Adaptive Synchronization of Fractional-Order Output-Coupling Neural Networks via Quantized Output Control

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Abstract—This article focuses on the adaptive synchronization for a class of fractional-order coupled neural networks (FCNNs) with output coupling. The model is new for output coupling item in the FCNNs that treat FCNNs with state coupling as its particular case. Novel adaptive output controllers with logarithm quantization are designed to cope with the stability of the fractional-order error systems for the first attempt, which is also an effective way to synchronize fractional-order complex networks. Based on fractional-order Lyapunov functionals and linear matrix inequalities (LMIs) method, sufficient conditions rather than algebraic conditions are built to realize the synchronization of FCNNs with output coupling. A numerical simulation is put forward to substantiate the applicability of our results.

Index Terms—Fractional order, neural networks, output coupling, quantized control, synchronization.

I. INTRODUCTION

FOR several decades, quantities of researchers have devoted themselves to neural networks due to their fruitful applications, including associative memories, pattern recognition, optimization, image processing, robotic manipulators, and so forth [1]–[3]. In particular, coupled neural networks have been paid fascinated massive attention due to the fact that the brain can be seen as being coupled with a lot of neurons. Research of dynamics and synchronization of coupled neural networks is also a vital step in understanding human brain science [4], [5].

Up to now, state coupling and output coupling are the two main types of coupling forms in coupled neural networks and complex networks. The coupled means includes linear coupling and nonlinear coupling. In coupled neural networks with state coupling, the neuron state is directly affected by its own state and other neurons' states. It is worth pointing out that most of the synchronization study in the literature is the ones with state coupling [6]–[8]. Chen *et al.* [7] obtained some simple and generic criteria for coupled delay neural networks by designing suitable coupling matrix and the inner linking matrix. Cao *et al.* [8] investigated the problem of synchronization for more general coupled neural networks with hybrid coupling, while the results on output coupling are only a few. However, in real neural networks, it is impossible or difficult to obtain and measure all the neurons' states because of the factors of sensors saturation, package loss, stochastic disturbances, and so forth. The investigation of dynamics and synchronization for coupled neural networks and complex networks with output coupling is of significance and necessity. Under this circumstance, a complex network model with output coupling was first proposed by Jiang *et al.* [9], and some sufficient synchronization conditions were given based on the Lyapunov stability and state observer design. Sufficient criteria were established to ascertain the complex network with output coupling to achieve exponential mean square synchronization [10]. Wang and Wu [11] investigated local and global exponential output synchronization for a class of complex dynamical networks with output coupling.

On the other hand, fractional-order calculus has recently received increasing attention for its superiority in describing infinite memory and hereditary properties of system models in the fields of bioengineering, neural networks, fluid mechanics, and so on [12], [13]. As we know, the long-term memory property of synapses is neglected in integer-order neural networks. Moreover, existing research shows the conductivity of the biological cell membrane is fractional-order [14]. Thus, it is more appropriate and precise to employ fractional-order differential equations to model the real neural networks and study the dynamics, such as stability [15]–[17], stabilization [18], and Hopf bifurcations [19], [20].

It should be pointed out that for fractional-order neural networks, most existing works about synchronization are the results of fractional-order neural networks without coupling [21]–[27]. There are only a few works about the synchronization of fractional-order neural networks with state coupling [28], [29]. Until now, there are no results of fractional-order coupled neural networks (FCNNs) with

output coupling. Here, we mainly aim to investigate the synchronization of FCNNs with output coupling. For simplicity and clarity, we consider linear output coupling in this article.

Considering the limited network communication capacity, different kinds of quantized control methods were proposed to effectively make use of the bandwidth and decrease the network transmission pressure [30]–[32]. For instance, Brockett and Liberzon [33] proposed a quantized feedback control approach to stabilize the linear systems. Zhang *et al.* [31] built less conservative conditions for inertial neural networks with the aid of quantized sampled-data control. Yang *et al.* [32] proposed a mode-dependent quantized control theme to realize the synchronization of coupled reaction–diffusion neural networks under Markovian switching topologies. There are also some good results about the topic of quantized state estimation [34]–[36].

To the best of our knowledge, most works relative to quantized control are state control, while, in real network control environment and applications, all the neurons' states information is difficult or too expensive to be directly measured. Output control was proposed to overcome the difficulty in achieving all the system states information [37]–[39]. It is noted that studies about quantized output control for integer-order systems are only a few [40], [41], not mention to the results of FCNNs. Inspired by the idea of the output control, we develop an output quantized control method for FCNNs with output coupling. Therefore, the method adopted in this article can be considered as the first attempt on quantized output control of the synchronization of FCNNs with output coupling.

Motivated by the abovementioned reasons, we investigate the adaptive synchronization problem of FCNNs with output coupling. The novelties of this article are listed as follows.

1) A new FCNN model with output coupling is proposed, which contains the model with state coupling as a special case.
2) The synchronization criteria are first established for FCNNs with output coupling in terms of linear matrix inequalities (LMIs) that are different from algebraic conditions. Several kinds of sufficient conditions are also given to ascertain the realization of synchronization by means of quantized control and output control.
3) The method of quantized control is not only first adopted in fractional-order neural networks but also developed to quantized output control.
4) The approaches in this article can also be used to study the synchronization of fractional-order complex networks.

This article is organized as follows. In Section II, we will give some definitions and lemmas together with the model description. Sufficient conditions are built for FCNNs with output coupling and quantized output control is Section III. Section IV gives an example to show the effectiveness of the proposed synchronization methods. Conclusions are drawn in Section V.

II. PROBLEM STATEMENT

In this section, we introduce mathematical models of FCNNs with output coupling and present some notations, definitions, and lemmas used in this article.

Notation: Throughout this article, n and N denote positive integers. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. "T" represents the transposition of a matrix or a vector. I_n is the $n \times n$ identify matrix. $|\cdot|$ is the Euclidean norm in $\mathbb{R}^n$. $*$ denotes the item induced by symmetry in a matrix. $P > 0$ $(P \geq 0)$ denotes P is a positive (semipositive) symmetric matrix. $P^s = 0.5(P + P^T)$. $\otimes$ denotes the Kronecher product.

