TSINGHUA SCIENCE AND TECHNOLOGY
ISSN 1007-0214 13/14 pp965–975
DOI: 10.26599/TST.2022.9010037
Volume 28, Number 5, October 2023

Autonomous Vehicles Testing Considering Utility-Based Operable Tasks

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Abstract: Virtual simulation testing of Autonomous Vehicles (AVs) is gradually being accepted as a mandatory way to test the feasibility of driving strategies for AVs. Mainstream methods focus on improving testing efficiency by extracting critical scenarios from naturalistic driving datasets. However, the criticalities defined in their testing tasks are based on fixed assumptions, the obtained scenarios cannot pose a challenge to AVs with different strategies. To fill this gap, we propose an intelligent testing method based on operable testing tasks. We found that the driving behavior of Surrounding Vehicles (SVs) has a critical impact on AV, which can be used to adjust the testing task difficulty to find more challenging scenarios. To model different driving behaviors, we utilize behavioral utility functions with binary driving strategies. Further, we construct a vehicle interaction model, based on which we theoretically analyze the impact of changing the driving behaviors on the testing task difficulty. Finally, by adjusting SV's strategies, we can generate more corner cases when testing different AVs in a finite number of simulations.

Key words: Autonomous Vehicle (AV); intelligence testing; operable tasks

1 Introduction

Recently, due to advances in cognitive planning and learning-based algorithms, the development of artificial intelligence systems has grown rapidly[1, 2]. Autonomous Vehicle (AV), as a representative AI system, is believed to achieve mass production at this stage.

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Manuscript received: 2022-05-31; revised: 2022-08-18; accepted: 2022-09-14

The prevailing view is that AVs should learn human strategies to continuously improve their intelligence. Obviously, a learned driving strategy, which is used to perform driving tasks in AVs, must be well tested before it can be deployed to AVs[3]. To achieve this, researchers have mainly focused on how to design the safety-related testing tasks that AVs need to complete[4–7]. These tasks are combined with critical scenarios that may be impossible for existing AVs.

A question that cannot be escaped is: "how to design testing tasks and sample critical scenarios?"

To find an answer, Zhao et al.[8] proposed a testing method of inspiring, i.e., accelerated evaluation based on naturalistic driving datasets. The method significantly improves testing efficiency by finding the distribution of scenario model parameters that fit the hypothesis[9]. Based on their study, Feng et al.[10] and Yan et al.[11] further improved the testing efficiency by 10 000 times.

Despite the significant improvement in testing efficiency, current approaches still suffer from two shortcomings. First, the testing scenarios sampled by the fixed vehicle model cannot be guaranteed to be challenging for all AVs. Second, we cannot adjust the testing scenarios for different AVs. It means that in every test, we need to sacrifice huge computing power and time to search for challenging scenarios.

To this end, Feng et al.[12] improved the accelerated evaluation by proposing an adversarial-based approach. They search for challenging scenarios by changing the behavior of the vehicles through Reinforcement Learning (RL) methods. Though using learning-based algorithms like RL can dramatically reduce empirical error in searching for a critical scenario[13, 14], we need to continue asking: "how to ensure that the testing results provide a strong reference for the development of AV?"

It should be noted that these solutions still fail to solve the lack of operability of the scenarios[15]. Additionally, it is also difficult to explain the specific reasons for the failure of the AV. We cannot make reasonable predictions and interpretations about the specific capabilities of AVs, the boundaries of their capabilities, and the scenarios, under which AVs will come across a challenge.

Such problems prompt the need for setting interpretable and operable testing tasks. It can help researchers explain why AVs fail when performing testing tasks and provide a strong basis for subsequent updates to the driving strategy of AVs[16]. To solve this, we notice that many factors in the real environment can easily lead to the inability of AVs to complete the task, such as sudden changes in weather, imperfect road facilities, unpredictable Surrounding Vehicle (SV) behaviors, etc. It is almost impossible to construct challenging scenarios that affect the safety of AVs by traversing all these potential factors, which is a well-known NP-hard problem[17]. However, researchers have made many useful attempts on finding some critical factors. Wang et al.[18] pointed out that the interaction between SVs and AVs can bring driving risks to AVs. Later, Zhao et al.[19] explored the difference between vehicles with different driving strategies in the natural environment when interacting with others.

On these bases, in this paper, we propose an approach for testing AVs by manipulating testing tasks to find more challenging scenarios. We focus on the changes brought by the driving strategies and behaviors of SVs on the AV. These changes also result in different task difficulties. For brevity and clarity, we define the number of challenging events in a testing scenario as the task difficulty. Besides, the challenging events encountered by the AV are reflected by the behavior of the AV. To model the driving behavior of SVs, we give behavioral utility with binary strategies. Though there are many approaches to modeling vehicle behaviors[20, 21], the behavioral utility function has two advantages for AV testing. First, given the high complexity of behavior, the utility function can model different behaviors by emphasizing the results of the vehicle's choices under different conditions. Second, the behavioral utility function has better algorithmic transparency compared to black-box algorithms such as neural networks. The designing tasks can be interpretable and we can better explain the evaluation results.

