Deep Learning Enabled Multicast Beamforming With Movable Antenna Array

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***Abstract*—Beamforming with movable antenna (MA) array has recently attracted increasing attention as an enabling technology for next-generation wireless communications, e.g., 6G. In this letter, we consider a multicast scenario where a base station (BS) equipped with a linear MA array broadcasts common information to multiple users, each equipped with a single fixed-position antenna. Our objective is to jointly optimize the antenna position vector (APV) and antenna weight vector (AWV) by maximizing the minimum beamforming gain for the users, which is challenging to tackle analytically due to nonconvexity. To effectively and intelligently break through such challenge, we propose a novel deep learning (DL) model with three modules, namely, feature extractor, APV optimizer, and AWV optimizer. An effective training strategy for the proposed DL model is also developed in an unsupervised manner with a customized loss function. The superiority and effectiveness of the proposed scheme are confirmed through simulation results.**

***Index Terms*—Beamforming, deep learning, movable antenna, multicast, 6G.**

I. Introduction

MOVABLE antenna (MA) (or fluid antenna) array, which is able to favorably reconfigure propagation environments or channel conditions by adaptively adjusting positions of all antennas via flexible cables, has recently drawn an upsurge of research interest as a key enabling technology for upcoming wireless networks, e.g., 6G [1], [2], [3]. In [4], the performance gain of wireless communication with MA over the fixed-position antenna (FPA) counterpart has been analyzed for deterministic and stochastic field-response channel models. Adjustable or flexible beamforming is also an appealing and promising application of MA array, with which remarkable performance improvements have been achieved over the conventional beamforming with FPA array, especially in multi-user multi-antenna communication scenarios [6], [7], [8], [9], [10], [11], [12], [13].

Particularly, MA array-enabled beamforming designs have been investigated in the literature for downlink scenarios [5], [6], [7], [8], uplink scenarios [9], [10], [11], and multiple-input multiple-output (MIMO) systems [12], [13], under various criteria. In [6], the tradeoff between maximizing the beamforming gains over desired directions and minimizing the interference leakages over undesired directions was studied in terms of multi-beam forming design with MA array. For downlink multiple-input single-output (MISO) multi-user systems with a base station (BS) having MA array and multiple users, each having a single FPA, the antenna position vector (AVP) and antenna weight vector (AWV) at the BS were jointly optimized to minimize the total transmit power [7] and to maximize the effective received signal-to-noise ratio with coordinate multi-point (CoMP) reception [8]. For uplink single-input multiple-output (SIMO) multi-user systems in the same configuration, the APV, receive beamforming at the BS, and users' transmit power were jointly optimized to minimize the total transmit power of the users [9] and to maximize the minimum achievable rate among the users [10]; whereas, in [11], the APV and users' transmit power were jointly optimized to minimize the total transmit power by adopting the zero-forcing receiver at the BS. For point-to-point MIMO systems, the positions of MAs at both the transmitter and receiver were optimized jointly with the covariance matrix of the transmit signals to maximize the achievable rate under the assumptions of instantaneous channel state information (CSI) [12] and partial CSI [13]. However, the beamforming designs in [6], [7], [8], [9], [10], [11], [12], [13] need to be conducted through complicated iterative algorithms with high computational complexities, which hinders their applicability in stringent real-time systems.

In this letter, we present a new beamforming design methodology for MA array based on deep learning (DL), which efficiently computes (or predicts) the optimal design strategy through very simple operations such as matrix-vector multiplications and nonlinear activation functions. Although several effective DL techniques have been developed in the literature [14], [15], [16] to address the position (or port) selection problems in fluid antenna multiple access (FAMA) systems, these techniques are not applicable to the beamforming design with MA array because the optimization problems for the port selection and beamforming design are different. To the best of our knowledge, our work is the first to exploit the DL technique for the beamforming design with MA array. In this letter, we consider a multicast scenario where a BS equipped with a linear MA array broadcasts common information to multiple users, each equipped with a single FPA. In this practical scenario, we aim to maximize the minimum beamforming gain over the users by jointly optimizing the APV and AWV. The main contributions of this letter are summarized as follows:

- We propose a novel and high-performing DL model for the multicast beamforming with MA array, which is composed of three learnable modules: feature extractor, APV optimizer, and AWV optimizer. These three DL modules are constructed by feedforward neural networks (FNNs) such that useful features for the MA array-enabled multicast beamforming as well as the optimized APV and AWV can be effectively and efficiently learned.
- Moreover, we devise an effective training strategy for the proposed DL model in an unsupervised manner such

