# An Enhanced MILP Model for Multistage Reliability-Constrained Distribution Network **Expansion Planning**

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Abstract-Reliability is an essential factor in distribution networkt expansion planning. However, standard distribution reliability assessment techniques rely on quantifying the impact of a pre-specified set of events on service continuity through the simulation of component outages, one at a time. Due to such a simulation-based nature, the incorporation of reliability into distribution network expansion planning has customarily required the application of heuristic and metaheuristic approaches. Recently, alternative mixed-integer linear programming (MILP) models have been proposed for distribution network expansion planning considering reliability. Nonetheless, such models suffer from either low computational efficiency or over-simplification. To overcome these shortcomings, this paper proposes an enhanced MILP model for multistage reliability-constrained distribution network expansion planning. Leveraging an efficient, yet accurate reliability evaluation model, proposing a customized technique for effectively imposing radial operation, as well as utilizing pragmatic measures to model

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reliability-related costs are the salient features of this work. In this respect, practical reliability-related costs are considered based on reliability incentive schemes and the revenue lost due to undelivered energy during customer outages. The proposed planning approach is tested on four networks with 24, 54, 86, and 138 nodes to illustrate its efficiency and applicability.

to efficiency and approaching.			
${\it Index\ Terms} \hbox{Distribution\ network\ expansion\ planning,\ mixed-integer\ linear\ programming,\ multistage,\ reliability.}$			
Nomenclature			
Acronyms			
EENS	Expected energy not served.		
KCL	Kirchhoff's current law.		
KVL	Kirchhoff's voltage law.		
MILP	Mixed-integer linear programming.		
SAIDI	System average interruption duration index.		
SAIFI	System average interruption frequency in-		
	dex.		
Indices			
b	Index for blocks of the load-duration curve.		
k	Index for investment options.		
$l,l',ar{l}$	Indices for feeder sections.		
m	Index for paths.		
n	Index for nodes.		
$t, \tau$	Indices for time stages of the planning hori-		
	zon.		
Sets			
$K^l$	Set of investment options for feeder section		
	l.		
$K^n$	Set of investment options for transformers		
	in the substation located at node $n$ .		
$L_{\perp}$	Set of feeder sections.		
$L^F, L^R$	Subsets of $L$ containing existing fixed and		
	existing replaceable feeder sections.		
$L^N$	Subset of $L$ containing newly added feeder		
- CIV	sections.		
$L^{SW}$	Subset of $L$ containing feeder sections that		

are switchable under normal operation.

Index set of blocks of the load-duration

LD

SL	Subset of L containing feeder sections di-	$\Delta_b$	Duration of block b of the load-duration
	rectly connected to substation nodes.		curve.
T	Index set of planning stages.	$\theta_n$	Binary parameter, which is equal to 1 for
$\Omega_{-}$	Set of nodes.		existing substations, being 0 otherwise.
$\Omega^D$	Subset of $\Omega$ containing load nodes.	$\lambda_l$	Failure rate per unit length for existing
$\Omega^S$	Subset of $\Omega$ containing substation nodes.		feeder section $l$ .
$\Psi_{l,ar{l}}$	Set of all paths between branches $l$ and $\bar{l}$ .	$\lambda_{l,k}$	Failure rate per unit length for option $k$ of
Parameters			feeder section $l$ .
		$\mu_l^r, \mu_l^{sw}$	Repair and switching times of existing
$B_t^D, B_t^F$	Benchmarks for SAIDI and SAIFI at stage		feeder section $l$ .
~~	t.	$\mu_{l,k}^r, \mu_{l,k}^{sw}$	Repair and switching times of option $k$ of
$CC_{l,k}$	Construction cost coefficient for option $k$ of		feeder section $l$ .
-	feeder section $l$ .	$\xi_{l',m}$	Binary parameter, which is equal to 1 if
$D_{n,b,t}$	Demand at node $n$ for load block $b$ at stage		feeder section $l'$ is in path $m$ , being 0 other-
n Ana	t.		wise.
$D_{n,t}^{Avg}$	Average demand at node $n$ at stage $t$ .	$\chi_{n,l}$	Binary parameter, which is $-1$ or $+1$ if
$EC_n$	Coefficient for the construction cost of a		feeder section $l$ is connected to node $n$ and
	new substation or the expansion cost of the		the predetermined arbitrary flow direction is
	existing substation at node $n$ .		toward or away from node $n$ , respectively,
$ER_t$	Expected revenue per unit of energy deliv-		being 0 otherwise.
_	ered to the customers at stage $t$ .	Variables	
$rac{ar{f}_l}{ar{f}_{l,k}}$	Flow capacity for existing feeder section $l$ .		
$f_{l,k}$	Flow capacity for option $k$ of feeder section	$BEENS_{l,t}$	EENS share of feeder section $l$ at stage $t$ .
	l.	$BIDI_{l,t}, BIFI_{l,t}$	Customer interruption duration and number
$G_n^{Ini}$	Initial capacity of the substation at node $n$ .		of customer interruptions caused by the fail-
$G_{n,k}$	Capacity of transformer option $k$ for the		ure of feeder section $l$ at stage $t$ .
	substation located at node $n$ .	$d_{l,t}$	Average demand downstream of feeder sec-
$I_t^D, I_t^F$	Incentive rates for SAIDI and SAIFI at stage		tion $l$ at stage $t$ .
	t.	$d_{l,t}^+, d_{l,t}^-$	Non-negative variables used to model the
$IC_{n,k}, MC_{n,k}$	Investment and maintenance cost coeffi-		absolute value of $d_{l,t}$ .
	cients of transformer option $k$ for the sub-	$EENS_t$	Expected energy not served at stage $t$ .
	station located at node $n$ .	$f_{l,b,t}$	Power flow across feeder section $l$ for load
M	Sufficiently large positive number.		block $b$ at stage $t$ .
$N_{n,t}$	Number of customers connected to node $n$	$g_{n,b,t}$	Electric power injection of the substation
	at stage $t$ .	, ,	located at node $n$ for load block $b$ at stage $t$ .
$OC_l$	Operating cost coefficient of existing feeder	$g_{n,t}^d, g_{n,t}^h$	Average demand and total number of cus-
	section $l$ .		tomers supplied by the substation connected
$OC_{l,k}$	Operating cost coefficient for option $k$ of		to node $n$ at stage $t$ .
	feeder section $l$ .	$h_{l,t}$	Number of customers connected to the
pf	System power factor.		nodes downstream of feeder section $l$ at
$Pr_{n,b,t}$	Substation electricity price at node $n$ for	- D - E	stage $t$ .
	load block $b$ at stage $t$ .	$Inc_t^D, Inc_t^F$	Costs of the incentive schemes based on
r	Interest rate.		SAIDI and SAIFI at stage $t$ .
$rr_k, UL_k$	Capital recovery rate and useful lifetime for	$Inv_t, Op_t$	Investment and operating costs at stage $t$ .
	option $k$ .	$LPM_{n,t}$	Binary variable, which is equal to 1 if node $n$
$rr_n, UL_n$	Capital recovery rate and useful lifetime of		is connected to the network at stage $t$ , being
	the assets other than transformers in the		0 otherwise.
	substation located at node $n$ .	$RL_t$	Revenue lost due to unserved energy at stage
$RC_{l,k}$	Replacement cost coefficient for option $k$ of		t.
_	feeder section $l$ .	$RRC_t$	Reliability-related cost at stage $t$ .
$rac{V}{Z^B}, S^B$	Lower and upper bounds for nodal voltages.	$SAIDI_t, SAIFI_t$	
	Base impedance magnitude and base power.		and system average interruption frequency
$Z_l$	Impedance magnitude per unit length for		index at stage $t$ .
	existing feeder section $l$ .	$u_{l,t}$	Binary-valued continuous variable for the
$Z_{l,k}$	Impedance magnitude per unit length for		utilization of feeder section $l$ at stage $t$ .
	option $k$ of feeder section $l$ .	$V_{n,b,t}$	Voltage magnitude at node $n$ for load block
$\alpha_l$	Length of feeder section $l$ .		b at stage $t$ .

$x_{l,k,t}$	Binary variable for the investment in option
	k of feeder section $l$ at stage $t$ .
$x_{n,k,t}$	Binary variable for the investment in trans-
, ,	former option $k$ in the substation located at
	node $n$ at stage $t$ .
$x_{n,t}$	Binary variable for the construction of a new
70,0	substation or the expansion of the existing
	substation at node $n$ at stage $t$ .
$y_{l,k,t}$	Binary variable for the utilization of option
- , ,	k of feeder section $l$ at stage $t$ .
$y_{l,t}$	Binary variable for the utilization of existing
<b>5</b> 1/1	feeder section $l$ at stage $t$ .
$z_{l,ar{l},t}$	Binary-valued continuous variable, which is
0,0,0	equal to 1 if feeder section $l$ is in a feeder
	whose first branch is $\bar{l}$ and the switch of
	feeder section <i>l</i> is closed, being 0 otherwise.
$\delta_{l,t}$	Average demand upstream of feeder section
-,-	l at stage $t$ .
$\eta_{l,t}$	Number of customers connected to the
70,0	nodes upstream of feeder section $l$ at stage
	t.

