Adaptive Event-Triggered Synchronization of Reaction–Diffusion Neural Networks

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Abstract—This article focuses on the design of an adaptive event-triggered sampled-data control (ETSDC) mechanism for synchronization of reaction–diffusion neural networks (RDNNs) with random time-varying delays. Different from the existing ETSDC schemes with predetermined constant thresholds, an adaptive ETSDC mechanism is proposed for RDNNs. The adaptive ETSDC mechanism can be promptly adaptively adjusted since the threshold function is based on the current sampled and latest transmitted signals. Thus, the adaptive ETSDC mechanism can effectively save communication resources for RDNNs. By taking the influence of uncertain factors, the random time-varying delays are considered, which belongs to two intervals in a probabilistic way. Then, by constructing an appropriate Lyapunov–Krasovskii functional (LKF), new synchronization criteria are derived for RDNNs. By solving a set of linear matrix inequalities (LMIs), the desired adaptive ETSDC gain is obtained. Finally, the merits of the adaptive ETSDC mechanism and the effectiveness of the proposed results are verified by one numerical example.

Index Terms—Adaptive event-triggered mechanism, random time-varying delays, reaction–diffusion neural networks (RDNNs), sampled-data control.

I. INTRODUCTION

BECAUSE of the superiorities in learning algorithms and estimating data, neural networks (NNs) have provoked the increasing interests of many researchers. In recent years,

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NNs have been successfully applied to diverse areas including cryptography, decryption, stock market prediction, signal processing, and model identification [1]–[7]. As an extension of NNs, reaction–diffusion NNs (RDNNs) have attracted much attention since they have been successfully applied to secure communication, population control, and artificial locomotion [8]–[12]. For example, reaction–diffusion cellular NNs have been successfully applied to artificial locomotion [10]. In [11], RDNNs have been applied to image encryption. In [12], RDNNs can effectively describe the different population densities of species, which are useful for better protecting and controlling the population. By taking the influence of reaction and diffusion of neurons into account, RDNNs can be represented by partial differential equations (PDEs), in which the neuron states are dependent on both time and space. Compared with traditional NNs, RDNNs exhibit more complicated and unpredictable behaviors. Hence, it is of great importance to study the dynamical behaviors of RDNNs.

Up until now, various dynamical behaviors of RDNNs, such as passivity [13], stability [14], and synchronization [15], have been investigated. Since the surprising discovery of synchronization in chaotic systems [16], synchronization has aroused substantial attention. It is motivated by the fact that the synchronization is ubiquitous in real systems and has many important engineering applications, including mechatronic systems, power systems, and secure communication [16]–[18]. Meanwhile, due to congestions of signal transmission and the finite switching speeds of amplifiers, time delays [19] inevitably exist in RDNNs. However, time delays have a great influence on the dynamical behaviors of RDNNs since they may result in oscillation, poor performance, or even instability. Moreover, due to the influence of uncertain factors and the limitations of equipment, random time-varying delays are often existent in natural and artificial systems. For example, in a shipboard gun fire control system, missynchronization between samples of the actual position and the inertial sensor and radar data samples results in the random time-varying delays [20]. Because of the asynchronous time-division-multiplexed networks, randomly varying distributed delays have been considered for the flight control system [21]. Using the statistical technique, random time-varying delays have been taken into account for power systems [22]. In recent years, random time-varying delays have been widely studied for many systems [23]–[25]. Thus, it is necessary to introduce random time-varying delays to RDNNs, and the investigation of synchronization control of RDNNs with random time-varying delays is particularly important.

Up until now, many control approaches have been proposed for synchronization of RDNNs with time delays, which includes impulsive control [26], pinning impulsive control [27], adaptive control [28], and feedback control [29]. For instance, in [27], by pinning impulsive control, the exponential synchronization problem has been considered for RDNNs with time delays. In [28], the antisynchronization of RDNNs with time delays has been studied by distributed adaptive control. In [29], by state feedback control, the asymptotic synchronization has been investigated for Markovian RDNNs with time delays. With the rapid growing of digital and communication technologies, sampled-data control is in the spotlight [30]–[32]. Compared with the forenamed control schemes, sampled-data control shows more merits, such as high reliability, easy installation, and low control cost. Thus, based on sampled-data control, the synchronization of RDNNs with time delays has been extensively investigated, and many interesting results are available in the literature [33]–[35]. Notice that all the aforesaid sampled-data control schemes are based on a time-triggered mechanism, which is assumed that all the signals are transmitted to the controllers without any data processing. However, the communication resources and computing capability of RDNNs are often limited. Hence, it is very important to save communication resources for RDNNs.

Recently, wide attention has been drawn to event-triggered sampled-data control (ETSDC) [36]–[39], where the sampled signals are not transmitted only if a predetermined threshold condition is violated. Thus, ETSDC can effectively reduce the number of transmission signals. Nowadays, adaptive event-triggered control has fascinated considerable research interests [40]–[43]. For example, in [40], an adaptive event-triggered mechanism with continuous control input signals has been considered for multiagent systems, which needs to test the triggered condition continuously. In [41], an adaptive ETSDC method has been proposed for networked control systems, where the threshold is a time-dependent continuous function. Similar scheme is adopted in [42]. However, the adaptive ETSDC schemes in [41] and [42] may not be appropriate since only sampled signals can be input the scheme. In [43], a new adaptive ETSDC mechanism has been given for network-based power systems, where the threshold function is dependent on the discrete sampled signals. Notice that the scheme in [43] is based on the past and latest transmission signals, which may not update the threshold promptly. Thus, how to improve the existing adaptive ETSDC mechanism is still challenging. These issues motivate this work.

The objective of this study is to develop a novel adaptive ETSDC mechanism for the synchronization of RDNNs with time delays. The main contributions are as follows.

1) An improved adaptive ETSDC mechanism is proposed for RDNNs for the first time. The adaptive ETSDC mechanism is based on the current sampled and latest transmitted signals, which can be promptly adaptively adjusted and effectively save the communication resources for RDNNs.

2) Taking the influence of uncertain factors, the random time-varying delays are considered for RDNNs, which make the derived results more applicable.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. Notations

Let I_n denote the $n \times n$ identity matrix, 0_n and $0_{n,m}$ the $n \times n$ and $n \times m$ zero matrices, $\Re^n$ the n-dimensional Euclidean space, and $\Re^{n \times n}$ the set of all $n \times n$ real matrices. $\mathrm{diag}\{\cdots\}$ and $\mathrm{col}\{\cdots\}$ mean a block-diagonal matrix and a column vector, respectively. $\lambda_{\min}(\cdot)$ stands for the minimum eigenvalue of a real symmetric matrix. $\mathscr{E}\{\cdot\}$ represents the mathematical expectation with respect to the given probability measure $\mathbb{P}$. $\mathrm{Sym}\{\mathcal{H}\} = \mathcal{H} + \mathcal{H}^T$. $C([-d_2, 0] \times \Omega, \Re^n)$ means all continuous functions from $[-d_2, 0] \times \Omega$ to $\Re^n$. For $\psi \in C([-d_2, 0] \times \Omega, \Re^n)$, the norm is defined by $\|\psi(s, x)\| = \sup_{-d_2 \le s \le 0}\{(\int_{\Omega} \psi^T(s, x)\psi(s, x)dx)^{(1/2)}\}$. For the vector $\varpi(t, x) \in \Re^n$, let $\|\varpi(t, x)\|_{L^2} = (\int_{\Omega} \varpi^T(t, x)\varpi(t, x)dx)^{(1/2)}$.

B. Problem Formulation

Consider the following RDNNs with a random time-varying delay:

$$\frac{\partial \varpi(t, x)}{\partial t} = \sum_{l=1}^{m} \frac{\partial}{\partial x_l}\left(\mathcal{A}_l \frac{\partial \varpi(t, x)}{\partial x_l}\right) - \mathcal{C}\varpi(t, x)$$
$$+ \mathcal{W}_1 g(\varpi(t, x)) + \mathcal{W}_2 g(\varpi(t - d(t), x))$$
$$+ J, \quad (t, x) \in [t_0^*, +\infty) \times \Omega \quad (1)$$
$$\varpi(t, x) = 0, \quad (t, x) \in [t_0^* - d_2, +\infty) \times \partial\Omega \quad (2)$$
$$\varpi(s + t_0^*, x) = \psi_1(s, x) \in C([-d_2, 0] \times \Omega, \Re^n) \quad (3)$$

where $l = 1, 2, \ldots, m$, and $x = \mathrm{col}\{x_1, x_2, \ldots, x_m\} \in \Omega$. $\Omega \triangleq \{x | \underline{\varrho}_l \le x_l \le \bar{\varrho}_l\}$ with $\partial\Omega$ being its boundary, and $\underline{\varrho}_l$ and $\bar{\varrho}_l$ are constants. $\mathcal{A}_l = \mathrm{diag}\{a_{l1}, a_{l2}, \ldots, a_{ln}\}$, in which $a_{li} > 0$ $(i = 1, 2, \ldots, n)$ stands for the transmission diffusion coefficient. $\mathcal{C} = \mathrm{diag}\{c_1, c_2, \ldots, c_n\}$ with $c_i > 0$ $(i = 1, 2, \ldots, n)$. $\mathcal{W}_1 \in \Re^{n \times n}$ and $\mathcal{W}_2 \in \Re^{n \times n}$ are the nondelayed and delayed connection weight matrices, respectively. $\varpi(t, x) = \mathrm{col}\{\varpi_1(t, x), \varpi_2(t, x), \ldots, \varpi_n(t, x)\} \in \Re^n$ is the state vector at time t and in space x. $g(\varpi(\cdot, x)) = \mathrm{col}\{g_1(\varpi_1(\cdot, x)), g_2(\varpi_2(\cdot, x)), \ldots, g_n(\varpi_n(\cdot, x))\} \in \Re^n$ is the neuron activation function. $J \in \Re^n$ is an external input vector. $d(t)$ is the random time-varying delay being in $[0, d_1]$ or $(d_1, d_2]$ with

$$\mathbb{P}\{d(t) \in [0, d_1]\} = \gamma, \quad \mathbb{P}\{d(t) \in (d_1, d_2]\} = 1 - \gamma \quad (4)$$

where $0 \le d_1 \le d_2$, $0 \le \gamma \le 1$, and d_1 and d_2 are constants. Equations (2) and (3) are, respectively, the Dirichlet boundary condition and the initial condition.

