Modeling the Kriging-Aided Spatial Spectrum Sharing Over Log-Normal Channels

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***Abstract*—In this letter, we first show that the error in the received signal power estimation with an optimally interpolated radio environment map (REM) over a shadowing environment can be modeled using a correlation coefficient, similar to that of the imperfect channel state information over multipath fading channels. After the estimation error characteristic is applied for modeling the typical spatial spectrum sharing, we verify the validity of the model via numerical simulations of the REM-assisted spectrum sharing.**

***Index Terms*—Interference constraints, radio spectrum management, radio propagation, log-normal distribution.**

I. Introduction

A RADIO environment map (REM) can enhance the communication efficiency in spatial spectrum sharing, such as communications over television white space (TVWS) [1] and heterogeneous networks [2]. REM is often defined as a map storing the spatial distribution of the average received signal power, which is estimated by a path-loss model or by a measurement dataset. Here, although the path-loss-based method can easily construct the REM, the method has poor accuracy because of uncertainties in the locality of the radio propagation. On the other hand, it is well known that the measurement-based REM using Kriging interpolation, a geostatistical technique [3], can realize a highly accurate REM [4]. Because the measurement-based technique can acquire site-specific propagation affected by shadowing, the performance of perfect REM-aided spectrum sharing can be obtained by the conventional works assuming that the secondary user (SU) has perfect knowledge of the shadowing factor (e.g., [5]). On the other hand, in a realistic situation, the REM has an estimation error based on the number of datasets and the propagation characteristics. Because the estimation error strongly affects the communication efficiency, the practical performance of the REM-assisted spectrum sharing needs to be discussed with respect to this error.

Such a problem can be categorized as spectrum sharing with imperfect channel state information (CSI). It is well known that the imperfection in multipath fading channels can be explained with a correlation factor between the true and estimated CSI [6]. On the other hand, few studies have been conducted to model such an error characteristic in the shadowing environment. The influence of the quantization of REM is discussed in [7]. However, the purpose of this letter is to evaluate the influence of the grid size, i.e., the difference between the average received power in the grid and the value at an arbitrary point in the grid. Thus, the discussion cannot be applied to spatial interpolation-aided situations.

In this letter, we discuss the model of spectrum sharing with practical REM. The main contributions of this letter are summarized as follows.

- We show that the error in the received signal power estimation with optimally interpolated REM over shadowing can be modeled using a correlation coefficient, similar to those of multipath fading channels (Section III).
- Considering the error model and discussions in [4], we model the typical spectrum-sharing environment. The ergodic capacity of REM-aided spatial spectrum sharing is then analyzed approximately (Section IV).
- The validity of the model is verified via numerical simulations of the REM-based spectrum sharing (Section V).

These contributions enable further study of the performance of Kriging-aided spatial spectrum sharing without complicated simulations that actually apply Kriging.

II. Overview of Kriging Interpolation

Based on typical discussions on the REM, we assume that the REM is stored in the spectrum database. The database first collects the information on the primary users (PUs), such as transmission power, location, and spectrum sharing criteria [1]. By querying the database, the SUs can acquire these information.[1] We define REM as a map that has two-dimensional coordinate information of the average received signal power focusing on a fixed primary transmitter. In a typical REM construction, multiple nodes first measure the signal strength from a transmitter and report the data to the database. However, because the number of nodes and the area where observations can be performed are limited, the REM is missing teeth; we need to thus interpolate for the missing-tooth information. Kriging is often utilized for REM construction.

Let us consider a situation where we construct an REM concerning a fixed transmitter and the signals are measured over a two-dimensional space. If multiple nodes measure the signal strength from the transmitter, we obtain the dataset vector $\boldsymbol{y} = (P(\boldsymbol{x}_1), P(\boldsymbol{x}_2), \ldots, P(\boldsymbol{x}_N))^{\mathrm{T}}$, where N is the number

[1] If these information are not available from the PUs, the received signal strength (RSS)-based joint localization and model identification will be a practical option (e.g., [8]).