#### I. INTRODUCTION

7 ITH a share of over 90%, distribution systems are responsible for the majority of electric power outages [1]. Thus, in order to curb this issue in future power systems, several regulatory policies have been introduced to control the service reliability of distribution companies [2], [3]. Under such policies, considering reliability in multistage distribution network expansion planning is crucial to improve the quality of service provision to consumers [4], [5]. The optimal expansion decisions over the planning horizon are determined based on the trade-off between the investment costs incurred to enhance the continuity of supply and the benefits from an improved reliability level. Such benefits include 1) reducing the loss of revenue due to unserved energy during customer interruptions, 2) decreasing the financial penalties imposed by distribution system regulators in the case of poor service reliability, and even 3) receiving bonuses offered by the regulators if the reliability level is higher than a specific threshold [2], [3], [6]. In order to estimate these financial benefits, i.e., the reliability worth, the reliability level of the distribution network must be quantified. Various indices are employed to that end [7], among which system average interruption duration index (SAIDI), system average interruption frequency index (SAIFI), and expected energy not served (EENS) are the most used [2], [3], [6].

Based upon the principles introduced in [8], SAIDI, SAIFI, and EENS are generally calculated using various analytical methods relying on event trees [9], ordered sets of nodes [10], branch sections [11], and minimum cut sets [12], to name a few. Unfortunately, these approaches require individually simulating the impact of all component outages under consideration and/or modeling the network topology, which is an outcome of the expansion planning problem, as a given parameter. Both

aspects prevent the application of standard mathematical programming to multistage reliability-constrained distribution network expansion planning. Thus, researchers and practitioners have typically adopted iterative approaches wherein the network topology is known at each iteration, thereby enabling the use of simulation-based reliability assessment [13]–[20]. Among others, these approaches include 1) the heuristic applied in [14] and [19] wherein economic- and reliability-related aspects were used sequentially to drive the optimization process, and 2) metaheuristics such as artificial immune systems [13], particle swarm optimization [15], tabu search [16], global-based harmony search [17], and genetic algorithms [18], [20]. However, such optimization approaches neither ascertain solution quality nor feature guaranteed convergence to optimality.

Alternatively, researchers have begun to focus on nonsimulation-based reliability assessment wherein the suitability for exact techniques is gained at the expense of modeling inaccuracy and/or computational cost. The first explicit mathematical formulation for reliability assessment was presented in [21] to characterize the effect of repair interruptions on reliability indices. An extended model featuring partial post-fault network reconfiguration was proposed in [22], wherein a fictitioussystem-based formulation relying on the enumeration of all component outages was used. The dimensionality issue of [22] was overcome in [23] by a set of algebraic expressions while also handling partial post-fault network reconfiguration. Unlike the load-node-based models in [22] and [23], a system-oriented perspective was adopted in [24] with a modeling capability similar to that in [21]. In [25], additional post-fault network reconfiguration considering the operation of tie switches was modeled by a scenario-based formulation, thereby featuring a dimensionality issue similar to that of [22]. A model equivalent to those described in [22] and [23] was presented in [26], wherein the topological dependence of reliability indices was characterized by variables rather than parameters while considering a system-oriented framework. Recently, the scenario-based model described in [25] was extended in [27] to consider the effect of switchgear locations on reliability.

The above non-simulation-based models pave the way for the application of standard mathematical programming to reliability-constrained distribution problems. Network reconfiguration was addressed in [21] and [26]. The scenario-based model with full post-fault network reconfiguration presented in [25] was applied to static switchgear investment planning [27] and static network expansion planning [28]. Based on [26], switchgear placement without tie line operation was considered in [29]. Such a model was extended in [30] to account for tie switchgear placement and operation. Within the context of the problem addressed in this paper, namely multistage reliability-constrained distribution network expansion planning, relevant examples are [24] and [31]–[33].

In the pioneering reference [31], optimization variables are used for the first time to explicitly incorporate reliability assessment into multistage distribution network expansion planning. To that end, the idea presented in [22] is borrowed, thereby giving rise to a reliability-constrained planning model cast as an instance of MILP. Nonetheless, this enumeration-based reliability

evaluation model relies on the explicit characterization of system operation under every outage, which requires numerous extra variables and constraints. The increased dimension of the resulting planning problem substantially rises the computational burden and can even cause intractability for large-scale distribution networks.

In [24], a MILP-based formulation is also described for the dynamic planning problem at hand. As a salient modeling feature, a non-enumeration-based reliability assessment model is proposed, which improves upon the approach described in [22] in terms of computational efficiency. However, such computational benefits are attained at the expense of disregarding the impact of switching interruptions, thereby potentially leading to optimistic reliability indices [26].

In [32], multistage reliability-constrained distribution network expansion planning with an explicit formulation of reliability assessment is addressed within the context of active distribution systems. The resulting MILP model extends that presented in [24] by accounting for distributed generation. However, similar to [24], the topology-variable-based reliability assessment neglects switching interruptions, which is a far drastic modeling simplification that may have a significant impact on solution accuracy [26].

In [33], the reliability model described in [25] is leveraged for multistage distribution network expansion planning. Interestingly, the impact of tie lines on post-fault network reconfiguration is captured. On the other hand, practical modeling aspects related to network topology, transfer nodes, and demand are disregarded. More importantly, the resulting model features the same major drawback as the conceptually identical static version presented in [28], namely the reliance on the explicit consideration of all operational constraints for every outage scenario. Thus, similar to [31], such an enumeration-based approach gives rise to high-dimensional optimization problems that are computationally prohibitive even for moderately sized networks.

As an alternative, this paper presents a novel MILP-based approach for multistage reliability-constrained distribution network expansion planning that does not feature the lack of convergence properties and quality certification of conventional approximate methods [13]–[20]. Note that reliability is explicitly incorporated in the problem formulation including the effect of switching interruptions neglected in [21] without featuring the limitation of [22], [23], [26], i.e., the a priori knowledge of the network topology is not required, and the computational shortcoming of contingency-constrained models [22] and [25]. Thus, unlike previous planning works [24], [31]–[33], the proposed approach comprises a non-enumeration-based model for analytical reliability assessment including partial post-fault network reconfiguration while accounting for switching interruptions, radial operation under the normal state, transfer nodes, and the chronological aspect of demand for network operation. The proposed approach relies on 1) new MILP expressions for reliability indices based upon the findings of [26], and 2) a new formulation to impose radial operation based on graph theory [34]. As major enhancements corroborated by our numerical experience, the dimensionality issue of [31] and [33]

TABLE I

COMPARISON OF MODELS FOR MULTISTAGE RELIABILITY-CONSTRAINED

DISTRIBUTION NETWORK EXPANSION PLANNING

Approach	Technique	Reliability assessment	Post-fault network reconfiguration
[13], [15]	Metaheuristic	Simulation	_
[14], [19]	Heuristic	Simulation	Partial
[16]–[18], [20]	Metaheuristic	Simulation	Full
[24], [32]	Mathematical programming	Non-enumeration- based formulation	_
[31]	Mathematical programming	Enumeration-based formulation	Partial
[33]	Mathematical programming	Enumeration-based formulation	Full
Proposed approach	Mathematical programming	Non-enumeration- based formulation	Partial

is overcome without resorting to the modeling simplifications of [24], [32], and [33]. Thus, not only EENS, as done in [31], but also SAIFI and SAIDI are effectively characterized to precisely account for reliability-related costs. According to current industry practice relying on incentive-based regulations [2], [3], [6], reliability-related costs are expressed in terms of reward-penalty schemes and the revenue lost due to undelivered energy. Note that leveraging the reliability-oriented model presented in this paper allows for accurately capturing the impact of such incentive schemes on network expansion planning. Moreover, compared to the state-of-the-art radiality model [35], the proposed graph-theory-based formulation significantly improves the computational performance.

This work features a major methodological novelty, namely the adoption of a new mixed-integer linear programming approach suitable for the application of the state-of-the-art branchand-cut algorithm [36]. The proposed technique is different from existing approaches as follows:

- Unlike conventional methods relying on heuristics or metaheuristics [13]–[20], the branch-and-cut algorithm is an exact technique that guarantees finite convergence to optimality while providing a measure of the distance to the optimal solution along the iterative process. Moreover, effective off-the-shelf software is readily available [37], which is advantageous for practical implementation purposes.
- 2) In contrast to recent applications of mixed-integer linear programming [24], [31]–[33], the proposed approach features the following major distinctive aspects: 1) compared to [31], the number of variables and constraints is significantly reduced with identical modeling capability, thereby giving rise to substantial computational superiority, as empirically evidenced, 2) the issues associated with [24] and [32] are overcome by precisely handling switching interruptions, which is of utmost relevance in terms of solution quality [26], and 3) unlike [33], computational tractability is gained without resorting to simplifications related to radiality, transfer nodes, and demand characterization.

Table I summarizes the main differences between this work and the state of the art of multistage reliability-constrained distribution network expansion planning [13]–[20], [24], [31]–[33].

The main contributions of this paper are twofold:

- On the modeling side, an alternative and computationally efficient formulation is presented for multistage reliability-constrained distribution network expansion planning wherein analytical reliability assessment including the effect of switching interruptions, radial operation, and reliability incentive schemes is explicitly cast in terms of the optimization variables.
- 2) Methodologically, a novel approach based on mixed-integer linear programming is presented. Thus, multi-stage reliability-constrained distribution network expansion planning is solved with guaranteed finite convergence to optimality. Moreover, in order to speed up the performance of the state-of-the-art branch-and-cut algorithm, a reduced set of variables and constraints is devised. The beneficial effect of the proposed enhancement is backed by the substantial computational savings that are attained without sacrificing solution accuracy.

The proposed approach provides valuable information to a centralized planner responsible for system-wide investment decisions. Within a decentralized decision-making framework, such information may be used by planners and regulators to determine suitable investments in distribution assets that would require appropriate incentives for investors for their eventual installation. Note that the design of such incentive mechanisms is beyond the scope of this paper.

The remainder of this paper is organized as follows. Section II presents the planning problem under consideration including the new formulation for radial operation. Section III is devoted to the proposed reliability assessment model, where MILP expressions for EENS, SAIDI, and SAIFI are described. Numerical experience is reported in Section IV. Finally, Section V concludes the paper.