Definition 1 [12], [13]: The fractional integral of order α for a function h is defined as

$$I^\alpha h(v) = \frac{1}{\Gamma(\alpha)} \int_{v_0}^{v} (v - s)^{\alpha-1} h(s) ds$$

where $v \geq v_0$ and $\alpha > 0$, $\Gamma(\alpha) = \int_0^\infty e^{-s} t^{\alpha-1} ds$ and v_0, v, and s are the lower limit, upper limit, and integral variable of the fractional integral, respectively.

We adopt Caputo fractional derivative in this article due to the physical interpretation of its initial conditions.

Definition 2 [12], [13]: Caputo's derivative of order α for a function $h \in C^k([v_0, +\infty), \mathbb{R})$ is defined by

$$_{v_0}D_v^\alpha h(v) = \frac{1}{\Gamma(k - \alpha)} \int_{v_0}^{v} (v - s)^{k-\alpha-1} h^{(k)}(s) ds$$

where $v \geq v_0$ and k is a positive integer such that $k - 1 < \alpha < k$. In particular, when $0 < \alpha < 1$, $_{v_0}D_v^\alpha q(v) = (1/\Gamma(1 - \alpha)) \int_{v_0}^{v} (v - s)^{-\alpha} h'(s) ds$.

For brevity, $D^\alpha h(v)$ is used to denote $_{v_0}D_v^\alpha h(v)$.

Consider a class of FCNNs with output coupling consisting of N identical networks and the dynamics of the ith network are described by the following equation:

$$\begin{cases} D^\alpha x_i(t) = -A x_i(t) + B f(x_i(t)) \\ \qquad\qquad + c \sum_{j=1}^{N} l_{ij} \Pi y_j(t) + u_i(t) \qquad (1) \\ y_i(t) = H x_i(t) \end{cases}$$

where $0 < \alpha < 1$, $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in \mathbb{R}^n$, $(i = 1, 2, \ldots, N)$ denotes the state of the ith network; $A = \text{diag}\{a_1, a_2, \ldots, a_n\} > 0$ and $B = [b_{ij}]_{n \times n}$, $f(x_i(t)) = (f_1(x_{i1}(t)), f_2(x_{i2}(t)), \ldots, f_n(x_{in}(t)))^T$, denote the connection weight matrix and neuron activation function, respectively. $c > 0$ is the coupling strength. $L = [l_{ij}]_{N \times N}$ is the coupling configuration matrix representing the topological structure of the networks that is irreducible and satisfies $l_{ij} \geq 0$, $i \neq j$, $l_{ii} = -\sum_{j=1, j \neq i}^{N} l_{ij}$. $\Pi = [\pi_{jk}]_{n \times m}$ represents constants inner coupling matrix between the jth network and the kth network, $y_j(t) \in \mathbb{R}^m$ and $u_i(t) \in \mathbb{R}^n$ are output vectors and the controller, respectively, and $H \in \mathbb{R}^{m \times n}$.

For the purpose of achieving synchronization, we define the set $S = \{x = (x_1^T(t), x_2^T(t), \ldots, x_N^T(t))^T : x_i(s) \in C([t_0, +\infty], \mathbb{R}^n), x_i(s) = x_j(s), i, j = 1, 2, \ldots, N\}$ as the synchronization manifold for system (1). Also, the

synchronization state is governed by the following equation:

$$\begin{cases} D^\alpha s(t) = -As(t) + Bf(s(t)) \\ z(t) = Hs(t) \end{cases} \tag{2}$$

where $s(t)$ could be an equilibrium point, a cycle or a chaotic orbit of system (2).

Define the synchronization error and output tracking error as $e_i(t) = x_i(t) - s(t)$ and $\theta_i(t) = y_i(t) - z(t)$ $(i = 1, 2, \ldots, N)$. It is obvious that $\theta_i(t) = He_i(t)$. Also, the error system is governed by the following equation:

$$\begin{cases} D^\alpha e_i(t) = -Ae_i(t) + B\tilde{f}(e_i(t)) \\ \qquad + c \sum_{j=1}^{N} l_{ij} \Pi \theta_j(t) + u_i(t) \\ \theta_i(t) = He_i(t) \end{cases} \tag{3}$$

where $\tilde{f}(e_i(t)) = f(x_i(t)) - f(s(t))$ $(i = 1, 2, \ldots, N)$.

Definition 3: The FCNNs with output coupling are said to be globally synchronized if $\forall i \in \{1, 2, \ldots, N\}$, $\lim_{t \to \infty} |e_i(t)| = 0$, i.e., $\lim_{t \to \infty} x_i(t) = s(t)$ $(i = 1, 2, \ldots N)$.

The existence and uniqueness of solutions of system (1) [13] are pledged by the following assumption.

Assumption 1: The activation function f is Lipschitz continuous, that is, there exists a positive constant l_f such that

$$|f(x_i) - f(x_j)| \le l_f |x_i - x_j|$$

for $\forall x_i, x_j \in \mathbb{R}^n$, and $\forall i, j \in \{1, 2, \ldots, N\}$.

The network output is quantized before transmitting on networks. The adaptive quantized controller $u_i(t)$ is designed as follows:

$$\begin{cases} u_i(t) = -d_i(t) H^T q(\theta_i(t)) \\ D^\alpha d_i(t) = \beta \|\theta_i(t)\|^2 \end{cases} \tag{4}$$

where β is a positive constant and $d_i(t)$ $(i = 1, 2, \ldots, N)$ are the control gains.

