A Competitive Scheduling Algorithm for Online Demand Response in Islanded Microgrids

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***Abstract*—A routine task faced by Microgrid (MG) operators is to optimally allocate incoming power demand requests while accounting for the underlying power distribution network and the associated constraints. Typically, this has been formulated as an offline optimization problem for day-ahead scheduling, assuming perfect forecasting of the demands. In practice, however, these loads are often requested in an ad-hoc manner and the control decisions are to be computed without any foresight into future inputs. With this in view, the present work contributes to the modeling and algorithmic foundations of real-time load scheduling problem in a demand response (DR) program. We model the problem within an AC Optimal Power Flow (OPF) framework and design an efficient online algorithm that outputs scheduling decisions provided with information on past and present inputs solely. Furthermore, a rigorous theoretical bound on the competitive ratio of the algorithm is derived. Practicality of the proposed approach is corroborated through numerical simulations on two benchmark MG systems against a representative greedy algorithm.**

***Index Terms*—Online demand response, real-time load scheduling, discrete demand requests, competitive online algorithm, combinatorial optimization, optimal power flow, microgrid.**

NOMENCLATURE

Sets and Indices

$\mathcal{T}$	Set of time intervals indexed by $t = 1, \ldots m$
$\mathcal{N}$	Set of customers indexed by $k = 1, \ldots, n$
$\mathcal{N}_i$	Set of customers located at bus i
$\mathcal{I}_{\text{S}}$	Set of customers with δ-Small demand requests
$\mathcal{I}_{\text{L}}$	Set of customers with δ-Large demand requests
$\mathcal{G}$	Graph of MG topology
$\mathcal{V}$	Set of nodes (buses) indexed by i or j
$\mathcal{V}_+$	Set of buses excluding the slack bus 0
$\mathcal{E}$	Set of edges (power lines) indexed by (i, j)

Variables

S_k	[VA Apparent power demand of customer k
$S^{\min}$	[VA] Minimum load of a customer
$S^{\max}$	[VA] Maximum load of a customer
T_k	Customer k's preferred scheduling interval
$T^{\max}$	Maximum duration of a scheduling interval
u_k	Utility (valuation) of customer k
$u^{\max}$	Highest utility among customers
$u^{\min}$	Lowest utility among customers
$a^{\max}$	Maximum demand to utility ratio
$a^{\min}$	Minimum demand to utility ratio
C_t	[VA] Net generation capacity at time t
$C^{\min}$	[VA] Lowest generation capacity over $\mathcal{T}$
$z_{i,j}$	$[\Omega/km]$ Impedance of power line (i, j)
V_i^t	[V] Voltage on bus i at time t
v_i^t	Squared voltage magnitude on bus i at time t
$v^{\min}$	Lower bound on squared voltage magnitude
$v^{\max}$	Upper bound on squared voltage magnitude
$I_{i,j}^t$	[A] Current flowing through line (i, j) at time t
$\ell_{i,j}^t$	Squared magnitude of current passing through line (i, j) at time t
$\widehat{S}_{i,j}^t$	[VA] Apparent power flowing along the line (i, j)
$\bar{s}_j^t$	[VA] Aggregate load on bus j at instant t
x_k	Binary scheduling decision for customer k's load request (1 = Schedule, 0 = Discard)
y	Solution of the primal minimization problem returned by the primal-dual schema

Parameters and Constants

θ	$[^\circ]$ Maximum angle between any pair of demand vectors
α	Rounding probability factor
δ	Demand categorization threshold
γ	Quadratic term in utility calculation
η	Linear term in utility calculation
ψ	Constant term in utility calculation

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I. Introduction

CENTRAL to the Smart Grid (SG) vision is the expansive modularization of the legacy power grid with notably smaller (typically MVAs, not GVAs) self-administered MGs [1]. As such, MGs are medium-to-low voltage distribution networks (often embodying a simple radial topology) reinforced with on-site Distributed Generators (DGs) and capability to operate both in grid-tied and standalone (islanded) modes [2]. Of distinctive interest, however, is their projected potential to facilitate intensive penetration of renewable-based DGs [3] and foster ubiquitous *integration of evolving load forms* (e.g., electro-mobility) [4]. With MGs dispersed as interfacing modules, the envisioned architecture promises a smooth and cost-effective transition into a more resilient, optimized and sustainable grid.

Yet, the outlined operational philosophy of MGs witnesses a drastic departure from that in conventional distribution systems with predominantly deterministic generation and predictable demand [5], [6]. Specifically, MGs' performance is hindered with colossal uncertainties stemming from the inherent volatility of renewable energy (RE) and frequent fluctuations in electrical load. Without proper intervention, these sizable variations threaten MGs with potential overloads, power congestions and voltage deviations [7], leaving their performance degraded and stability jeopardized. While intermittency of renewables is to some extent circumventable (by absorbing into storage reserves), precise forecasting of future electricity demand is questionable in practice [8], especially in light of looming electro-mobility. Nonetheless, in the related works, MGs' energy scheduling has been often carried out in an offline fashion, *presuming perfect prediction of customers' set and loads*.