Manuscript received August 6, 2018; revised December 5, 2018; accepted December 14, 2018. Date of publication January 1, 2019; date of current version June 19, 2019. This work was supported by the JSPS KAKENHI under Grant 16KK0124 and Grant 18H01439. The associate editor coordinating the review of this paper and approving it for publication was D. Tarchi. *(Corresponding author: Koya Sato.)*

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Digital Object Identifier 10.1109/LWC.2018.2890644

of measurements, $\boldsymbol{x}_i$ is the measurement location, and $P(\boldsymbol{x}_i)$ is the received signal power. The task of Kriging is to interpolate the received signal power at a location $\boldsymbol{x}_0$ from $\boldsymbol{y}$. If the effect of multipath fading can be removed by averaging the instantaneous received signals, the averaged received signal power at a given location $\boldsymbol{x}$ can be modeled as

$$P(\boldsymbol{x}) = \underbrace{\overbrace{(P_{\mathrm{Tx}} - L_{\mathrm{c}})}^{\triangleq P_{\mathrm{c}}} - 10\eta\log_{10}||\boldsymbol{x} - \boldsymbol{x}_{\mathrm{Tx}}||}_{\triangleq \overline{P}(\boldsymbol{x})} + W(\boldsymbol{x}), \quad (1)$$

where $\boldsymbol{x}$ is the received signal location, $\boldsymbol{x}_{\mathrm{Tx}}$ is the transmitter location, P_{Tx} [dBm] is the transmission power, L_{c} [dB] is the offset that includes the effect of carrier frequency, η is the path-loss index, and W [dB] is the shadowing factor that follows a normal distribution with zero mean and standard deviation σ [dB]. In addition, it is empirically known that shadowing has a spatial correlation [9]. Therefore, by taking a weighted average and by assigning suitable weight factors, we can implement a highly accurate spatial interpolation.

A. Semivariogram Estimation

Kriging interpolation preliminarily needs to estimate the spatial covariance structure of the random process from the datasets. This letter explains the maximum likelihood (ML)-based semivariogram modeling. This method assumes that the measurement vector $\boldsymbol{y}$ follows a Gaussian process, described as $\boldsymbol{y} \sim \mathcal{N}(\boldsymbol{X\beta}, \boldsymbol{M}(\boldsymbol{\theta}))$. In the channel model defined in Eq. (1), $\boldsymbol{X}$ and $\boldsymbol{\beta}$ can be defined as follows:

$$\boldsymbol{X} = \begin{pmatrix} 1 & -10\log_{10}(d_1) \\ 1 & -10\log_{10}(d_2) \\ \vdots & \vdots \\ 1 & -10\log_{10}(d_N) \end{pmatrix}, \boldsymbol{\beta} = \begin{pmatrix} P_{\mathrm{c}} \\ \eta \end{pmatrix}, \quad (2)$$

where $d_i \triangleq ||\boldsymbol{x}_i - \boldsymbol{x}_{\mathrm{Tx}}||$ [m]. $\boldsymbol{M}(\boldsymbol{\theta})$ is the variance–covariance matrix defined as $\boldsymbol{M}(\boldsymbol{\theta}) = \alpha_n^2\boldsymbol{I} + \alpha_s^2\boldsymbol{H}(\alpha_r)$; its elements consist of several covariance functions. $\alpha_n^2,\ \alpha_s^2,\ \alpha_r$, the parameters of the covariance function, are called *nugget, sill, range*, and the parameter vector $\boldsymbol{\theta}$ consists of these parameters. $\boldsymbol{I}$ is the $N \times N$ identity matrix and $\boldsymbol{H}(\alpha_r)$ is an $N \times N$ correlation matrix that follows a theoretical correlation function. Considering conventional discussions on the shadowing (e.g., [9]), each correlation can be written as $H_{i,j} = \exp\left(-d_{i,j}/\alpha_r\right)$ where $d_{i,j} \triangleq ||\boldsymbol{x}_i - \boldsymbol{x}_j||$. Then, the log-likelihood function of the measurement datasets can be formulated as

$$\begin{aligned} l(\boldsymbol{y}|\boldsymbol{\beta},\boldsymbol{\theta}) = &-\frac{N}{2}\ln 2\pi - \frac{1}{2}\ln|\boldsymbol{M}(\boldsymbol{\theta})| \\ &-\frac{1}{2}(\boldsymbol{y} - \boldsymbol{X\beta})^{\mathrm{T}}\boldsymbol{M}(\boldsymbol{\theta})^{-1}(\boldsymbol{y} - \boldsymbol{X\beta}). \end{aligned} \quad (3)$$