Based on the behavioral utility model, we construct a vehicle interaction utility model to theoretically demonstrate that different strategies of SVs can affect AV behavior and thus the difficulty of the testing scenario for AV. In addition, we use a typical lane change scenario as an initial scenario for the simulation. The overall framework of our method is shown in Fig. 1.

To better describe our approach, the paper is organized

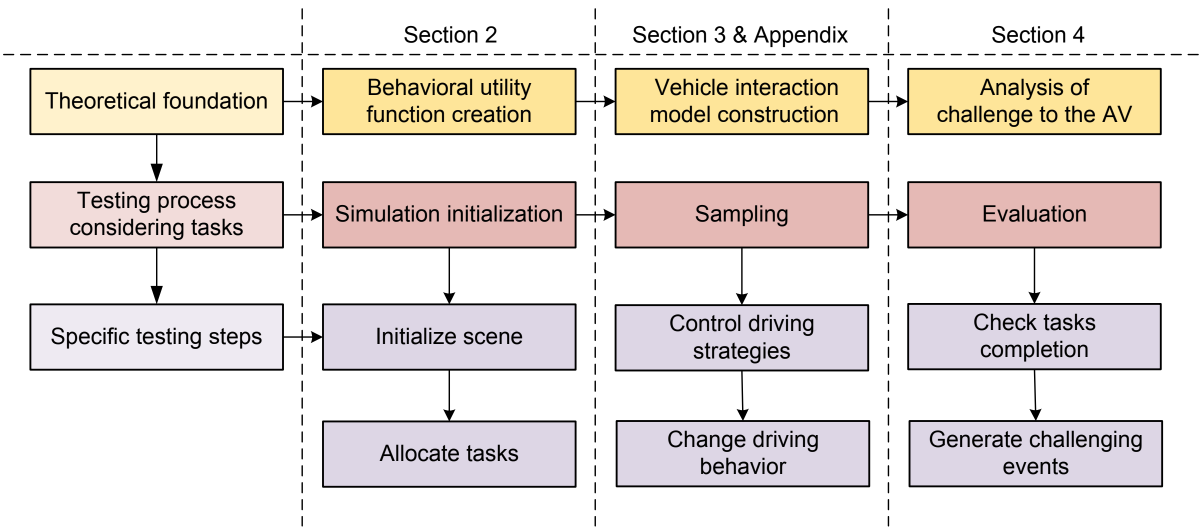

Fig. 1 Overall framework of the method.

as follows: Section 2 gives the related work on behavioral utility function and intelligence testing for AV; in Section 3, the testing tasks are explained, as well as the construction of the vehicle behavioral utility function with binary driving strategies; Section 4 models vehicle interaction model; Section 5 validates the proposed method; and Section 6 concludes this paper.

2 Related Work

2.1 Behavioral utility function

The utility function, which compares different choices, was first applied in economics to measure consumer welfare or satisfaction[22, 23]. The application of the utility function in intelligent transportation systems is equally widespread. In macroscopic traffic simulation, for example, we employ them for traffic volume allocation analysis, while in microscopic traffic simulation they are used for driver behavior analysis, etc.[24, 25]

The successful construction of behavioral utility functions to analyze driving behavior dates back to 2003, when Toledo et al.[26] constructed integrated lane-changing behavior based on utility function. Altendorf and Flemisch[27] further investigated the behavioral utility function and argued that the advantage of the behavioral utility function over other behavioral models is that it highlights the driver's choice to distinguish between different types of driver behavior. They also gave a basic behavioral utility function that considers only the vehicle state information, including position and speed, that is readily available to the ego vehicle and other vehicles around it. Besides, Li et al.[2] pointed out that in addition to the vehicle state information, refined traffic information, such as the average speed of different lanes, the time interval between surrounding vehicles, etc., is also beneficial for a driver to make the correct behavior when changing lanes in reality.

Based on this, we used the behavioral utility function in Ref. [28] to construct the behavior of SVs in autonomous vehicle testing, emphasizing the decisive influence of behavioral utility on whether SVs choose to make a lane change at a certain time, as a test of the intelligence of the AV when AV faces heterogeneous SVs. However, the shortcoming is that considering the behavioral utility of a single SV is not enough to simulate the dynamic interaction between vehicles.

2.2 Autonomous vehicle testing

To sample critical testing scenarios, researchers have worst-case scenario generation methods. For example, Ma and Peng[29] generated scenarios that are most likely to cause AV rollover or emergent braking by introducing game theory; Kou[30] obtained the most challenging scenario by introducing rolling time-domain optimization methods to test the safety performance of AV. However, extreme scenarios are likely to be rare in nature, which are not equivalent to natural scenarios.

