.10	1.042	0.961	1.036
Average TVD	—	—	0.0536	0.0521	0.0512
Average loss	—	—	0.1021	0.1103	0.0958

Table 6 Results of IEEE 14 with voltage. (p.u)

Buses	Large peak values				Small peak values			
	Initial	MACA	HIGM	FCSTA	Initial	MACA	HIGM	FCSTA
4	0.8460	0.9395	0.9237	0.9541	0.9969	1.0290	1.0341	1.0172
5	0.8466	0.9304	0.9721	0.9213	1.0020	1.0314	1.0213	1.0108
7	0.9070	1.0538	0.9910	0.9782	1.0351	1.0477	1.0765	1.0531
9	0.8356	1.1534	1.0910	1.0124	1.0116	1.0678	1.0432	1.0329
10	0.8446	1.1310	1.0010	0.9981	1.0111	1.0579	1.0721	1.0210
11	0.9412	1.0887	0.9987	0.9543	1.0351	1.0590	1.0218	1.0923
12	0.9974	1.0418	0.9870	1.0201	1.0459	1.0516	1.0810	1.0117
13	0.9624	1.0478	1.0321	1.0134	1.0360	1.0470	1.0112	1.0379
14	0.8025	1.0714	0.9623	0.9798	0.9946	1.0426	1.0219	1.0218

a range of 0.90 p.u. to 1.10 p.u. The lower and upper limits of all bus voltages are 0.90 p.u. and 1.10 p.u. And the reactive power control equipments are allowed to switch ten times per day. The on-load tap changers are allowed to switch three times in adjacent time intervals.

To further compare the results of this work with those of other researchers, we also choose the multi-population HIGM^[25] and MACA^[18] as the comparison methods. The initial APL for the whole day is 0.307 p.u.

The obtained results are separately compared with those of the 24h MACA and HIGM reactive power optimization results. The power loss curves (Fig. 12) show that all these three methods can reduce the power loss. It can be observed in Fig.12 that the power losses obtained by FCSTA for the whole day are the least, except for those at the 1st and 2nd hours. The switching number of shunt capacitors at buses C9, C10, C13, and C14, settings of transformers T4-7, T4-9, and T5-6 on the whole day, APL, and ATVD are all shown in Table 8. It can be observed from Table 8 that the MACA, HIGM, and FCSTA meet all switching number constraints. The control variable switching number of FCSTA is close to that of MACA. But the whole day APL of FCSTA is 0.1900 p.u., which is lower than that of MACA (0.2060 p.u). Moreover, the whole day ATVD of FCSTA is 0.0568 p.u., which is also the least. These results also show the efficiency and superior optimization ability of the FCSTA.

The TVD curves (Fig. 13) show that the TVD

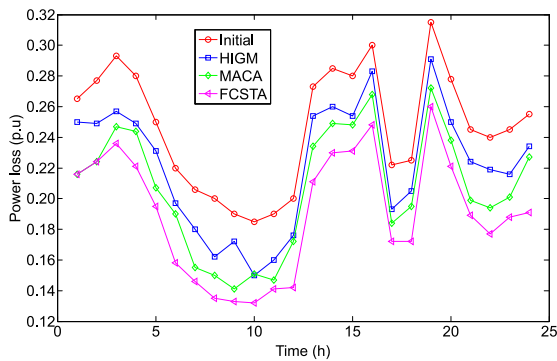


Fig. 12 Power loss curves for one day.

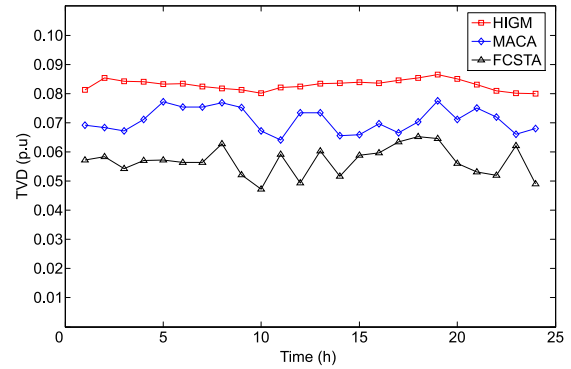


Fig. 13 TVD curves for one day.

obtained by HIGM for the whole day has the minimum fluctuations, but its TVD are all larger than those of the other two methods. The FCSTA obtained the lowest TVD.