By maximizing Eq. (3), we can obtain $\boldsymbol{\theta}$ and $\boldsymbol{\beta}$.

B. Ordinary Kriging

We consider Ordinary Kriging for the interpolation. Ordinary Kriging is assumed to be constrained in the local neighborhood of the estimation point, i.e., $\mathrm{E}[Z(\boldsymbol{x})] = const.$ for a random variable Z, for each nearby data value. In the radio environment according to Eq. (1), the interpolation at $\boldsymbol{x}_0$ can be implemented by

$$\begin{aligned} \hat{P}(\boldsymbol{x}_0) = &\hat{P}_{\mathrm{c}} - 10\hat{\eta}\log_{10}||\boldsymbol{x}_0 - \boldsymbol{x}_{\mathrm{Tx}}|| \\ &+ \sum_{i=1}^{N}\omega_i\big(P(\boldsymbol{x}_i) - (\hat{P}_{\mathrm{c}} - 10\hat{\eta}\log_{10}||\boldsymbol{x}_i - \boldsymbol{x}_{\mathrm{Tx}}||)\big), \end{aligned} \quad (4)$$

where $\hat{\eta}$ is the estimated η, $\hat{P}_{\mathrm{c}}$ is the estimated P_{c}, and ω_i is the weight factor. Kriging determines the optimum weights that minimize the variance of the estimation error $\sigma_k^2 = \mathrm{Var}[\hat{P}(\boldsymbol{x}_0) - P(\boldsymbol{x}_0)]$. In order to achieve the best linear unbiased estimator (BLUE), this method determines the weights under the constraint $\sum_{i=1}^{N}\omega_i = 1$. Using the method of the Lagrange multiplier, the objective function can be written as $\phi(\omega_i, \mu) = \sigma_k^2 - 2\mu(\sum\omega_i - 1)$, where μ is the Lagrange multiplier. Here, σ_k^2 can be written as follows [3]:

$$\sigma_k^2 = -\gamma(d_{0,0}) - \sum_{i=1}^{N}\sum_{j=1}^{N}\omega_i\omega_j\gamma(d_{i,j}) + 2\sum_{i=1}^{N}\omega_i\gamma(d_{i,0}), \quad (5)$$

where γ is the semivariogram and is defined as $\gamma(||\boldsymbol{x}_i - \boldsymbol{x}_j||) = \frac{1}{2}\mathrm{Var}[P(\boldsymbol{x}_i) - P(\boldsymbol{x}_j)]$. From the partial derivatives in $\phi(\omega_i, \mu)$, we can obtain $N + 1$ simultaneous equations:

$$\begin{pmatrix} \gamma(d_{1,1}) & \cdots & \gamma(d_{1,N}) & 1 \\ \gamma(d_{2,1}) & \cdots & \gamma(d_{2,N}) & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \gamma(d_{N,1}) & \cdots & \gamma(d_{N,N}) & 1 \\ 1 & \cdots & 1 & 0 \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_N \\ \mu \end{pmatrix} = \begin{pmatrix} \gamma(d_{1,0}) \\ \gamma(d_{2,0}) \\ \vdots \\ \gamma(d_{N,0}) \\ 1 \end{pmatrix}. \quad (6)$$

From the above simultaneous equations, the weights that minimize σ_k^2 can be derived. Here, the minimized σ_k^2 is called the *Kriging variance*.

