

Energy Efficient Maximal Throughput Resource Scheduling Strategy in Satellite Networks

Jiacheng Shuai¹, Yu Liu¹, and Yumei Wang¹

Abstract—In this letter, a greedy routing strategy is proposed for resource scheduling in Earth observation mission to achieve the maximum throughput and improve the energy efficiency. Resource time-expanded graph (RTEG) is introduced to depict the time-evolving topology and resources of satellite networks. Based on RTEG, an energy efficient resource allocation algorithm (EERA) is designed to reduce the total energy consumption and maximize the throughput in data transmission by properly scheduling the transceiver resource, storage resource and power resource of satellites. Simulation results show that EERA achieves preferred energy efficiency when reaching the maximum throughput.

Index Terms—Satellite networks, time-expanded graph, resource scheduling, energy-efficiency.

I. INTRODUCTION

SATELLITE networks have been widely used in Earth observation missions, such as disaster detection, loss assessment, and reconnaissance. Due to the high dynamic topology of the satellite networks, the efficient representation of its time-varying resources is attracting the attention of researchers [1], [2]. In addition, in satellite networks, the link discontinuity and limited transceiver and power resources bring severe difficulties of ensuring qualified network performance [3], [4].

Up to now, many works have focused on the resource allocation problem in satellite networks. Reference [5] used time-expanded graph (TEG) to depict the time-varying topology of satellite networks and allocated transceiver resource of satellites to reach the network's maximum throughput. Reference [6] proposed a resource allocation strategy based on task priority, which improved the resource utilization. In [7], a resource scheduling method for virtual cluster based on rough set was proposed to improve the cluster resource utilization. Reference [8] investigated the resource allocation problem in data relay satellite systems from the perspective of users' behavior analysis to effectively decrease the resource conflicts.

However, transmission energy consumption has not been taken into account in their works. The power resource provided

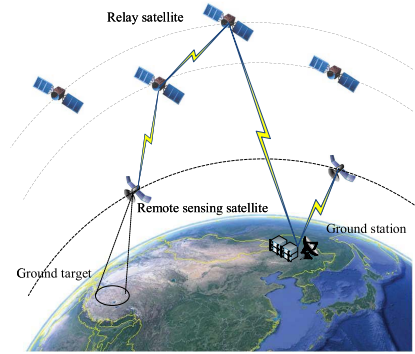


Fig. 1. Satellite network for Earth observation mission.

by solar panel is also a scarce resource in satellite networks. In [9], a satellite routing algorithm with minimum energy consumption was proposed based on time-expanded graph, which reduced the energy consumption of the network, but failed to maximize the network throughput. Reference [10] achieved energy efficiency for multicast in satellite networks without guarantee for the maximum network throughput. Although the above studies reduce the total transmission energy cost in satellite networks, the maximum throughput of the networks is compromised.

Therefore, it's indispensable to design a routing strategy to meet the demand of massive data transmission with energy-efficiency in satellite networks. In this letter, we study the energy-efficient routing based on temporal graph in Earth observation. Due to the limited resources, our routing strategy jointly scheduling transceiver resource, storage resource and power resource of satellites to reach the maximum throughput of the network with low energy cost by searching for the minimum energy cost path for each traffic flow. Simulation results show that the strategy achieves the expected performance.

II. SYSTEM MODEL

A. Network Model

This letter focuses on data transmission in the Earth observation mission shown in Fig. 1. The total number of satellites is R . Among them, there are R_1 relay satellites and R_2 remote sensing satellites. After the remote sensing satellites obtain data from the ground target S , they can choose to directly transmit the data to the ground station D , or to transmit the data to the relay satellites and then to the ground station D . The relay satellites can only obtain data from the remote sensing satellites and cannot directly obtain data from the ground target S .

Manuscript received 17 July 2022; revised 7 October 2022; accepted 16 November 2022. Date of publication 25 November 2022; date of current version 10 February 2023. This work was supported by the National Key Research and Development Program of China under Grant 2019YFB1803103. The associate editor coordinating the review of this article and approving it for publication was T. De Cola. (Corresponding author: Yu Liu.)

Jiacheng Shuai and Yumei Wang are with the School of Artificial Intelligence, Beijing University of Posts and Telecommunications, Beijing 100088, China (e-mail: shuaijiacheng@bupt.edu.cn; ymwang@bupt.edu.cn).