# II. MULTISTAGE RELIABILITY-CONSTRAINED DISTRIBUTION NETWORK EXPANSION PLANNING MODEL

Multistage reliability-constrained distribution network expansion planning aims to determine the least-cost set of investment actions on capital-intensive network assets, i.e., substations and feeder sections, in anticipation of growing demands over the planning horizon [4], [5]. As for reliability, the well-known analytical assessment [4], [38] discussed in [14] is implemented. Thus, the reliability assessment relies on the following practical assumptions, which are customarily adopted for the sake of tractability [13]–[15], [19], [22], [23], [26], [31]:

- Only sustained interruptions due to single branch outages are considered. Branch outages are characterized by failure rates and interruption durations.
- 2) A balanced radial or meshed albeit radially operated distribution system is considered wherein each branch connected to a substation is equipped with a circuit breaker without a recloser at the output of the substation. Moreover, all branches are equipped with a switch that enables

partial restoration [38], i.e., the isolation of the part of the system downstream of the fault in order to meet the demand of the healthy portion of the system. Thus, once a sustained fault has occurred, the first circuit breaker upstream of the fault trips, thereby curtailing all downstream load demands. Subsequently, the system topology is reconfigured by operating switches and circuit breakers to reduce the non-supplied energy. To that end, the first switch upstream of the fault is opened in order to isolate the fault. Then the circuit breaker is closed so that the supply to all load demands between the circuit breaker and the switch is restored. Finally, once the isolated fault is cleared, the corresponding switch is closed and complete service is reestablished.

Load nodes are thus affected by interruptions of two types: 1) repair-and-switching interruptions, for which the supply is not restored until the damage is repaired, and 2) switching-only interruptions, which are associated with the network reconfiguration implemented to clear a faulty component.

Admittedly, a complete assessment of reliability should consider temporary faults, line overloading, and additional post-fault network reconfiguration to restore the service for load nodes downstream of the fault. This generalization would, however, render the problem essentially intractable through optimization, thereby requiring the use of heuristics or repeated simulations [16]–[18], [20]. These modeling limitations notwith-standing, the solution of the multistage reliability-constrained expansion planning problem, albeit ignoring those practical aspects, is acceptable for distribution planning purposes [13]–[15], [19], [24], [31], [32] and provides the planner with a first estimate of a cost-effective and reliable expansion scheme.

A detailed description of the proposed approach is provided next. As sketched in Fig. 1, multistage reliability-constrained distribution network expansion planning is addressed by formulating an instance of mixed-integer linear programming that is effectively solved by off-the-shelf software based on the state-of-the-art branch-and-cut algorithm [37].

#### A. Objective Function

Based on industry practice [4] and relevant references on multistage reliability-constrained distribution network expansion planning [13]–[15], [19], [20], [24], [31]–[33], the goal is to minimize the present value of the total cost including investment, operational, as well as reliability-related costs, as formulated in (1). Present value factors are set based on an infinite perpetuity, where it is assumed that each asset is replaced with an identical piece of equipment at the end of its lifetime, whereas the operational and reliability-related costs of the last stage are repeated in the following years [31].

$$\min \sum_{t \in T} \left[ \frac{1}{r(1+r)^t} Inv_t + \frac{1}{(1+r)^t} \left( Op_t + RRC_t \right) \right] + \frac{1}{r(1+r)^{|T|}} \left( Op_{|T|} + RRC_{|T|} \right). \tag{1}$$

#### **Input Data**

- Technical system data (e.g., nodal demands, line impedances, and operational limits)
- Economic system data (e.g., investment and operational costs of equipment and energy prices)
- Technical reliability data (failure rates, repair times, and switching times of components)
- Economic reliability data (parameters of the incentive reliability schemes and the expected revenue losses)

## **Proposed MILP Model**

Minimize the total cost

Subject to:

- Investment and utilization constraints
- Kirchhoff's current and voltage laws
- Operating limits of equipment
- · Radiality constraints
- Reliability assessment expressions

Branch-and-Cut Algorithm

#### **Optimal Expansion Plan**

Fig. 1. Proposed reliability-constrained expansion planning approach.

The investment cost at stage t comprises terms associated with adding new feeder sections, replacing the existing ones, constructing or expanding substations, and adding substation transformers, as expressed in (2):

$$Inv_{t} = \sum_{l \in L^{N}} \sum_{k \in K^{l}} rr_{k} CC_{l,k} x_{l,k,t} + \sum_{l \in L^{R}} \sum_{k \in K^{l}} rr_{k} RC_{l,k} x_{l,k,t}$$

$$+ \sum_{n \in \Omega^{S}} \left[ rr_{n} E C_{n} x_{n,t} + \sum_{k \in K^{n}} rr_{k} I C_{n,k} x_{n,k,t} \right]; \forall t \in T$$
(2)

where capital recovery rates are defined as  $rr_k = \frac{r(1+r)^{UL_k}}{(1+r)^{UL_k}-1}, \forall k \in K^l \cup K^n;$  and  $rr_n = \frac{r(1+r)^{UL_n}}{(1+r)^{UL_n}-1}, \forall n \in \Omega^S.$  In the absence of decision variables modeling the investment in protection devices, their installation costs are included in the investment cost coefficients for feeder sections.

The operational cost for each stage is formulated in (3), which includes the operating cost of feeder sections, the maintenance cost of substation transformers, as well as the cost of electrical power from substations:

$$Op_{t} = \sum_{l \in L^{F} \cup L^{R}} OC_{l}y_{l,t} + \sum_{l \in L^{R} \cup L^{N}} \sum_{k \in K^{l}} OC_{l,k}y_{l,k,t}$$
$$+ \sum_{n \in \Omega^{S}} \left[ \sum_{k \in K^{n}} MC_{n,k} \sum_{\tau=1}^{t} x_{n,k,\tau} + \sum_{b \in LD} pf\Delta_{b}Pr_{n,b,t}g_{n,b,t} \right];$$
$$\forall t \in T. \tag{3}$$

Finally, the reliability-related cost at stage t is modeled based on 1) the revenue lost due to undelivered energy to the consumers during the network failures, and 2) the financial consequences

of incentive reliability schemes as follows:

$$RRC_t = RL_t + Inc_t^D + Inc_t^F; \forall t \in T$$
 (4)

where the expected revenue loss  $RL_t$  is formulated as:

$$RL_t = ER_t EENS_t; \forall t \in T \tag{5}$$

and the costs associated with the reliability incentive schemes are calculated using (6) and (7):

$$Inc_t^D = \left(SAIDI_t - B_t^D\right)I_t^D; \forall t \in T \tag{6}$$

$$Inc_t^F = \left(SAIFI_t - B_t^F\right)I_t^F; \forall t \in T. \tag{7}$$

Expressions (6) and (7) model linear incentive schemes based on SAIDI and SAIFI, respectively. In such schemes, a reliability index better (smaller) than the corresponding benchmark, i.e.,  $B_t^D$  or  $B_t^F$ , results in a financial reward, modeled as a negative cost in (6) and (7) [2]. On the other hand, if the reliability index exceeds the benchmark, a financial penalty is incurred. In this model,  $I_t^D$  and  $I_t^F$  are incentive rates, which indicate the amount of financial incentive for a unit change in the corresponding reliability index [2], [6].

## B. Investment and Utilization Constraints

Investment decisions are subject to the following constraints:

$$\sum_{t \in T} \sum_{k \in K^l} x_{l,k,t} \le 1; \forall l \in L^R \cup L^N$$
(8)

$$\sum_{t \in T} x_{n,t} \le 1; \forall n \in \Omega^S$$
(9)

$$\sum_{t \in T} \sum_{k \in K^n} x_{n,k,t} \le 1; \forall n \in \Omega^S$$
 (10)

$$x_{n,k,t} \le \sum_{\tau=1}^{t} x_{n,\tau}; \forall n \in \Omega^{S}, \forall k \in K^{n}, \forall t \in T$$
 (11)

$$x_{l,k,t} \in \{0,1\}; \forall l \in L^R \cup L^N, \forall k \in K^l, \forall t \in T$$
 (12)

$$x_{n,t} \in \{0,1\}; \forall n \in \Omega^S, \forall t \in T$$

$$\tag{13}$$

$$x_{n,k,t} \in \{0,1\}; \forall n \in \Omega^S, \forall k \in K^n, \forall t \in T.$$
 (14)

As per (8)–(10), only one investment is allowed to be performed on each asset over the planning horizon [31]. Expressions (11) prevent the installation of a new substation transformer prior to the expansion of the corresponding substation. The binary nature of the investment variables is set in (12)–(14).

Constraints on the utilization of the network assets are cast as (15)–(21):

$$y_{l,t} = 1; \forall l \in L^F \setminus L^{SW}, \forall t \in T$$
 (15)

$$y_{l,t} = 1 - \sum_{\tau=1}^{t} \sum_{k \in K^l} x_{l,k,\tau}; \forall l \in L^R \backslash L^{SW}, \forall t \in T$$
 (16)

$$y_{l,k,t} = \sum_{\tau=1}^{t} x_{l,k,\tau}; \forall l \in (L^R \cup L^N) \setminus L^{SW}, \forall k \in K^l, \forall t \in T \quad (17)$$

$$y_{l,t} \le 1 - \sum_{\tau=1}^{t} \sum_{k \in K^l} x_{l,k,\tau}; \forall l \in L^R \cap L^{SW}, \forall t \in T$$

$$\tag{18}$$

$$y_{l,k,t} \leq \sum_{\tau=1}^{t} x_{l,k,\tau}; \forall l \in (L^R \cup L^N) \cap L^{SW}, \forall k \in K^l, \forall t \in T \quad (19)$$

$$y_{l,t} \in \{0,1\}; \forall l \in L^F \cup L^R, \forall t \in T$$

$$(20)$$

$$y_{l,k,t} \in \{0,1\}; \forall l \in L^R \cup L^N, \forall k \in K^l, \forall t \in T.$$
 (21)

Expressions (15)–(17) set the utilization variables for feeder sections that are not switchable under normal operation, and, therefore, cannot be used for network reconfiguration. Existing non-switchable feeder sections are available at all stages. Hence, their utilization variables must be equal to 1, as set in (15). The existing replaceable feeder sections must be utilized prior to any replacement action, as modeled in (16). Finally, according to (17), each option for replaceable and newly added feeder sections must be utilized from the stage at which the corresponding investment is performed onward.