III. Characteristics of Interpolation Error

This section discusses and models the error characteristic of Kriging. If the received signal power follows a log-normal distribution, the estimation error follows a log-normal distribution with a median value of zero [dB] [3]: the estimation error $\varepsilon \triangleq \boldsymbol{P}(\boldsymbol{x}_0) - \hat{P}(\boldsymbol{x}_0)$ [dB] follows a normal distribution with zero median. Additionally, because the estimated and true values are highly correlated in the logarithmic domain, $P(\boldsymbol{x}_0)$ and $\hat{P}(\boldsymbol{x}_0)$ follow a bivariate normal distribution with median $\overline{P}(\boldsymbol{x}_0)$ and the variance–covariance matrix,

$$\begin{pmatrix} \sigma^2 & \rho\sigma\sigma_{\hat{P}} \\ \rho\sigma_{\hat{P}}\sigma & \sigma_{\hat{P}}^2 \end{pmatrix}, \quad (7)$$

where $\sigma_{\hat{P}}$ is the standard deviation of $\hat{P}(\boldsymbol{x}_0)$, and ρ is the correlation coefficient between $P(\boldsymbol{x}_0)$ and $\hat{P}(\boldsymbol{x}_0)$. Note that $\mathrm{Var}[P(\boldsymbol{x}_0)] = \mathrm{Var}[W(\boldsymbol{x}_0)] = \sigma^2$. The probability density function (PDF) of ε can be derived from the bivariate normal distribution. Because ε consists of the difference of two variables following the bivariate normal distribution, its variance can be calculated by $\sigma_\varepsilon^2 = \sigma^2 + \sigma_{\hat{P}}^2 - 2\rho\sigma\sigma_{\hat{P}}$. Here, this equation is minimized when $\sigma_{\hat{P}} = \rho\sigma$, and Kriging determines $\hat{P}$ such that $\sigma_{\hat{P}}^2$ is minimized under $\sum\omega_i = 1$. Considering these facts, we approximate $\sigma_{\hat{P}}$ by $\rho\sigma$: σ_ε^2 can be thus derived as $\sigma_\varepsilon^2 \approx \sigma^2(1 - \rho^2)$. This equation shows that ρ can directly

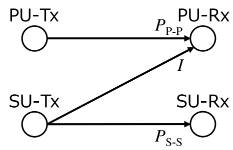


Fig. 1. Spectrum sharing model.

express the accuracy of REM. Assigning parameters σ and ρ to this equation, we can easily emulate the Kriging-based radio-propagation estimation without complicated simulations. In Sections IV and V, we treat σ_ε as the function of ρ: i.e., $\sigma_\varepsilon(\rho)$.

IV. Modeling Kriging-Aided Spectrum Sharing

We model the spectrum sharing using Kriging-interpolated REM. In [4], we showed that $\sigma_\varepsilon(\rho)$ in the practical situation can be estimated by σ_k directly, and the use of σ_k enables the design of the transmission power at the SUs. Based on this result and the discussion in Section III, we construct a simple spectrum sharing model that consists of ρ. After modeling, we analyze the ergodic capacity of the secondary link.

A. Spectrum Sharing Model

We consider a typical spatial spectrum sharing where a secondary link shares a spectrum with a primary link, as shown in Fig. 1. We discuss the interference power constraint at the communication protection area defined by the primary transmitter PU-Tx. The primary receiver PU-Rx is assumed to be on the nearest edge of the area from the secondary transmitter SU-Tx, and the location is available at SU-Tx. To evaluate the spectrum-sharing efficiency, we calculate the allowable transmission power at SU-Tx for the given channel information. SU-Tx acquires radio propagation characteristics via the spectrum database that has the REM of PU-Tx and the path-loss model on SU-Tx. Additionally, we assume that the channel state in our discussion consists of the path loss and the shadowing. Because our purpose is found in the spatial coverage design, multipath fading is not considered.