The quantizer $q(\cdot) : R^n \to \mathcal{V}$ is defined as follows: $q(\theta_i(t)) = (\tilde{q}(\theta_{i_1}(t)), \tilde{q}(\theta_{i_2}(t)), \ldots, \tilde{q}(\theta_{i_n}(t)))^T$

$$\tilde{q}(v) = \begin{cases} \xi_i, & \dfrac{1}{1+\delta}\xi_i < v \le \dfrac{1}{1-\delta}\xi_i \\ 0, & v = 0 \\ -\tilde{q}(-v), & v < 0 \end{cases}$$

where $\mathcal{V} = \{\pm\xi_i : \xi_i = \rho^i \xi_0, i = 0, \pm 1, \pm 2, \ldots, \} \cup \{0\}$ with $\xi_0 > 0$, and $\delta = (1 - \rho/1 + \rho)$, $0 < \rho < 1$. Based on the theory of Filippov, $q(v)$ can be denoted as $q(v) = (1 + \zeta)v$, $v \in \mathbb{R}^n$, $\zeta \in [-\delta, \delta]$.

Remark 1: The controller (4) is first designed with the aid of adaptive control, output control, and quantized control. The main features and advantages of this control type are as follows: 1) it can reduce the control cost; 2) the difficulty in obtaining all the neurons' states is avoided; and 3) it can effectively make use of the bandwidth and decrease the network transmission pressure.

To carry out the proof of synchronization criteria, the following lemmas are needed.

Lemma 1 [42]: If $h(t) \in \mathcal{C}^1([t_0, \infty], \mathbb{R}^n)$, $Q > 0$, then the following inequality holds:

$$\frac{1}{2} D^\alpha h^T(t) Q h(t) \le h^T(t) Q D^\alpha h(t), \quad 0 < \alpha \le 1.$$

Lemma 2 [43]: Suppose that A, B, C, and D are matrices with appropriate dimensions for algebraic operations, κ is a real constant, and then, the following properties for Kronecher product hold.

1) $(\kappa A) \otimes B = A \otimes (\kappa B)$.
2) $(A + B) \otimes C = A \otimes C + B \otimes C$.
3) $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$.

Lemma 3 [44]: $\forall a, b \in \mathbb{R}^n$, $\forall \gamma > 0$, the following inequality holds:

$$a^T b \le \frac{\gamma}{2} a^T a + \frac{1}{2\gamma} b^T b.$$

Lemma 4 (Schur Complement [45]): For a given matrix

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} < 0$$

is equivalent to any one of the following conditions.

1) $P_{22} < 0$, $P_{11} - P_{12} P_{22}^{-1} P_{12}^T < 0$.
2) $P_{11} < 0$, $P_{22} - P_{12}^T P_{11}^{-1} P_{12} < 0$.

III. MAIN RESULTS

We devote to give some theorems and corollaries for the synchronization of FCNNs in this section.

Theorem 1: Suppose that Assumption 1 holds; if there exist two positive constants ϵ and γ and selecting d^* such that $\epsilon - (1 - \delta)d^* < 0$ and the following LMI holds, then the FCNNs with output coupling (1) achieve globally asymptotical synchronization under the adaptive quantization controller (4)

$$\Omega = \begin{bmatrix} \Omega_{11} & I_N \otimes (l_f B^T) \\ * & -2\gamma I_{Nn} \end{bmatrix} < 0 \tag{5}$$

where $\Omega_{11} = -I_N \otimes A + (\gamma/2)I_{Nn} + c(L \otimes (\Pi H))^s - \epsilon I_N \otimes (H^T H)$ and I_N and I_{Nn} are $N \times N$ and $Nn \times Nn$ identify matrices, respectively.

Proof: Let $x_i(t)$ $(i = 1, 2, \ldots, N)$ and $s(t)$ are the solutions of system (1) and (2) with different initial conditions $x_i(t_0)$ and $s(t_0)$, respectively, and then, $e_i(t_0) \ne 0$. Suppose that $e_i(t)$ $(i = 1, 2, \ldots, N)$ are the solutions of the system (3) with the controller (4) satisfying $e_i(t_0) \ne 0$.

Construct a Lyapunov function of the following form:

$$V(t) = \frac{1}{2} \sum_{i=1}^{N} e_i^T(t) e_i(t) + \frac{1-\delta}{2\beta} \sum_{i=1}^{N} (d_i(t) - d^*)^2 \tag{6}$$

where d^* is an adaptive constant to be determined later.

Using Lemma 1 and computing the fractional derivative of $V(t)$ along the solution of (3), one obtains that

$$D^\alpha V(t)$$
$$\le \sum_{i=1}^{N} e_i^T(t) D^\alpha e_i(t)$$
$$+ \frac{1-\delta}{\beta} \sum_{i=1}^{N} (d_i(t) - d^*) D^\alpha d_i(t)$$

$$= \sum_{i=1}^{N} e_i^T(t) \left[-Ae_i(t) + B\tilde{f}(e_i(t)) + c\sum_{j=1}^{N} l_{ij}\Pi\theta_j(t) \right]$$

$$- \sum_{i=1}^{N} e_i^T(t)d_i(t)H^T q(\theta_i(t))$$

$$+ (1-\delta)\sum_{i=1}^{N}(d_i(t) - d^*)\|\theta_i(t)\|^2. \tag{7}$$

By defining $e(t) = (e_1^T(t), e_2^T(t), \ldots, e_N^T(t))^T$ and using Lemmas 2 and 3, one has

$$- \sum_{i=1}^{N} e_i^T(t)Ae_i(t) = -e^T(t)(I_N \otimes A)e(t) \tag{8}$$

and

$$\sum_{i=1}^{N} e_i^T(t)B\tilde{f}(e_i(t))$$

$$\leq \frac{\gamma}{2}e^T(t)e(t) + \frac{1}{2\gamma}\sum_{i=1}^{N}\tilde{f}^T(e_i(t))B^T B\tilde{f}(e_i(t))$$

$$\leq \frac{\gamma}{2}e^T(t)e(t) + \frac{1}{2\gamma}\sum_{i=1}^{N} e_i^T(t)l_f B^T l_f Be_i(t)$$

$$= \frac{\gamma}{2}e^T(t)e(t) + \frac{1}{2\gamma}e^T(t)(I_N \otimes \bar{B})e(t) \tag{9}$$

where $\bar{B} = l_f^2 B^T B$.