Analogous to (16) and (17), expressions (18) and (19) model the utilization of replaceable and new feeder sections that are switchable, and, hence, can be employed for reconfiguration purposes. These constraints avoid the utilization of feeder section options that are not installed in the network by setting their utilization variables to 0. On the other hand, for already installed options, constraints (18) and (19) impose no limits on the utilization variables. Thus, these options can be employed to optimize the configuration of the network. In addition, expressions (20) and (21) model the binary nature of the utilization variables.

# C. Kirchhoff's Laws and Operational Limits

Network operation is subject to various technical constraints formulated in (22)–(29):

$$\sum_{l \in I} \chi_{n,l} f_{l,b,t} + D_{n,b,t} = 0; \forall n \in \Omega^D, \forall b \in LD, \forall t \in T$$
 (22)

$$\sum_{l \in I} \chi_{n,l} f_{l,b,t} - g_{n,b,t} = 0; \forall n \in \Omega^S, \forall b \in LD, \forall t \in T$$
(23)

$$-M(1-y_{l,t})\!\leq\!\alpha_lZ_lf_{l,b,t}\!-\!Z^BS^B\sum_{n\in\Omega}\!\chi_{n,l}V_{n,b,t}\!\leq\!$$

$$M(1 - y_{l,t}); \forall l \in L^F \cup L^R, \forall b \in LD, \forall t \in T$$
 (24)

$$-M(1-y_{l,k,t})\!\leq\!\alpha_{l}Z_{l,k}f_{l,b,t}-Z^{B}S^{B}\sum_{n\in\Omega}\!\chi_{n,l}V_{n,b,t}\!\leq\!$$

$$M\!(1\!-\!y_{l,k,t}); \forall l\!\in\!L^R \cup L^N, \forall k\!\in\!K^l, \forall b\!\in\!LD, \forall t\!\in\!T \tag{25}$$

$$0 \le g_{n,b,t} \le G_n^{Ini} + \sum_{k \in K^n} \sum_{\tau=1}^t G_{n,k} x_{n,k,\tau};$$

$$\forall n \in \Omega^S, \forall b \in LD, \forall t \in T \tag{26}$$

$$-\bar{f}_l y_{l,t} \le f_{l,b,t} \le \bar{f}_l y_{l,t}; \forall l \in L^F \cup L^R, \forall b \in LD, \forall t \in T \quad (27)$$

$$-\bar{f}_{l,k}y_{l,k,t} \le f_{l,b,t} \le \bar{f}_{l,k}y_{l,k,t};$$

$$\forall l \in L^R \cup L^N, \forall k \in K^l, \forall b \in LD, \forall t \in T$$
(28)

$$\underline{V} \le V_{n,b,t} \le \overline{V}; \forall n \in \Omega, \forall b \in LD, \forall t \in T.$$
(29)

Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL) are modeled based on the linear approximation proposed in [39]. KCL for load nodes and substation nodes is cast by (22) and (23), respectively. Expressions (24) and (25) model KVL for various types of branches. Limits on substation injections and power flows across existing and new feeder sections are imposed in (26)–(28), respectively. Lower and upper bounds for nodal voltages are set in (29).

#### D. Radiality Constraints

This section describes how graph theory can be leveraged to effectively impose radial operation in distribution network expansion planning, which constitutes a relevant modeling novelty over both conventional non-topology-variable-based approaches [13]–[20] and recent topology-variable-based works [24], [31]–[33]. According to [34], a connected graph has no loops if the number of nodes is equal to the number of branches plus 1. This finding can be extended for a graph comprising a disjoint union of connected sub-graphs with no loops, for which the number of nodes is equal to the number of branches plus the number of sub-graphs.

This result is useful to impose radial operation in multistage distribution network expansion planning, which is characterized by two distinctive aspects. First, at each stage, a number of nodes may be isolated, and, hence, should not be counted. Secondly, the number of sub-graphs of the network is equal to the number of in-service substations since each set of connected load nodes must be supplied by a single substation. According to the above general result of graph theory, the number of in-service branches is equal to the number of connected nodes (i.e., the number of connected load nodes plus the number of connected substation nodes) minus the number of sub-graphs. As the number of connected substation nodes is equal to the number of sub-graphs, radial operation is ensured if the number of in-service branches is equal to the number of connected load nodes. Thus, in contrast to the model described in [35], the characterization of the number of connected substation nodes is not required, which is beneficial for practical implementation purposes.

This graph-theory-based condition for radial operation is formulated in (30)–(36):

$$u_{l,t} = y_{l,t}; \forall l \in L^F, \forall t \in T$$
(30)

$$u_{l,t} = y_{l,t} + \sum_{k \in K^l} y_{l,k,t}; \forall l \in L^R, \forall t \in T$$
(31)

$$u_{l,t} = \sum_{k \in K^l} y_{l,k,t}; \forall l \in L^N, \forall t \in T$$
(32)

$$0 \le u_{l,t} \le M |d_{l,t}|; \forall l \in L, \forall t \in T$$
(33)

$$u_{l,t} \le LPM_{n,t}; \forall n \in \Omega^D, \forall l \in L \mid \chi_{n,l} \ne 0, \forall t \in T$$
 (34)

$$LPM_{n,t} \le 1; \forall n \in \Omega^D, \forall t \in T$$
 (35)

$$\sum_{n \in \Omega^D} LPM_{n,t} = \sum_{l \in L} u_{l,t}; \forall t \in T$$
(36)

where  $LPM_{n,t}$  is a binary-valued continuous variable that equals 1 if node n is connected to at least a feeder section at stage t, being 0 otherwise; and  $u_{l,t}$  is a binary-valued continuous variable that is equal to 1 if feeder section l is in service at stage t, being 0 otherwise.

Expressions (30)–(32) model  $u_{l,t}$  for existing fixed, existing replaceable, and new feeder sections, respectively. According to (33), variables  $u_{l,t}$  are set to 0 for unused branches. Note that preventing the utilization of branches with zero flow is a necessary condition to guarantee network radiality [32]. In (33),  $d_{l,t}$  is the average demand served through feeder section l at stage t, whereas the absolute value is used due to the lack of knowledge of the network topology. In Section III, variables  $d_{l,t}$  are modeled and the linearization of the absolute value is presented. For the non-isolated nodes, i.e., for those nodes for which the utilization variables  $u_{l,t}$  of at least one of the branches connected to them is 1, the binary-valued continuous variables  $LPM_{n,t}$  are set to 1 as per (34) and (35). On the other hand, for those nodes for which none of the connected branches is in service, expressions (34)– (36) set  $LPM_{n,t}$  to 0. Note that, for such nodes, (34) and (35) ensure that  $LPM_{n,t}$  take non-negative values less than or equal to 1. Finally, the above-mentioned graph-theory-based radiality condition is modeled in (36). Considering that a connected graph with no loops has the least number of branches [34], expressions (36) can only be satisfied in case  $LPM_{n,t}$  is equal to 0 for every isolated node.

# III. RELIABILITY EVALUATION METHOD

This section is devoted to the description of the MILP-based expressions proposed for the calculation of the widely used distribution system reliability indices, namely EENS, SAIDI, and SAIFI. These expressions substantially differ from the reliability models used in both conventional approaches [13]–[20] and recent works [21]–[25], [27]–[33]. Moreover, this model is a nontrivial extension of that presented in [26] featuring major salient aspects, namely 1) the consideration of investment options for network assets through extra binary variables and new nonlinear expressions, 2) the characterization of substation availability through the original imposition of KCL on a fictitious system, and 3) the equivalent linearization of the new mixed-integer nonlinear expressions.