Yu Liu is with the School of Artificial Intelligence, Beijing University of Posts and Telecommunications, Beijing 100088, China, and also with the Department of Broadband Communication, Peng Cheng Laboratory, Shenzhen 518066, China (e-mail: liuy@bupt.edu.cn).

Digital Object Identifier 10.1109/LWC.2022.3224770

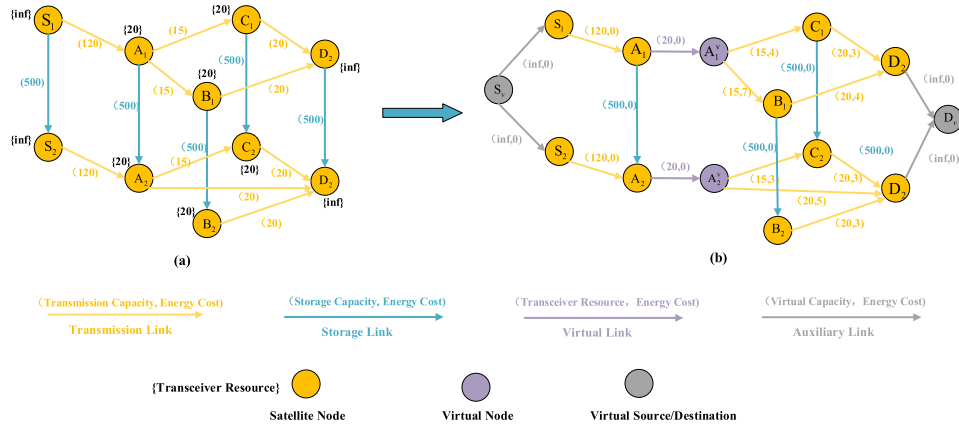


Fig. 2. The proposed resource time-expanding graph (RTEG). (a) TEG in two time slots; (b) RTEG in two time slots.

B. Energy Consumption

In satellite networks, the transmission through inter-satellite links (ISL) is mainly affected by free space loss [11]. P_R denotes the received power which is computed as follows.

$$P_R = \frac{G_R G_T P_T}{L_P} \quad (1)$$

where P_T denotes transmission power, G_T represents the transmitting antenna gain and G_R denotes the receiving antenna gain. L_P is the free space loss which is given by:

$$L_P = \left(\frac{4\pi d}{\lambda} \right)^2 = \left(\frac{4\pi f}{c} \right)^2 \quad (2)$$

where d denotes the propagation distance, λ denotes the operating wavelength, c denotes the speed of light, and f denotes the operating frequency. According to (1) and (2), we have the satellite transmitting power P_T as follows.

$$P_T = \frac{16\pi^2 P_R d^2}{G_R G_T \lambda^2} \quad (3)$$

Assuming that the operating frequency f , the receiving gain G_R and transmitting gain G_T of all satellites are the same and remain unchanged over time. The corresponding transmitting power of the satellite nodes must increase with the transmission distance of two satellites to have the same receiving power. Hence, the transmission energy consumption e in unit time τ is:

$$e = P_T \tau = \frac{16\pi^2 P_R \tau}{G_R G_T \lambda^2} d^2 = k \tau d^2 \quad (4)$$

which is only related to transmission time τ and the distance d between satellites. k remains constant in our system.

C. Resource Time-Expanded Graph

In order to depict the relationship between transceiver resources, storage capacity and energy consumption in the network in terms of time and space dimension, we construct the resource time-expanding graph (RTEG) which is shown in Fig. 2.

For a certain period of time $T = (t_1, t_h]$, we equally divide it into h time slots $\{t_1, \dots, t_{i-1}, t_i, \dots, t_h\}$. The interval of each time slot is a constant number τ . The topology in each

time slot is regarded as a static snapshot of the network. RTEG = $\{\mathcal{V}, \mathcal{A}, \mathcal{C}, \mathcal{E}, T\}$ consists of node set \mathcal{V} , link set \mathcal{A} , capacity set \mathcal{C} , energy consumption set \mathcal{E} , and time horizon T . RTEG introduces the concept of transmission energy consumption and use virtual nodes and links to depict the limited transceiver resource. The virtual links between satellite nodes and their corresponding virtual nodes represent the transceiver resource constraint. Besides, the storage capacity is represented by vertical links in the graph which connects the same satellite in adjacent time slot. By vertical links in RTEG, the sequence of snapshots are connected to depict the dynamic topology. RTEG is formulated as follows.