Also, we have

$$c\sum_{i=1}^{N} e_i^T(t)\sum_{j=1}^{N} l_{ij}\Pi\theta_j(t) = c\sum_{i=1}^{N} e_i^T(t)\sum_{j=1}^{N} l_{ij}\Pi He_j(t)$$

$$= e^T(t)(c(L \otimes (\Pi H))^s)e(t) \tag{10}$$

and

$$- \sum_{i=1}^{N} e_i^T(t)d_i(t)H^T q(\theta_i(t))$$

$$+ (1-\delta)\sum_{i=1}^{N}(d_i(t) - d^*)\|\theta_i(t)\|^2$$

$$\leq -(1-\delta)\sum_{i=1}^{N} d_i(t)\theta_i^T(t)\theta_i(t)$$

$$+ (1-\delta)\sum_{i=1}^{N}(d_i(t) - d^*)\|\theta_i(t)\|^2$$

$$= -(1-\delta)d^*\sum_{i=1}^{N} e_i^T(t)H^T He_i(t)$$

$$= e^T(t)(\epsilon - (1-\delta)d^*)I_N \otimes (H^T H)e(t)$$

$$- \epsilon e^T(t)(I_N \otimes (H^T H))e(t). \tag{11}$$

Properly choosing d^* such that $(\epsilon - (1-\delta)d^*)I_N \otimes (H^T H) \leq 0$, and combining (7)–(11), we have

$$D^\alpha V(t) \leq e^T(t)\left[-I_N \otimes A + \frac{\gamma}{2}I_{Nn} + \frac{1}{2\gamma}I_N \otimes \bar{B} \right.$$

$$\left. + c(L \otimes (\Pi H))^s - \epsilon I_N \otimes (H^T H) \right]$$

$$\times e(t). \tag{12}$$

By Lemma 4, $-I_N \otimes A + (\gamma/2)I_{Nn} + (1/2\gamma)I_N \otimes \bar{B} + c(L \otimes (\Pi H))^s - \epsilon I_N \otimes (H^T H) < 0$ that is equivalent to the inequality (5) holds. Hence, we get

$$D^\alpha V(t) \triangleq g(t, e(t)) \leq -\lambda_{\min}(\Omega)e^T(t)e(t)$$

$$= -\lambda_{\min}(\Omega)\sum_{i=1}^{N} e_i^T(t)e_i(t)$$

$$= -2\lambda_{\min}(\Omega)W(t) < 0 \tag{13}$$

where $W(t) = (1/2)\sum_{i=1}^{N} e_i^T(t)e_i(t)$. From Definition 1, we get

$$V(t) - V(t_0) = \frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-s)^{\alpha-1}g(s, e(s))ds < 0. \tag{14}$$

Therefore, $V(t) \leq V(t_0)$, $t \geq t_0$. From the definition of (6), $e_i(t)$ and $d_i(t)$ are bounded for $t \geq t_0$. Therefore, there exists a positive constant M satisfying $|D^\alpha V(t)| \leq M$, $t \geq t_0$. We then conclude that $\lim_{t\to\infty} W(t) = 0$. The proof of $\lim_{t\to\infty} W(t) = 0$ is similar to [26, Proof of Theorem 1], hence omitted here. From the definition of $W(t)$, we have $\lim_{t\to\infty} e_i(t) = 0$. Therefore, FCNNs with output coupling are globally asymptotical synchronized based on the controller (4). This completes the proof.

Remark 2: The theory and methods of integer-order differential equations cannot be directly used to investigate fractional-order systems. It should be pointed out that how to design suitable quantized output controllers for FCNNs is not an easy job and 2) how to give easily checked synchronization criteria in terms of LMIs for FCNNs which will introduce some difficulties.

Remark 3: Compared with the results in [28] and [29], the model in this article has the item of output coupling, and thus, our models are new and more general. What is more, the results in this article are in terms of LMIs and easily checked than those algebraic conditions [28]. Although Wang *et al.* [29] investigated synchronization of FCNNs, the coupling item was state coupling and the controller was state-feedback controller not adaptive quantized output controller. This is the first time to use an adaptive quantized output control method to investigate the synchronization of FCNNs with output coupling or state coupling.

If $m = n$ and $H = I$, we can get the following FCNNs with state coupling, error system, quantized controller and results:

$$D^\alpha x_i(t) = -Ax_i(t) + Bf(x_i(t))$$

$$+ c\sum_{j=1}^{N} l_{ij}\Pi x_j(t) + u_i(t) \tag{15}$$

$$D^\alpha e_i(t) = -Ae_i(t) + B\tilde{f}(e_i(t))$$

$$+ c\sum_{j=1}^{N} l_{ij}\Pi e_j(t) + u_i(t) \tag{16}$$

$$\begin{cases} u_i(t) = -d_i(t)q(e_i(t)) \\ D^\alpha d_i(t) = \beta\|e_i(t)\|^2. \end{cases} \tag{17}$$

Theorem 2: Under Assumptions 1, the network (15) is globally synchronized under the quantized controller (17),

if there exist positive constants ϵ and γ, and selecting d^* such that

$$\Phi = \begin{bmatrix} \Phi_{11} & I_N \otimes (l_f B)^T \\ * & -2\gamma I_{Nn} \end{bmatrix} < 0 \qquad (18)$$

and $\epsilon - (1 - \delta)d^* < 0$ hold, where $\Phi_{11} = -I_N \otimes A + (\gamma/2)I_{Nn} + c(L \otimes \Pi)^s - \epsilon I_{Nn}$.

Proof: If $x_i(t)$ $(i = 1, 2, \ldots, N)$ and $s(t)$ are the solutions of system (15) and (2), respectively, with different initial conditions $x_i(t_0)$ and $s(t_0)$, then $e_i(t_0) \neq 0$. Suppose that $e_i(t)$ $(i = 1, 2, \ldots, N)$ are the solutions of the system (16) with the controller (17) satisfying $e_i(t_0) \neq 0$.