# A. EENS

EENS quantifies the amount of annual energy not served due to network failures. This index can be expressed as the summation of the annual energy curtailed owing to the failure of each feeder section, denoted by  $BEENS_{l,t}$ , as formulated in (37):

$$EENS_t = \sum_{l \in L} BEENS_{l,t}; \forall t \in T.$$
 (37)

For the existing fixed branches,  $BEENS_{l,t}$  can be readily expressed as in (38), where the outage duration for the demands downstream of feeder section l is equal to the repair time  $\mu_l^r$ , whereas upstream loads experience shorter outages lasting for

the switching time  $\mu_l^{sw}$ :

$$BEENS_{l,t} = \alpha_l \lambda_l (\mu_l^r | d_{l,t} | + \mu_l^{sw} \delta_{l,t}); \forall l \in L^F, \forall t \in T.$$
 (38)

For the existing replaceable and newly added feeder sections, the failure rate and repair time vary for each option k. Thus, binary utilization variables are employed in (39) and (40) to characterize the reliability parameters for the replaceable and newly added feeder sections, respectively:

$$BEENS_{l,t} = \sum_{k \in K^l} \alpha_l \lambda_{l,k} y_{l,k,t} \left( \mu_{l,k}^r \left| d_{l,t} \right| + \mu_{l,k}^{sw} \delta_{l,t} \right)$$

$$+ \alpha_l \lambda_l y_{l,t} \left( \mu_l^r \left| d_{l,t} \right| + \mu_l^{sw} \delta_{l,t} \right); \forall l \in L^R, \forall t \in T$$
 (39)

$$BEENS_{l,t} = \sum_{k \in K^l} \alpha_l \lambda_{l,k} y_{l,k,t} \left( \mu_{l,k}^r | d_{l,t} | + \mu_{l,k}^{sw} \delta_{l,t} \right);$$

$$\forall l \in L^N, \forall t \in T.$$

$$(40)$$

In order to obtain the total demand downstream of each feeder section, i.e.,  $d_{l,t}$ , KCL is applied to a fictitious, topologically identical, lossless network whose nodal loads are equal to the average annual nodal demands (41)–(44):

$$\sum_{l \in L} \chi_{n,l} d_{l,t} + D_{n,t}^{Avg} = 0; \forall n \in \Omega^D, \forall t \in T$$
 (41)

$$\sum_{l \in L} \chi_{n,l} d_{l,t} - g_{n,t}^d = 0; \forall n \in \Omega^S, \forall t \in T$$
 (42)

$$-Mu_{l\,t} < d_{l\,t} < Mu_{l\,t}; \forall l \in L, \forall t \in T \tag{43}$$

$$g_{n,t}^{d} \leq M \left( \theta_{n} + \sum_{k \in K^{n}} \sum_{\tau=1}^{t} x_{n,k,\tau} \right); \forall n \in \Omega^{S}, \forall t \in T.$$
 (44)

Power balance equations at demand and substation nodes of the fictitious network are formulated in (41) and (42), respectively. Expressions (43) and (44) set to 0 both the fictitious flow of non-utilized feeder sections as well as the fictitious generation of non-utilized substations.

The total demand upstream of feeder section l at stage t,  $\delta_{l,t}$ , is equal to the difference between the downstream demand of the first branch  $\bar{l}$  of the feeder to which feeder section l belongs, i.e.,  $d_{\bar{l},t}$ , and the downstream demand of feeder section l itself, i.e.,  $d_{l,t}$ . Nonetheless, since the radial network topology is an outcome of the optimization problem, the first branch  $\bar{l}$  associated with a given branch l, is not known a priori. Thus, based upon the concepts developed in [26], we consider a binary-valued continuous variable  $z_{l,\bar{l},t}$ , which becomes 1 if feeder section  $\bar{l}$  is the first branch of the feeder associated with feeder section l at stage t, being 0 otherwise. Variables  $z_{l,\bar{l},t}$  are related to variables  $u_{l,t}$  as follows:

$$z_{l\,\bar{l}\,t} > 0; \forall l \in L \setminus SL, \forall \bar{l} \in SL, \forall t \in T$$
 (45)

$$\sum_{\bar{l} \in SL} z_{l,\bar{l},t} = u_{l,t}; \forall l \in L \setminus SL, \forall t \in T$$
 (46)

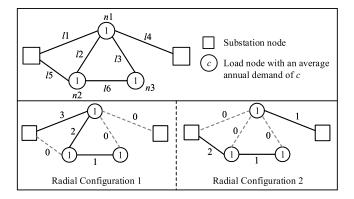


Fig. 2. Illustrative example – Fictitious flows for the feeder sections of two possible radial configurations.

$$z_{l,\bar{l},t} \ge 1 + \sum_{l' \in L} \xi_{l',m} (u_{l',t} - 1); \forall l \in L \setminus SL, \forall \bar{l} \in SL,$$

$$\forall t \in T, \forall m \in \Psi_{l,\bar{l}}. \tag{47}$$

The non-negativity of  $z_{l,\bar{l},t}$  is modeled in (45). As per (46), if a feeder section l is in service at stage t, there must be a unique corresponding first feeder section  $\bar{l}$ . Expressions (47) indicate that feeder section  $\bar{l}$  corresponds to feeder section l only if the utilization variables of all the feeder sections in path m, which includes feeder sections l and  $\bar{l}$ , are equal to 1. Thus, these expressions can consistently yield the desired values of  $z_{l,\bar{l},t}$ .

Employing variables  $z_{l,\bar{l},t}$ , the demands upstream of the feeder sections that are not directly connected to substation nodes are cast in (48). For the branches directly connected to substation nodes, the respective upstream demands are equal to 0, as expressed in (49).

$$\delta_{l,t} = \sum_{\bar{l} \in SL} z_{l,\bar{l},t} \left( \left| d_{\bar{l},t} \right| - \left| d_{l,t} \right| \right); \forall l \in L \setminus SL, \forall t \in T$$
 (48)

$$\delta_{l,t} = 0; \forall l \in SL, \forall t \in T. \tag{49}$$

In order to illustrate the behavior of (41)–(49), let us consider that the simple meshed network depicted in Fig. 2 represents an investment plan for a given time stage t. As can be seen, this network has two substation nodes, three load nodes with average demands all equal to 1 MW, and six feeder sections, of which three are directly connected to a substation node, hence,  $L = \{l1 - l6\}$  and  $SL = \{l1, l4, l5\}$ . Fig. 2 also displays two possible radial network topologies referred to as Radial Configuration 1 and Radial Configuration 2. As per Fig. 2, the first feeder section of each feeder and even the number of feeders are both topology dependent.

For the two radial configurations in Fig. 2, the fictitious flows  $d_{l,t}$  resulting from (41)–(44) are shown next to the corresponding feeder sections. As can be observed, the fictitious flow of the first feeder section of each feeder is equal to the total demand of that feeder.

For the calculation of  $\delta_{l,t}$ , since |SL|=3, we consider three auxiliary variables  $z_{l,\bar{l},t}$  for each of the three other feeder sections  $l2,\,l3,\,l6$  not directly connected to the substation nodes, namely  $z_{l2,l1,t},\,z_{l2,l4,t},\,z_{l2,l5,t},\,z_{l3,l1,t},\,z_{l3,l4,t},\,z_{l3,l5,t},\,z_{l6,l1,t},\,z_{l6,l4,t},\,z_{l6,l5,t}.$  For instance, for feeder section l6 in Radial Configuration 1, expressions (45)–(47) yield  $z_{l6,l1,t}=1$  and  $z_{l6,l4,t}=z_{l6,l5,t}=0$ . Hence, using such values of  $z_{l,\bar{l},t}$  in (48) gives rise to:

$$\delta_{l6,t} = z_{l6,l1,t} (|d_{l1,t}| - |d_{l6,t}|) + z_{l6,l4,t} (|d_{l4,t}| - |d_{l6,t}|)$$

$$+ z_{l6,l5,t} (|d_{l5,t}| - |d_{l6,t}|) = 1 \times (3-1) + 0 \times (0-1)$$

$$+ 0 \times (3-1) = 2,$$

which is equal to the total demand upstream of feeder section l6, as desired. Analogously, for feeder section l6 in Radial Configuration 2,  $z_{l6,l5,t} = 1$  and  $z_{l6,l1,t} = z_{l6,l4,t} = 0$  as per (45)–(47). Thus, from (48):

$$\delta_{l6,t} = z_{l6,l1,t} (|d_{l1,t}| - |d_{l6,t}|) + z_{l6,l4,t} (|d_{l4,t}| - |d_{l6,t}|)$$

$$+ z_{l6,l5,t} (|d_{l5,t}| - |d_{l6,t}|) = 0 \times (0-1) + 0 \times (0-1)$$

$$+ 1 \times (2-1) = 1.$$

The EENS model formulated in (37)–(49) features two sources of nonlinearity, namely 1) the absolute value terms  $|d_{l,t}|$  and  $|d_{\bar{l},t}|$  in (38)–(40) and (48), and 2) the product terms  $y_{l,t}|d_{l,t}|$ ,  $y_{l,t}\delta_{l,t},\ y_{l,k,t}|d_{l,t}|,\ y_{l,k,t}\delta_{l,t},\$ and  $z_{l,\bar{l},t}(|d_{\bar{l},t}|-|d_{l,t}|)$  in (39), (40), and (48). Note that the absolute value terms  $|d_{l,t}|$  can also be found in (33). The effect of the absolute value operators in (33), (38)–(40), and (48) can be equivalently modeled by replacing  $|d_{l,t}|$  with the sum of two auxiliary non-negative variables,  $d_{l,t}^+$  and  $d_{l,t}^-$ , and incorporating (50)–(52) into the model:

$$d_{l,t} = d_{l,t}^{+} - d_{l,t}^{-}; \forall l \in L, \forall t \in T$$
 (50)

$$d_{l\,t}^{+} \ge 0; \forall l \in L, \forall t \in T \tag{51}$$

$$d_{l,t}^{-} \ge 0; \forall l \in L, \forall t \in T. \tag{52}$$

Note that, according to (4) and (5), the objective function (1) is monotonically increasing with respect to  $EENS_t$ , and, consequently, with respect to  $d_{l,t}^+ + d_{l,t}^-$ . Therefore, there is no need to explicitly impose the logical constraint preventing  $d_{l,t}^+$  and  $d_{l,t}^-$  from simultaneously taking a non-zero value.