Define stochastic variable $\beta(t) = \begin{cases} 1, & d(t) \in [0, d_1] \\ 0, & d(t) \in (d_1, d_2]. \end{cases}$ We find

$$\mathbb{P}\{\beta(t) = 1\} = \mathbb{P}\{d(t) \in [0, d_1]\} = \gamma$$
$$\mathbb{P}\{\beta(t) = 0\} = \mathbb{P}\{d(t) \in (d_1, d_2]\} = 1 - \gamma. \quad (5)$$

Let the time-varying delays $d_i(t)$ ($i = 1, 2$) satisfy $d(t) =$
$$\begin{cases} d_1(t), & d(t) \in [0, d_1] \\ d_2(t), & d(t) \in (d_1, d_2], \end{cases}$$ and $\dot{d}_i(t) \leq \mu_i$ ($i = 1, 2$). Then,
RDNN (1) can be rewritten as

$$\frac{\partial \varpi(t, x)}{\partial t} = \sum_{l=1}^{m} \frac{\partial}{\partial x_l} \left(\mathcal{A}_l \frac{\partial \varpi(t, x)}{\partial x_l} \right) - \mathcal{C}\varpi(t, x)$$
$$+ \mathcal{W}_1 g(\varpi(t, x)) + \beta(t)\mathcal{W}_2 g(\varpi(t - d_1(t), x))$$
$$+ (1 - \beta(t))\mathcal{W}_2 g(\varpi(t - d_2(t), x)) + J. \quad (6)$$

Viewing system (6) as the drive system, the corresponding controlled response system is described as

$$\frac{\partial z(t, x)}{\partial t} = \sum_{l=1}^{m} \frac{\partial}{\partial x_l} \left(\mathcal{A}_l \frac{\partial z(t, x)}{\partial x_l} \right) - \mathcal{C}z(t, x)$$
$$+ \mathcal{W}_1 g(z(t, x)) + \beta(t)\mathcal{W}_2 g(z(t - d_1(t), x))$$
$$+ (1 - \beta(t))\mathcal{W}_2 g(z(t - d_2(t), x))$$
$$+ \mathcal{U}(t, x) + J, \quad (t, x) \in [t_0^{**}, +\infty) \times \Omega$$
$$(7)$$
$$z(t, x) = 0, \quad (t, x) \in [t_0^{**} - d_2, +\infty) \times \partial\Omega \quad (8)$$
$$z(s + t_0^{**}, x) = \psi_2(s, x) \in C([-d_2, 0] \times \Omega, \Re^n) \quad (9)$$

where $\mathcal{U}(t, x) \in \Re^n$ is the control signal.

Let the error signal $\vartheta(t, x) = z(t, x) - \varpi(t, x) = $
$\text{col}\{\vartheta_1(t, x), \vartheta_2(t, x), \ldots, \vartheta_n(t, x)\} \in \Re^n$. From (6) and (7), we have the error system as

$$\frac{\partial \vartheta(t, x)}{\partial t} = \sum_{l=1}^{m} \frac{\partial}{\partial x_l} \left(\mathcal{A}_l \frac{\partial \vartheta(t, x)}{\partial x_l} \right) - \mathcal{C}\vartheta(t, x)$$
$$+ \mathcal{W}_1 f(\vartheta(t, x)) + \beta(t)\mathcal{W}_2 f(\vartheta(t - d_1(t), x))$$
$$+ (1 - \beta(t))\mathcal{W}_2 f(\vartheta(t - d_2(t), x))$$
$$+ \mathcal{U}(t, x), \quad (t, x) \in [\tilde{t}_0, +\infty) \times \Omega \quad (10)$$
$$\vartheta(t, x) = 0, \quad (t, x) \in [\tilde{t}_0 - d_2, +\infty) \times \partial\Omega \quad (11)$$
$$\vartheta(s + \tilde{t}_0, x) = \psi(s, x) \in C([-d_2, 0] \times \Omega, \Re^n) \quad (12)$$

where $f(\vartheta(t, x)) = g(z(t, x)) - g(\varpi(t, x))$ and $\tilde{t}_0 = \max\{t_0^*, t_0^{**}\}$.

In order to save the limited communication resources of RDNNs, an adaptive event-triggered generator with aperiodic sampled-data signals will be introduced, in which the threshold function can be promptly adaptively adjusted based on the current sampled and latest transmitted signals.

Let the input signals be generated by a zero-order-hold (ZOH) function with the holding times $0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots$. The nonuniform sampling interval h_k satisfies the following condition:

$$h_k = t_{k+1} - t_k \leq \tilde{h}$$

where $\tilde{h} > 0$ is a constant.

Then, we propose the following adaptive event-triggered communication mechanism:

$$i_p = t_{\sigma_{p-1}}, \quad p \in \mathbb{N}$$
$$\sigma_{p-1} = \begin{cases} 0, & p = 0 \\ \sum_{k=0}^{p-1} \tau_k, & p \in \mathbb{N}^+ \end{cases}$$

$$\tau_k = \inf \left\{ \tau \mid \delta(t_{\sigma_{k-1}+\tau}) \left\| \mathcal{B}_1^{\frac{1}{2}} \vartheta(t_{\sigma_{k-1}}, x) \right\|_{L^2}^2 \right.$$
$$\left. < \left\| \mathcal{B}_2^{\frac{1}{2}} e(t_{\sigma_{k-1}+\tau}, x) \right\|_{L^2}^2, \ \tau \in \mathbb{N}^+ \right\}, \quad k \in \mathbb{N}$$
$$(13)$$

to determine whether the newly sampled signal should be transmitted, where $\mathcal{B}_1 > 0 \in \Re^{n \times n}$ and $\mathcal{B}_2 > 0 \in \Re^{n \times n}$ are weighting matrices, and $e(t_{\sigma_{k-1}+\tau}, x)$ is defined as $e(t_{\sigma_{k-1}+\tau}, x) \triangleq \vartheta(t_{\sigma_{k-1}+\tau}, x) - \vartheta(t_{\sigma_{k-1}}, x)$, which is the state error between the current instant and latest transmitted instant. The event-triggered transmission sequence is described by $\beth_1 = \{i_0, i_1, i_2, \ldots, i_p, \ldots\} \subseteq \beth_2 = \{t_0, t_1, t_2, \ldots, t_k, \ldots\}$.

It is noted that the threshold $\delta(t_{\sigma_{k-1}+\tau})$ in (13) is time-varying, which can be adaptively adjusted based on the current sampled and latest transmitted signals, that is, for $\tau \in \mathbb{N}^+$

$$\delta(t_{\sigma_{k-1}+\tau}) = (1 + \alpha(t_{\sigma_{k-1}+\tau}))\delta(t_{\sigma_{k-1}+\tau-1}) \quad (14)$$

where

$$\alpha(t_{\sigma_{k-1}+\tau})$$
$$= \begin{cases} \dfrac{\delta_0}{\delta(t_{\sigma_{k-1}+\tau-1})} - 1, & \|\vartheta(t_{\sigma_{k-1}+\tau}, x)\|_{L^2} \geq \|\vartheta(t_{\sigma_{k-1}}, x)\|_{L^2} \\ \theta\dfrac{\|\vartheta(t_{\sigma_{k-1}}, x)\|_{L^2} - \|\vartheta(t_{\sigma_{k-1}+\tau}, x)\|_{L^2}}{\|\vartheta(t_{\sigma_{k-1}}, x)\|_{L^2}}, & \text{otherwise} \end{cases}$$

where θ is a positive scalar.

Proposition 1: Give the initial value of $\delta(t)$ as $\delta(t_0) = \delta_0$. For $\forall \tau \in \mathbb{N}^+, k \in \mathbb{N}$, the inequalities

$$\delta(t_{\sigma_{k-1}+\tau}) \geq \delta_0$$

hold.

Proof: From $\delta(t_{\sigma_{k-1}+\tau}) = (1 + \alpha(t_{\sigma_{k-1}+\tau}))\delta(t_{\sigma_{k-1}+\tau-1})$ and the definition of $\alpha(t_{\sigma_{k-1}+\tau})$ in (14), one gets

$$\delta(t_1) = \delta_0 \quad \text{or} \quad \delta(t_1) > \delta_0$$

which implies that $\delta(t_1) \geq \delta_0$. To continue this process, one can derive for $\forall \tau \in \mathbb{N}^+, k \in \mathbb{N}$

$$\delta(t_{\sigma_{k-1}+\tau}) \geq \delta_0.$$

This completes the proof. ∎

Remark 1: It is worth mentioning that the adaptive event-triggered communication mechanism (13) is newly designed for RDNNs. By adaptively adjusting the threshold $\delta(t_{\sigma_{k-1}+\tau})$, the mechanism (13) can effectively reduce the number of transmission signals. When $\|\vartheta(t_{\sigma_{k-1}+\tau}, x)\|_{L^2} \geq \|\vartheta(t_{\sigma_{k-1}}, x)\|_{L^2}$, one can derive $\delta(t_{\sigma_{k-1}+\tau}) = \delta_0$, which implies that the mechanism (13) uses the smaller threshold δ_0 to produce a higher transmission frequency to strengthen the control. When $\|\vartheta(t_{\sigma_{k-1}+\tau}, x)\|_{L^2} < \|\vartheta(t_{\sigma_{k-1}}, x)\|_{L^2}$, one finds $\delta(t_{\sigma_{k-1}+\tau}) > \delta(t_{\sigma_{k-1}+\tau-1})$, which implies that the mechanism (13) uses the larger value of $\delta(t_{\sigma_{k-1}+\tau})$ to produce a lower transmission frequency to attenuate the control.

Remark 2: In [40], the adaptive event-triggered scheme is designed with continuous input signals, which requires to test the trigger condition continuously. In [41] and [42], a continuous time-dependent threshold function is introduced to the adaptive event-triggered schemes. However, these schemes

may not be appropriate since only discrete sampled information can be used. In [43], based on the past and latest transmission signals, an adaptive event-triggered scheme subject to periodic sampling is considered. It should be pointed out that the adaptive event-triggered communication mechanism (13) is based on the current sampled and latest transmitted signals. Thus, compared with the adaptive event-triggered schemes with continuous threshold functions in [40]–[42], the adaptive event-triggered communication mechanism (13) with discrete threshold function needs less calculation cost. Compared with the scheme in [43], (13) is more effective to promptly adjust the threshold.

Based on the communication mechanism (13), the adaptive event-triggered sampled-data controller is designed by

$$\mathcal{U}(t, x) = \mathcal{K}\vartheta(i_p, x), \quad i_p \leq t < i_{p+1} \tag{15}$$

where $\mathcal{K} \in \Re^{n \times n}$ is the feedback gain to be designed.