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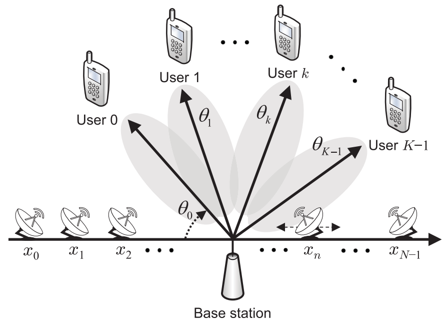

Fig. 1. A multicast transmission system with a linear MA array.

that the minimum beamforming gain for the users is customized as the loss function, and then, the three DL modules are jointly trained to maximize such a customized loss function.

- We present extensive simulation results to demonstrate the superiority and effectiveness of the proposed scheme compared to baseline schemes. The computational complexities of the proposed and baseline schemes are also analyzed.

II. System Model and Problem Formulation

As shown in Fig. 1, we consider a multicast scenario where a BS delivers common information to a set of K users. Each user $k \in \mathcal{K} \triangleq \{0, 1, \ldots, K-1\}$ is equipped with a single FPA, while the BS is equipped with a linear MA array of size N, i.e., the positions of N antennas at the BS are adjustable within a line segment of length D. Let x_n, $n \in \{0, 1, \ldots, N-1\}$, denote the nth MA's position. Without loss generality, suppose that $0 \leq x_0 < x_1 < \cdots < x_{N-1} \leq D$. The steering vector of the MA array with respect to (w.r.t.) an arbitrary steering angle $\theta \in [0, \pi)$ can be expressed as [6]

$$\boldsymbol{h}(\boldsymbol{x}, \theta) = \left[e^{j\frac{2\pi}{\lambda}x_0\cos(\theta)}, e^{j\frac{2\pi}{\lambda}x_1\cos(\theta)}, \ldots, e^{j\frac{2\pi}{\lambda}x_{N-1}\cos(\theta)}\right]^{\mathsf{T}} \tag{1}$$

where $\boldsymbol{x} \triangleq [x_0, x_1, \ldots, x_{N-1}]^{\mathsf{T}}$ denotes the APV and λ is the wavelength. Then, denoting the AWV (or digital transmit beamforming) of the BS by $\boldsymbol{w} \in \mathbb{C}^{N\times 1}$, the resulting beamforming gain can be written as $|\boldsymbol{w}^{\mathsf{H}}\boldsymbol{h}(\boldsymbol{x}, \theta)|^2$.

Our goal in this letter is to maximize the minimum beamforming gain over the users by jointly optimizing the APV $\boldsymbol{x}$ and the AWV $\boldsymbol{w}$ under practical constraints as follows:

$$\text{(P1)}: \quad \underset{\boldsymbol{x},\boldsymbol{w}}{\text{maximize}} \quad \min_{k\in\mathcal{K}} \left|\boldsymbol{w}^{\mathsf{H}}\boldsymbol{h}(\boldsymbol{x}, \theta_k)\right|^2$$
$$\text{subject to} \quad x_0 \geq 0, \quad x_{N-1} \leq D, \tag{2a}$$
$$x_n - x_{n-1} \geq d, \quad n = 1, \ldots, N-1, \tag{2b}$$
$$\|\boldsymbol{w}\|^2 \leq 1, \tag{2c}$$

where θ_k denotes the steering angle to where user k is located. Also, $d > 0$ in (2b) denotes the minimum distance between any two adjacent MAs required to avoid the coupling effect and (2c) denotes the transmit power constraint at the BS. Note that problem (P1) is nonconvex since the objective function is not jointly concave over the whole optimization variables $\boldsymbol{x}$ and $\boldsymbol{w}$. In particular, even for fixed $\boldsymbol{w}$, (P1) remains still nonconvex w.r.t. $\boldsymbol{x}$. Thus, it is infeasible to tackle problem (P1) directly due to the NP-hardness. To break through these technical challenges effectively and intelligently, in the next section, we propose a new and innovative solution to (P1) via the construction of a novel DL model.