Moreover, an equivalent linear formulation for (39) is given in (53)–(55):

$$BEENS_{l,t} \ge \alpha_l \lambda_l \left( \mu_l^r \left( d_{l,t}^+ + d_{l,t}^- \right) + \mu_l^{sw} \delta_{l,t} \right)$$

$$- M \left( 1 - y_{l,t} \right); \forall l \in L^R, \forall t \in T$$
(53)

$$BEENS_{l,t} \ge \alpha_l \lambda_{l,k} \left( \mu_{l,k}^r \left( d_{l,t}^+ + d_{l,t}^- \right) + \mu_{l,k}^{sw} \delta_{l,t} \right)$$

$$- M \left( 1 - y_{l,k,t} \right); \forall l \in L^R, \forall k \in K^l, \forall t \in T$$

$$(54)$$

$$BEENS_{l,t} \ge 0; \forall l \in L^R, \forall t \in T.$$
 (55)

Note that (53) and (54) respectively get deactivated for  $y_{l,t} = 0$  and  $y_{l,k,t} = 0$ , as the right-hand sides of these expressions

become a large negative value. Hence, the prevailing lower bound for  $BEENS_{l,t}$  is equal to 0 as per (55). Conversely, for  $y_{l,t}=1$  and  $y_{l,k,t}=1$ , expressions (53) and (54) respectively set the lower bound for  $BEENS_{l,t}$  to the right-hand side of (39). Thus, as the objective function (1) is monotonically increasing with respect to  $EENS_t$ ,  $BEENS_{l,t}$  becomes the desired value.

In the same fashion, expressions (40) and (48) are linearized as follows:

$$0 \leq BEENS_{l,t} \geq \alpha_l \lambda_{l,k} \left( \mu_{l,k}^r \left( d_{l,t}^+ + d_{l,t}^- \right) + \mu_{l,k}^{sw} \delta_{l,t} \right)$$
$$- M \left( 1 - y_{l,k,t} \right); \forall l \in L^N, \forall k \in K^l, \forall t \in T$$
 (56)

$$0 \le \delta_{l,t} \ge \left(d_{\bar{l},t}^+ + d_{\bar{l},t}^-\right) - \left(d_{l,t}^+ + d_{l,t}^-\right) - M\left(1 - z_{l,\bar{l},t}\right);$$

$$\forall l \in L \setminus SL, \forall \bar{l} \in SL, \forall t \in T. \tag{57}$$

#### B. SAIDI

This index represents the average annual power interruption duration experienced by distribution system customers. Denoting by  $BIDI_{l,t}$  the total annual customer interruption duration caused by the failures of feeder section l at stage t, this reliability index can be formulated as:

$$SAIDI_{t} = \frac{\sum_{l \in L} BIDI_{l,t}}{\sum_{n \in \Omega^{D}} N_{n,t}}; \forall t \in T.$$
 (58)

Expressions (59)–(61) model  $BIDI_{l,t}$  for various types of feeder sections. Such expressions are respectively identical to (38)–(40), with  $h_{l,t}$  and  $\eta_{l,t}$  playing the role of  $d_{l,t}$  and  $\delta_{l,t}$ , respectively.

$$BIDI_{l,t} = \alpha_l \lambda_l \left( \mu_l^r | h_{l,t} | + \mu_l^{sw} \eta_{l,t} \right); \forall l \in L^F, \forall t \in T \quad (59)$$

$$BIDI_{l,t} = \sum_{k \in K^{l}} \alpha_{l} \lambda_{l,k} y_{l,k,t} \left( \mu_{l,k}^{r} | h_{l,t} | + \mu_{l,k}^{sw} \eta_{l,t} \right)$$
$$+ \alpha_{l} \lambda_{l} y_{l,t} \left( \mu_{l}^{r} | h_{l,t} | + \mu_{l}^{sw} \eta_{l,t} \right); \forall l \in L^{R}, \forall t \in T \quad (60)$$

$$BIDI_{l,t} = \sum_{k \in K^l} \alpha_l \lambda_{l,k} y_{l,k,t} \left( \mu_{l,k}^r \left| h_{l,t} \right| + \mu_{l,k}^{sw} \eta_{l,t} \right);$$

$$\forall l \in L^N, \forall t \in T. \tag{61}$$

In order to obtain  $h_{l,t}$ , i.e., the number of customers connected to the nodes downstream of feeder section l at stage t, KCL is applied to a fictitious, topologically identical, lossless network whose nodal demands are equal to the numbers of connected customers, as formulated in (62)–(65), which are analogous to (41)–(44):

$$\sum_{l \in I} \chi_{n,l} h_{l,t} + N_{n,t} = 0; \forall n \in \Omega^D, \forall t \in T$$
 (62)

$$\sum_{l \in I} \chi_{n,l} h_{l,t} - g_{n,t}^h = 0; \forall n \in \Omega^S, \forall t \in T$$
 (63)

$$-Mu_{l,t} \le h_{l,t} \le Mu_{l,t}; \forall l \in L, \forall t \in T$$

$$\tag{64}$$

$$g_{n,t}^{h} \le M \left( \theta_{n} + \sum_{k \in K^{n}} \sum_{\tau=1}^{t} x_{n,k,\tau} \right); \forall n \in \Omega^{S}, \forall t \in T.$$
 (65)

Finally, similar to (48) and (49), the number of customers connected to the nodes upstream of feeder section l at stage t,  $\eta_{l,t}$ , is modeled as follows:

$$\eta_{l,t} = \sum_{\bar{l} \in SL} z_{l,\bar{l},t} \left( \left| h_{\bar{l},t} \right| - \left| h_{l,t} \right| \right); \forall l \in L \setminus SL, \forall t \in T$$
 (66)

$$\eta_{l,t} = 0; \forall l \in SL, \forall t \in T.$$
 (67)

The model devised for SAIDI in (58)–(67) is nonlinear owing to the absolute value terms  $|h_{l,t}|$  and  $|h_{\bar{l},t}|$  in (59)–(61) and (66) as well as the product terms  $y_{l,t}|h_{l,t}|$ ,  $y_{l,t}\eta_{l,t}$ ,  $y_{l,k,t}|h_{l,t}|$ ,  $y_{l,k,t}\eta_{l,t}$ , and  $z_{l,\bar{l},t}(|h_{\bar{l},t}|-|h_{l,t}|)$  in (60), (61), and (66). These sources of nonlinearity can also be linearized following the procedure explained for the EENS model.

#### C. SAIFI

This index represents the average number of annual interruptions experienced by distribution network customers. Due to the structural similarity of SAIFI to SAIDI, its model can be readily derived as follows:

$$SAIFI_{t} = \frac{\sum_{l \in L} BIFI_{l,t}}{\sum_{n \in \Omega^{D}} N_{n,t}}; \forall t \in T$$
(68)

$$BIFI_{l,t} = \alpha_l \lambda_l \left( |h_{l,t}| + \eta_{l,t} \right); \forall l \in L^F, \forall t \in T$$
 (69)

$$BIFI_{l,t} = \sum_{k \in K^{l}} \alpha_{l} \lambda_{l,k} y_{l,k,t} \left( |h_{l,t}| + \eta_{l,t} \right)$$

$$+ \alpha_l \lambda_l y_{l,t} (|h_{l,t}| + \eta_{l,t}); \forall l \in L^R, \forall t \in T \quad (70)$$

$$BIFI_{l,t} = \sum_{k \in K^l} \alpha_l \lambda_{l,k} y_{l,k,t} (|h_{l,t}| + \eta_{l,t}); \forall l \in L^N, \forall t \in T.$$
 (71)

Expressions (68)–(71) are respectively analogous to (58)–(61), where  $BIFI_{l,t}$  plays the role of  $BIDI_{l,t}$ , whereas  $\mu_l^r$ ,  $\mu_l^{sw}$ ,  $\mu_{l,k}^r$ , and  $\mu_{l,k}^{sw}$  are dropped. Using the above-described linearization scheme, (68)–(71) can be modeled by an equivalent set of linear expressions.

#### IV. CASE STUDIES

This section presents the results associated with the application of the proposed model to four test distribution networks with 24, 54, 86, and 138 nodes. For the sake of reproducibility and a comprehensive analysis of results, the interested reader is referred to [40] for a complete description of input data, which have been specially designed for the case studies without using a specific source of information. Simulations have been run on a Fujitsu CELSIUS W530 Power PC with an Intel Xeon E3-1230 processor at 3.30 GHz and 32 GB of RAM using the branch-and-cut algorithm implemented in CPLEX 12.6 under GAMS 24.9 [37]. The optimality gap was set to 1E-7 for the 24-and 54-node networks, and 1E-2 for the 86- and 138-node test systems, which ensure that the branch-and-cut algorithm converges when high-quality near-optimal solutions are attained.

# A. 24-Node Test Network

This network comprises 20 load nodes, 4 substation nodes, and 33 feeder sections of which 2 are existing fixed, 2 are existing

TABLE II		
24-NODE TEST NETWORK -	RESULTS	

S	implified	Proposed
	model	model
Investment cost (M\$)	4.339	4.830
Operating cost (M\$)	191.736	191.766
Revenue lost due to undelivered energy (M\$)	0.077	0.050
Cost of SAIDI-based reliability incentive scheme (M\$)	1.891	-4.819
Cost of SAIFI-based reliability incentive scheme (M\$)	0.099	-0.560
Total cost (M\$)	198.142	191.267
Average EENS (MWh/year)	86.213	54.113
Average SAIDI (hours/customer/year)	3.807	2.413
Average SAIFI (interruptions/customer/year)	1.086	0.603

replaceable, and 29 are candidates for future installation. Four and two investment options are considered for feeder sections and substation transformers, respectively. The planning horizon comprises three stages and all feeder sections are considered switchable. For illustration purposes, we have also implemented a simplified version of the proposed model wherein the costs associated with reliability incentive schemes are disregarded.

Table II summarizes the results for both models. For the sake of a fair comparison, the total cost reported for the simplified model includes the costs associated with the reliability incentive schemes, which are computed *ex post*. As can be seen, the revenue lost due to the undelivered energy is negligible as compared to the investment and operating costs. Hence, this lost revenue *per se* does not provide the distribution company with sufficient motivation to enhance system reliability. As a result, for the simplified model, the average values of the reliability indices over the planning horizon are higher, thus representing a lower reliability level.