Divide the holding interval $[i_p, i_{p+1})$ into the subintervals as

$$[i_p, i_{p+1}) = \bigcup_{j=0}^{\tau_p - 1} F_j$$

where $F_j = [t_{\sigma_{p-1}+j}, t_{\sigma_{p-1}+j+1})$, $j = 0, 1, \ldots, \tau_p - 1$, and $p \in \mathbb{N}$. Then, the controller (15) is changed into

$$\begin{aligned}
\mathcal{U}(t, x) &= \mathcal{K}(\vartheta(t_{\sigma_{p-1}+j}, x) - e(t_{\sigma_{p-1}+j}, x)) \\
&= \mathcal{K}(\vartheta(t - h(t), x) - e(t - h(t), x)), \quad t \in F_j
\end{aligned} \tag{16}$$

where $h(t) = t - t_{\sigma_{p-1}+j}$ with $0 \leq h(t) \leq h_{\sigma_{p-1}+j} \leq \tilde{h}$.

Then, from (10) and (16), one gets

$$\begin{aligned}
\frac{\partial \vartheta(t, x)}{\partial t} &= \sum_{l=1}^{m} \frac{\partial}{\partial x_l}\left(\mathcal{A}_l \frac{\partial \vartheta(t, x)}{\partial x_l}\right) - \mathcal{C}\vartheta(t, x) \\
&+ \mathcal{W}_1 f(\vartheta(t, x)) + \beta(t)\mathcal{W}_2 f(\vartheta(t - d_1(t), x)) \\
&+ (1 - \beta(t))\mathcal{W}_2 f(\vartheta(t - d_2(t), x)) \\
&+ \mathcal{K}(\vartheta(t - h(t), x) - e(t - h(t), x)) \\
&\qquad\qquad t \in F_j, \quad x \in \Omega
\end{aligned} \tag{17}$$

$$\vartheta(t, x) = 0, \quad (t, x) \in [\tilde{t}_0 - d_2, +\infty) \times \partial\Omega \tag{18}$$

$$\vartheta(s + \tilde{t}_0, x) = \psi(s, x) \in C([-d_2, 0] \times \Omega, \Re^n). \tag{19}$$

C. Assumption and Lemmas

Assumption 1 [29]: For any $y_1, y_2 \in \Re$, there exist scalars l_i^- and l_i^+, $i = 1, 2, \ldots, n$, such that the activation function $g_i(\cdot)$ in (1) satisfies $l_i^- \leq ((g_i(y_1) - g_i(y_2))/(y_1 - y_2)) \leq l_i^+$, and $y_1 \neq y_2$.

Lemma 1 [44]: For a matrix $\mathscr{D} \geq 0 \in \Re^{n \times n}$ and all continuously differentiable function y in $[\varsigma_a, \varsigma_b] \rightarrow \Re^n$, the following inequality holds: $\int_{\varsigma_a}^{\varsigma_b} \dot{y}^T(s)\mathscr{D}\dot{y}(s)ds \geq (1/(\varsigma_b - \varsigma_a))(Y_1^T \mathscr{D}Y_1 + 3Y_2^T \mathscr{D}Y_2)$, where $Y_1 = y(\varsigma_b) - y(\varsigma_a)$ and $Y_2 = y(\varsigma_b) + y(\varsigma_a) - (2/(\varsigma_b - \varsigma_a))\int_{\varsigma_a}^{\varsigma_b} y(s)ds$.

Lemma 2 [45]: For $\mathscr{N} \geq 0 \in \Re^{n \times n}$ and $u \in C(\Omega, \Re^n)$ with $u(x)|_{\partial\Omega} = 0$, and $l = 1, 2, \ldots, m$, it is true that $\int_{\Omega} u^T(x)\mathscr{N}u(x)dx \leq (((\bar{\varrho}_l - \underline{\varrho}_l)^2)/\pi^2) \int_{\Omega} (\partial u(x)/\partial x_l)^T \mathscr{N}(\partial u(x)/\partial x_l)dx$.

Remark 3: Under Assumption 1 and $\mathcal{A}_l > 0$ $(l = 1, 2, \ldots, m)$, similar to the proofs in [46], the existence and uniqueness of the solution to system (17) can be ensured by the semigroup theory and contraction mapping principle.

III. MAIN RESULTS

In this section, by designing the sampled-data controller (16) subject to the adaptive event-triggered mechanism (13), we study the synchronization of the RDNNs (1) and (7). We first derive sufficient conditions to synchronize RDNNs (1) and (7), which is equivalent to the stability of error system (17). Then, we propose a design method for the adaptive event-triggered sampled-data controller (16).

A. Synchronization for Reaction–Diffusion Neural Networks With Adaptive Event-Triggered Sampled-Data Control

In this section, by introducing an appropriate Lyapunov–Krasovskii functional (LKF), a new synchronization criterion will be established for RDNNs (1) and (7). Denote $\mathcal{I}_i = [0_{n,(i-1)n} \quad I_n \quad 0_{n,(10-i)n}]$, $(i = 1, \ldots, 10)$, $\mathcal{F}^- = \text{diag}\{l_1^-, l_2^-, \ldots, l_n^-\}$, $\mathcal{F}^+ = \text{diag}\{l_1^+, l_2^+, \ldots, l_n^+\}$, $\xi_3(t, x) =$
$$\begin{cases} (1/(t - t_{\sigma_{p-1}+j}))\int_{t_{\sigma_{p-1}+j}}^{t} \vartheta(s, x)ds, & t \neq t_{\sigma_{p-1}+j} \\ \vartheta(t_{\sigma_{p-1}+j}, x), & t = t_{\sigma_{p-1}+j}, \end{cases}$$ and

$$\begin{aligned}
\eta(t, x) = \text{col}\Big\{ &\vartheta(t, x), \ \vartheta(t - h(t), x), \ f(\vartheta(t, x)), \ \frac{\partial\vartheta(t, x)}{\partial t}, \\
&\xi_3(t, x), \ \vartheta(t - d_1(t), x), \ \vartheta(t - d_2(t), x), \\
&f(\vartheta(t - d_1(t), x)), \ f(\vartheta(t - d_2(t), x)), \\
&e(t - h(t), x)\Big\}.
\end{aligned}$$

Theorem 1: For given scalars $0 \leq \gamma \leq 1$, μ_i $(i = 1, 2)$, $\tilde{h} > 0$, δ_0, κ, and and controller gain $\mathcal{K} \in \Re^{n \times n}$, the response RDNN (7) is asymptotically synchronized to the drive RDNN (1) if there exist matrices $\mathcal{P} > 0 \in \Re^{n \times n}$, $\mathcal{Q}_i > 0 \in \Re^{2n \times 2n}$ $(i = 1, 2)$, $\mathcal{T} > 0 \in \Re^{n \times n}$, $\mathcal{R} > 0 \in \Re^{n \times n}$, and $\mathcal{B}_i > 0 \in \Re^{n \times n}$ $(i = 1, 2)$, diagonal matrices $\mathcal{D}_i > 0 \in \Re^{n \times n}$ $(i = 1, 2, 3)$, and any matrices $\mathcal{S} \in \Re^{n \times n}$, $\mathcal{H}_1 \in \Re^{n \times 2n}$, and $\mathcal{H}_2 \in \Re^{n \times 3n}$ satisfying $\mathcal{S}\mathcal{A}_l \geq 0$ and

$$\Xi(0; 0, 0) < 0 \tag{20}$$

$$\Xi(0; \tilde{h}, 0) < 0 \tag{21}$$

$$\begin{bmatrix} \Xi(0; \tilde{h}, \tilde{h}) & \sqrt{\tilde{h}}[\mathcal{I}_1^T, \mathcal{I}_2^T]\mathcal{H}_1^T & \sqrt{3\tilde{h}}[\mathcal{I}_1^T, \mathcal{I}_2^T, \mathcal{I}_5^T]\mathcal{H}_2^T \\ * & -\mathcal{R} & 0 \\ * & * & -\mathcal{R} \end{bmatrix}$$
$$< 0 \tag{22}$$

where $\Xi(\rho; h_{\sigma_{p-1}+j}, h(t)) = \sum_{i=1}^{5} \Xi_i(\rho; h_{\sigma_{p-1}+j}, h(t))$ with

$\Xi_1(\rho; h_{\sigma_{p-1}+j}, h(t))$
$$= \delta_0(\mathcal{I}_2 - \mathcal{I}_{10})^T \mathcal{B}_1(\mathcal{I}_2 - \mathcal{I}_{10}) - \mathcal{I}_{10}^T \mathcal{B}_2 \mathcal{I}_{10},$$

$\Xi_2(\rho; h_{\sigma_{p-1}+j}, h(t))$
$$\begin{aligned}
&= \text{Sym}\{(\mathcal{I}_3 - \mathcal{F}^- \mathcal{I}_1)^T \mathcal{D}_1(\mathcal{F}^+ \mathcal{I}_1 - \mathcal{I}_3)\} \\
&+ \text{Sym}\{(\mathcal{I}_8 - \mathcal{F}^- \mathcal{I}_6)^T \mathcal{D}_2(\mathcal{F}^+ \mathcal{I}_6 - \mathcal{I}_8)\} \\
&+ \text{Sym}\{(\mathcal{I}_9 - \mathcal{F}^- \mathcal{I}_7)^T \mathcal{D}_3(\mathcal{F}^+ \mathcal{I}_7 - \mathcal{I}_9)\}
\end{aligned}$$

$\Xi_3(\rho; h_{\sigma_{p-1}+j}, h(t))$

$$= \text{Sym}\{\mathcal{I}_1^T \mathcal{P} \mathcal{I}_4\} + \gamma \left[\mathcal{I}_1^T, \mathcal{I}_3^T\right] \mathcal{Q}_1 \left[\mathcal{I}_1^T, \mathcal{I}_3^T\right]^T$$
$$- \gamma (1 - \mu_1)\left[\mathcal{I}_6^T, \mathcal{I}_8^T\right] \mathcal{Q}_1 \left[\mathcal{I}_6^T, \mathcal{I}_8^T\right]^T$$
$$+ (1 - \gamma)\left[\mathcal{I}_1^T, \mathcal{I}_3^T\right] \mathcal{Q}_2 \left[\mathcal{I}_1^T, \mathcal{I}_3^T\right]^T$$
$$- (1 - \gamma)(1 - \mu_2)\left[\mathcal{I}_7^T, \mathcal{I}_9^T\right] \mathcal{Q}_2 \left[\mathcal{I}_7^T, \mathcal{I}_9^T\right]^T$$