The following Lyapunov function is constructed:

$$V(t) = \frac{1}{2} \sum_{i=1}^{N} e_i^T(t) e_i(t) + \frac{1 - \delta}{2\beta} \sum_{i=1}^{N} (d_i(t) - d^*)^2. \qquad (19)$$

Calculating its fractional-order derivative along the solution of (16), we obtain that

$$D^\alpha V(t)$$
$$\leq \sum_{i=1}^{N} e_i^T(t) D^\alpha e_i(t)$$
$$+ \frac{1 - \delta}{\beta} \sum_{i=1}^{N} (d_i(t) - d^*) D^\alpha d_i(t)$$
$$= \sum_{i=1}^{N} e_i^T(t) \left[-A e_i(t) + B \tilde{f}(e_i(t)) + c \sum_{j=1}^{N} l_{ij} \Pi e_j(t) \right]$$
$$- \sum_{i=1}^{N} e_i^T(t) d_i(t) q(e_i(t))$$
$$+ (1 - \delta) \sum_{i=1}^{N} (d_i(t) - d^*) \|e_i(t)\|^2$$
$$\leq e^T(t) \left[-I_N \otimes A + \frac{\gamma}{2} I_{Nn} + \frac{1}{2\gamma} I_N \otimes \bar{B} \right.$$
$$\left. + c(L \otimes \Pi)^s - \epsilon I_{Nn} \right] e(t)$$
$$+ (\epsilon - (1 - \delta)d^*) e^T(t) e(t). \qquad (20)$$

By choosing suitable d^* satisfying $\epsilon - (1 - \delta)d^* < 0$ and using Lemma 4, we get

$$D^\alpha V(t) \leq -\lambda_{\min}(\Phi) e^T(t) e(t)$$
$$= -\lambda_{\min}(\Phi) \sum_{i=1}^{N} e_i^T(t) e_i(t)$$
$$= -2\lambda_{\min}(\Phi) W(t) < 0. \qquad (21)$$

The rest of the proof is similar to that of Theorem 1 and thus omitted here. This proof is completed.

Theorem 3: Under Assumptions 1, the network (15) is globally synchronized under the quantized controller (17), if there exist positive constants ϵ and γ, and selecting a suitable constant d^*, such that

$$\Lambda = \begin{bmatrix} -I_N \otimes A + c(L \otimes \Pi)^s & \frac{1}{2} I_N \otimes B \\ * & -\epsilon I_{Nn} \end{bmatrix} < 0 \qquad (22)$$

$$\epsilon - (1 - \delta)d^*/l_f^2 < 0. \qquad (23)$$

Proof: If $x_i(t)$ $(i = 1, 2, \ldots, N)$ and $s(t)$ are the solutions of system (15) and (2), respectively, with different initial conditions $x_i(t_0)$ and $s(t_0)$, then $e_i(t_0) \neq 0$. Suppose that $e_i(t)$ $(i = 1, 2, \ldots, N)$ are the solutions of the system (16) with the controller (17) satisfying $e_i(t_0) \neq 0$.

Choosing the same Lyapunov functional (19), the following inequality can be derived:

$$D^\alpha V(t)$$
$$\leq \sum_{i=1}^{N} e_i^T(t) D^\alpha e_i(t)$$
$$+ \frac{1 - \delta}{\beta} \sum_{i=1}^{N} (d_i(t) - d^*) D^\alpha d_i(t)$$
$$= \sum_{i=1}^{N} e_i^T(t) \left[-A e_i(t) + B \tilde{f}(e_i(t)) + c \sum_{j=1}^{N} l_{ij} \Pi e_j(t) \right]$$
$$- \sum_{i=1}^{N} e_i^T(t) d_i(t) q(e_i(t))$$
$$+ (1 - \delta) \sum_{i=1}^{N} (d_i(t) - d^*) \|e_i(t)\|^2$$
$$\leq e^T(t)[-I_N \otimes A + c(L \otimes \Pi)^s] e(t)$$
$$+ \sum_{i=1}^{N} e_i(t)^T B \tilde{f}(e_i(t))$$
$$- (1 - \delta) \sum_{i=1}^{N} d_i(t) e_i^T(t) e_i(t)$$
$$+ (1 - \delta) \sum_{i=1}^{N} (d_i(t) - d^*) \|e_i(t)\|^2$$
$$= e^T(t)[-I_N \otimes A + c(L \otimes \Pi)^s] e(t)$$
$$+ \sum_{i=1}^{N} e_i(t)^T B \tilde{f}(e_i(t))$$
$$- (1 - \delta)d^* \sum_{i=1}^{N} e_i^T(t) e_i(t)$$
$$\leq e^T(t)[-I_N \otimes A + c(L \otimes \Pi)^s] e(t)$$
$$+ \frac{1}{2} e^T(t)(I_N \otimes B^T) F(t)$$
$$+ \frac{1}{2} F^T(t)(I_N \otimes B) e(t) - (1 - \delta)d^* e^T(t) e(t)$$
$$\leq e^T(t)[-I_N \otimes A + c(L \otimes \Pi)^s] e(t)$$
$$+ \frac{1}{2} e^T(t)(I_N \otimes B^T) F(t)$$
$$+ \frac{1}{2} F^T(t)(I_N \otimes B) e(t) - \epsilon F^T(t) F(t)$$
$$+ [\epsilon - (1 - \delta)d^*/l_f^2] F^T(t) F(t) \qquad (24)$$

where $F(t) = [\tilde{f}_1^T(t), \ldots, \tilde{f}_n^T(t)]^T$.