- *Node set \mathcal{V}* : Source set $\mathcal{V}_S = \{S^i \mid 1 \leq i \leq h\}$ that denotes h replicas of the ground target. Relay satellite node set $\mathcal{V}_{R_1} = \{r_1^i \mid 1 \leq i \leq h, 1 \leq l \leq R_1\}$ denotes replicas of relay satellites in h time slots. Remote sensing satellite node set $\mathcal{V}_{R_2} = \{r_2^i \mid 1 \leq i \leq h, 1 \leq m \leq R_2\}$ denotes replicas of remote sensing satellites in h time slots. Destination set $\mathcal{V}_D = \{D^i \mid 1 \leq i \leq h\}$ denotes h replicas of ground station. Virtual node set $\mathcal{V}_R = \{r_n^i \mid 1 \leq i \leq h, 1 \leq n \leq R\}$ denotes replicas of virtual nodes of the corresponding satellite nodes. In addition, the node set also includes virtual source S_v and virtual destination D_v , in this way the multi-source-multi-sink problem can be transformed into a single-source-single-sink problem.
- *Link set \mathcal{A}* : Transmission link set $\mathcal{A}_t = \{(v_p^i, v_q^i) \mid p \neq q, 1 \leq i \leq h, v_p^i \in \mathcal{V} - \mathcal{V}_D, v_q^i \in \mathcal{V} - \mathcal{V}_S - \mathcal{V}_{R_3}\}$ denotes inter-satellite links or satellite-to-Earth links at the same time slot. Storage link set $\mathcal{A}_s = \{(v_p^i, v_p^{i+1}) \mid 1 \leq i \leq h, v_p^i \in \mathcal{V}_{R_1} \cup \mathcal{V}_{R_2}, v_p^{i+1} \in \mathcal{V}_{R_1} \cup \mathcal{V}_{R_2}\}$ denotes the storage capacity of satellite nodes across the adjacent time slots. Virtual link set $\mathcal{A}_v = \{(v_p^i, v_q^i) \mid 1 \leq i \leq h, v_p^i \in \mathcal{V}_{R_1} \cup \mathcal{V}_{R_2}, v_q^i \in \mathcal{V}_{R_3}\}$ denotes the link between virtual node and its corresponding node. Auxiliary link set $\mathcal{A}_a = \{(S_v, S^i), (D^i, D_v) \mid 1 \leq i \leq h\}$ denotes the link between virtual source, virtual destination and their corresponding node.
- *Capacity set \mathcal{C}* = $\{c_{v_p^i, v_q^j} \mid i \neq j \mid p \neq q, v_p^i \in \mathcal{V}, v_q^j \in \mathcal{V}\}$ denotes the link capacity.
- *Energy consumption set \mathcal{E}* = $\{e_{v_p^i, v_q^j} \mid i \neq j \mid p \neq q, v_p^i \in \mathcal{V}, v_q^j \in \mathcal{V}\}$ denotes the transmission energy consumption.

The $e_{v_p^i, v_q^j}$ of storage links \mathcal{A}_s , virtual links \mathcal{A}_v and auxiliary links \mathcal{A}_a are set to 0. The $c_{v_p^i, v_q^j}$ of virtual links \mathcal{A}_v and auxiliary links \mathcal{A}_a are set to infinity.

Fig. 2b shows an example of RTEG in two time slots. The capacity of virtual links between virtual node and satellite node represent the transceiver resource constraint. Besides, the transmission energy consumption is taken into consideration to achieve energy efficiency. Due to the limited resources and dynamic topology, when traffic flow arrives, satellites can transmit it to other satellites in current time slot or to store the data until there is an energy efficient path to transmit the traffic flow.

III. ENERGY-EFFICIENT RESOURCE ALLOCATION ALGORITHM BASED ON RTEG

A. Problem Formulation

In this letter, we aim to find a jointly resource scheduling strategy that maximizes network traffic and reduces energy consumption. Within time T , the problem can be described as follows.

$$E_{sum} = \min \sum_{v_p^i, v_q^j \in \mathcal{V}} e_{v_p^i, v_q^j} \cdot f_{v_p^i, v_q^j} \quad (5)$$

$$s.t. \quad F_{max} = \sum_{v_q^j \in \mathcal{V}} f_{S_v, v_q^j} = \sum_{v_p^i \in \mathcal{V}} f_{v_p^i, D_v} \quad (6)$$

where $e_{v_p^i, v_q^j}$ denotes the transmission energy consumption between v_p^i and v_q^j . $f_{v_p^i, v_q^j}$ denotes the traffic flow between v_p^i and v_q^j . F_{max} denotes the maximum flow that the network can possibly carry. f_{S_v, v_q^j} denotes the outgoing traffic flow of S_v . $f_{v_p^i, D_v}$ denotes the incoming traffic flow of D_v . $f_{v_p^i, v_q^j}$ subjects to constraints as follows.