III. Proposed DL Model

A. Network Architecture

The network architecture of the proposed DL model is presented in Fig. 2, which is composed of three learnable modules: the feature extractor, APV optimizer, and AWV optimizer. The proposed DL model takes the steering angles $\{\theta_k\}_{k=0}^{K-1}$ of the users as inputs. To avoid a bias and assure a constant level of deviation, the inputs are normalized to have zero mean and unit variance such that $\hat{\theta}_k = \frac{\theta_k - \mathbb{E}[\theta_k]}{\sqrt{\mathbb{E}[\theta_k^2] - \mathbb{E}[\theta_k]^2}}$, $\forall k \in \mathcal{K}$.[1] Thereafter, passing through the three modules, the proposed DL model finally predicts the jointly optimized APV $\boldsymbol{x}$ and AWV $\boldsymbol{w}$ as outputs.

1) Feature Extractor: For better learning of the optimized $\boldsymbol{x}$ and $\boldsymbol{w}$, useful features are extracted from the normalized steering angles $\{\hat{\theta}_k\}_{k=0}^{K-1}$. To this end, we construct an effective feature extractor by stacking four dense layers (i.e., fully-connected FNNs), where the number of output nodes in the lth dense layer is set as $\frac{48}{2^{l-1}}N$ for $l = 1, 2, 3, 4$. In the first three dense layers, the activation function at each output node is chosen as the exponential linear unit (ELU), i.e., $f(a) = e^a - 1$ for $a \leq 0$ and $f(a) = a$ for $a > 0$; whereas, in the last dense layer, the activation function is set to the linear function, i.e., $f(a) = a$. The extracted feature (i.e., the output of the feature extractor), denoted by $\boldsymbol{\mu} \in \mathbb{R}^{6N\times 1}$, is then fed into both the subsequent APV and AWV optimizers.

2) APV Optimizer: In the proposed DL model, the APV optimizer is designed to (indirectly) learn the optimized $\boldsymbol{x}$ from the feature $\boldsymbol{\mu}$ through a variable transformation technique (rather than directly learning the optimized $\boldsymbol{x}$). This is inspired by the fact that the variables $\{x_i\}_{i=1}^{N-1}$ fulfilling the minimum distance constraints in (2b) can be written in terms of x_0 and non-negative variables $\{\Delta_i\}_{i=1}^{N-1}$ (indicating the relative distances between two successive MAs) as follows:

$$\begin{aligned} x_1 &= x_0 + d + \Delta_1, \\ x_2 &= x_1 + d + \Delta_2 = x_0 + 2d + \Delta_1 + \Delta_2, \\ &\vdots \\ x_{N-1} &= x_{N-2} + d + \Delta_{N-1} = x_0 + (N-1)d + \sum_{i=1}^{N-1}\Delta_i. \end{aligned} \tag{3}$$

Meanwhile, let $(\boldsymbol{x}^\star, \boldsymbol{w}^\star)$ denote an optimal solution to (P1). Then it follows that

$$\begin{aligned} \left|(\boldsymbol{w}^\star)^{\mathsf{H}}\boldsymbol{h}(\boldsymbol{x}^\star, \theta)\right|^2 &= \left|(\boldsymbol{w}^\star)^{\mathsf{H}}\boldsymbol{h}(\boldsymbol{x}^\star - x_0^\star\mathbf{1}_N, \theta)e^{j\frac{2\pi}{\lambda}x_0\cos(\theta)}\right|^2 \\ &= \left|(\boldsymbol{w}^\star)^{\mathsf{H}}\boldsymbol{h}(\boldsymbol{x}^\star - x_0^\star\mathbf{1}_N, \theta)\right|^2, \end{aligned} \tag{4}$$

[1] Note that this normalization does not affect the optimization in (P1), and thus, the outputs of the proposed DL model (i.e., predicted APV and AWV) can be directly used as the solution to problem (P1) without any further transformation.