Selecting an approximate d^* to ensure $\epsilon - (1 - \delta)d^*/l_f^2 < 0$, then
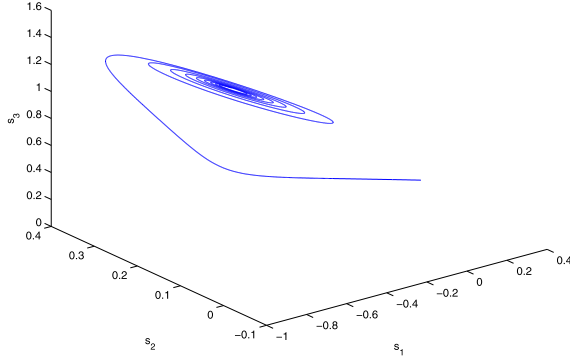
$$D^\alpha V(t) \leq \xi^T(t) \Lambda \xi(t) \leq -\lambda_{\min}(\Lambda) \xi^T(t) \xi(t)$$
$$\leq -\lambda_{\min}(\Lambda) e^T(t) e(t) \qquad (25)$$

Fig. 1.   Chaotic attractors of (2).

Fig. 2.   Synchronization errors of $x_{i1} - s_1$.

Fig. 3.   Synchronization errors of $x_{i2} - s_2$.

Fig. 4.   Synchronization errors of $x_{i3} - s_3$.

Fig. 5.   Total synchronous error.

where $\xi(t) = (e^T(t), F^T(t))^T$. The rest of proof is similar to that of Theorem 1 and hence omitted here. This completes the proof.

When there is no quantization of the network output, the adaptive output controller $u_i(t)$ becomes the following form:

$$\begin{cases} u_i(t) = -d_i(t)H^T\theta_i(t) \\ D^\alpha d_i(t) = \beta\|\theta_i(t)\|^2. \end{cases} \quad (26)$$

Corollary 1: Under Assumption 1, the network (1) is globally synchronized under the controller (26) if there exist positive constants ϵ, γ, and properly choosing d^* such that the inequality (5) and $(\epsilon - d^*)I_N \otimes (H^T H) \leq 0$ hold.

Corollary 2: Under the condition $m = n$, $H = I$, and Assumption 1, the network (15) is globally synchronized under the controller

$$\begin{cases} u_i(t) = -d_i(t)e_i(t) \\ D^\alpha d_i(t) = \beta\|e_i(t)\|^2 \end{cases} \quad (27)$$

if there exist positive constants ϵ and γ such that the inequality (18) and $d^* > \epsilon$ hold.

Corollary 3: Under the condition $m = n$, $H = I$ and Assumption 1, the networks (15) is globally synchronized under the controller (27), if there exist positive constants ϵ and γ, such that the inequality (22) and $d^* > l_f^2\epsilon$ hold.
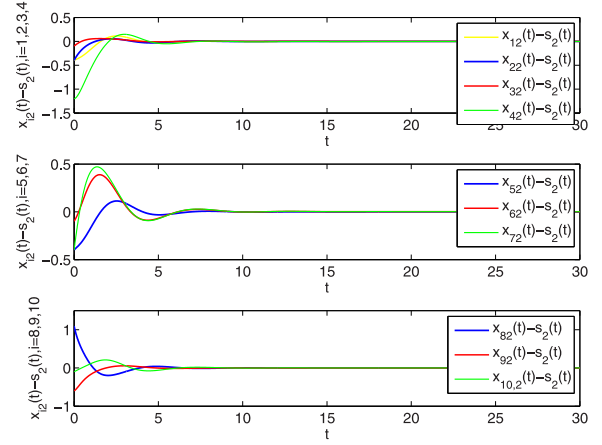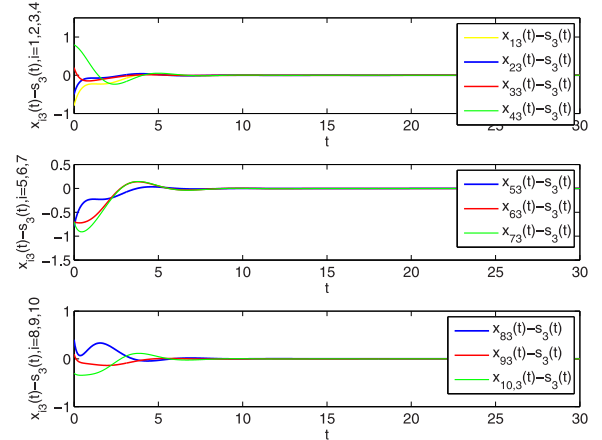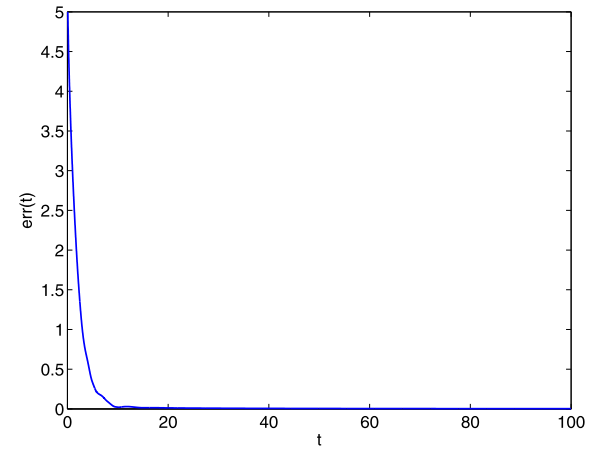
Remark 4: The results in Corollaries 1–3 are also new since the existing results about synchronization of fractional-order neural networks [21]–[27] are the ones without coupling item.

IV. NUMERICAL EXAMPLE

In this section, a numerical example is employed to illustrate the effectiveness of the obtained theoretical results.