- Capacity Constraint: The traffic flow between two nodes cannot exceed the capacity of the link.

$$0 \leq f_{v_p^i, v_q^j} \leq c_{v_p^i, v_q^j}, (v_p^i, v_q^j) \in \mathcal{A} \quad (7)$$

- Flow Conservation Constraint: the incoming traffic flow of each node should be equal to its outgoing flow.

$$\sum_{v_q^j \in \mathcal{V}} f_{v_p^i, v_q^j} = \sum_{v_r^k \in \mathcal{V}} f_{v_r^k, v_p^i}, v_p^i \in \mathcal{V} - S_v - D_v \quad (8)$$

The optimization objective is to reduce transmission energy consumption when the network traffic reaches the maximum by scheduling resources to find routing path which cost least amount of energy. By RTEG, the problem can be described as a max-flow problem with the objective (5) and constraints (6)-(8).

B. RTEG Based Energy Efficient Resource Allocation Algorithm

Based on RTEG, we propose an energy efficient resource scheduling algorithm (EERA) illustrated in Algorithm 1. EERA is a greedy iterative flow augmenting algorithm to find the minimum energy consumption resource scheduling strategy without degradation of network traffic throughput. When

Algorithm 1 RTEG Based EERA Algorithm

Input: Simulation time T , interval of time slot τ , energy consumption $e_{v_p^i, v_q^j}$, link capacity $c_{v_p^i, v_q^j}$.

Output: A maximum flow minimum cost resource scheduling strategy F based on RTEG;

- 1: Construct the RTEG = $\{(\mathcal{V}, \mathcal{A}, \mathcal{C}, \mathcal{E}, T)\}$;
- 2: **while** Minimum cost path P_{min_cost} exists **do**
- 3: **for** All links in P_{min_cost} **do**
- 4: **if** $f_{v_p^i, v_q^j} \leq c_{v_p^i, v_q^j}$ **then**
- 5: $\Delta f_{v_p^i, v_q^j} = c_{v_p^i, v_q^j} - f_{v_p^i, v_q^j}$.
- 6: **end if**
- 7: **end for**
- 8: Find the minimum $\Delta f_{v_p^i, v_q^j}$ of all edges as Δf_P
- 9: $F \leftarrow F + \Delta f_P$
- 10: **for** All links **do**
- 11: **if** $0 < f_{v_p^i, v_q^j} < c_{v_p^i, v_q^j}$ **then**
- 12: $e_{v_q^j, v_p^i} = -e_{v_p^i, v_q^j}$
- 13: $c_{v_q^j, v_p^i} = f_{v_p^i, v_q^j}$
- 14: **else if** $f_{v_p^i, v_q^j} = c_{v_p^i, v_q^j}$ **then**
- 15: $c_{v_q^j, v_p^i} = 0$
- 16: $e_{v_q^j, v_p^i} = \infty$
- 17: **end if**
- 18: **end for**
- 19: $\backslash\backslash$ Adding reverse links and update RTEG;
- 20: **end while**

return the final allocation strategy F .

scheduling resources, EERA always gives priority to find the path with less energy consumption, and distributes the satellites' transceiver resource to transmit data through this path. When there's no suitable path in current time slot, the algorithm will utilize storage resource to store the data in the satellite to waiting for paths in subsequent time slot.

In the algorithm, we initially construct RTEG according to the topology of the network. The energy consumption set \mathcal{E} is generated by calculating the distance between every satellites in each time slot.

First of all, the algorithm begins to search a minimum energy consumption path P_{min_cost} from the virtual source S_v to the virtual destination D_v . After finding P_{min_cost} , the algorithm distribute resources to this path to transmit data. The feasible flow $\Delta f_{v_p^i, v_q^j}$ of link (v_p^i, v_q^j) is defined as

$c_{v_p^i, v_q^j} - f_{v_p^i, v_q^j} \cdot f_{v_p^i, v_q^j}$ is the flow delivered by link (v_p^i, v_q^j) . The feasible flow Δf_P transmitted through P_{min_cost} depends on the minimum feasible flow of all links in P_{min_cost} .