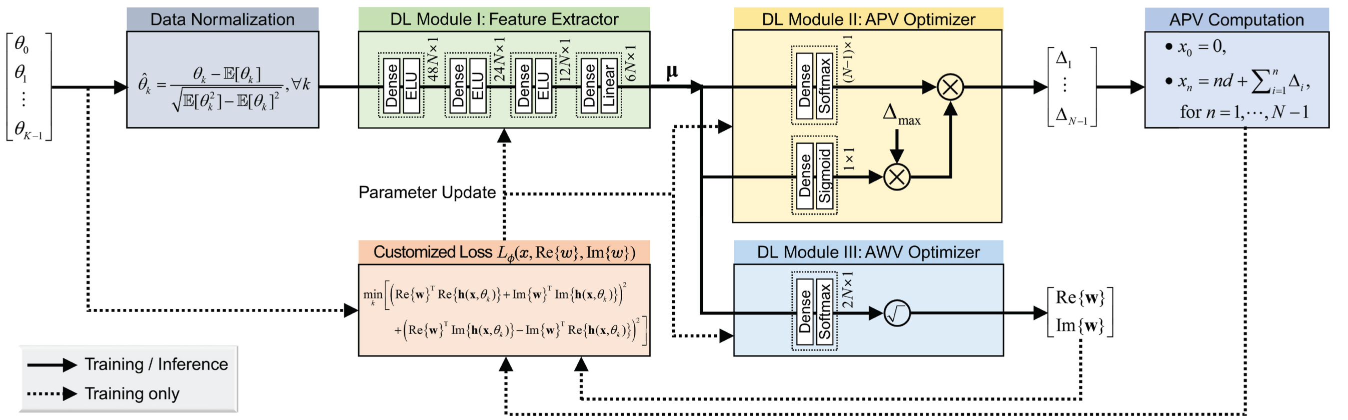

Fig. 2. Proposed DL model to learn and optimize the APV $\boldsymbol{x}$ and AWV $\boldsymbol{w}$ for the multicast beamforming with MA array in (P1).

implying that $(\boldsymbol{x}^\star - x_0^\star \mathbf{1}_N, \boldsymbol{w}^\star)$ is also an optimal solution to (P1),[2] where $\mathbf{1}_N$ denotes an all-one vector of size N. Therefore, one can determine $x_0 = 0$ without loss of any optimality. Consequently, from (3) and (4), we have

$$\begin{cases} x_0 = 0, \\ x_n = nd + \sum_{i=1}^{n} \Delta_i, \quad n = 1, \dots, N-1. \end{cases} \tag{5}$$

With (5), the length constraint $x_{N-1} \le D$ in (2a) is equivalent to

$$\sum_{i=1}^{N-1} \Delta_i \le \Delta_{\max} \tag{6}$$

where $\Delta_{\max} \triangleq D - (N-1)d$.[3]

According to the proposed transformation in (5), optimizing $\boldsymbol{x}$ is equivalent to optimizing $\Delta_i \ge 0$, $i = 1, \dots, N-1$. Thus, only the optimized values of $\{\Delta_n\}_{n=1}^{N-1}$ need to be learned with the sum constraint in (6). To this end, as depicted in Fig. 2, we construct the APV optimizer using two parallel (i.e., upper and lower) dense layers: one with $(N-1)$ output nodes that are fed into the softmax function (i.e., $f(a_n) = \frac{e^{a_n}}{\sum_i e^{a_i}}$) and the other with one output node that is fed into the sigmoid function (i.e., $f(a) = \frac{e^a}{1+e^a}$), followed by being multiplied with $\Delta_{\max}$. Then the outputs of these two parallel dense layers are multiplied such that the optimized $\{\Delta_n\}_{n=1}^{N-1}$ can be inferred as follows:

$$\Delta_n = \left(\frac{e^{\boldsymbol{\mu}^\mathsf{T} \boldsymbol{a}_n^{(1)} + b_n^{(1)}}}{\sum_{i=1}^{N-1} \boldsymbol{\mu}^\mathsf{T} \boldsymbol{a}_i^{(1)} + b_i^{(1)}} \right) \left(\frac{\Delta_{\max} e^{\boldsymbol{\mu}^\mathsf{T} \boldsymbol{a}^{(2)} + b^{(2)}}}{1 + e^{\boldsymbol{\mu}^\mathsf{T} \boldsymbol{a}^{(2)} + b^{(2)}}} \right) \tag{7}$$

for $n = 1, \dots, N-1$, where the weights and biases (i.e., learnable parameters) of the upper dense layer are denoted by $\boldsymbol{a}_n^{(1)} \in \mathbb{R}^{6N \times 1}$ and $b_n^{(1)} \in \mathbb{R}$, $\forall n$, respectively. Also, $\boldsymbol{a}^{(2)} \in \mathbb{R}^{6N \times 1}$ and $b^{(2)} \in \mathbb{R}$ denote the learnable parameters of the lower dense layer. It is worth noting that the first term in (7) (i.e., the output of the upper dense layer) corresponds to the proportion of Δ_n to the summation $\sum_{i=1}^{N-1} \Delta_i$, whereas the second term (i.e., the output of the lower dense layer) corresponds to the value of the summation $\sum_{i=1}^{N-1} \Delta_i$. Once $\{\Delta_n\}_{n=1}^{N-1}$ are determined as in (7), the APV $\boldsymbol{x}$ can be readily obtained using (5).