First, $P_{\text{P-P}}$ [dBm] is defined as the received signal power at PU-Rx from PU-Tx. We assume the value fluctuates according to the path loss and log-normal shadowing as,

$$P_{\text{P-P}} = \underbrace{\left(P_{\text{P,Tx}} - L_{\text{P,c}}\right) - L_{\text{P-P}}}_{\triangleq \overline{P}_{\text{P-P}}} + W_{\text{P-P}} \quad [\text{dBm}], \qquad (8)$$

where $P_{\text{P, Tx}}$ [dBm] is the transmission power, $L_{\text{P, c}}$ [dB] is the offset factor, $L_{\text{P-P}}$ [dB] is the path loss, and $W_{\text{P-P}}$ [dB] is the shadowing gain following the i.i.d. normal distribution with zero mean and standard deviation $\sigma_{\text{P-P}}$ [dB]. Similarly, SU-Tx interferes with PU-Rx with the following power:

$$I = \underbrace{\left(P_{\text{S,Tx}} - L_{\text{S,c}}\right) - L_{\text{P-S}}}_{\triangleq \overline{I}} + W_{\text{P-S}} \quad [\text{dBm}], \qquad (9)$$

where $P_{\text{S,Tx}}$ [dBm] is the transmission power, $L_{\text{S,c}}$ [dB] is the offset factor, $L_{\text{P-S}}$ [dB] is the path loss, and $W_{\text{P-S}}$ [dB] is the shadowing gain that follows i.i.d. normal distribution with zero mean and standard deviation $\sigma_{\text{P-S}}$ [dB]. In addition, the secondary receiver SU-Rx receives the signal from SU-Tx with

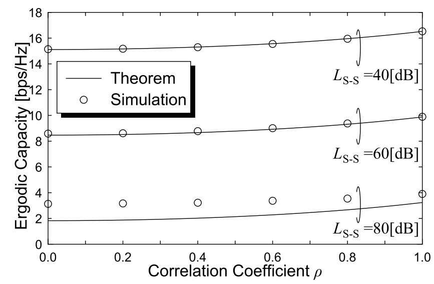


Fig. 2. Ergodic capacity in Fig. 1, where $N_0 = -100.0$ [dBm], $\overline{P}_{\text{P-P}} = -90.0$ [dBm], $\sigma_{\text{P-P}} = \sigma_{\text{P-S}} = \sigma_{\text{S-S}} = 8.0$ [dB], $L_{\text{P-S}} = 100.0$ [dB], $L_{\text{S,c}} = 0.0$ [dB], $\Gamma_{\text{d}} = 10.0$ [dB] and $p_{\text{out}} = 0.10$.

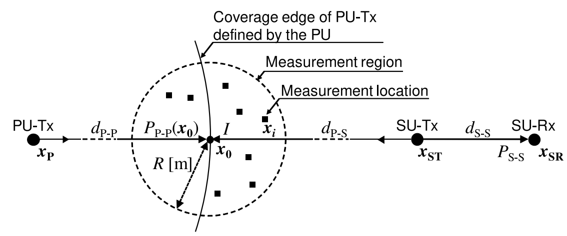


Fig. 3. Simulation model.

the received signal power $P_{\text{S-S}} = (P_{\text{S,Tx}} - L_{\text{S,c}}) - L_{\text{S-S}} + W_{\text{S-S}}$ [dBm], where $L_{\text{S-S}}$ [dB] is the path loss, and $W_{\text{S-S}}$ [dB] is the shadowing that follows an i.i.d. normal distribution with zero mean and standard deviation $\sigma_{\text{S-S}}$ [dB].[2]

Under the above condition, we utilize the signal-to-interference power ratio (SIR) as the protection criterion for the primary communication according to a typical rule of spectrum sharing (e.g., [1]). Here, the SIR at PU-Rx can be written as $\Gamma = P_{\text{P-P}} - I$ [dB]. If the PU requires the desired SIR Γ_{d} [dB], the outage event can be formulated as $\Gamma_{\text{d}} > \Gamma$. Thus, considering the desired protection probability $1-p_{\text{out}}$, the SU must satisfy the protection probability at PU-Rx, given by $\Pr[\Gamma \geq \Gamma_{\text{d}}] \geq 1-p_{\text{out}}$. Using the channel information in both the primary link and the interference link, SU-Tx estimates the allowable transmission power $\max P_{\text{S,Tx}} \triangleq P_{\text{S, max}}$ [dBm] under the constraint for p_{out}. We assume that SU-Tx estimates $P_{\text{P-P}}$ via the Kriging-based REM and can acquire $\overline{I}$, $\sigma_{\text{P-S}}$, $\sigma_\varepsilon(\rho)$, p_{out} and Γ_{d} from the spectrum database. Note that ρ is the correlation between $P_{\text{P-P}}$ and $\hat{P}_{\text{P-P}}$ in this case.