Then, the RTEG will be updated. For any link $(v_p^i, v_q^j) \in \mathcal{A}$ with flow $f_{v_p^i, v_q^j} = f$ passing through, a reverse link (v_q^j, v_p^i) with a capacity of f is added. The energy consumption of this link is set to $-e_{v_p^i, v_q^j}$. By adding reverse links, we can adjust the previously distributed flow by distribute flows to reverse links.

After the RTEG is updated, the algorithm starts searching new minimum energy consumption path in RTEG and enters iteration. The algorithm runs until there is no minimum energy consumption path from virtual source S_v to virtual destination D_v to distribute new flow. Finally, the energy efficient resource scheduling strategy is obtained.

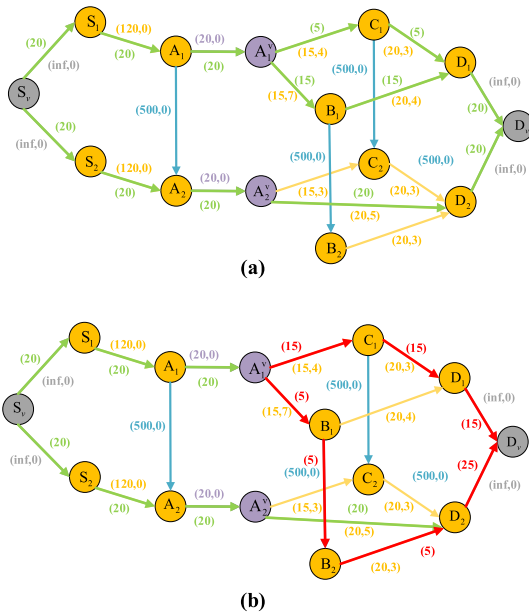


Fig. 3. The comparison of the allocation strategy of EERA algorithm and Ford-Fulkerson algorithm. a) Allocation strategy of Ford-Fulkerson; $F = 40$, $E = 300$ b) Allocation strategy of EERA; $F = 40$, $E = 255$.

Fig. 3 shows the comparison of the allocation strategy of EERA algorithm and Ford-Fulkerson algorithm [12]. The green edge indicates that the flow passes through the link, and the green number indicates the size of the flow. Red edges and red numbers represent the different resource scheduling choices of EERA algorithm from Ford-Fulkerson algorithm.

Ford-Fulkerson is a traditional algorithm to solve max-flow problems in graph theory. We can see in Fig. 3(a), in the resource scheduling strategy obtained by Ford-Fulkerson algorithm, the amount of flow passes through the network is 40 ($f_{S_1, S_1} + f_{S_2, S_2}$), and the total energy consumption is 300. While in Fig. 3(b), the amount of flow passed through the network is also 40. However, the transmission energy consumption is reduced to 255.

IV. SIMULATIONS

To evaluate the performance of the proposed algorithm, we conduct our simulation in a Low Earth Orbit (LEO) constellation based on Iridium constellation [13]. There are 28 relay satellites arbitrarily selected from Iridium constellation which are orbiting at an altitude of 780 km with an inclination of 86.4° in 6 orbit planes. Besides, two remote sensing satellites operate in a sun-synchronous orbit at an altitude of 631 km with an inclination of 97.9° . The ground target S and the ground station D are located at Rome ($41.8^\circ\text{N}, 12.5^\circ\text{E}$) and Beijing ($39.9^\circ\text{N}, 166.4^\circ\text{E}$) respectively. Remote sensing satellites obtain data from ground target and transmit the data to ground station through relay satellites. We utilize satellite topology simulation software to generate network topology and construct RTEG. The simulation time T is set to 2 hours and the time slot interval is set to 60 seconds.