[2]Thus, for the linear MA array, the relative distances between MAs are key design factors, rather than their absolute coordinates.

[3]Thus, for problem (P1) to be feasible, it is required that $D \ge (N-1)d$.

3) AWV Optimizer: To learn the optimized AWV $\boldsymbol{w}$ with the power constraint $\|\boldsymbol{w}\|^2 = 1$,[4] we construct the AWV optimizer by employing a single dense layer with $2N$ output nodes that are fed into the softmax function. Then we take the square root of each output of the softmax function such that the real and imaginary parts of the optimized $\boldsymbol{w}$ are predicted as follows:

$$\begin{bmatrix} \mathsf{Re}\{\boldsymbol{w}\} \\ \mathsf{Im}\{\boldsymbol{w}\} \end{bmatrix} = \sqrt{\frac{e^{\boldsymbol{A}\boldsymbol{\mu} + \boldsymbol{b}}}{\mathbf{1}_{2N}^\mathsf{T} e^{\boldsymbol{A}\boldsymbol{\mu} + \boldsymbol{b}}}} \tag{8}$$

where $\boldsymbol{A} \in \mathbb{R}^{2N \times 6N}$ and $\boldsymbol{b} \in \mathbb{R}^{2N \times 1}$ denote the weight matrix and bias vector, respectively. Once $\mathsf{Re}\{\boldsymbol{w}\}$ and $\mathsf{Im}\{\boldsymbol{w}\}$ are determined as in (8), the AWV $\boldsymbol{w}$ can be computed via $\mathsf{Re}\{\boldsymbol{w}\} + j\mathsf{Im}\{\boldsymbol{w}\}$.

B. Training Methodology

Through the training process, we jointly train the three DL modules, the feature extractor, APV optimizer, and AWV optimizer, of the proposed DL model in an unsupervised manner such that the joint optimization of $\boldsymbol{x}$ and $\boldsymbol{w}$ in (P1) is carried out. To this end, we adopt the minimum beamforming gain (i.e., objective function of (P1)) as the loss function for the training via the following customization: as shown in (10), shown at the bottom of the next page, using the facts that $\|\boldsymbol{c}\|^2 = \mathsf{Re}\{\boldsymbol{c}\}^2 + \mathsf{Im}\{\boldsymbol{c}\}^2$ and $\mathsf{Im}\{\boldsymbol{c}^\mathsf{H}\} = -\mathsf{Im}\{\boldsymbol{c}\}^\mathsf{T}$ for an arbitrary complex-valued vector $\boldsymbol{c}$, we derive an explicit expression of the loss function in terms of $\boldsymbol{x}$, $\mathsf{Re}\{\boldsymbol{w}\}$, and $\mathsf{Im}\{\boldsymbol{w}\}$ (i.e., outputs of the proposed DL model), denoted by $L_\phi(\boldsymbol{x}, \mathsf{Re}\{\boldsymbol{w}\}, \mathsf{Im}\{\boldsymbol{w}\})$, where ϕ denotes the set of all learnable parameters of the three modules. Then ϕ is iteratively updated such that the customized loss function $L_\phi(\cdot)$ in (10) is maximized via the stochastic gradient ascent method as follows:

$$\phi \leftarrow \phi + \alpha \nabla_\phi L_\phi(\boldsymbol{x}, \mathsf{Re}\{\boldsymbol{w}\}, \mathsf{Im}\{\boldsymbol{w}\}) \tag{9}$$

where $\alpha > 0$ is the learning rate. Also, $\nabla_\phi L_\phi(\cdot)$ is the gradient of the loss function w.r.t. ϕ, which is derived in (11), shown at the bottom of the next page, where ∇_a denotes the gradient operator w.r.t. a variable a and $I(\cdot)$ denotes the indicator function that is equal to one if its argument is true and zero otherwise.

[4]It can be readily shown that the inequality constraint $\|\boldsymbol{w}\|^2 \le 1$ in (2c) must be fulfilled with equality at the optimal point of $\boldsymbol{w}$ in (P1).