The obtained optimal values of control variables from the proposed FCSTA method are compared with those of PSO and STA, using the same DRPO model. The resulting APL, ATVD, and optimal control variables are presented in Table 9. It can be observed from Table 9 that the obtained power loss from the proposed FCSTA approach is 0.190 p.u., which is less than that of the PSO average results (0.231 p.u) by 0.041 p.u and that of STA (0.252 p.u) by 0.062 p.u. The value of TVD improved

Table 9 Comparison of simulation results for IEEE 30. (p.u)

Control Variable	Limit		Algorithm		
	Lower	Upper	STA	PSO	FCSTA
V_1	1.00	1.10	1.021	1.009	1.043
V_2	1.00	1.10	1.002	1.011	1.002
V_5	1.00	1.10	1.023	1.023	1.012
V_8	1.00	1.10	1.034	1.012	1.051
V_{11}	1.00	1.10	1.028	1.074	1.002
V_{13}	1.00	1.10	1.008	0.943	1.017
T6-9	0.90	1.10	0.910	1.032	0.951
T6-10	0.90	1.10	1.024	0.917	1.010
T4-12	0.90	1.10	1.012	1.035	0.986
T28-27	0.90	1.10	0.965	1.001	0.991
Average power loss	—	—	0.252	0.231	0.190
Average TVD	—	—	0.0617	0.0702	0.0568

Table 8 Results of switching number.

Method	Switching number of the whole day														APL (p.u)	ATVD (p.u)
	C12	C15	C18	C19	C21	C24	C26	C28	C30	T6-9	T6-10	T4-12	T28-27	Total		
MACA	3	5	8	6	5	6	7	5	6	4	3	3	5	66	0.2060	0.0709
HIGM	4	7	8	9	4	7	6	8	5	5	6	7	5	81	0.2220	0.0830
FCSTA	2	5	7	3	5	5	6	4	5	3	2	3	3	53	0.1900	0.0568

for the FCSTA-based approach is reported as 0.0568, which is lower than those of STA and PSO.

6 Conclusion

To solve the large-scale, multi-period, and strongly coupled nonlinear mixed-integer programming DRPO problem, a two-objective DRPO optimized by FCSTA is proposed. The first objective is to minimize the real power loss and the TVDs, and the second objective is to minimize incremental system loss. The effectiveness of the proposed FCSTA is demonstrated by a 24-hour test on Ward & Hale 6 bus, IEEE 14 bus, and IEEE 30 bus test power systems.

The optimal results of FCSTA are better than those of the HIGM and MACA methods. In addition, FCSTA can obtain better optimal results than PSO and STA. These all indicate the superiority of the proposed approach in solving DRPO problems. Therefore, it can be concluded that the two-objective DRPO optimized by FCSTA is a useful technique for dynamic reactive power optimization of an interconnected power sources. Thus, the FCSTA algorithm may be recommended as a very promising tool for solving some more complex engineering optimization problems.

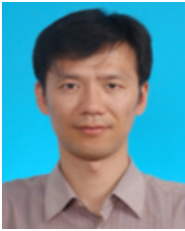
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References