$$\Xi_4(\rho; h_{\sigma_{p-1}+j}, h(t))$$
$$= (h_{\sigma_{p-1}+j} - h(t))\text{Sym}\{(\mathcal{I}_1 - \mathcal{I}_2)^T \mathcal{T} \mathcal{I}_4\}$$
$$- (\mathcal{I}_1 - \mathcal{I}_2)^T \mathcal{T} (\mathcal{I}_1 - \mathcal{I}_2)$$
$$+ (h_{\sigma_{p-1}+j} - h(t))\mathcal{I}_4^T \mathcal{R} \mathcal{I}_4$$
$$- \text{Sym}\{(\mathcal{I}_1 - \mathcal{I}_2)^T \mathcal{H}_1 \left[\mathcal{I}_1^T, \mathcal{I}_2^T\right]^T\}$$
$$+ \rho h(t)\left[\mathcal{I}_1^T, \mathcal{I}_2^T\right] \mathcal{H}_1^T \mathcal{R}^{-1} \mathcal{H}_1 \left[\mathcal{I}_1^T, \mathcal{I}_2^T\right]^T$$
$$- 3\text{Sym}\{(\mathcal{I}_1 + \mathcal{I}_2 - 2\mathcal{I}_5)^T \mathcal{H}_2 \left[\mathcal{I}_1^T, \mathcal{I}_2^T, \mathcal{I}_5^T\right]^T\}$$
$$+ 3\rho h(t)\left[\mathcal{I}_1^T, \mathcal{I}_2^T, \mathcal{I}_5^T\right] \mathcal{H}_2^T \mathcal{R}^{-1} \mathcal{H}_2 \left[\mathcal{I}_1^T, \mathcal{I}_2^T, \mathcal{I}_5^T\right]^T$$

$$\Xi_5(\rho; h_{\sigma_{p-1}+j}, h(t))$$
$$= \text{Sym}\{(\mathcal{I}_4^T + \kappa \mathcal{I}_1^T)\mathcal{S}(-\mathcal{I}_4 - \mathcal{C}\mathcal{I}_1 + \mathcal{W}_1 \mathcal{I}_3 + \gamma \mathcal{W}_2 \mathcal{I}_8$$
$$+ (1 - \gamma)\mathcal{W}_2 \mathcal{I}_9)\} - 2\kappa \sum_{l=1}^{m} \frac{\pi^2}{(\bar{\varrho}_l - \underline{\varrho}_l)^2} \mathcal{I}_1^T \mathcal{S} \mathcal{A}_l \mathcal{I}_1$$
$$+ \text{Sym}\{(\mathcal{I}_4^T + \kappa \mathcal{I}_1^T)\mathcal{S}\mathcal{K}(\mathcal{I}_2 - \mathcal{I}_{10})\}.$$

Proof: Choose an LKF candidate for system (17) as

$$\mathcal{V}(t) = \sum_{i=1}^{5} \mathcal{V}_i(t), \quad t \in F_j \tag{23}$$

where

$$\mathcal{V}_1(t) = \int_{\Omega} \vartheta^T(t, x)\mathcal{P}\vartheta(t, x)dx$$

$$\mathcal{V}_2(t) = \gamma \int_{\Omega} \int_{t-d_1(t)}^{t} \xi_1^T(s, x)\mathcal{Q}_1\xi_1(s, x)dsdx$$
$$+ (1 - \gamma) \int_{\Omega} \int_{t-d_2(t)}^{t} \xi_1^T(s, x)\mathcal{Q}_2\xi_1(s, x)dsdx$$

$$\mathcal{V}_3(t) = (h_{\sigma_{p-1}+j} - h(t)) \int_{\Omega} \xi_2^T(t, x)\mathcal{T}\xi_2(t, x)dx$$

$$\mathcal{V}_4(t) = \int_{\Omega} \sum_{l=1}^{m} \frac{\partial \vartheta^T(t, x)}{\partial x_l}\mathcal{S}\mathcal{A}_l\frac{\partial \vartheta(t, x)}{\partial x_l}dx$$

$$\mathcal{V}_5(t) = (h_{\sigma_{p-1}+j} - h(t))$$
$$\cdot \int_{\Omega} \int_{t_{\sigma_{p-1}+j}}^{t} \frac{\partial \vartheta^T(s, x)}{\partial s}\mathcal{R}\frac{\partial \vartheta(s, x)}{\partial s}dsdx$$

with $\xi_1(t, x) = \text{col}\{\vartheta(t, x), f(\vartheta(t, x))\}$ and $\xi_2(t, x) = \vartheta(t, x) - \vartheta(t - h(t), x)$.

It is noted that $\lim_{t \to t_{\sigma_{p-1}+j}} \mathcal{V}_i(t) = 0$ $(i = 3, 5)$, which means that $\mathcal{V}_i(t)$ $(i = 3, 5)$ vanish before and after $t_{\sigma_{p-1}+j}$. Thus, $\mathcal{V}(t)$ is continuous in time. In the meantime, one finds that

$$\mathcal{V}(t_{\sigma_{p-1}+j}) = \sum_{i=1,2,4} \mathcal{V}_i(t_{\sigma_{p-1}+j}) \geq 0. \tag{24}$$

Let $\mathcal{L}$ be the infinitesimal operator [47] along error system (17). Then, $\mathcal{L}\{\mathcal{V}(t)\}$ is calculated as

$$\mathcal{E}\{\mathcal{L}\mathcal{V}(t)\} = \sum_{i=1}^{5} \mathcal{E}\{\mathcal{L}\mathcal{V}_i(t)\} \tag{25}$$

where $\mathcal{E}\{\mathcal{L}\mathcal{V}_i(t)\}$ $(i = 1, 2, \ldots, 5)$ are listed as

$$\mathcal{E}\{\mathcal{L}\mathcal{V}_1(t)\}$$
$$= \mathcal{E}\left\{2 \int_{\Omega} \vartheta^T(t, x)\mathcal{P}\frac{\partial \vartheta(t, x)}{\partial t}dx\right\} \tag{26}$$

$$\mathcal{E}\{\mathcal{L}\mathcal{V}_2(t)\}$$
$$\leq \mathcal{E}\left\{\gamma \int_{\Omega} \xi_1^T(t, x)\mathcal{Q}_1\xi_1(t, x)dx - \gamma(1 - \mu_1)\right.$$
$$\cdot \int_{\Omega} \xi_1^T(t - d_1(t), x)\mathcal{Q}_1\xi_1(t - d_1(t), x)dx$$
$$+ (1 - \gamma) \int_{\Omega} \xi_1^T(t, x)\mathcal{Q}_2\xi_1(t, x)dx - (1 - \gamma)(1 - \mu_2)$$
$$\left. \cdot \int_{\Omega} \xi_1^T(t - d_2(t), x)\mathcal{Q}_2\xi_1(t - d_2(t), x)dx\right\} \tag{27}$$

$$\mathcal{E}\{\mathcal{L}\mathcal{V}_3(t)\}$$
$$= \mathcal{E}\left\{2(h_{\sigma_{p-1}+j} - h(t)) \int_{\Omega} \xi_2^T(t, x)\mathcal{T}\frac{\partial \vartheta(t, x)}{\partial t}dx\right.$$
$$\left. - \int_{\Omega} \xi_2^T(t, x)\mathcal{T}\xi_2(t, x)dx\right\} \tag{28}$$

$$\mathcal{E}\{\mathcal{L}\mathcal{V}_4(t)\}$$
$$= \mathcal{E}\left\{2 \int_{\Omega} \sum_{l=1}^{m} \frac{\partial^2 \vartheta^T(t, x)}{\partial x_l \partial t}\mathcal{S}\mathcal{A}_l\frac{\partial \vartheta(t, x)}{\partial x_l}dx\right\} \tag{29}$$

$$\mathcal{E}\{\mathcal{L}\mathcal{V}_5(t)\}$$
$$= \mathcal{E}\left\{-\int_{\Omega} \int_{t_{\sigma_{p-1}+j}}^{t} \frac{\partial \vartheta^T(s, x)}{\partial s}\mathcal{R}\frac{\partial \vartheta(s, x)}{\partial s}dsdx\right.$$
$$\left. + (h_{\sigma_{p-1}+j} - h(t)) \int_{\Omega} \frac{\partial \vartheta^T(t, x)}{\partial t}\mathcal{R}\frac{\partial \vartheta(t, x)}{\partial t}dx\right\}. \tag{30}$$

When the current sampled signal is not transmitted to the controller, from (13) and Proposition 1, one gets the following inequality:

$$\delta_0 \left\|\mathcal{B}_1^{\frac{1}{2}}(\vartheta(t - h(t), x) - e(t - h(t), x))\right\|_{L^2}^2$$
$$- \left\|\mathcal{B}_2^{\frac{1}{2}}e(t - h(t), x)\right\|_{L^2}^2 \geq 0 \tag{31}$$

which is equivalent to

$$\delta_0 \int_{\Omega} (\vartheta(t - h(t), x) - e(t - h(t), x))^T \mathcal{B}_1$$
$$\cdot (\vartheta(t - h(t), x) - e(t - h(t), x))dx$$
$$- \int_{\Omega} e^T(t - h(t), x)\mathcal{B}_2 e(t - h(t), x)dx \geq 0. \tag{32}$$

According to Lemma 1, from (30), we have

$$- \int_{\Omega} \int_{t_{\sigma_{p-1}+j}}^{t} \frac{\partial \vartheta^T(s, x)}{\partial s}\mathcal{R}\frac{\partial \vartheta(s, x)}{\partial s}dsdx$$
$$\leq -\frac{1}{h(t)} \int_{\Omega} \xi_2^T(t, x)\mathcal{R}\xi_2(t, x)dx$$
$$- \frac{3}{h(t)} \int_{\Omega} \xi_4^T(t, x)\mathcal{R}\xi_4(t, x)dx \tag{33}$$

where $\xi_4(t, x) = \vartheta(t, x) + \vartheta(t - h(t), x) - 2\xi_3(t, x)$.

For any matrices $\mathcal{H}_1 \in \Re^{n \times 2n}$ and $\mathcal{H}_2 \in \Re^{n \times 3n}$, we find that

$$\int_{\Omega} \frac{1}{h(t)}(\mathcal{R}\xi_2(t, x) - h(t)\mathcal{H}_1\xi_5(t, x))^T \mathcal{R}^{-1}$$

$$\cdot (\mathcal{R}\xi_2(t,x) - h(t)\mathcal{H}_1\xi_5(t,x))dx \geq 0 \tag{34}$$

$$\int_\Omega \frac{1}{h(t)}(\mathcal{R}\xi_4(t,x) - h(t)\mathcal{H}_2\xi_6(t,x))^T\mathcal{R}^{-1}$$
$$\cdot (\mathcal{R}\xi_4(t,x) - h(t)\mathcal{H}_2\xi_6(t,x))dx \geq 0 \tag{35}$$

from which one can obtain

$$-\frac{1}{h(t)}\int_\Omega \xi_2^T(t,x)\mathcal{R}\xi_2(t,x)dx$$
$$\leq -2\int_\Omega \xi_2^T(t,x)\mathcal{H}_1\xi_5(t,x)dx$$
$$+ h(t)\int_\Omega \xi_5^T(t,x)\mathcal{H}_1^T\mathcal{R}^{-1}\mathcal{H}_1\xi_5(t,x)dx \tag{36}$$

$$-\frac{3}{h(t)}\int_\Omega \xi_4^T(t,x)\mathcal{R}\xi_4(t,x)dx$$
$$\leq -6\int_\Omega \xi_4^T(t,x)\mathcal{H}_2\xi_6(t,x)dx$$
$$+ 3h(t)\int_\Omega \xi_6^T(t,x)\mathcal{H}_2^T\mathcal{R}^{-1}\mathcal{H}_2\xi_6(t,x)dx \tag{37}$$

where $\xi_5(t,x) = \text{col}\{\vartheta(t,x), \vartheta(t-h(t),x)\}$ and $\xi_6(t,x) = \text{col}\{\vartheta(t,x), \vartheta(t-h(t),x), \xi_3(t,x)\}$.