For the proposed model, the reliability incentive schemes give rise to increased investments in network assets to yield a more reliable network. Consequently, the resulting reliability indices are significantly improved. It is worth mentioning that negative values for the SAIDI- and SAIFI-based reliability costs reflect the financial bonuses received by the distribution company for the attainment of reliability indices lower than the corresponding benchmarks. As a result, the solution to the proposed model yields a 3.47% reduction in total cost compared to the solution to the simplified formulation.

Figs. 3 and 4 respectively depict the resulting network configurations at the last stage of the planning horizon for both models. These figures also show the type and timing of investment decisions. In addition to major investment- and operation-related topological differences, note that, unlike the solution to the simplified model, which relies on options A1 and A2 (Fig. 3), the expansion plan for the proposed model features the more reliable, albeit more expensive, options A3 and A4 (Fig. 4). For the sake of completeness, the resulting operational topologies for every stage of the planning horizon are available in [40].

The proposed approach required 1.61 hours to attain the above-described solution. In order to illustrate the computational superiority of the proposed approach over the state of the art, we have implemented three modified, albeit equivalent, versions of our proposed planning model respectively featuring a reliability formulation based on that presented in [31], a reliability characterization relying on the model formulated in [33], and a set

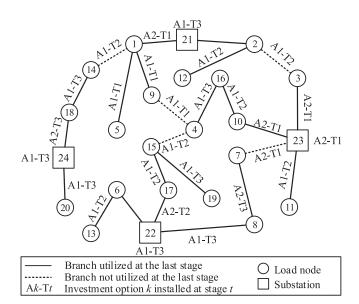


Fig. 3. 24-node test network – Topology at the last stage of the planning horizon for the simplified model.

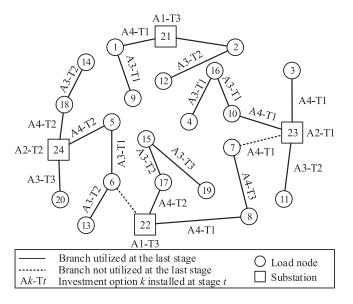


Fig. 4. 24-node test network – Topology at the last stage of the planning horizon for the proposed model.

of constraints for radial operation using the findings of [35]. As expected, the costs and reliability indices of the solution identified by the benchmark based on [31] were identical to those reported in Table II. However, such a model required 2.23 hours, representing a 38.5% increase in the computational burden. The adoption of the contingency-dependent reliability assessment model of [33] gave rise to an intractable optimization problem that ran out of memory. As for the benchmark relying on [35], the simulation was stopped after 27 hours without the solver being able to reduce the optimality gap under 1.61%. These results clearly back the computational benefits of the new formulation.

Aiming to investigate the impact of reliability incentive schemes on the planned network reliability, a sensitivity analysis is carried out on the incentive rates. To that end, the incentive

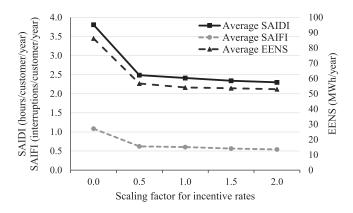


Fig. 5. 24-node test network – Sensitivity of the reliability indices to the incentive rates.

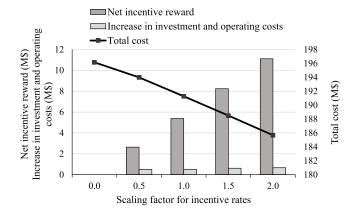


Fig. 6. 24-node test network – Sensitivity of different economic terms to the incentive rates.

rates for SAIDI and SAIFI are multiplied by a scaling factor ranging between 0 and 2 with a step of 0.5. The main results of this sensitivity analysis are summarized in Figs. 5 and 6. Fig. 5 illustrates the resulting average values of SAIDI, SAIFI, and EENS. Fig. 6 shows the resulting values for the net reward from both reliability incentive schemes, the increase in investment and operating costs over the case with a scaling factor equal to 0, and the total cost. As depicted in Fig. 5, increasing the incentive rates improves service reliability due to the enhanced financial motivations. However, evidenced by the results presented in Fig. 6, only a relatively small share of the financial incentives is invested in reliability enhancement. This could be due to the fact that the incremental cost of reliability enhancement is lower than the incremental benefits. Such counter-intuitive observations demonstrate the significance of a proper design of the regulatory incentives to effectively motivate distribution companies to enhance their service reliability.

This case study is also useful to substantiate the use of the approximate network model. In this regard, load flow results provided by the proposed approach for the high load level at the last time stage have been compared with those achieved with a full ac load flow model. The average and maximum errors are respectively 1.42% and 3.69% for branch current flows, 3.18% and 3.99% for injections at substations, and 0.32% and 1.00% for nodal voltage magnitudes. These results, which

TABLE III 54-, 86-, AND 138-NODE TEST NETWORKS – RESULTS

	54 nodes	86 nodes	138 nodes
Investment cost (M\$)	2.079	2.760	3.271
Operating cost (M\$)	27.988	440.477	419.820
Revenue lost due to undelivered energy (M\$)	0.021	0.060	0.059
Cost of SAIDI-based reliability incentive scheme (M\$)	9.917	-12.511	-6.463
Cost of SAIFI-based reliability incentive scheme (M\$)	1.699	-0.472	-0.179
Total cost (M\$)	41.704	430.313	416.508
Average EENS (MWh/year)	52.216	37.423	35.403
Average SAIDI (hours/customer/year)	7.307	1.275	1.121
Average SAIFI (interruptions/customer/year)	2.281	1.018	1.121

are consistent with the experience reported previously [31], [39], validate the suitability of the linearized network model for planning purposes.

# B. 54-, 86-, and 138-Node Test Networks

In order to investigate its scalability, the proposed approach has been applied to three larger test systems with 54, 86, and 138 nodes. The 54-node test system consists of 50 load nodes, 4 substation nodes, and 63 branches comprising 9 existing fixed, 8 existing replaceable, and 46 prospective feeder sections. The 86-and 138-node networks both comprise 3 substation nodes, while the former supplies 83 load nodes through 94 feeder sections and the latter has 135 load nodes and 151 feeder sections. A ten-stage planning horizon is considered for each of these networks.

Table III presents the results for the proposed model. It should be noted that the positive values for the SAIDI- and SAIFI-based reliability cost terms indicate that the reliability indices are worse than the benchmarks and consequently penalties are imposed. On the other hand, negative values for these terms, e.g., for the 86- and 138-node systems, represent the rewards gained from reaching reliability indices less (better) than the corresponding benchmarks. Again, as per Table III, the relatively negligible amounts of the revenue lost due to the undelivered energy to the customers for the three networks reveal that this term cannot sufficiently motivate investing in service reliability. This result stresses the significance of the reliability incentive schemes in guaranteeing an acceptable reliability level in the power distribution sector. For further details, the interested reader is referred to [40], where the corresponding operational topologies under normal conditions are available for the 54-node network.

As for computational performance, the proposed model respectively required 9.10, 0.17, and 5.65 hours for the 54-, 86-, and 138-node test systems, which are reasonable computing times for a planning setting. By contrast, the simulations of the models based on the formulations described in [31] and [35] were stopped after 10 days without reaching the pre-specified optimality tolerances, whereas the model relying on [33] either failed to attain solutions with the prescribed quality or exceeded our computing capabilities. These results corroborate that the proposed approach computationally outperforms the state of the art with acceptable solution quality for planning purposes.

#### V. CONCLUSION

In this paper, a novel model has been presented to explicitly incorporate analytical reliability assessment into the formulation of multistage expansion planning of distribution networks. According to industry practice, the reliability worth is estimated based on the costs associated with reliability incentive schemes as well as the loss of revenue due to the unserved energy during contingencies. In order to properly account for reliability-related costs, innovative MILP-based expressions have been devised for widely used reliability indices, namely SAIDI, SAIFI, and EENS. The reported numerical experience allows drawing two main conclusions: 1) the computational effort required by the proposed approach to attain high-quality near-optimal solutions is acceptable, and 2) the proposed model overcomes the dimensionality issue featured by the state-of-the-art formulations without relying on their modeling simplifications.

The proposed approach features three main limitations, namely 1) temporary faults are neglected, thereby ignoring the role played by reclosers, 2) post-fault network reconfiguration to supply load nodes downstream of the fault is disregarded, which does not allow leveraging the operational benefits of tie switches, and 3) a linear network model is adopted whereby the effect of reactive power is approximately accounted for. Further work will address such limitations along with the consideration of additional practical modeling aspects such as distributed energy resources, investments in protection devices, alternative assumptions on equipment deployment, the availability of probabilistic information, line overloading under contingency, and unbalanced systems. Particular attention will be paid to the application of the recently proposed convexified network models that would improve the accuracy of the characterization of reactive power. Future research will also be devoted to the analysis of larger case studies. In this regard, an interesting avenue of research is the examination of the computational savings that may be gained from the development of efficient solution algorithms and the use of tighter formulations. Finally, research effort will also be conducted to characterize the conflicting nature of investment cost minimization and reliability maximization through a multiobjective optimization framework.