The natural testing scenario generation method was proposed in Refs. [7] and [31], which is based on large Naturalistic Driving Data (NDD). However, the disadvantage of the method is that it cannot solve the serious inefficiency existing in on-road testing. For this reason, Zhao et al.[8, 9] and Zhao[32] proposed an accelerated evaluation method. Accelerated evaluation is to select some parametric scenario model as a priori information and learn the distribution of model parameters in the dataset. Scenarios with low sampling frequency can be better obtained by Importance Sampling (IS).

However, the accelerated evaluation relies too much on the priori information. Feng et al.[10] improved the accelerated evaluation by proposing an adversarial-based RL method to help search for critical testing scenarios. The method constructs a natural adversarial driving environment, which improves the testing efficiency by 10 000 times while guaranteeing the same accident rate as in a natural dataset. However, the problem of poor interpretability of RL still cannot be avoided which is not conducive to the evaluation of AV.

Li et al.[3] proposed that testing the intelligence of AV could be accomplished by constructing suitable testing tasks. They considered the tasks as the connection between two current methods, i.e., scenario-based testing and function-based testing. However, it is still worth thinking about how to set up operable testing tasks that sample more challenging scenarios while satisfying the need for interpretability.

3 Problem Formulation

3.1 Testing tasks

Testing tasks are defined as activities that AV needs to complete within a limited time frame in a testing scenario. According to the definition, those activities should be testable as an indispensable link between the testing environment and AV and is a prerequisite for the evaluation[15]. The PAC testing theory[17] also points out that by sampling several tasks, a complete evaluation of the AVs can be made.

Thus, we can split up testing tasks to fit different testing requirements. Before testing, we need to set the testing requirements, which can be a single type of intelligence, such as intelligence safety, task execution efficiency, comfort level, and other capabilities. Then, the set of testing tasks can be represented as several task compositions, i.e., $\Psi = \{\Psi_1, \Psi_2, \ldots, \Psi_n\}$, while each task can be divided into several subtasks in a limited time frame with a defined space, i.e., $\psi_i = \{\psi_{i1}, \psi_{i2}, \ldots, \Psi_{in}\}$, as is shown in Fig. 2. The testing requirement differs from testing modules. Take safety testing of AV as an example, there are mainly two modules to test the intelligence of AV considering safety, i.e., safety functionality testing and safety performance testing. In safety functionality testing, the testing task can be decomposed into testing different modules of AV, such as the sensing module, decision module, control module, etc. Different subtasks are designed for different functionality modules. In safety performance testing, the testing task can be divided into subtasks used to examine safety-related performance, such as recognizing traffic lights, deciding whether to change lanes, and following the car ahead. In this paper, we focus on the safety performance testing of AVs.

3.2 Behavioral utility based on driving strategies

To generate utility-based operable tasks, we set semantic driving strategies and utilize these strategies to describe various SV's driving behaviors. Further, we focus on characterizing the heterogeneous output of the behavioral utility function by adjusting the hyperparameters in utility functions.

To be specific, the behavioral utility function takes the external environment and its own parameter design as inputs and outputs the utility value of the vehicle's behavior at that moment. We assume that the inputs to the utility function to be a set of independent factors: $x_1, x_2, \ldots, x_n$. As for any participant $V^{(i)}$, we define U_i as its behavior utility. We consider the behavioral utility as a weighted sum of these independent factors, then the basic formula of the utility function can be written as

$$U_i = \sum_{k=1}^{n} \alpha_{k,i} x_k \tag{1}$$

where $\alpha_{k,i}$ is the weight component.

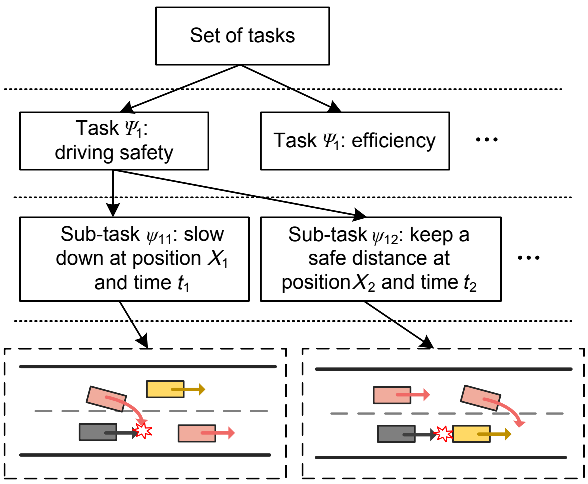

Fig. 2 Assignment of the testing tasks.

The parameters of the function can be obtained by training through learning-based methods or set by expert experience. The different parameters reflect the rich behavioral characteristics of the vehicles. Due to the space limitation of this paper, we will explain the design of our behavioral utility function in detail in a subsequent paper.