From the error system (17), one has the following zero equality:

$$0 = \mathscr{E}\left\{2\int_\Omega \left[\frac{\partial\vartheta(t,x)}{\partial t} + \kappa\vartheta(t,x)\right]^T \mathcal{S}\right.$$
$$\times \left[-\frac{\partial\vartheta(t,x)}{\partial t} + \sum_{l=1}^m \mathcal{A}_l\frac{\partial^2\vartheta(t,x)}{\partial x_l^2} - \mathcal{C}\vartheta(t,x)\right.$$
$$+ \mathcal{W}_1 f(\vartheta(t,x)) + \gamma\mathcal{W}_2 f(\vartheta(t-d_1(t),x))$$
$$+ (1-\gamma)\mathcal{W}_2 f(\vartheta(t-d_2(t),x))$$
$$\left.\left. + \mathcal{K}(\vartheta(t-h(t),x) - e(t-h(t),x))\right]dx\right\}. \tag{38}$$

Based on the Green formula, the Dirichlet boundary condition (18), and (38), we can find that

$$2\int_\Omega \frac{\partial\vartheta^T(t,x)}{\partial t}\mathcal{S}\sum_{l=1}^m \mathcal{A}_l\frac{\partial^2\vartheta(t,x)}{\partial x_l^2}dx$$
$$= 2\int_{\partial\Omega}\sum_{l=1}^m \frac{\partial\vartheta^T(t,x)}{\partial t}\mathcal{S}\mathcal{A}_l\frac{\partial\vartheta(t,x)}{\partial x_l}dx$$
$$- 2\int_\Omega\sum_{l=1}^m \frac{\partial^2\vartheta^T(t,x)}{\partial x_l\partial t}\mathcal{S}\mathcal{A}_l\frac{\partial\vartheta(t,x)}{\partial x_l}dx$$
$$= -2\int_\Omega\sum_{l=1}^m \frac{\partial^2\vartheta^T(t,x)}{\partial x_l\partial t}\mathcal{S}\mathcal{A}_l\frac{\partial\vartheta(t,x)}{\partial x_l}dx. \tag{39}$$

Similarly, one derives from Lemma 2 that

$$2\kappa\int_\Omega \vartheta^T(t,x)\mathcal{S}\sum_{l=1}^m \mathcal{A}_l\frac{\partial^2\vartheta(t,x)}{\partial x_l^2}dx$$
$$= -2\kappa\int_\Omega\sum_{l=1}^m \frac{\partial\vartheta^T(t,x)}{\partial x_l}\mathcal{S}\mathcal{A}_l\frac{\partial\vartheta(t,x)}{\partial x_l}dx$$
$$\leq -2\kappa\int_\Omega\sum_{l=1}^m \frac{\pi^2}{(\bar{\varrho}_l - \underline{\varrho}_l)^2}\vartheta^T(t,x)\mathcal{S}\mathcal{A}_l\vartheta(t,x)dx. \tag{40}$$

According to Assumption 1, for any diagonal matrices $\mathcal{D}_i > 0 \in \Re^{n\times n}(i = 1, 2, 3)$, one obtains

$$\int_\Omega 2(f(\vartheta(t,x)) - \mathcal{F}^-\vartheta(t,x))^T\mathcal{D}_1$$
$$\cdot (\mathcal{F}^+\vartheta(t,x) - f(\vartheta(t,x)))dx \geq 0$$
$$\int_\Omega 2(f(\vartheta(t-d_1(t),x)) - \mathcal{F}^-\vartheta(t-d_1(t),x))^T\mathcal{D}_2$$
$$\cdot (\mathcal{F}^+\vartheta(t-d_1(t),x) - f(\vartheta(t-d_1(t),x)))dx \geq 0$$
$$\int_\Omega 2(f(\vartheta(t-d_2(t),x)) - \mathcal{F}^-\vartheta(t-d_2(t),x))^T\mathcal{D}_3$$
$$\cdot (\mathcal{F}^+\vartheta(t-d_2(t),x) - f(\vartheta(t-d_2(t),x)))dx \geq 0. \tag{41}$$

Combining (25)–(41), for $t \in F_j$, we obtain

$$\mathscr{E}\{\mathscr{L}\mathscr{V}(t)\}$$
$$\leq \mathscr{E}\left\{\int_\Omega \eta^T(t,x)\Xi(1; h_{\sigma_{p-1}+j}, h(t))\eta(t,x)dx\right\}$$
$$= \mathscr{E}\left\{\int_\Omega \eta^T(t,x)\left[\frac{h_{\sigma_{p-1}+j} - h(t)}{\tilde{h}}\Xi(1;\tilde{h},0) + \frac{h(t)}{\tilde{h}}\right.\right.$$
$$\left.\left. \times \Xi(1;\tilde{h},\tilde{h}) + \frac{\tilde{h} - h_{\sigma_{p-1}+j}}{\tilde{h}}\Xi(1;0,0)\right]\eta(t,x)dx\right\}. \tag{42}$$

Using the Schur complement to (22) and from (20)–(22) and (42), one finds that

$$\mathscr{E}\{\mathscr{L}\mathscr{V}(t)\} \leq 0, \quad t \in F_j \tag{43}$$

from which we derive

$$\mathscr{E}\{\mathscr{L}\mathscr{V}(t)\} \leq -\varepsilon\mathscr{E}\left\{\int_\Omega \eta^T(t,x)\eta(t,x)dx\right\}$$
$$\leq -\varepsilon\mathscr{E}\{\|\vartheta(t,x)\|_{L^2}^2\}, \quad t \in F_j \tag{44}$$

where $\varepsilon = \min\{\lambda_{\min}(-\Xi(1;\tilde{h},0)), \lambda_{\min}(-\Xi(1;\tilde{h},\tilde{h}), \lambda_{\min}(-\Xi(1;0,0))\}$.

For $\eta(t,x) \neq 0$, we have from (24) and (43) that

$$\mathscr{V}(t) > \mathscr{V}(t_{\sigma_{p-1}+j+1}^-) = \mathscr{V}(t_{\sigma_{p-1}+j+1}) > 0$$
$$t \in F_j, \quad j = 0, 1, 2, \ldots, \tau_p - 1 \tag{45}$$

which implies that $\mathscr{V}(t)$ is positive definite.

By the Itô's formula, one gets

$$\mathscr{E}\{\mathscr{V}(t)\} - \mathscr{E}\{\mathscr{V}(\tilde{t}_0)\} = \mathscr{E}\left\{\int_{\tilde{t}_0}^t \mathscr{L}\mathscr{V}(s)ds\right\}. \tag{46}$$

Then, from (23) and (44)–(46), there exists a scalar $\iota > 0$ such that

$$\iota\mathscr{E}\{\|\vartheta(t,x)\|_{L^2}^2\} \leq \mathscr{E}\{\mathscr{V}(t)\}$$
$$= \mathscr{E}\{\mathscr{V}(\tilde{t}_0)\} + \mathscr{E}\left\{\int_{\tilde{t}_0}^t \mathscr{L}\mathscr{V}(s)ds\right\}$$
$$\leq \mathscr{E}\{\mathscr{V}(\tilde{t}_0)\} - \varepsilon\int_{\tilde{t}_0}^t \mathscr{E}\{\|\vartheta(s,x)\|_{L^2}^2\}ds. \tag{47}$$

Thus, the error system (17) is asymptotically stable. This completes the proof. ∎

Remark 4: Based on the Lyapunov stability theory, it is crucial to choose an appropriate LKF for deriving stability criteria. In this article, (23) is chosen as the LKF. $\mathcal{V}_1(t)$ is the basic term. $\mathcal{V}_4(t)$ is constructed to counteract the reaction–diffusion term $2\int_\Omega ((\partial \vartheta^T(t,x))/\partial t)\mathcal{S}\sum_{l=1}^m \mathcal{A}_l((\partial \vartheta^2(t,x))/\partial x_l^2)dx$ in (38). It is well known that delay information and sampling information is effective to reduce the conservatism of stability criteria. Thus, $\mathcal{V}_2(t)$ is introduced to capture the information of the time delay $d(t)$. $\mathcal{V}_3(t)$ and $\mathcal{V}_5(t)$ are used to capture the information of sampling.

B. Adaptive Event-Triggered Sampled-Data Controller Design

When the controller gain $\mathcal{K}$ is not given, based on Theorem 1, the adaptive event-triggered sampled-data controller (16) is designed for synchronization of the RDNNs (1) and (7) as follows.