# REFERENCES

- [1] U.S. Department of Energy, "Quadrennial energy review Transforming the Nation's electricity system: The second installment of the QER," Washington, DC, USA, Jan. 2017. Accessed: Jul. 2021. [Online]. Available: https://www.energy.gov/sites/prod/files/2017/02/f34/Quadrennial% 20Energy%20Review--Second%20Installment%20(Full%20Report).pdf
- [2] E. Fumagalli, L. Lo Schiavo, and F. Delestre, Service Quality Regulation in Electricity Distribution and Retail. Berlin, Germany: Springer, 2007.
- [3] "6th CEER benchmarking report on the quality of electricity and gas supply - 2016," Council of European Energy Regulators (CEER), Brussels, Belgium, Aug. 2016. Accessed: Jul. 2021. [Online]. Available: https://www.ceer.eu/documents/104400/-/-/d064733a-9614-e320a068-2086ed27be7f
- [4] H. L. Willis, Power Distribution Planning Reference Book, 2nd ed. New York, NY, USA: Marcel Dekker, 2004.
- [5] P. S. Georgilakis and N. D. Hatziargyriou, "A review of power distribution planning in the modern power systems era: Models, methods and future research," *Elect. Power Syst. Res.*, vol. 121, pp. 89–100, Apr. 2015.
- [6] R. Billinton and Z. Pan, "Historic performance-based distribution system risk assessment," *IEEE Trans. Power Deliv.*, vol. 19, no. 4, pp. 1759–1765, Oct. 2004.

- [7] IEEE Guide for Electric Power Distribution Reliability Indices, IEEE Standard 1366-2012 (Revision of IEEE Standard 1366-2003), May 2012.
- [8] R. Billinton and R. N. Allan, Reliability Evaluation of Power Systems. New York, NY, USA: Plenum Press, 1984.
- [9] S. Kazemi, M. Fotuhi-Firuzabad, and R. Billinton, "Reliability assessment of an automated distribution system," *IET Gener. Transm. Distrib.*, vol. 1, no. 2, pp. 223–233, Mar. 2007.
- [10] S. Conti, R. Nicolosi, and S. A. Rizzo, "Generalized systematic approach to assess distribution system reliability with renewable distributed generators and microgrids," *IEEE Trans. Power Deliv.*, vol. 27, no. 1, pp. 261–270, Jan. 2012.
- [11] K. Xie, J. Zhou, and R. Billinton, "Fast algorithm for the reliability evaluation of large-scale electrical distribution networks using the section technique," *IET Gener. Transm. Distrib.*, vol. 2, no. 5, pp. 701–707, Sep. 2008.
- [12] R. N. Allan, R. Billinton, and M. F. De Oliveira, "An efficient algorithm for deducing the minimal cuts and reliability indices of a general network configuration," *IEEE Trans. Reliab.*, vol. R- 25, no. 4, pp. 226–233, Oct. 1976.
- [13] E. G. Carrano, F. G. Guimarães, R. H. C. Takahashi, O. M. Neto, and F. Campelo, "Electric distribution network expansion under load-evolution uncertainty using an immune system inspired algorithm," *IEEE Trans. Power Syst.*, vol. 22, no. 2, pp. 851–861, May 2007.
- [14] R. C. Lotero and J. Contreras, "Distribution system planning with reliability," *IEEE Trans. Power Deliv.*, vol. 26, no. 4, pp. 2552–2562, Oct. 2011.
- [15] I. Ziari, G. Ledwich, A. Ghosh, and G. Platt, "Optimal distribution network reinforcement considering load growth, line loss, and reliability," *IEEE Trans. Power Syst.*, vol. 28, no. 2, pp. 587–597, May 2013.
- [16] B. R. Pereira, Jr., A. M. Cossi, J. Contreras, and J. R. S. Mantovani, "Multiobjective multistage distribution system planning using tabu search," *IET Gener. Transm. Distrib.*, vol. 8, no. 1, pp. 35–45, Jan. 2014.
- [17] M. Shivaie, M. T. Ameli, M. S. Sepasian, P. D. Weinsier, and V. Vahidinasab, "A multistage framework for reliability-based distribution expansion planning considering distributed generations by a self-adaptive global-based harmony search algorithm," *Reliab. Eng. Syst. Saf.*, vol. 139, pp. 68–81, Jul. 2015.
- [18] S. Nejadfard-Jahromi, M. Rashidinejad, and A. Abdollahi, "Multistage distribution network expansion planning under smart grids environment," *Int. J. Electr. Power Energy Syst.*, vol. 71, pp. 222–230, Oct. 2015.
- [19] G. Muñoz-Delgado, J. Contreras, and J. M. Arroyo, "Multistage generation and network expansion planning in distribution systems considering uncertainty and reliability," *IEEE Trans. Power Syst.*, vol. 31, no. 5, pp. 3715–3728, Sep. 2016.
- [20] S. Heidari, M. Fotuĥi-Firuzabad, and M. Lehtonen, "Planning to equip the power distribution networks with automation system," *IEEE Trans. Power* Syst., vol. 32, no. 5, pp. 3451–3460, Sep. 2017.
- [21] J. C. López, M. Lavorato, and M. J. Rider, "Optimal reconfiguration of electrical distribution systems considering reliability indices improvement," *Int. J. Electr. Power Energy Syst.*, vol. 78, pp. 837–845, Jun. 2016.
- [22] G. Muñoz-Delgado, J. Contreras, and J. M. Arroyo, "Reliability assessment for distribution optimization models: A non-simulation-based linear programming approach," *IEEE Trans. Smart Grid*, vol. 9, no. 4, pp. 3048–3059, Jul. 2018.
- [23] A. Tabares, G. Muñoz-Delgado, J. F. Franco, J. M. Arroyo, and J. Contreras, "An enhanced algebraic approach for the analytical reliability assessment of distribution systems," *IEEE Trans. Power Syst.*, vol. 34, no. 4, pp. 2870–2879, Jul. 2019.
- [24] M. Jooshaki, A. Abbaspour, M. Fotuhi-Firuzabad, H. Farzin, M. Moeini-Aghtaie, and M. Lehtonen, "A MILP model for incorporating reliability indices in distribution system expansion planning," *IEEE Trans. Power Syst.*, vol. 34, no. 3, pp. 2453–2456, May 2019.
- [25] Z. Li, W. Wu, B. Zhang, and X. Tai, "Analytical reliability assessment method for complex distribution networks considering post-fault network reconfiguration," *IEEE Trans. Power Syst.*, vol. 35, no. 2, pp. 1457–1467, Mar. 2020.
- [26] A. A. Jooshaki et al., "Linear formulations for topology-variable-based distribution system reliability assessment considering switching interruptions," *IEEE Trans. Smart Grid*, vol. 11, no. 5, pp. 4032–4043, Sep. 2020.
- [27] Z. Li, W. Wu, X. Tai, and B. Zhang, "Optimization model-based reliability assessment for distribution networks considering detailed placement of circuit breakers and switches," *IEEE Trans. Power Syst.*, vol. 35, no. 5, pp. 3991–4004, Sep. 2020.
- [28] Z. Li, W. Wu, B. Zhang, and X. Tai, "Feeder-corridor-based distribution network planning model with explicit reliability constraints," *IET Gener. Transm. Distrib.*, vol. 14, no. 22, pp. 5310–5318, Nov. 2020.

- [29] M. Jooshaki, S. Karimi-Arpanahi, M. Lehtonen, R. J. Millar, and M. Fotuhi-Firuzabad, "Electricity distribution system switch optimization under incentive reliability scheme," *IEEE Access*, vol. 8, pp. 93455–93463, 2020.
- [30] M. Jooshaki, S. Karimi-Arpanahi, M. Lehtonen, R. J. Millar, and M. Fotuhi-Firuzabad, "Reliability-oriented electricity distribution system switch and tie line optimization," *IEEE Access*, vol. 8, pp. 130967–130978, 2020.
- [31] G. Muñoz-Delgado, J. Contreras, and J. M. Arroyo, "Distribution network expansion planning with an explicit formulation for reliability assessment," *IEEE Trans. Power Syst.*, vol. 33, no. 3, pp. 2583–2596, May 2018.
- [32] M. Jooshaki, A. Abbaspour, M. Fotuhi-Firuzabad, M. Moeini-Aghtaie, and M. Lehtonen, "MILP model of electricity distribution system expansion planning considering incentive reliability regulations," *IEEE Trans. Power* Syst., vol. 34, no. 6, pp. 4300–4316, Nov. 2019.
- [33] Z. Li, W. Wu, X. Tai, and B. Zhang, "A reliability-constrained expansion planning model for mesh distribution networks," *IEEE Trans. Power Syst.*, vol. 36, no. 2, pp. 948–960, Mar. 2021.
- [34] B. Bollobás, Modern Graph Theory, vol. 184. Berlin, Germany: Springer Science & Business Media, 2013.
- [35] M. Lavorato, J. F. Franco, M. J. Rider, and R. Romero, "Imposing radiality constraints in distribution system optimization problems," *IEEE Trans. Power Syst.*, vol. 27, no. 1, pp. 172–180, Feb. 2012.
- [36] G. L. Nemhauser and L. A. Wolsey, Integer and Combinatorial Optimization. New York, NY, USA: Wiley, 1999.
- [37] IBM ILOG CPLEX. (2021). IBM CPLEX Optimizer. Accessed: Jul. 2021. [Online]. Available: https://www.ibm.com/analytics/cplex-optimizer
- [38] R. E. Brown, Electric Power Distribution Reliability, 2nd ed. Boca Raton, FL, USA: CRC Press, 2008.
- [39] S. Haffner, L. F. A. Pereira, L. A. Pereira, and L. S. Barreto, "Multistage model for distribution expansion planning with distributed generation-Part I: Problem formulation," *IEEE Trans. Power Deliv.*, vol. 23, no. 2, pp. 915–923, Apr. 2008.
- [40] A. A. Jooshaki et al., "Test data and results for reliability-oriented distribution system expansion planning," *IEEE Dataport*. Accessed: Feb. 2021. [Online]. Available: https://dx.doi.org/10.21227/81n1-zp95



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