The driving strategies of human drivers can generally be classified as aggressive or conservative based on the observations[19]. We consider that the SV behavior is governed by binary strategies, i.e., aggressive strategy and conservative strategy, to demonstrate the "internal realism" of the vehicle. The aggressive driving strategy is referred to as a competitive driving strategy. Under this strategy, traffic participants usually focus more on improving their driving gains and consider others as rational traffic participants[19]. The conservative driving strategy is referred to as the defensive driving strategy. The core task of conservative strategy is to "prevent all potential danger against uncertaintie". Thus, we can define the combination of operable binary driving strategies as

$$\text{strategy\_set} = \{\text{aggressive},\ \text{conservarive}\} \tag{2}$$

Driving behavior based on different driving strategies can be controlled by modulating the output through a family of hyperparameters, which is $H = \{h_1, h_2, \ldots, h_m\}$, $m \in \mathbf{Z}^+$. For example, we consider the utility functions of lane-change behavior. Referring to the Ref. [26], the decision on whether to change lanes and the distance after changing lanes are important characterization quantities for measuring vehicle safety, which also reflect different strategies. From this, we set h_α to be the probability of choosing different lanes after receiving input from the current environment. By controlling h_α, we can construct different lane-change behavior based on the above two strategies.

3.3 Testing metrics and evaluation

The two main types of testing metrics are Boolean type and numerical type[3]. In intelligent safety testing, we choose the Boolean type for determining whether the vehicle successfully completes each assigned task within the appropriate spatial-temporal range. In this paper, we set the threshold value of Time To Collision (TTC) to determine the value of the Boolean quantity. TTC, as the most commonly used safety performance metric, can effectively reflect the safety level of the AV in the spatial-temporal range. If the TTC exceeds the threshold, there will be a hazardous event and the Boolean quantity $B = 0$, otherwise, $B = 1$.

According to the above, the testing task difficulty varies from vehicle to vehicle, which requires us to continuously adapt the operable tasks based on the evaluation results to find more targeted corner cases. The Appendix theoretically demonstrates adapting the behavior of the SVs can find more corner cases.

Based on the safety performance metric considered in this paper, i.e., collisions, we define the challenging event (e.g., a collision) as A. Then, we set the probability of a challenging event occurring in the testing scenario O_T as

$$P\left(A \mid O_T\left(\text{Scene}^t, \Psi, V, C, M, E\right)_{t \in T}\right) \tag{3}$$

where Scene^t is the testing scene at time t, V means vehicle under testing, $C = \{C_i\}$, $i = 2, 3, \ldots, m$, is the set of preset constants that reflect SVs' driving strategies, M means the testing metric, E shows the testing evaluation to the AV used to adjust the tasks[28], and T is the duration of the scenario.

4 Vehicle Interaction Behavioral Utility Modeling

We construct a vehicle interaction behavioral utility to show the interactions between the AV and the SVs, which is also used for analyzing the task difficulty.

To describe an interaction that is more reflective of the drivers' psychology, we depict the correlation of driving strategies between vehicles. We assume that AV cannot foresee danger coming from SVs until SVs make dangerous movements. This assumption matches the reality where drivers usually find it difficult to distinguish the driving strategies of other drivers until they make movements that reflect their strategies. We also do not consider that the collaboration between vehicles, i.e., traffic participants are not influenced by others or roadside, they make decisions and plane routes independently, based on their perceptual information.

To facilitate the analysis, we set the AV under test as $V^{(1)}$, and SVs as $V^{(2)}, V^{(3)}, \ldots, V^{(m)}$. Assuming that these influences x_k constitute a Gaussian distribution. In addition, these influences are measurable on a standard scale, i.e., they have zero mean and unit variance. Then the prior distribution of the behavioral utility of the vehicle $V^{(i)}$ is

$$p\left(U_i\right) \propto \phi\left(\frac{U_i}{\sqrt{\sum_{k=1}^{n} \alpha_{k,i} x_k}}\right) \tag{4}$$

Then the behavioral utility of all traffic participants in the test scenario can be formed as

$$p\left(U_1, U_2, \ldots, U_n\right) = \mathcal{N}(U, 0, \Sigma) \tag{5}$$

where $U = [U_1, U_2, \ldots, U_n]^{\mathrm{T}}$ and

$$\Sigma = \begin{bmatrix} 1 & \rho_{1,2} & \cdots & \rho_{1,N} \\ \rho_{2,1} & \cdots & \cdots & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{N,1} & \cdots & \rho_{N,N-1} & 1 \end{bmatrix}$$

with $\rho_{(k,l)}$ to be the connection between the utility of $V^{(k)}$ and the utility of $V^{(1)}$.

It is usually valid to assume that it is an undifferentiated group formed by all traffic participants, i.e., each participant has the same preferences when selecting factor x_i, so that $\rho_{k,l} = \rho, \forall k, i$.

It reveals that the AV knows that there are some different strategies of SVs and the approximate overlap, but does not further know which vehicles are consistent with itself.

When a vehicle uses only its own information to determine each step of the decision, it's usually unperfect that such information is with some noise $v_{(k,i)}$,

$$\hat{x}_{k,i} = x_k + v_{k,i} \tag{6}$$

where x_k is the measured value, and $\hat{x}_{k,i}$ is the real value.