Theorem 2: For given scalars $0 \le \gamma \le 1$, μ_i $(i = 1, 2)$, $\tilde{h} > 0$, δ_0, and κ, the response RDNN (7) is asymptotically synchronized to the drive RDNN (1) if there exist matrices $\mathcal{P} > 0 \in \Re^{n\times n}$, $\mathcal{Q}_i > 0 \in \Re^{2n\times 2n}$ $(i = 1, 2)$, $\mathcal{T} > 0 \in \Re^{n\times n}$, $\mathcal{R} > 0 \in \Re^{n\times n}$, and $\mathcal{B}_i > 0 \in \Re^{n\times n}$ $(i = 1, 2)$, diagonal matrices $\mathcal{D}_i > 0 \in \Re^{n\times n}$ $(i = 1, 2, 3)$, and any matrices $\mathcal{S} \in \Re^{n\times n}$, $\mathcal{H}_1 \in \Re^{n\times 2n}$, $\mathcal{H}_2 \in \Re^{n\times 3n}$, and $\mathcal{K}^* \in \Re^{n\times n}$ satisfying $\mathcal{S}\mathcal{A}_l \ge 0$ and

$$\Xi^*(0; 0, 0) < 0 \tag{48}$$

$$\Xi^*(0; \tilde{h}, 0) < 0 \tag{49}$$

$$\begin{bmatrix} \Xi^*(0;\tilde{h},\tilde{h}) & \sqrt{\tilde{h}}[\mathcal{I}_1^T,\mathcal{I}_2^T]\mathcal{H}_1^T & \sqrt{3\tilde{h}}[\mathcal{I}_1^T,\mathcal{I}_2^T,\mathcal{I}_5^T]\mathcal{H}_2^T \\ * & -\mathcal{R} & 0 \\ * & * & -\mathcal{R} \end{bmatrix}$$
$$< 0 \tag{50}$$

where $\Xi^*(\rho; h_{\sigma_{p-1}+j}, h(t)) = \sum_{i=1}^4 \Xi_i(\rho; h_{\sigma_{p-1}+j}, h(t)) + \Xi_5^*(\rho; h_{\sigma_{p-1}+j}, h(t))$ with

$$\Xi_5^*(\rho; h_{\sigma_{p-1}+j}, h(t))$$
$$= \text{Sym}\{(\mathcal{I}_4^T + \kappa\mathcal{I}_1^T)\mathcal{S}(-\mathcal{I}_4 - \mathcal{C}\mathcal{I}_1 + \mathcal{W}_1\mathcal{I}_3 + \gamma\mathcal{W}_2\mathcal{I}_8$$
$$+ (1-\gamma)\mathcal{W}_2\mathcal{I}_9)\} - 2\kappa\sum_{l=1}^m \frac{\pi^2}{(\bar{\varrho}_l - \underline{\varrho}_l)^2}\mathcal{I}_1^T\mathcal{S}\mathcal{A}_l\mathcal{I}_1$$
$$+\text{Sym}\{(\mathcal{I}_4^T + \kappa\mathcal{I}_1^T)\mathcal{K}^*(\mathcal{I}_2 - \mathcal{I}_{10})\}$$

and other notations are defined in Theorem 1. The adaptive event-triggered sampled-data controller gain of (16) is

$$\mathcal{K} = \mathcal{S}^{-1}\mathcal{K}^*. \tag{51}$$

Proof: Let $\mathcal{S}\mathcal{K} = \mathcal{K}^*$. From Theorem 1, we find that (48)–(50) hold.

C. Synchronization for Reaction–Diffusion Neural Networks With Event-Triggered Sampled-Data Control

When the event-triggered mechanism (13) is not adaptive, the threshold functions $\delta(t_{0+\tau})$ and $\delta(t_{\sigma_{k-1}+\tau})$ are reduced to a determined constant δ^*. Following the similar proof of Theorem 2, we can obtain the following results.
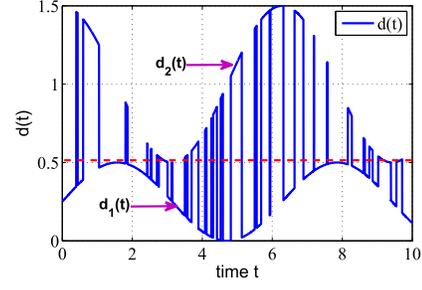
Fig. 1. Random time-varying delay $d(t)$.

Theorem 3: For given scalars $0 \le \gamma \le 1$, μ_i $(i = 1, 2)$, $\tilde{h} > 0$, δ^*, and κ, the response RDNN (7) is asymptotically synchronized to the drive RDNN (1) if there exist matrices $\mathcal{P} > 0 \in \Re^{n\times n}$, $\mathcal{Q}_i > 0 \in \Re^{2n\times 2n}$ $(i = 1, 2)$, $\mathcal{T} > 0 \in \Re^{n\times n}$, $\mathcal{R} > 0 \in \Re^{n\times n}$, and $\mathcal{B}_i > 0 \in \Re^{n\times n}$ $(i = 1, 2)$, diagonal matrices $\mathcal{D}_i > 0 \in \Re^{n\times n}$ $(i = 1, 2, 3)$, and any matrices $\mathcal{S} \in \Re^{n\times n}$, $\mathcal{H}_1 \in \Re^{n\times 2n}$, $\mathcal{H}_2 \in \Re^{n\times 3n}$, and $\mathcal{K}^* \in \Re^{n\times n}$ satisfying $\mathcal{S}\mathcal{A}_l \ge 0$ and

$$\Xi^{**}(0; 0, 0) < 0 \tag{52}$$

$$\Xi^{**}(0; \tilde{h}, 0) < 0 \tag{53}$$

$$\begin{bmatrix} \Xi^{**}(0;\tilde{h},\tilde{h}) & \sqrt{\tilde{h}}[\mathcal{I}_1^T,\mathcal{I}_2^T]\mathcal{H}_1^T & \sqrt{3\tilde{h}}[\mathcal{I}_1^T,\mathcal{I}_2^T,\mathcal{I}_5^T]\mathcal{H}_2^T \\ * & -\mathcal{R} & 0 \\ * & * & -\mathcal{R} \end{bmatrix}$$
$$< 0 \tag{54}$$

where $\Xi^{**}(\rho; h_{\sigma_{p-1}+j}, h(t)) = \sum_{i=2}^4 \Xi_i(\rho; h_{\sigma_{p-1}+j}, h(t)) + \Xi_1^*(\rho; h_{\sigma_{p-1}+j}, h(t)) + \Xi_5^*(\rho; h_{\sigma_{p-1}+j}, h(t))$ with

$$\Xi_1^*(\rho; h_{\sigma_{p-1}+j}, h(t))$$
$$= \delta^*(\mathcal{I}_2 - \mathcal{I}_{10})^T\mathcal{B}_1(\mathcal{I}_2 - \mathcal{I}_{10}) - \mathcal{I}_{10}^T\mathcal{B}_2\mathcal{I}_{10}$$

and other notations are defined in Theorems 1 and 2. Furthermore, the event-triggered sampled-data controller gain is given as

$$\mathcal{K} = \mathcal{S}^{-1}\mathcal{K}^*. \tag{55}$$

D. Synchronization for Reaction–Diffusion Neural Networks With Sampled-Data Control

When the adaptive event-triggered mechanism (13) is not considered, the controller (15) is reduced to the following sampled-data controller:

$$\mathcal{U}(t, x) = \mathcal{K}\vartheta(t_k, x), \quad t_k \le t < t_{k+1} \tag{56}$$

and the error system (17) is transformed into

$$\frac{\partial \vartheta(t,x)}{\partial t} = \sum_{l=1}^m \frac{\partial}{\partial x_l}\left(\mathcal{A}_l\frac{\partial \vartheta(t,x)}{\partial x_l}\right) - \mathcal{C}\vartheta(t,x)$$
$$+ \mathcal{W}_1 f(\vartheta(t,x)) + \beta(t)\mathcal{W}_2 f(\vartheta(t - d_1(t), x))$$
$$+ (1 - \beta(t))\mathcal{W}_2 f(\vartheta(t - d_2(t), x))$$
$$+ \mathcal{K}\vartheta(t_k, x), \ t \in [t_k, t_{k+1}), \ x \in \Omega \tag{57}$$

$$\vartheta(t, x) = 0, \quad (t, x) \in [\tilde{t}_0 - d_2, +\infty) \times \partial\Omega \tag{58}$$

$$\vartheta(s + \tilde{t}_0, x) = \psi(s, x) \in C([-d_2, 0] \times \Omega, \Re^n). \tag{59}$$
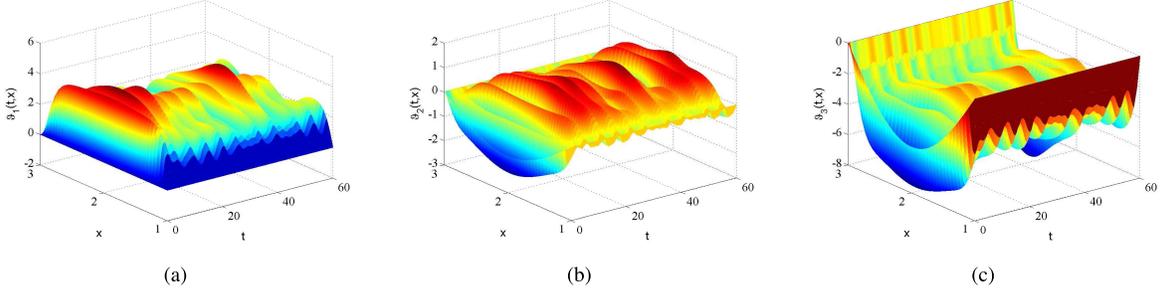
Fig. 2. Time-space behaviors of error system (10) with $\mathcal{U}(t,x) = 0$. (a) $\vartheta_1(t,x)$ with $\psi(s,x) = 0$. (b) $\vartheta_2(t,x)$ with $\psi(s,x) = -1.5\Lambda(s,x)$. (c) $\vartheta_3(t,x)$ with $\psi(s,x) = -5\Lambda(s,x)$.
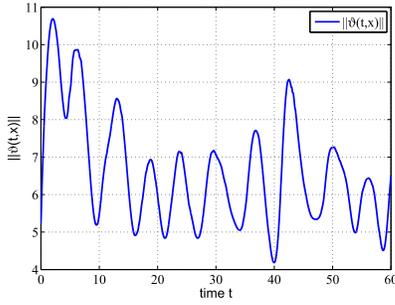
Fig. 3. Time responses of error signal $\|\vartheta(t,x)\|_{L^2}$ with $\mathcal{U}(t,x) = 0$.
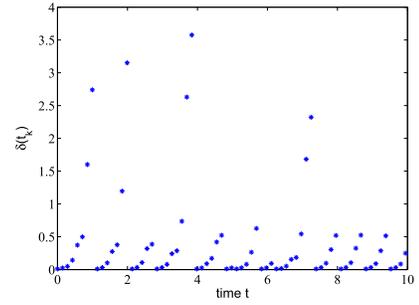
Fig. 4. Threshold function $\delta(t_k)$ of the adaptive event-triggered communication mechanism (13).

Denote $\hat{\mathcal{I}}_i = [0_{n,(i-1)n} \quad I_n \quad 0_{n,(9-i)n}], (i = 1, \ldots, 9)$,
$\hat{h}(t) = t - t_k, \hat{\xi}_3(t,x) = \begin{cases} (1/(t - t_k)) \int_{t_k}^t \vartheta(s,x)ds, & t \neq t_k, \\ \vartheta(t_k,x), & t = t_k, \end{cases}$
and $\hat{\eta}(t,x) = \text{col}\{\vartheta(t,x), \vartheta(t - \hat{h}(t),x),$
$f(\vartheta(t,x)), (\partial\vartheta(t,x)/\partial t), \hat{\xi}_3(t,x), \vartheta(t - d_1(t),x), \vartheta(t - d_2(t),x), f(\vartheta(t - d_1(t),x)), f(\vartheta(t - d_2(t),x))\}$. Similar to the proof of Theorem 2, one can derive the following results.