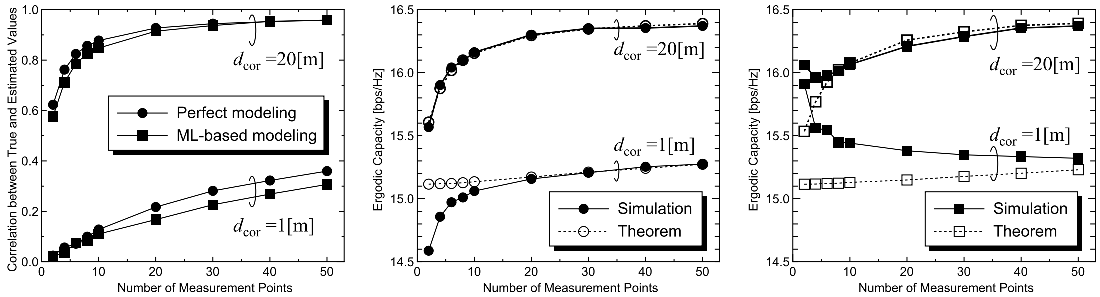
B. Allowable Transmission Power

We derive the allowable transmission power of SU-Tx $P_{\text{S,max}}$ that satisfies the interference constraint. Because $P_{\text{P-P}}$ is estimated by the Kriging-based method, the SIR follows a normal distribution with median $(\hat{P}_{\text{P-P}} - \overline{I})$ and standard deviation $\sigma_{\text{SIR}}(\rho) = \sqrt{\sigma_\varepsilon^2(\rho) + \sigma_{\text{P-S}}^2} \approx \sqrt{\sigma_{\text{P-P}}^2(1-\rho^2) + \sigma_{\text{P-S}}^2}$. Therefore, the allowable transmission power can be written as

$$P_{\text{S,max}} = L_{\text{S,c}} + L_{\text{P-S}} + \hat{P}_{\text{P-P}} - \Gamma_{\text{d}} - \sigma_{\text{SIR}}(\rho)\sqrt{2}\,\text{erf}^{-1}(1-2p_{\text{out}}), \qquad (10)$$

where $\text{erf}^{-1}(\cdot)$ is the inverse error function.

[2] We do not consider the effect of the interference from PU-Tx to SU-Rx because SU-Rx is far from PU-Tx in the typical spatial spectrum sharing.



(a) (b) (c)

Fig. 4. Simulation results where $R = 100$ [m], $L_{\text{S-S}} = 40$ [dB], and $\eta = 3.5$. The other parameters follow the same conditions as Fig. 2. (a) Correlation between $P_{\text{P-P}}(\boldsymbol{x}_0)$ and $\hat{P}_{\text{P-P}}(\boldsymbol{x}_0)$. (b) Ergodic capacity in the case of perfect semivariogram modeling. (c) Ergodic capacity in the case of ML-based semivariogram modeling. We calculate the theoretical ergodic capacity in (b) and (c) by substituting the correlation from (a) into Eq. (11).