Alternatively, the estimated behavioral utility of the vehicle $v_{(i)}$ can be rewritten as

$$\hat{U}_i = \sum_{k=1}^{n} \alpha_{k,i} \hat{x}_k = U_k + \sum_{k=1}^{n} \alpha_{k,i} v_{k,i} \tag{7}$$

Without losing any generality, we consider that

$$E\left(v_{k,i}\right) = 0, \operatorname{var}\left(v_{k,i}\right) = \varepsilon^2 \tag{8}$$

where ε is the ambient noise level associated with the utility. Then we can get the conditional probability density distribution of $V_{(i)}$'s behavioral utility,

$$p\left(\hat{U}_i \mid U_i\right) = \phi\left(\frac{\hat{U}_i - U_i}{\varepsilon}\right) \tag{9}$$

Referring to the above equations, we can have

$$\operatorname{cov}(U_k, U_l) = \operatorname{cov}(\hat{U}_k, U_l) = \rho + \delta_{k,l}(1 - \rho) \tag{10}$$

$$\mathrm{cov}(U_k, U_l) = \mathrm{cov}(\hat{U}_k, \hat{U}_l) = \rho + \delta_{k,l}(1 + \varepsilon^2 - \rho) \tag{11}$$

where $\delta_{k,l}$ is the Kronecker delta function.

Clearly, if we traverse and combine all the SVs' behaviors, it is possible to cover all possible challenging events to obtain different testing tasks. However, as mentioned above, it is an NP-hard problem and to the best of our knowledge cannot be solved by now. Consequently, we give an alternative solution based on sampling SVs' behavior with semantic driving strategies, where we consider changing the ratio of SVs with different strategies by using vehicle interaction behavioral utility.

To theoretically achieve this, the probability of the occurrence of a challenging event is set to be $P\left(A \mid \mathrm{Scene}_{\mathrm{origin}}, \Psi, \hat{U}_1, C, M, E\right)$ based on Eq. (1), where $\hat{U}_1$ is the random variable that reflects the observed utility of AV.

Then the probability of AV's behavior can be expressed as $P\left(U_1 \mid \mathrm{Scene}_{\mathrm{origin}}, \Psi, \hat{U}_1, C, M, E\right)$. It shows the probability of AV's behavior from the initial scene $\mathrm{Scene}_{\mathrm{origin}}$ when completing the preset testing tasks Ψ. It is essential to consider the driving behavior of the AV for two reasons: (1) it shows AV's performance when AV carries out testing tasks; and (2) it also reflects the difficulty of the performed tasks in the scenario.

By utilizing the interaction behavioral utility model, it is easy to deduce the probability of AV's behavior if the behavior is only determined by the information estimated by AV itself,

$$p\left(U_1 \mid \hat{U}_1\right) \propto \phi\left(U_1\right) \phi\left(\frac{\hat{U}_1 - U_1}{\varepsilon}\right) \tag{12}$$

However, if we consider the impact of one SV on AV, which means we adjust the testing task difficulty by changing the driving strategy of one SV like $V^{(2)}$, Formula (3) can be rewritten as (see Appendix)

$$P\left(A \mid C_2, \widehat{U}_1\right) = \sum P\left(U_1 \mid C_2, \widehat{U}_1\right) P\left(A \mid U_1, C_2, \widehat{U}_1\right) \tag{13}$$

where the probability of AV's behavior is

$$p\left(U_1 \mid \widehat{U}_1, C_2\right) \propto \phi\left(U_1\right) \phi\left(\frac{\widehat{U}_1 - U_1}{\varepsilon}\right) \Phi\left(\frac{C_2 U_1 \rho}{\sqrt{1 + \varepsilon^2 - \rho}}\right) \tag{14}$$

Similarly, if we adjust the testing task by changing the driving strategy of two SVs, the probability of a challenging event occurring in the testing scenario is

$$P\left(A \mid C_2, C_3, \widehat{U}_1\right) = \sum P\left(U_1 \mid C_2, C_3, \widehat{U}_1\right) P\left(A \mid U_1, C_2, C_3, \widehat{U}_1\right) \tag{15}$$

where the probability of AV's behavior in Eq. (15) is

$$p\left(U_1 \mid C_2, C_3, \hat{U}_1\right) \propto \phi\left(U_1\right) \phi\left(\frac{\widehat{U}_1 - U_1}{\varepsilon}\right) \cdot \int_{-\infty}^{\widehat{U}^*_{2\to1}} \int_0^{+\infty} N\left(\begin{bmatrix} \widehat{U}_2 \\ \widehat{U}_3 \end{bmatrix}, \rho \begin{bmatrix} U_1 \\ U_1 \end{bmatrix}, \begin{bmatrix} 1+\varepsilon^2 & \rho-\rho^2 \\ \rho-\rho^2 & 1+\varepsilon^2 \end{bmatrix}\right) \mathrm{d}\widehat{U}_3 \mathrm{d}\widehat{U}_2 \tag{16}$$

where $\widehat{U}^*_{2\to1}$ is the strategy option bounclary of $\widehat{U}_2$.