Theorem 4: For given scalars $0 \leq \gamma \leq 1$, μ_i $(i = 1, 2)$, $\tilde{h} > 0$, and κ, the response RDNN (7) is asymptotically synchronized to the drive RDNN (1) if there exist matrices $\mathcal{P} > 0 \in \Re^{n \times n}$, $\mathcal{Q}_i > 0 \in \Re^{2n \times 2n}$ $(i = 1, 2)$, $\mathcal{T} > 0 \in \Re^{n \times n}$, and $\mathcal{R} > 0 \in \Re^{n \times n}$, diagonal matrices $\mathcal{D}_i > 0 \in \Re^{n \times n}$ $(i = 1, 2, 3)$, and any matrices $\mathcal{S} \in \Re^{n \times n}$, $\mathcal{H}_1 \in \Re^{n \times 2n}$, $\mathcal{H}_2 \in \Re^{n \times 3n}$, and $\mathcal{K}^* \in \Re^{n \times n}$ satisfying $\mathcal{S}\mathcal{A}_l \geq 0$ and

$$\hat{\Xi}(0; 0, 0) < 0 \tag{60}$$

$$\hat{\Xi}(0; \tilde{h}, 0) < 0 \tag{61}$$

$$\begin{bmatrix} \hat{\Xi}(0; \tilde{h}, \tilde{h}) & \sqrt{\tilde{h}}[\hat{\mathcal{I}}_1^T, \hat{\mathcal{I}}_2^T]\mathcal{H}_1^T & \sqrt{3\tilde{h}}[\hat{\mathcal{I}}_1^T, \hat{\mathcal{I}}_2^T, \hat{\mathcal{I}}_5^T]\mathcal{H}_2^T \\ * & -\mathcal{R} & 0 \\ * & * & -\mathcal{R} \end{bmatrix}$$
$$< 0 \tag{62}$$

where $\hat{\Xi}(\rho; h_{\sigma_{p-1}+j}, h(t)) = \sum_{i=2}^5 \hat{\Xi}_i(\rho; h_k, \hat{h}(t))$ with

$\hat{\Xi}_2(\rho; h_k, \hat{h}(t))$
$= \text{Sym}\{(\hat{\mathcal{I}}_3 - \mathcal{F}^-\hat{\mathcal{I}}_1)^T\mathcal{D}_1(\mathcal{F}^+\hat{\mathcal{I}}_1 - \hat{\mathcal{I}}_3)\}$
$+ \text{Sym}\{(\hat{\mathcal{I}}_8 - \mathcal{F}^-\hat{\mathcal{I}}_6)^T\mathcal{D}_2(\mathcal{F}^+\hat{\mathcal{I}}_6 - \hat{\mathcal{I}}_8)\}$
$+ \text{Sym}\{(\hat{\mathcal{I}}_9 - \mathcal{F}^-\hat{\mathcal{I}}_7)^T\mathcal{D}_3(\mathcal{F}^+\hat{\mathcal{I}}_7 - \hat{\mathcal{I}}_9)\}$

$\hat{\Xi}_3(\rho; h_k, \hat{h}(t))$

$= \text{Sym}\{\hat{\mathcal{I}}_1^T\mathcal{P}\hat{\mathcal{I}}_4\} + \gamma[\hat{\mathcal{I}}_1^T, \hat{\mathcal{I}}_3^T]\mathcal{Q}_1[\hat{\mathcal{I}}_1^T, \hat{\mathcal{I}}_3^T]^T$
$- \gamma(1 - \mu_1)[\hat{\mathcal{I}}_6^T, \hat{\mathcal{I}}_8^T]\mathcal{Q}_1[\hat{\mathcal{I}}_6^T, \hat{\mathcal{I}}_8^T]^T$
$+ (1 - \gamma)[\hat{\mathcal{I}}_1^T, \hat{\mathcal{I}}_3^T]\mathcal{Q}_2[\hat{\mathcal{I}}_1^T, \hat{\mathcal{I}}_3^T]^T$
$- (1 - \gamma)(1 - \mu_2)[\hat{\mathcal{I}}_7^T, \hat{\mathcal{I}}_9^T]\mathcal{Q}_2[\hat{\mathcal{I}}_7^T, \hat{\mathcal{I}}_9^T]^T$

$\hat{\Xi}_4(\rho; h_k, \hat{h}(t))$
$= (h_k - \hat{h}(t))\text{Sym}\{(\hat{\mathcal{I}}_1 - \hat{\mathcal{I}}_2)^T\mathcal{T}\hat{\mathcal{I}}_4\}$
$- (\hat{\mathcal{I}}_1 - \hat{\mathcal{I}}_2)^T\mathcal{T}(\hat{\mathcal{I}}_1 - \hat{\mathcal{I}}_2) + (h_k - \hat{h}(t))\hat{\mathcal{I}}_4^T\mathcal{R}\hat{\mathcal{I}}_4$
$- \text{Sym}\{(\hat{\mathcal{I}}_1 - \hat{\mathcal{I}}_2)^T\mathcal{H}_1[\hat{\mathcal{I}}_1^T, \hat{\mathcal{I}}_2^T]^T\}$
$+ \rho\hat{h}(t)[\hat{\mathcal{I}}_1^T, \hat{\mathcal{I}}_2^T]\mathcal{H}_1^T\mathcal{R}^{-1}\mathcal{H}_1[\hat{\mathcal{I}}_1^T, \hat{\mathcal{I}}_2^T]^T$
$- 3\text{Sym}\{(\hat{\mathcal{I}}_1 + \hat{\mathcal{I}}_2 - 2\hat{\mathcal{I}}_5)^T\mathcal{H}_2[\hat{\mathcal{I}}_1^T, \hat{\mathcal{I}}_2^T, \hat{\mathcal{I}}_5^T]^T\}$
$+ 3\rho\hat{h}(t)[\hat{\mathcal{I}}_1^T, \hat{\mathcal{I}}_2^T, \hat{\mathcal{I}}_5^T]\mathcal{H}_2^T\mathcal{R}^{-1}\mathcal{H}_2[\hat{\mathcal{I}}_1^T, \hat{\mathcal{I}}_2^T, \hat{\mathcal{I}}_5^T]^T$

$\hat{\Xi}_5(\rho; h_k, \hat{h}(t))$
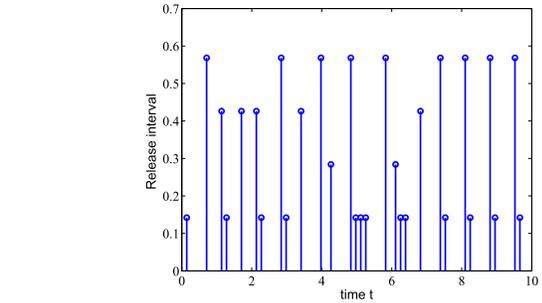
Fig. 5. Transmission instants and release intervals with $\delta(t_k)$.

Fig. 6. Spatial behaviors of error signal $\vartheta(t, x)$ with $\mathcal{U}(t, x)$. (a) $\vartheta_1(t, x)$. (b) $\vartheta_2(t, x)$. (c) $\vartheta_3(t, x)$.

$$= \mathrm{Sym}\left\{ (\hat{\mathcal{I}}_4^T + \kappa \hat{\mathcal{I}}_1^T) \mathcal{S}(-\hat{\mathcal{I}}_4 - \mathcal{C}\hat{\mathcal{I}}_1 + \mathcal{W}_1 \hat{\mathcal{I}}_3 + \gamma \mathcal{W}_2 \hat{\mathcal{I}}_8 \right.$$
$$\left. + (1 - \gamma) \mathcal{W}_2 \hat{\mathcal{I}}_9) \right\} - 2\kappa \sum_{l=1}^{m} \frac{\pi^2}{(\bar{\varrho}_l - \underline{\varrho}_l)^2} \hat{\mathcal{I}}_1^T \mathcal{S} \mathcal{A}_l \hat{\mathcal{I}}_1$$
$$+ \mathrm{Sym}\left\{ (\hat{\mathcal{I}}_4^T + \kappa \hat{\mathcal{I}}_1^T) \mathcal{K}^* \hat{\mathcal{I}}_2 \right\}.$$

Moreover, the sampled-data controller gain of (56) is

$$\mathcal{K} = \mathcal{S}^{-1} \mathcal{K}^*. \tag{63}$$

IV. NUMERICAL EXAMPLE

In this section, a numerical example is provided to verify the effectiveness and superiorities of the analysis results. In order to show how to obtain the maximum sampling interval (MSI) $\tilde{h}$ and the corresponding solutions of matrices $\mathcal{P}$, $\mathcal{Q}_i$ ($i = 1, 2$), $\mathcal{T}$, and others in Theorem 2 (Theorem 3 or 4), Algorithm 1 is given as follows.

Algorithm 1: MSI Search and Feasible Solution Algorithm

Step 1: For given $0 \leq \gamma \leq 1$, μ_i ($i = 1, 2$), $\tilde{h} > 0$, δ_0, and κ, specify the ranges $\tilde{h}$ with increments $\Delta \tilde{h} > 0$. Set $\tilde{h} = \Delta \tilde{h}$.

Step 2: Use MATLAB LMI Toolbox to solve LMIs in Theorem 2 with specified $\tilde{h}$.

Step 3: If there exists a feasible solution, then let $\tilde{h} = \tilde{h} + \Delta \tilde{h}$, and go to Step 2. Otherwise, go to Step 4.

Step 4: If $\tilde{h} = \Delta \tilde{h}$, output "No feasible solution satisfying Theorem 2". Then reselect values of δ_0, κ, go to Step 1. Otherwise, go to Step 5.

Step 5: Output $\tilde{h} = \tilde{h} - \Delta \tilde{h}$, which is the MSI. With the output MSI $\tilde{h}$, and using MATLAB LMI Toolbox to solve the LMIs in Theorem 2, we get the corresponding solutions of matrices $\mathcal{P}$, $\mathcal{Q}_i$ ($i = 1, 2$), $\mathcal{T}$, and others.

Consider the 3-D RDNN (1) with the following parameters:

$$\mathcal{A}_1 = \mathrm{diag}\{0.01, 0.01, 0.01\}, \quad \mathcal{C} = I_3$$
$$\mathcal{W}_1 = \begin{bmatrix} 1.2 & -1.6 & 0 \\ 1.24 & 1 & 0.9 \\ 0 & 2.2 & 1.5 \end{bmatrix}$$
$$\mathcal{W}_2 = \begin{bmatrix} 0.1 & -0.5 & 0 \\ 0.24 & 0.21 & 0.2 \\ 0 & 1.2 & 0.6 \end{bmatrix}$$
$$\Omega = \{x | 1 \leq x \leq 3\}, \quad J = 0$$
$$g_i(\varpi_i(t, x)) = \frac{1}{2}(|\varpi_i(t, x) + 1| - |\varpi_i(t, x) - 1|)$$

where $i = 1, 2, 3$. It is obvious that $g_i(\varpi_i(t, x))$ satisfies Assumption 1 with $l_1^- = l_2^- = l_3^- = 0$ and $l_1^+ = l_2^+ = l_3^+ = 1$.