C. Ergodic Capacity Analysis

We analyze the ergodic capacity. If the interference power from PU-Tx can be ignored, the channel capacity at SU-Rx can be derived as $C_{\text{SU-Rx}} = \log_2(1 + 10^{\frac{P_{\text{S-S}} - N_0}{10}})$ [bps/Hz], where N_0 [dBm] is the noise floor. Here, $P_{\text{S,Rx}}$ follows a normal distribution because $\hat{P}_{\text{P-P}}$ with the Kriging-based method follows a normal distribution. In addition, because the channel capacity can be approximated as $C_{\text{SU-Rx}} \approx \log_2(10^{\frac{P_{\text{S-S}} - N_0}{10}})$ in a high-SNR region, $C_{\text{SU-Rx}}$ nearly follows a normal distribution. Thus, we can approximately derive the ergodic capacity as

$$\mathrm{E}[C_{\text{SU-Rx}}] \approx \frac{\log_2 10}{10}\Big(L_{\text{P-S}} + \overline{P}_{\text{P-P}} - \Gamma_{\text{d}} - \sigma_{\text{SIR}}(\rho)\sqrt{2}\text{erf}^{-1}(1 - 2p_{\text{out}}) - L_{\text{S-S}} - N_0\Big). \quad (11)$$

Figure 2 shows the theoretical and simulated ergodic capacities. In the high-SNR region, Eq. (11) shows good agreement with the simulation result. Using Eq. (11), we can predict the communication quality under a given accuracy of the REM.

V. Validation of the Spectrum Sharing Model

In order to validate the accuracy of the proposed model, we compare Eq. (11) with the simulation results for the typical REM construction. As shown in Fig. 3, PU-Tx, PU-Rx, SU-Tx, and SU-Rx are deployed at $\boldsymbol{x}_{\text{P}}$, $\boldsymbol{x}_0$, $\boldsymbol{x}_{\text{ST}}$, and $\boldsymbol{x}_{\text{SR}}$, respectively. We determine $d_{\text{P-P}} \triangleq ||\boldsymbol{x}_0 - \boldsymbol{x}_{\text{P}}||$ such that $\overline{P}_{\text{P-P}}(\boldsymbol{x}_0)$ is equal to the simulation parameter. Similarly, $d_{\text{P-S}} \triangleq ||\boldsymbol{x}_0 - \boldsymbol{x}_{\text{ST}}||$ and $d_{\text{S-S}} \triangleq ||\boldsymbol{x}_{\text{SR}} - \boldsymbol{x}_{\text{ST}}||$ are determined such that $L_{\text{P-S}}$ and $L_{\text{S-S}}$ satisfy the simulation parameters. After N terminals are randomly deployed in the measurement circle, except $\boldsymbol{x}_0$, based on a two-dimensional uniform distribution, each terminal measures the received signal power $P_{\text{P-P}}(\boldsymbol{x}_i)$. Next, $P_{\text{P-P}}(\boldsymbol{x}_0)$ and $P_{\text{S,max}}$ are estimated with the Kriging and Eq. (10). Additionally, we assume that shadowing correlation follows a typical model, $r_{i,j} = \exp(-||\boldsymbol{x}_i - \boldsymbol{x}_j||\ln 2/d_{\text{cor}})$, where d_{cor} [m] is the correlation distance [9]. Note that $r_{i,j}$ does not have direct relationship with the correlation factor discussed in Sections III and IV because ρ expresses the accuracy of the REM. Under the above condition, we evaluate both the perfect and the ML-based semivariogram modeling cases.

Figure 4(a) shows the correlation between $P_{\text{P-P}}(\boldsymbol{x}_0)$ and $\hat{P}_{\text{P-P}}(\boldsymbol{x}_0)$ over the logarithmic domain; the figure directly shows the interpolation accuracy. The ergodic capacity of each method is shown in Figs. 4(b) and 4(c). We calculate the theoretical ergodic capacity by substituting the correlation shown in Fig. 4(a) into Eq. (11). The use of the correlation can accurately model the Kriging-based spectrum sharing. Note that the simulated value in Fig. 4(c) slightly exceeds p_{out} because σ_k assumes that the semivariogram can be modeled perfectly and the ML-based semivariogram estimation contains the estimation error [4]. Thus, the simulated capacity also clearly exceeds Eq. (11) when N is small.

VI. Conclusion

We have shown that the error of the Kriging-aided radio-environment estimation can be explained by the correlation factor ρ, similar to that of the multipath fading channels. Additionally, according to the error model, we have modeled the Kriging-aided spatial spectrum sharing. The proposed model can simplify the design of REM-based systems.

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