5 Simulation

We design a typical lane-changing scenario without considering support from the roadside unit or other equipment (shown in Fig. 3) to validate our approach. The scenario consists of a one-way two-lane road, one AV, and three SVs where each lane is 4 m wide and vehicles are all 4 m long and 2 m wide. The distance between the starting position of the AV and the reference line is set as R, while the distance to the nearest lane line is set as R' (see Table 1).

In Fig. 3, the relationship description adopts the polar coordinate method with the measured vehicle as the axis and sets the distance between the four vehicles as (R_1, D_1), (R_2, D_2), and (R_3, D_3), where R_i denotes the absolute distance between the vehicle centers, and D_i denotes the angular deviation between the driving direction of $V^{(i+1)}$ and the driving direction of the AV.

We conduct experiments with the simulator presented in Refs. [33] and [34]. A pure tracking algorithm is used as the lateral control of the vehicle, referenced in Ref. [35], with the longitudinal model referenced in Ref. [36], which is used to generate continuous trajectories, also as an extension of the collision avoidance model. We assume that the AV at the back maintains a suitable and

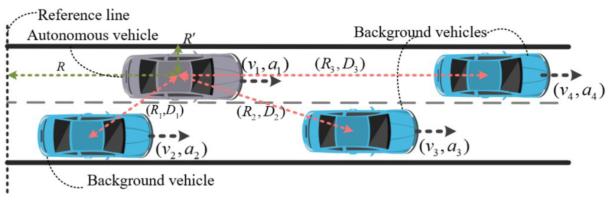

Fig. 3 Initial scene in the lane-changing scenario.

Table 1 Details on the participants in the initial scene.

Parameter	Value	Parameter	Value
R	22 m	R'	2 m
R_1	13.6 m	D_1	174.6°
R_2	13.6 m	D_2	5.4°
R_3	13.6 m	D_3	0°
v_1	5 m/s	a_1	0 m/s^2
v_2	5 m/s	a_2	0 m/s^2
v_3	5 m/s	a_1	0 m/s^2
v_4	5 m/s	a_2	0 m/s^2

adjustable distance from the SV in front.

The AV calculates the distance $L(t)$ in the following:

$$L(t) = x_{\text{lead}}(t) - x_{\text{follow}}(t) + \frac{v_{\text{lead}}^2(t) - v_{\text{follow}}^2(t)}{2a_{\max}} \tag{17}$$

where $x_{\text{lead}}(t)$ and $v_{\text{lead}}(t)$ indicate the location and speed of the vehicle in front, respectively, $x_{\text{follow}}(t)$ and $v_{\text{follow}}(t)$ are the location and speed of the following vehicle, respectively, and $a_{\max}$ is the max acceleration of the vehicle. The speed of the AV can be calculated as follows:

$$v_{\text{follow}}(t+T) = \begin{cases} \max\{0, v_{\text{follow}}(t) - a_{\max}T\}, & L(t) < G; \\ \min\{v_{\max}, v_{\text{follow}}(t) + a_{\max}T\}, & L(t) \geqslant G \end{cases} \tag{18}$$

where $v_{\max}$ is the max speed (14 m/s in this paper) and T is the time interval (0.1 s in this paper). The desired distance G is set to judge the final distance. If $L(t)$ is less than G, the vehicle behind will slow down. If $L(t)$ is greater than G, the vehicle will accelerate until it travels at maximum speed.

Besides, TTC is the testing metric for the simulation to determine that the AV encounters a challenging event when TTC is below the preset threshold θ_{TTC}. We define the relative distance and relative speed between $V^{(1)}$and $V^{(m)}$ as $R_m(t,x)$ and $\dot{R}_m(t,x)$ at time t and position x, respectively, the mathematical expression of TTC at time t is written as

$$\text{TTC}_m(t,x) = -\frac{R_m(t,x)}{\dot{R}_m(t,x)} \tag{19}$$

Then we can describe the safety tasks for AV in the simulation. The safety task is to safely driving and each subtask is to make sure that TTC is above the threshold at time T_i and position X_i,

$$B_{T_i,X_i} = \begin{cases} 1, & \text{TTC}_m(T_i, X_i) < \theta_{\text{TTC}}; \\ 0, & \text{otherwise} \end{cases} \tag{20}$$

In this paper, we set that the utility of lane-changing behavior is reflected in the probability of changing the lane at a certain scene. For vehicles with different strategies, they obtain different probabilities from the utility function.

The probabilities of whether lane-changing behavior occurs rely on different driving strategies. The semi-qualitative and semi-quantitative utility function with hyperparameters h_α is given. By adjusting the h_α we can generate lane-changing behaviors with different strategies.