The data acquisition rate of remote sensing satellites from the ground is set to 1200 Mbps. The transmission rate is set to 200 Mbps for the inter-satellite link capacity, and 300 Mbps for

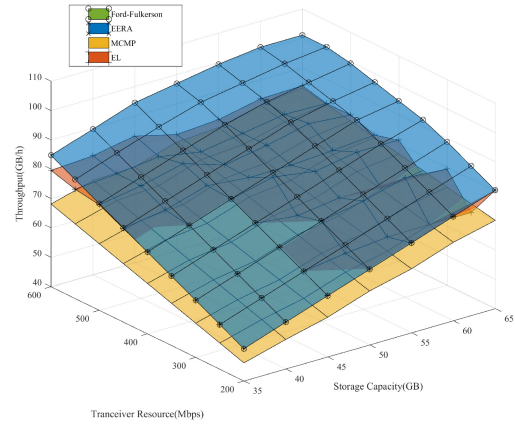


Fig. 4. Comparison of throughput among four algorithms.

the satellite-ground link capacity. In the simulation, the maximum length of the ISLs is around 6300 km, and we normalize the transmission energy consumption and set the energy consumption of satellite transmission with a transmission distance of 6300 km to 1.

We compare the performance of EERA with a minimum-cost constrained multipath algorithm (MCMP) in [9], a energy-limited routing algorithm (EL) and Ford-Fulkerson algorithm in [12].

- **MCMP** MCMP is a greedy iterative flow augmenting algorithm to search for the minimum energy cost routing strategy. MCMP greedily finds the least energy cost routing path and distributes flow in the path in each iteration. The difference between MCMP and EERA is that MCMP only focus on minimizing the energy cost while not maximizing network flow.
- **EL** EL takes both throughput and energy-efficiency into consideration. EL greedily searches for the minimum cost path to distribute flow until there are no possible path from source to destination. In the process, the energy consumption of a satellite is constraint with a limitation. When the accumulative energy consumption of a satellite in the path reaches the limitation, the transceiver resources of this satellite will be set to 0. Therefore, the traffic flow will be distributed to other satellites. Compared to MCMP, EL utilizes more satellites in the network (not just those with low transmission energy cost) to reach a higher network throughput.
- **Ford-Fulkerson** Ford-Fulkerson maximizes the network flow in time-expanding graph according to [5].

In the simulation, the transceiver resource varies from 200 Mbps to 600 Mbps. The storage capacity varies from 35 GB to 65 GB. We mainly focus on the following metrics to evaluate the performance of EERA.

- **Throughput** We evaluate the average network throughput in 2 hours. We can see in Fig. 4, the throughput performance of EERA always overlaps with the throughput of Ford-Fulkerson when resources change. As transceiver resource and storage capacity increase, the throughput of EERA increases stably. The performance of EL is the same with Ford-Fulkerson within a small range of the transmission resources and storage capacity. In most cases, EL

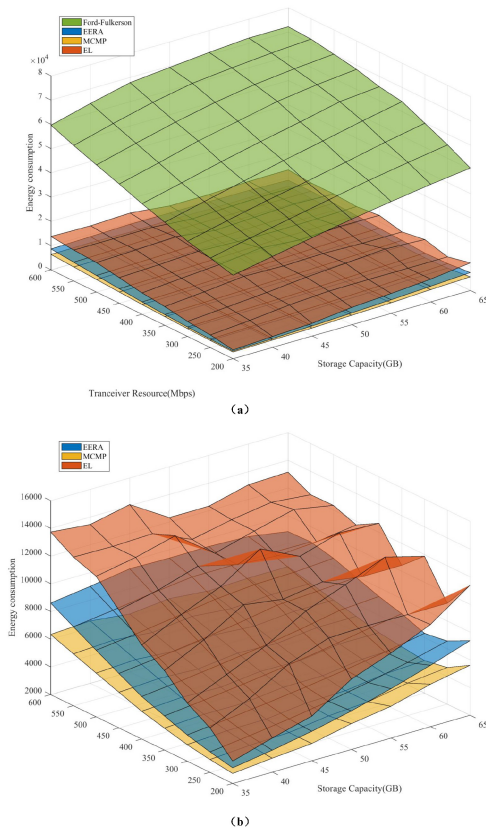


Fig. 5. a) Comparison of energy consumption among all four algorithms; b) Comparison of energy consumption among EERA, EL and MCMP.

has a smaller throughput than Ford-Fulkerson. While the throughput performance of MCMP is around 82% to 90% of performance of Ford-Fulkerson. This is mainly because EERA adds reverse links to the graph in the procedure to adjust the previous flow distribution for a better choice to reach the maximal flow.