Consider the 3-D FCNNs given by (1), with $n = 3$, $N = 10$, $x_i(t) = (x_{i1}(t), x_{i2}(t), x_{i3}(t))^T \in \mathbb{R}^3$, and $(i = 1, 2, \ldots, 10)$ denotes the state of the ith network. The synchronization
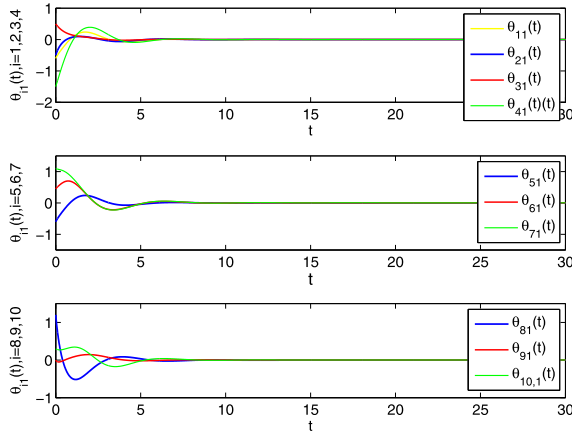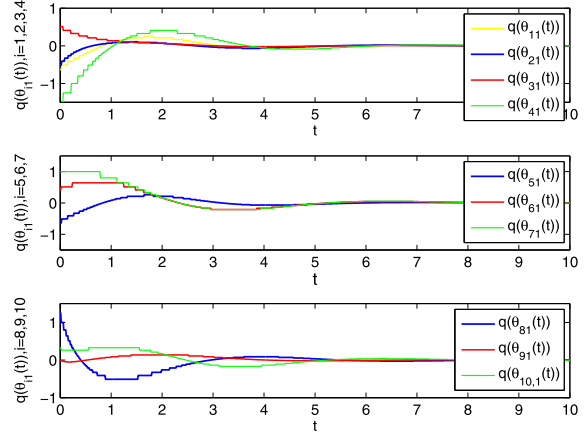
Fig. 6.   Output tracking errors of θ_{i1}.
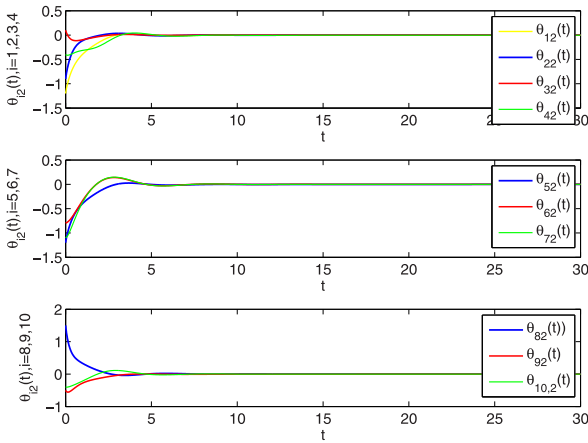

Fig. 7.   Output tracking errors of θ_{i2}.

Fig. 8.   Time evolutions of $q(\theta_{i1})$.

Fig. 9.   Time evolutions of $q(\theta_{i2})$.
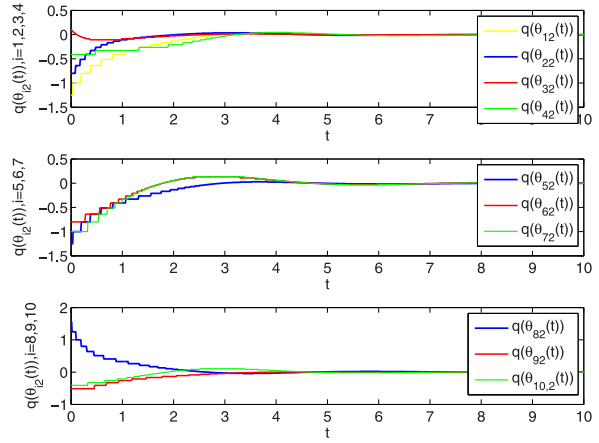

Fig. 10.   Quantized output controller u_{i1} in (4).

reference manifold $s(t)$ is defined by (2). The other parameters are given as follows:

$$A = \begin{bmatrix} 1.1 & 0 & 0 \\ 0 & 1.1 & 0 \\ 0 & 0 & 1.1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -1.2 & 0 \\ 1.5 & 2.71 & 1.15 \\ -2.75 & 0 & 1.1 \end{bmatrix}$$

$$\Pi = \begin{bmatrix} 1 & 0.2 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$L = \begin{bmatrix} -4 & 1 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -2 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -2 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & -4 & 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & -2 & 0 & 0 \\ 1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -3 & 0 \\ 1 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -3 \end{bmatrix}$$

$c = 0.5$, $\beta = 0.2$, $\alpha = 0.95$, $\delta = 0.8$, $\xi_0 = 1$, and $f(x_i) = (0.5x_{i1}, 0.5\tanh(x_{i2}), 0.5\tanh(x_{i3}))^T$.

We use the MATLAB LMI Control Toolbox to solve the LMI in (5) and obtain the following feasible solution with $\epsilon = 22.7061$ and $\gamma = 1.4165$. Choosing $d^* = 120$, and through simple computation, we get $\epsilon - (1 - \delta)d^* = -0.21 < 0$.
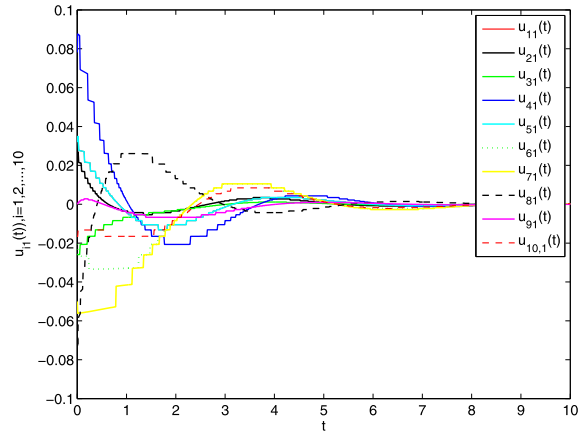
According to Theorem 1, it can be concluded that the system (1) is globally synchronized. Fig. 1 shows the trajectory of (2), which has a chaotic attractor. Figs. 2–4 show the synchronization errors of $x_{i1} - s_1$, $x_{i2} - s_2$, and $x_{i3} - s_3$, respectively. Obviously, the synchronization errors tend to zero, which has also confirmed that system (1) achieves asymptotical synchronization.