To compare the behaviors under different strategies more clearly, we also set a normal strategy. Normal strategy statistically reflects the driving style when we ignore the difference in driving strategies. As shown in Fig. 4, vehicles with aggressive strategies have a greater probability of changing lanes than vehicles with normal strategies, and vehicles with conservative strategies are the opposite when vehicles share the same utility. Despite the prevalence of aggressive and conservative drivers in everyday life, we still lack relative data sets that characterize these two types of drivers separately. Therefore, data on utility functions are mainly derived from our laboratory's previous experience in conducting research on AV[37].

We adapt the driving strategies of SVs in two testing scenarios, i.e., Scenarios 1 and 2 to obtain more difficult tasks. In Scenario 1, we only change the driving strategy of $V^{(2)}$, while in Scenario 2, we also change the strategy of $V^{(3)}$. Besides, we provide the benchmark scenario where we maintain a normal driving strategy for all SVs. In this case, we set $\theta_{\text{TTC}} = 2$ s. We represent the results of 200 replicate experiments in box plots, as shown in Fig. 5.

We emphasize that changing the strategy and behavior of different numbers of SVs can have different effects on the task difficulty, whereas previous work[28] emphasized changing the strategy of only one SV.

The results show that, by adapting the driving

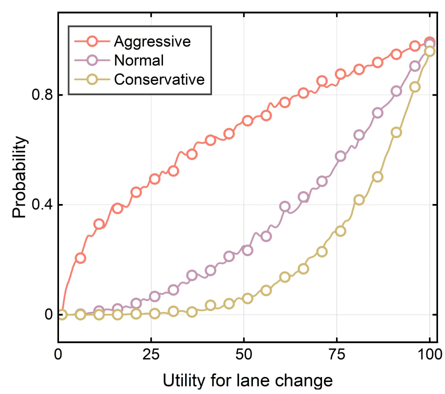

Fig. 4 Simulation on the behavioral utility for lane change.

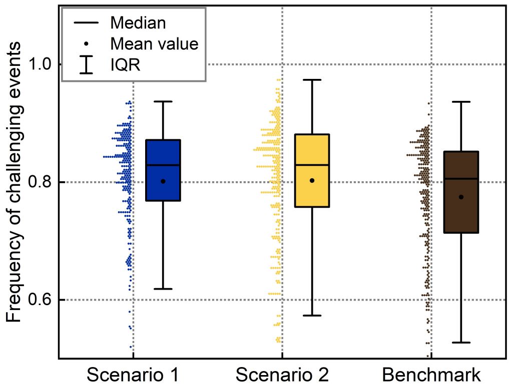

Fig. 5 Boxplots for the simulation: the frequency of challenging events in each scenario when TTC is less than 2 s (IQR is Iter-Quartile Range).

strategies of the SVs, we can obtain more challenging events, i.e., we generate higher difficulty levels for the AV under test. Furthermore, we prove that adapting to different numbers of SVs results in different task difficulties. Compared to the benchmark both Scenarios 1 and 2 are more challenging.

6 Conclusion

In this paper, we propose an intelligence testing method for AV. The method considers operable testing tasks with different difficulties. The testing task difficulty is tuned by the driving behavior of SVs. To express these behaviors, we utilize the behavioral utility functions and set a binary driving strategy, i.e., aggressive and conservative strategy to demonstrate the "inner reality" of SVs. Based on behavioral utility functions, we construct the vehicle interaction utility model and then theoretically demonstrate that changing the driving behavior of SVs results in changes on testing task difficulty. Finally, we simulate and validate the feasibility of our method. Compared to the benchmark, our method can obtain more challenging events for the AV under test, i.e., we can find more critical scenarios by generating difficult testing tasks.

Due to space limitations, this paper only focuses on testing a single performance of AVs, i.e., safety, and takes collision as the challenging event. However, the complete intelligence evaluation of AVs requires more comprehensive consideration of multi-dimensional metrics, including comfort, efficiency, etc. In the future work, first, we will give a comprehensive evaluation framework including various testing metrics and testing tasks for AVs; and second, we will also continue to build data sets for different driving strategies and further explore more realistic vehicle driving behaviors based on the binary strategy.

Appendix

In Appendix, we give the proof of Eqs. (13)–(16) to theoretically demonstrate that we can generate testing scenarios with different task difficulties by adapting the driving strategies of the SVs.

We can obtain Eq. (1) through the total probability theorem as follows:

$$P\left(A \mid \text{Scene}_{\text{origin}}, \Psi, C, \hat{U}_1, M, E\right) = \sum\left(P\left(U_1 \mid \text{Scene}_{\text{origin}}, \Psi, C, \widehat{U}_1, M, E\right) \cdot P\left(A \mid U_1, \text{Scene}_{\text{origin}}, \Psi, C, \widehat{U}_1, M, E\right)\right) \tag{A1}$$

Since we highlight the importance of SVs' behavior on the difficulty of the testing tasks, we assume that the initial tasks Ψ, metrics M, and initial scene are all preset to simplify the equations below.