Take $\gamma = 0.7$, $d_1(t) = 0.25 + 0.25 \sin(t)$, and $d_2(t) = 1 + 0.5 \cos(t)$, from which one obtains $\mu_1 = 0.25$, $\mu_2 = 0.5$, and $d_1 = 0.5$. Similar to [20] and [48], assume the random time-varying delay $d(t)$ be a Markov process with transition rate matrix $(1/\mathcal{T}) \times \begin{bmatrix} \gamma - 1 & 1 - \gamma \\ \gamma & -\gamma \end{bmatrix}$, where the updated period $\mathcal{T} = 0.1$. The time responses of random time-varying delay $d(t)$ are displayed in Fig. 1. From Fig. 1, one finds that the random time-varying delay $d(t)$ switches between $d_1(t)$ and $d_2(t)$.

Choose the initial conditions as

$$\psi_1(s, x) : \mathrm{col}\{-\Lambda(s, x), 0.5\Lambda(s, x), \Lambda(s, x)\}$$
$$\psi_2(s, x) : \mathrm{col}\{-\Lambda(s, x), -\Lambda(s, x), -4\Lambda(s, x)\}$$

where $\Lambda(s, x) = \cos(((2x - 4)\pi)/4)$. Then, when $\mathcal{U}(t, x) = 0$, the time-space behaviors of error system (10) are shown in Fig. 2, and the time responses of error signal $\|\vartheta(t, x)\|_{L^2}$ are plotted in Fig. 3, from which we can find that the synchronization of RDNNs (1) and (7) cannot be achieved if there is no control input.

Now, we show the effectiveness and advantages of Theorem 2. Take $\delta_0 = 0.01$, $\theta = 3$, $\gamma = 0.7$, $\kappa = 3$, $\mu_1 = 0.25$, $\mu_2 = 0.5$, and set $9\delta_0 \mathcal{B}_1 > \mathcal{B}_2$. By Algorithm 1, we find the MSI $\tilde{h} = 0.1421$ and obtain the solutions of matrices as (to save space, we only list some of the obtained matrices) the matrices can be derived, as shown at the bottom of next page. Then, from (51), the adaptive event-triggered sampled-data controller gain is

$$\mathcal{K} = \begin{bmatrix} -1.8999 & 0.0999 & 1.1411 \\ -0.8020 & -3.3004 & -0.7980 \\ 0.9529 & -2.3492 & -3.6566 \end{bmatrix}.$$

With the abovementioned parameters, by the adaptive event-triggered communication mechanism (13), the threshold function $\delta(t_k)$ is displayed in Fig. 4, which shows that $\delta(t_k)$ can be dynamically adjusted based on the current sampled and latest transmitted signals. The transmission instants and release intervals are plotted in Fig. 5. From Fig. 5, we find that, when $t = 10$, the number of transmitted signals (NTSs) is 29. Fig. 6 represents the controlled spatial behaviors of error signal $\vartheta(t, x)$, and the trajectories of the controlled signal $\|\vartheta(t, x)\|_{L^2}$ are displayed in Fig. 7. From Figs. 6 and 7, we find that the synchronization of RDNNs (1) and (7)

TABLE I

NTSs AND APs FOR VARIOUS SCHEMES

Scheme	NTSs	APs	Improvement rates (%)
Theorem 2	29	0.3448	
Theorem 3	42	0.2380	44.87%
Theorem 4	70	0.1428	141.45%

TABLE II

NTSs FOR VARIOUS θ'S

θ	0.1	0.3	0.5	1.0	2.0	3.0
NTS	38	37	32	26	28	29

Fig. 7. Trajectories of the error signal $\|\vartheta(t,x)\|_{L^2}$ with the adaptive event-triggered sampled-data controller $\mathcal{U}(t,x)$.

is realized under the adaptive event-triggered sampled-data controller $\mathcal{U}(t,x)$ in (15), where the adaptive event-triggered sampled-data controller $\mathcal{U}(t,x)$ is shown in Fig. 8.

By various schemes, the number of NTSs and average periods (APs) is listed in Table I. From Table I, one finds that, by the adaptive event-triggered scheme in Theorem 2, the event-triggered scheme in Theorem 3, and the sampled-data control scheme in Theorem 4, the NTSs are 29, 42, and 70, and the APs are 0.3448, 0.2380, and 0.1428, respectively. The AP of the adaptive event-triggered scheme in Theorem 2 improves 44.87% and 141.45% than that of

Theorems 3 and 4, respectively. Thus, compared with the schemes of Theorems 3 and 4, the adaptive event-triggered scheme in Theorem 2 can effectively reduce the transmission signals to save the communication resources. Meanwhile, by using MATLAB 8.0.0.783 (R2012b) running on a PC with 2.50-GHz Intel Core i7 CPU, 8-GB RAM, and Windows 10 64-bit Ultimate, the calculation costs of the adaptive event-triggered scheme in Theorem 2, the event-triggered scheme in Theorem 3, and the sampled-data control scheme in Theorem 4 are 15.254834, 12.020567, and 11.281892 s,

$$\mathcal{P} = \begin{bmatrix} 2.0576 & -0.7085 & 0.7470 \\ -0.7085 & 1.7663 & -0.3834 \\ 0.7470 & -0.3834 & 0.7375 \end{bmatrix}$$

$$\mathcal{Q}_1 = \begin{bmatrix} 2.6257 & -0.8719 & 0.5472 & -1.6348 & 0.9302 & 0.0430 \\ -0.8719 & 1.7606 & -0.9880 & -0.0913 & -1.5033 & 0.7556 \\ 0.5472 & -0.9880 & 0.6240 & 0.2623 & 0.8968 & -0.4241 \\ -1.6348 & -0.0913 & 0.2623 & 2.3361 & 0.0075 & -0.4813 \\ 0.9302 & -1.5033 & 0.8968 & 0.0075 & 3.3123 & -0.8309 \\ 0.0430 & 0.7556 & -0.4241 & -0.4813 & -0.8309 & 0.8681 \end{bmatrix}$$

$$\mathcal{Q}_2 = \begin{bmatrix} 5.2972 & -1.6710 & 1.0817 & -3.3399 & 1.7325 & 0.1344 \\ -1.6710 & 3.5694 & -1.9986 & -0.2521 & -3.0165 & 1.4823 \\ 1.0817 & -1.9986 & 1.2748 & 0.5970 & 1.7254 & -0.8616 \\ -3.3399 & -0.2521 & 0.5970 & 4.9238 & -0.0552 & -1.0963 \\ 1.7325 & -3.0165 & 1.7254 & -0.0552 & 6.0008 & -1.8674 \\ 0.1344 & 1.4823 & -0.8616 & -1.0963 & -1.8674 & 1.5572 \end{bmatrix}$$

$$\mathcal{T} = \begin{bmatrix} 3.3068 & -1.0311 & 1.2505 \\ -1.0311 & 3.3526 & -0.6795 \\ 1.2505 & -0.6795 & 1.3331 \end{bmatrix}$$

$$\mathcal{S} = \begin{bmatrix} 0.6559 & -0.2241 & 0.3114 \\ -0.2148 & 0.4708 & -0.1472 \\ 0.2439 & -0.1400 & 0.2236 \end{bmatrix}$$

$$\mathcal{K}^* = \begin{bmatrix} -0.7696 & 0.0737 & -0.2113 \\ -0.1097 & -1.2296 & -0.0827 \\ -0.1381 & -0.0391 & -0.4277 \end{bmatrix}$$

$$\mathcal{B}_1 = \begin{bmatrix} 64.3921 & 17.7032 & 8.1665 \\ 17.7032 & 155.2993 & 43.6707 \\ 8.1665 & 43.6707 & 56.9334 \end{bmatrix}$$

$$\mathcal{B}_2 = \begin{bmatrix} 5.3630 & 1.6145 & 0.6230 \\ 1.6145 & 13.6247 & 4.0925 \\ 0.6230 & 4.0925 & 5.0247 \end{bmatrix}$$
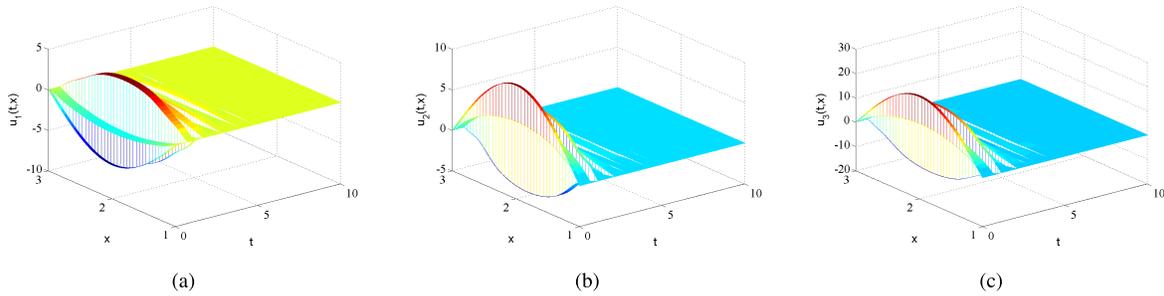
Fig. 8. Spatial behaviors of the adaptive event-triggered sampled-data controller $\mathcal{U}(t, x)$ in (15). (a) $u_1(t, x)$. (b) $u_2(t, x)$. (c) $u_3(t, x)$.

respectively. Compared with the control schemes in Theorems 3 and 4, the control scheme in Theorem 2 increases the calculation cost since its threshold function is time-varying and adaptive. Furthermore, for different values of θ, the NTSs of the adaptive event-triggered mechanism (13) are given in Table II, from which we find that the values of θ have some impact on the NTSs.

V. CONCLUSION

In this article, we have studied the synchronization of RDNNs with random time-varying delays. By designing an adaptive aperiodic ETSDC scheme and introducing an appropriate LKF, we have established some new synchronization criteria for RDNNs. Different from the existing ETSDC methods with constant thresholds, our method can be adaptively adjusted according to the current sampled and latest transmitted signals. In comparison with the existing control methods, our method can effectively save the communication resources for RDNNs. Taking the influence of uncertain factors, the random time-varying delays have been considered for RDNNs, which makes the obtained results more applicable. In the end, we have presented some simulations to show the superiorities of the adaptive ETSDC mechanism and the effectiveness of the obtained results.

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