- [1] D. S. Kirschen and H. P. Van Meeteren, MW/voltage control in linear programming based optimal power flow, *IEEE Transactions on Power Systems*, vol. 3, no. 2, pp. 481–489, 1988.
- [2] K. Y. Lee, Y. M. Park, and J. L. Ortiz, A united approach to optimal real and reactive power dispatch, *IEEE Transactions on Power Apparatus and Systems*, vol. 104, no. 5, pp. 1147–1153, 1985.
- [3] V. H. Quintana and M. Santos-Nieto, Reactive power-dispatch by successive quadratic programming, *IEEE Transactions on Energy Conversion*, vol. 4, no. 3, pp. 425–435, 1989.
- [4] S. Granville, Optimal reactive dispatch through interior point methods, *IEEE Transactions on Power Systems*, vol. 9, no. 1, pp. 136–146, 1994.
- [5] W. Yan, S. Lu, and D. C. Yu, A hybrid genetic algorithm-interior point method for optimal reactive power flow, *IEEE Transactions on Power Systems*, vol. 21, no. 3, pp. 1163–1209, 2006.
- [6] W. H. E. Liu, A. D. Papalexopoulos, and W. F. Tinney, Discrete shunt controls in a Newton optimal power flow, *IEEE Transactions on Power Systems*, vol. 17, pp. 1509–1518, 1992.
- [7] J. Y. Park, S. R. Nam, and J. K. Park, Control of a ULTC considering the dispatch schedule of capacitors in a distribution system, *IEEE Transactions on Power Systems*, vol. 22, no. 2, pp. 1349–1356, 2007.
- [8] M. B. Liu, C. M. Zhu and K. L. Qian, Dynamic reactive-power optimization algorithm incorporating action number constraints of control devices, (in Chinese), *Proceedings-Chinese Society of Electrical Engineering*, vol. 24, no. 3, pp. 34–40, 2004.
- [9] J. Y. Park, J. M. Sohn, and J. K. Park, Optimal capacitor allocation in a distribution system considering operation costs, *IEEE Transactions on Power Systems*, vol. 24, no. 1, pp. 462–468, 2009.
- [10] G. B. Liu, W. T. Yan, and W. B. Zhang, Optimization and dispatching method of dynamic reactive power in distribution network with distributed generators, (in Chinese), *Automation of Electric Power System*, vol. 39, no. 15, pp. 49–54, 2015.
- [11] B. Bhattacharyya and S. Raj, PSO based bio inspired algorithms for reactive power planning, *International Journal of Electrical Power & Energy Systems*, vol. 74, pp. 396–402, 2016.
- [12] B. Bhattacharyya and R. Babu, Teaching learning based optimization algorithm for reactive power planning, *International Journal of Electrical Power & Energy Systems*, vol. 81, pp. 248–253, 2016.
- [13] K. Ayan and U. Kürs, Artificial bee colony algorithm solution for optimal reactive power flow, *Applied Soft Computing*, vol. 12, no. 5, pp. 1477–1482, 2012.
- [14] R. P. Singh, V. Mukherjee, and S. P. Ghoshal, Optimal reactive power dispatch by particle swarm optimization with an aging leader and challengers, *Applied Soft Computing*, vol. 29, pp. 298–309, 2015.
- [15] J. Li, T. Q. Liu, X. Y. Li, and D. P. Xing, Application of improved particle swarm-tabu search algorithm in multi-objective reactive power optimization, (in Chinese), *Electric Power Automation Equipment*, vol. 34, no. 8, pp. 71–78, 2014.
- [16] G. M. Xie, X. J. Guo, and J. G. Liu, Multi-objective firefly algorithm for power system reactive optimization, (in Chinese), *Journal of Liaoning Technical University (Natural Science)*, vol. 35, no. 4, pp. 444–448, 2016.
- [17] T. Sun, P. Zou, and Z. F. Yang, A multi-stage solution approach for dynamic reactive power optimization, (in Chinese), *Power System Technology*, vol. 40, no. 6, pp. 1804–1810, 2016.
- [18] X. Zhou, H. Zhu, and A. Ma, Multi-objective dynamic reactive power optimization based on multi-population ant colony algorithm, (in Chinese), *Power System Technology*, vol. 36, no. 7, pp. 231–236, 2012.
- [19] T. Ding, Q. L. Guo, and R. Bo, Two-stage heuristic-correction for dynamic reactive power optimization based on relaxation-MPEC and MIQP, (in Chinese), *Proceedings of CSEE*, vol. 34, no. 13, pp. 2100–2107, 2014.
- [20] R. Fletcher, S. Leyffer, and P. L. Toint, A brief history of filter methods, *SIAM SIAG/OPT Views-and-News*, vol. 18, no. 1, pp. 2–12, 2006.

- [21] X. J. Zhou, C. H. Yang, and W. H. Gui, Initial version of state transition algorithm, in *Proc. 2nd Int. Conf. Digital Manufacturing & Automation*, Zhangjiajie, China, 2011, pp. 644–647.
- [22] X. J. Zhou, C. H. Yang, and W. H. Gui, State transition algorithm, *Journal of Industrial and Management Optimization*, vol. 8, no. 4, pp. 1039–1056, 2012.
- [23] C. H. Yang, X. L. Tang, X. J. Zhou, and W. H. Gui, A discrete state transition algorithm for traveling salesman problem, *Control Theory & Applications*, vol. 30, no. 8, pp. 1040–1046, 2013.
- [24] R. Fletcher, S. Leyffer, and P. L. Toint, On the global convergence of a filter—SQP algorithm, *SIAM Journal on Optimization*, vol. 13, no. 1, pp. 44–59, 2002.
- [25] F. Liu, C. Y. Chung, K. P. Wong, W. Yan, and G. Y. Xu, Hybrid immune genetic method for dynamic reactive power optimization, in *Proc. 2006 Int. Conf. Power System Technology*, Chongqing, China, 2006, pp. 1–6.
- [26] K. Ayan, Artificial bee colony algorithm solution for optimal reactive power flow, *Applied Soft Computing*, vol. 12, no. 5, pp. 1477–1482, 2012.



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