For example, we set the semantical requirements as testing the ability of AV to recognize traffic conditions and turn safely at intersections with 4 SVs, all of which use conservative strategies. Therefore, guided by the test requirements, we can regulate the location, attributes, and strategies of the SVs in the initial scene, and design specific tasks and testing metrics.

To this end, Eq. (13) can be simplified as

$$P\left(A \mid C, \widehat{U}_1\right) = \sum P\left(U_1 \mid C, \widehat{U}_1\right) P\left(A \mid U_1, C, \widehat{U}_1\right) \tag{A2}$$

When we adopt the behavioral utility of one vehicle like $V^{(2)}$, Eq. (14) can be written as

$$P\left(A \mid C_2, \widehat{U}_1\right) = \sum P\left(U_1 \mid C_2, \widehat{U}_1\right) P\left(A \mid U_1, C_2, \widehat{U}_1\right) \tag{A3}$$

From this, considering AV's behavior is in conjunction with the observation of its own utility and the utility of SVs. According to Bayes' Rule, $P\left(U_1 \mid C_2, \widehat{U}_1\right)$ can be rewritten as

$$P\left(U_1 \mid C_2, \widehat{U}_1\right) \propto P\left(U_1 \mid \widehat{U}_1\right) P\left(C_2 \mid U_1, \widehat{U}_1\right) \tag{A4}$$

Obviously, the $V^{(2)}$'s behavior observed by itself is influenced by its driving strategy, since we consider all strategies are binary, it satisfies that $C_2\widehat{U}_2 > 0$ or $C_2\widehat{U}_2 < 0$. Since C_2 is independent of $\widehat{U}_1$, we have

$$P\left(C_2 \mid U_1\right) = P\left(C_2\widehat{U}_2 > 0 \mid U_1\right) \tag{A5}$$

According to the vehicle interaction behavioral utility model above,

$$\widehat{U}_2 \mid U_1 \sim \mathcal{N}\left(\rho U_1, 1 + \varepsilon^2 - \rho\right) \tag{A6}$$

Then under the AV's behavior utility U_1, we have

$$P\left(C_2 \mid U_1, \widehat{U}_1\right) \propto \Phi\left(\frac{C_2 U_1 \rho}{\sqrt{1+\varepsilon^2-\rho}}\right) \tag{A7}$$

So the probability density function of the behavior made by the AV is

$$p\left(U_1 \mid \widehat{U}_1, C_2\right) \propto \phi\left(U_1\right) \phi\left(\frac{\widehat{U}_1-U_1}{\varepsilon}\right) \Phi\left(\frac{C_2 U_1 \rho}{\sqrt{1+\varepsilon^2-\rho}}\right) \tag{A8}$$

Similarly, if we adopt two SVs, the probability of a challenging event occurring in the testing scenario is

$$P\left(A \mid C_2, C_3, \widehat{U}_1\right) = \sum P\left(U_1 \mid C_2, C_3, \widehat{U}_1\right) P\left(A \mid U_1, C_2, C_3, \widehat{U}_1\right) \tag{A9}$$

According to the above assumption, $V^{(1)}$ shares the same ρ with $V^{(2)}$ and $V^{(3)}$, from the perspective of the AV, if it considers the behavioral utility of two SVs, the probability of its behavior can be rewritten as

$$p\left(U_1 \mid C_2, C_3, \widehat{U}_1\right) \propto p\left(U_1 \mid \widehat{U_1}\right) p\left(C_2, C_3 \mid U_1, \widehat{U}_1\right) \tag{A10}$$

where

$$p\left(C_2, C_3 \mid U_1, \widehat{U}_1\right) = \int_{-\infty}^{\widehat{U}^*_{2\to 1}} \int_0^{+\infty} N\left(\begin{bmatrix} \widehat{U}_2 \\ \widehat{U}_3 \end{bmatrix}, \rho \begin{bmatrix} U_1 \\ U_1 \end{bmatrix}, \begin{bmatrix} 1+\varepsilon^2 & \rho-\rho^2 \\ \rho-\rho^2 & 1+\varepsilon^2 \end{bmatrix}\right) \mathrm{d}\widehat{U}_3 \mathrm{d}\widehat{U}_2 \tag{A11}$$

Acknowledgment

This work was supported in part by the National Key Research and Development (No. 2021YFB2501200).

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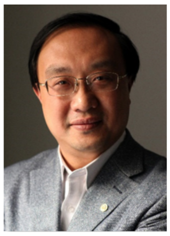

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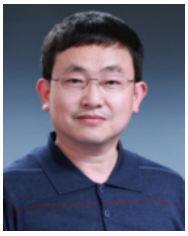

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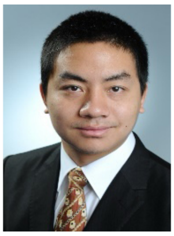

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