The total error of (3) is defined by $\text{err}(t) = \sum_{i=1}^{3}(\sum_{j=1}^{10}|s_i - x_{ji}|^2)^{1/2}$. Fig. 5 shows the total synchronization error and Figs. 6–12 show the output tracking
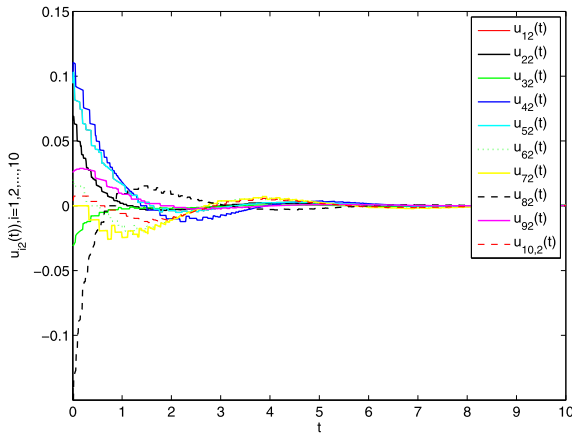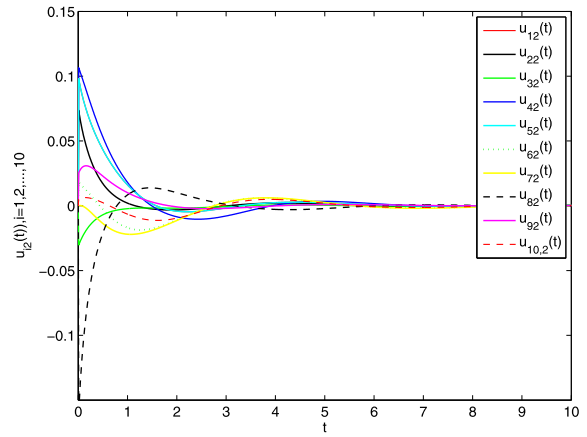
Fig. 11.   Quantized output controller u_{i2} in (4).
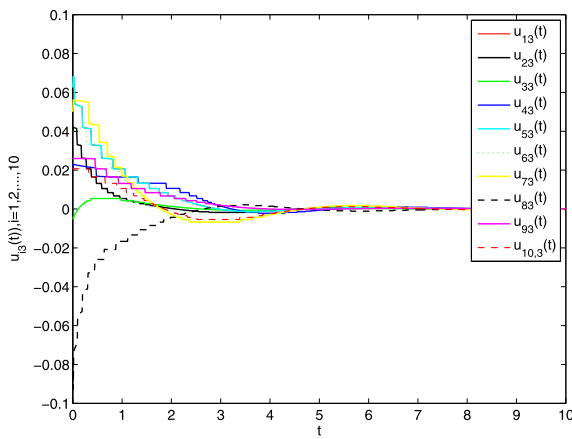

Fig. 12.   Quantized output controller u_{i3} in (4).

Fig. 13.   Output controller u_{i1} in (26).

Fig. 14.   Quantized output controller u_{i2} in (26).
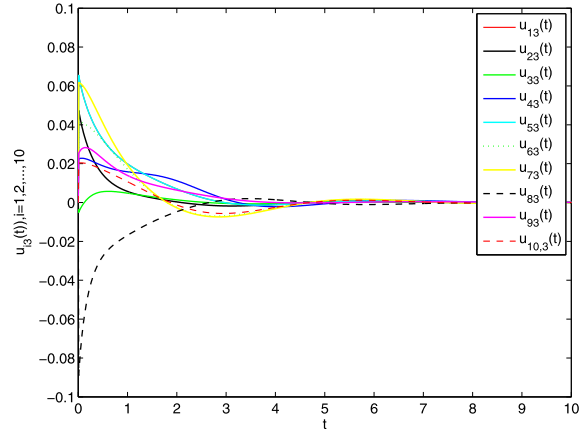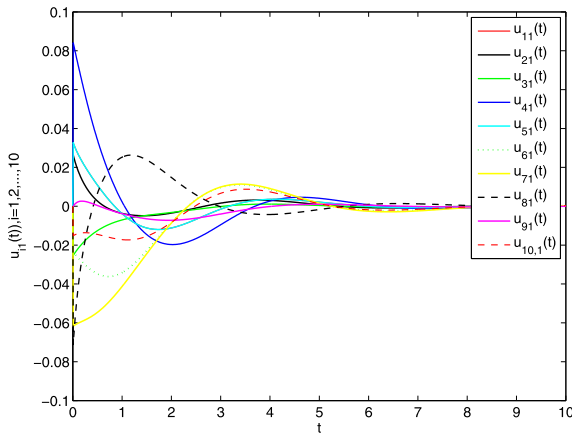

Fig. 15.   Quantized output controller u_{i3} in (26).

control cost and the communication burden in comparisons with Figs. 13–15.

V. CONCLUSION

The problem of synchronization of FCNNs with output coupling has been addressed with the help of fractional-order Lyapunov functions. New adaptive quantized output controllers are proposed to effectively decrease control fees and avoid communication channels congestion. Some new synchronization criteria are derived in forms of LMIs. An illustrated example is used to show the correctness of the obtained results. Further research works would extend our results to the ones of fractional-order coupled memristor-based neural networks with and without time delays.

errors θ_i, time evolutions of $q(\theta_i)$, and the quantized output controller $u_i(t)$ in (4), respectively.

It should be mentioned that the FCNNs (1) also achieve globally asymptotical synchronization with the output controller (26). For the sake of length, the trajectories of synchronization errors under output controller (26) are omitted. Figs. 13–15 shows the time evolutions of the output controller (26) for the same system (1). Obviously, in Figs. 10–12, the continuous signals are converted into piecewise continuous signals by quantized output controller (4) and this reduces

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