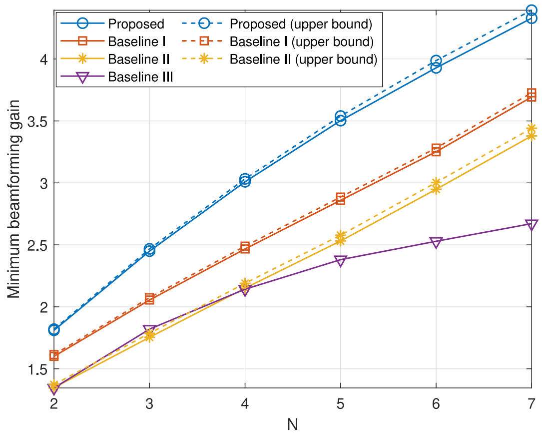

Fig. 3. Minimum beamforming gain versus the number N of MAs.

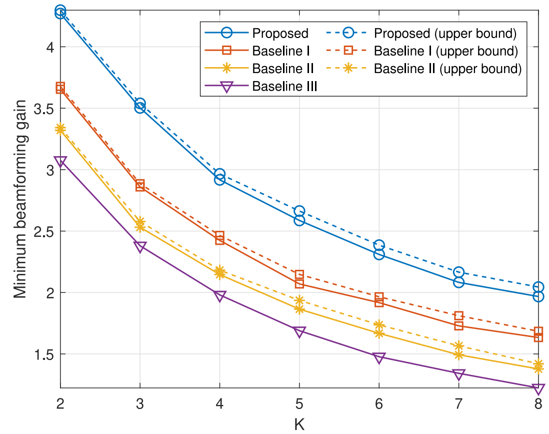

Fig. 4. Minimum beamforming gain versus the number K of users for $N = 5$.

IV. Results and Discussion

A. Performance Comparisons

In the simulations, we consider the following parameter setup as default unless specified otherwise: $N = 8$, $K = 3$, $d = 0.5\lambda$, $D = 10\lambda$; and $\{\theta_k\}_{k=0}^{K-1}$ are generated randomly according to the uniform distribution over $[0, \pi)$. The proposed DL model is trained using 10^7 training samples with a learning rate of $\alpha = 10^{-3}$ and a batch size of 10^3 for 500 epochs. Its performance is then evaluated using 3×10^6 test samples and compared to that of the following baseline schemes:

- *Baseline Scheme I:* The APV $\boldsymbol{x}$ and AWV $\boldsymbol{w}$ are jointly optimized in an alternating manner such as in [6]. Specifically, for given $\boldsymbol{w}$, $\boldsymbol{x}$ is optimized via successive convex approximation (SCA) [6] and for given $\boldsymbol{x}$, $\boldsymbol{w}$ is optimized via semi-definite relaxation (SDR) with rank-one approximation via Gaussian randomization [17].
- *Baseline Scheme II:* The AWV $\boldsymbol{w}$ is optimized via SDR with Gaussian randomization for a fixed APV $\boldsymbol{x}$ with $\{x_n\}_{n=0}^{N-1}$ evenly spaced over $[0, D]$ as $x_n = \frac{D}{N-1}n$, $\forall n$.
- *Baseline Scheme III:* The APV $\boldsymbol{x}$ and AWV $\boldsymbol{w}$ are randomly determined such that the constraints in (2a)–(2c) are fulfilled.

In Figs. 3 and 4, the performance of the various schemes is compared for different values of N and K, respectively. The performance of the SDR multicast beamforming for the MAs' positions predicted by the proposed scheme and that of the baseline schemes I and II without the rank-one approximation are also presented as performance upper bounds (or benchmarks). From Figs. 3 and 4, it can be seen that the proposed scheme considerably and consistently outperforms the baseline schemes (as well as their upper bounds) in terms of the minimum beamforming gain over the entire ranges of N and K, thereby validating the effectiveness of the proposed DL model for learning and optimizing the multicast beamforming with MA array. The performance gain is observed to be more pronounced in the regimes of moderate N and small K. Meanwhile, the performance gap between the proposed scheme and its upper bound gets larger as N or K increases, but the gap remains small, thereby evidently indicating that the AWV optimizer of the proposed DL model can learn the near-optimal solution even in the unsupervised manner.