- **Energy Consumption** Fig. 5(a) shows the total energy consumption of four algorithms. As we can see, MCMP, EL and EERA all reduce the network energy consumption significantly compared to Ford-Fulkerson. To have a clear vision of the difference among EL, MCMP and EERA, we leave out Ford-Fulkerson in Fig. 5(b). Among the three algorithms, the Energy consumption of EL is still nearly twice as large as MCMP. The energy consumption of EERA is 30% larger than MCMP because EERA gives priority to maximizing network flow. To achieve maximal flow, EERA may rather choose high energy cost path.

In summary, EERA achieves the same performance as the Ford-Fulkerson algorithm in terms of throughput and gets not far to the MCMP in terms of energy consumption.

Compared to EL, EERA performs better in both throughput and energy consumption. Overall, EERA jointly schedules resources to maximize the network throughput and reduce energy consumption.

V. CONCLUSION

In this letter, we introduced energy consumption to TEG and construct RTRG to represent the limited resources in satellite network. Based on RTEG, EERA algorithm was proposed to jointly schedule transceiver resource and storage resource to maximize network throughput and achieve energy efficiency. We evaluated the proposed algorithm in Earth observation satellite network. Simulation results showed that the proposed algorithm obtained the maximum network flow and reduced the energy consumption significantly.

REFERENCES

- [1] Q. Liu and L. Yao, "Satellite resource description and search based on hybrid granularity," in *Proc. IEEE Int. Conf. Netw. Sens. Control (ICNSC)*, 2018, pp. 1–5.
- [2] T. Zhang, J. Li, H. Li, S. Zhang, P. Wang, and H. Shen, "Application of time-varying graph theory over the space information networks," *IEEE Netw.*, vol. 34, no. 2, pp. 179–185, Mar./Apr. 2020.
- [3] H. He, D. Zhou, M. Sheng, and J. Li, "Mission structure learning-based resource allocation in space information networks," in *Proc. IEEE Int. Conf. Commun. (ICC)*, 2021, pp. 1–6.
- [4] J. Hu, G. Li, D. Bian, S. Shi, R. Ge, and L. Gou, "Energy-efficient cooperative spectrum sensing in cognitive satellite terrestrial networks," *IEEE Access*, vol. 8, pp. 161396–161405, 2020.
- [5] P. Wang, X. Zhang, S. Zhang, H. Li, and T. Zhang, "Time-expanded graph-based resource allocation over the satellite networks," *IEEE Commun. Lett.*, vol. 8, no. 2, pp. 360–363, Apr. 2019.
- [6] C. Fan, X. Zhao, L. Xie, Y. Ma, Y. Zhang, and X. Feng, "A resource mapping method in cloud-based satellite ground system," in *Proc. IEEE Int. Conf. Smart City (SmartCity)*, 2015, pp. 1163–1166.
- [7] C. Fan et al., "A resource scheduling method with rough set for virtual cluster," in *Proc. IEEE Int. Conf. Comput. Inf. Technol. (CIT)*, 2016, pp. 652–654.
- [8] L. Wang, C. Jiang, L. Kuang, S. Wu, H. Huang, and Y. Qian, "High-efficient resource allocation in data relay satellite systems with users behavior coordination," *IEEE Trans. Veh. Technol.*, vol. 67, no. 12, pp. 12072–12085, Dec. 2018.
- [9] F. Jiang, Q. Zhang, Z. Yang, and P. Yuan, "A space-time graph based multipath routing in disruption-tolerant Earth-observing satellite networks," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 55, no. 5, pp. 2592–2603, Oct. 2019.
- [10] K. Shi, X. Zhang, S. Zhang, and H. Li, "Time-expanded graph based energy-efficient delay-bounded multicast over satellite networks," *IEEE Trans. Veh. Technol.*, vol. 69, no. 9, pp. 10380–10384, Sep. 2020.
- [11] H. Keller and H. Salzwedel, "Link strategy for the mobile satellite system IRIDIUM," in *Proc. IEEE Veh. Technol. Conf.*, vol. 2, 1996, pp. 1220–1224.
- [12] L. R. Ford and D. R. Fulkerson, *Flows in Networks*. Princeton, NJ, USA: Princeton Univ. Press, 1962.
- [13] D. E. Sterling and J. E. Hatlelid, "The IRIDIUM system—a revolutionary satellite communications system developed with innovative applications of technology," in *Proc. IEEE Mil. Commun. Conf.*, vol. 2, 1991, pp. 436–440.