In Fig. 5, the probability density functions (PDFs) of the optimized MAs' positions x_1 and x_2 as well as their relative distances Δ_1 and Δ_2 predicted by the proposed DL model are plotted when $N = 3$. From Fig. 5, one can see that the shapes of PDFs of x_1 and x_2 look dissimilar, while those of Δ_1 and Δ_2 look similar. Overall, it turns out that evenly spaced or fixed antenna positions are never optimal; instead, the antenna positions must be adjusted according to the system requirements and propagation environments, e.g., d, D, and $\{\theta_k\}_{k=0}^{K-1}$.

$$
\begin{aligned}
&L_\phi(\boldsymbol{x}, \mathsf{Re}\{\boldsymbol{w}\}, \mathsf{Im}\{\boldsymbol{w}\}) = \min_{k\in\mathcal{K}} \left|\boldsymbol{w}^{\mathsf{H}}\boldsymbol{h}(\boldsymbol{x},\theta_k)\right|^2 = \min_{k\in\mathcal{K}} \left[\mathsf{Re}\left\{\boldsymbol{w}^{\mathsf{H}}\boldsymbol{h}(\boldsymbol{x},\theta_k)\right\}^2 + \mathsf{Im}\left\{\boldsymbol{w}^{\mathsf{H}}\boldsymbol{h}(\boldsymbol{x},\theta_k)\right\}^2\right] \\
&= \min_{k\in\mathcal{K}} \left[\left(\mathsf{Re}\{\boldsymbol{w}\}^{\mathsf{T}}\mathsf{Re}\{\boldsymbol{h}(\boldsymbol{x},\theta_k)\} + \mathsf{Im}\{\boldsymbol{w}\}^{\mathsf{T}}\mathsf{Im}\{\boldsymbol{h}(\boldsymbol{x},\theta_k)\}\right)^2 + \left(\mathsf{Re}\{\boldsymbol{w}\}^{\mathsf{T}}\mathsf{Im}\{\boldsymbol{h}(\boldsymbol{x},\theta_k)\} - \mathsf{Im}\{\boldsymbol{w}\}^{\mathsf{T}}\mathsf{Re}\{\boldsymbol{h}(\boldsymbol{x},\theta_k)\}\right)^2\right]. \qquad (10)
\end{aligned}
$$

$$
\begin{aligned}
&\nabla_\phi L_\phi(\boldsymbol{x}, \mathsf{Re}\{\boldsymbol{w}\}, \mathsf{Im}\{\boldsymbol{w}\}) = I\left(k = \arg\min_{i\in\mathcal{K}} \left|\boldsymbol{w}^{\mathsf{H}}\boldsymbol{h}(\boldsymbol{x},\theta_i)\right|^2\right)\Bigg\{2\left(\mathsf{Re}\{\boldsymbol{w}\}^{\mathsf{T}}\mathsf{Re}\{\boldsymbol{h}(\boldsymbol{x},\theta_k)\} + \mathsf{Im}\{\boldsymbol{w}\}^{\mathsf{T}}\mathsf{Im}\{\boldsymbol{h}(\boldsymbol{x},\theta_k)\}\right) \\
&\times \left[\left(\mathsf{Re}\{\boldsymbol{w}\}^{\mathsf{T}}\nabla_{\boldsymbol{x}}\mathsf{Re}\{\boldsymbol{h}(\boldsymbol{x},\theta_k)\} + \mathsf{Im}\{\boldsymbol{w}\}^{\mathsf{T}}\nabla_{\boldsymbol{x}}\mathsf{Im}\{\boldsymbol{h}(\boldsymbol{x},\theta_k)\}\right)\nabla_\phi\boldsymbol{x} + \mathsf{Re}\{\boldsymbol{h}(\boldsymbol{x},\theta_k)\}^{\mathsf{T}}\nabla_\phi\mathsf{Re}\{\boldsymbol{w}\} + \mathsf{Im}\{\boldsymbol{h}(\boldsymbol{x},\theta_k)\}^{\mathsf{T}}\nabla_\phi\mathsf{Im}\{\boldsymbol{w}\}\right] \\
&+ 2\left(\mathsf{Re}\{\boldsymbol{w}\}^{\mathsf{T}}\mathsf{Im}\{\boldsymbol{h}(\boldsymbol{x},\theta_k)\} - \mathsf{Im}\{\boldsymbol{w}\}^{\mathsf{T}}\mathsf{Re}\{\boldsymbol{h}(\boldsymbol{x},\theta_k)\}\right) \\
&\times \left[\left(\mathsf{Re}\{\boldsymbol{w}\}^{\mathsf{T}}\nabla_{\boldsymbol{x}}\mathsf{Im}\{\boldsymbol{h}(\boldsymbol{x},\theta_k)\} - \mathsf{Im}\{\boldsymbol{w}\}^{\mathsf{T}}\nabla_{\boldsymbol{x}}\mathsf{Re}\{\boldsymbol{h}(\boldsymbol{x},\theta_k)\}\right)\nabla_\phi\boldsymbol{x} + \mathsf{Im}\{\boldsymbol{h}(\boldsymbol{x},\theta_k)\}^{\mathsf{T}}\nabla_\phi\mathsf{Re}\{\boldsymbol{w}\} - \mathsf{Re}\{\boldsymbol{h}(\boldsymbol{x},\theta_k)\}^{\mathsf{T}}\nabla_\phi\mathsf{Im}\{\boldsymbol{w}\}\right]\Bigg\}. \qquad (11)
\end{aligned}
$$

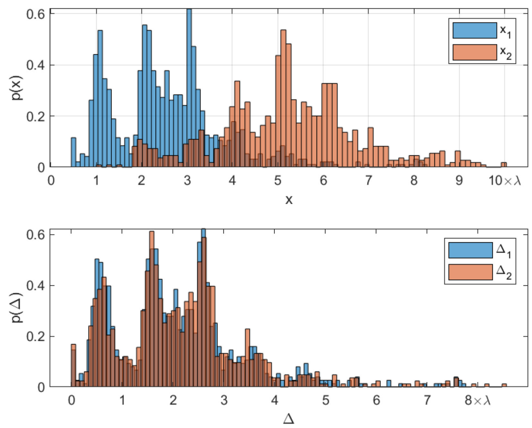

Fig. 5. PDFs of the optimized MAs' positions and their relative distances predicted by the proposed DL model.

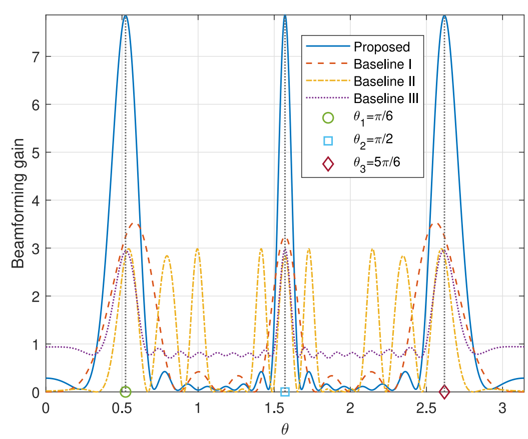

Fig. 6. Beam patterns of the proposed and baseline schemes.

Fig. 6 depicts the beam patterns of the proposed and baseline schemes when $\theta_1 = \frac{\pi}{6}$, $\theta_2 = \frac{\pi}{2}$, and $\theta_3 = \frac{5\pi}{6}$. It can be observed that the proposed scheme achieves much higher beamforming gains at θ_1, θ_2, and θ_3 while yielding the least amount of beam leakages compared to the baseline schemes. This clearly reveals the significant merit of leveraging the DL technique in jointly optimizing the antenna positions and weights for improvement of beamforming performance.

B. Computational Complexity Analysis

Note that the optimization of the APV $\boldsymbol{x}$ via SCA and that of the AWV $\boldsymbol{w}$ via SDR require computational complexities of $\mathcal{O}(N^3)$ and $\mathcal{O}((N^2+K)^{3.5})$, respectively [18]. Thus, the baseline schemes I and II requires high computational complexities of $\mathcal{O}((N^3+(N^2+K)^{3.5})N_{\text{iter}})$ and $\mathcal{O}((N^2+K)^{3.5})$, respectively, where N_{iter} is the numbers of iterations required for the baseline scheme I to converge. On the other hand, the baseline scheme III requires a low computational complexity of $\mathcal{O}(N)$ for randomly choosing $\boldsymbol{x}$ and $\boldsymbol{w}$. Also, after the (offline) training, the proposed DL model requires a low computational complexity of $\mathcal{O}(N^2)$ for predicting the jointly optimized $\boldsymbol{x}$ and $\boldsymbol{w}$ through the feedforward computations of the feature extractor, APV optimizer, and AWV optimizer, which is proportional to the dimension of learnable weights.

V. Conclusion

In this letter, we proposed the novel and effective DL model for the multicast beamforming with the linear MA array, which jointly optimized the APV and AWV by maximizing the minimum beamforming gain for the users through the unsupervised training procedure of the three modules, the feature extractor, APV optimizer, and AWV optimizer. The extensive simulation results demonstrated that the proposed DL model significantly surpassed the baseline schemes, thereby rendering it highly useful in practical applications. As interesting and important future works, it is deserved to investigate DL-based beamforming designs with MA arrays for broadcast scenarios, multiple-access scenarios, interference channels, etc.

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