In response to these hurdles, the extant demand side management (DSM) methods, which comprise a portfolio of measures designed to shape/influence customer's electricity consumption patterns and volumes, require a radical shift from traditionally offline to a *real-time (i.e., online) domain of operation* [6], [9]. This rationale rests on the remarkable flexibility of online algorithms allowing for real-time decision-making in convoluted dynamic environments with *little to no a priori knowledge* on the statistics of underlying stochastic processes. Particularly, in an online computation the input (e.g., in the current context the arrival, duration and demand of customers' loads) is not available in advance, as opposed to a standard offline problem, but is rather revealed in parts over time (e.g., once customers' loads connect to MG) and the respective decisions are to be attained forthwith. Furthermore, the continuous feedback from these decisions, in turn, can shed a constructive, timely *insight on evolving system dynamics* (e.g., operational conditions in MGs). On the other hand, under such stringent guidelines and paucity of input data, devising online numerical solvers or efficient algorithms with bounded worst-case performance ratio (i.e., *competitive*) remains a formidable challenge [10]. Thus, taken together, these factors substantiate the necessity of developing online DSM strategies with provable performance guarantees.

Among various DSM techniques, DR has proven particularly expedient for immediate purposes [11], proffering a manifold of programs for active monitoring and control at the distribution level. These programs (e.g., dynamic pricing [12], direct load control (DLC) [13], adaptive load shedding [14] etc.) aim at establishing a mutually beneficial interaction framework between the subscribed power consumers and providers where the former are incentivized (e.g., through monetary benefits or compensations) to amend their electricity demand to the latter's operational or economic circumstances. Alongside the target load modulations, DR enables refined frequency control [15], strengthened system security [16], enhanced voltage profile [17] as well as diminished operational and capital expenditures.

Although offline DR applications, being prevalent in the literature, have consolidated an eclectic arsenal of algorithmic and modeling techniques, as surveyed in [18], the efforts on real-time analogs are still in a relatively nascent stage. Indeed, most prior works in this vein of research, such as [8], [12], [19]–[23], resort to price-based plans, wherein end-users' electricity consumption is procured through the proxy of time-differentiated price signals. Despite the potential virtues, these schemes, however, might expose customers to exorbitant financial risks, thereby hampering their active participation, as well as exacerbate the variability in system load [24], [25]. Moreover, limited flexibility and meager controllability render price driven mechanisms imprudent for fully responsive and sufficiently reliable demand control at real-time scales [11], [26], [27], consequently questioning their viability in online DR for isolated MGs.

To this end, direct control strategies serve as a promising alternative towards efficient realization of online DR scheduling. In DLC, customers cede management of their devices to grid operators which then enact the corresponding load adjustments through binding signals. Leveraging this framework, the studies in [27]–[32] attempted to tackle the online DR problem in MGs, featuring different algorithmic methodologies and design aspects. However, for the sake of mathematical tractability, these approaches tended to cater just for the net balance between supply and demand, effectively abstracting away the underlying distribution network along with the associated power flow and operational constraints (e.g., Kirchhoff's laws, voltage bounds etc.). Whereas, as highlighted in [33], [34], such simplified models may lead to infeasible load management decisions in practice, therewith impairing credibility of the DR program.

Along these lines, in [35] the online energy management of MGs is cast as a stochastic OPF problem so as to capture the power flow equations and system operational constraints. A Lyapunov-inspired optimization method is developed and applied to a simulated MG system for numerical evaluation. More recently, the work in [36] presented an online algorithm to optimize the power distribution in MGs with a particular focus on the reactive power generation of DGs. The proposed scheme is relaxed to a convex problem and solved with a semidefinite programming based interior point method. In essence, both studies treat customer loads' *power consumption levels as continuous*, which parses the confronted optimization problem convex and therefore computationally conducive. Meanwhile in practice, it is often necessary to consider *discrete power injections*. For instance, a spectrum of household electrical appliances, such as vacuum cleaner or TV, require a particular supply of electricity to function properly (i.e., are either switched on with a fixed power consumption rate or turned off). These discrete operating points can only be represented by *binary decision variables*, hence conferring upon DR management a *combinatorial structure with NP-hard computational complexity*.[1]

Crucially, most of the above-reported works appeal to heuristic optimization techniques, which, per se, are devoid of any optimality guarantees or theoretical guidance. However, such an approach precludes the analytic component from assessment frameworks therein, leaving them reliant solely upon the chosen case studies and experimental settings. Besides, no hint is provided concerning the extent to which further performance improvement is still achievable.

Against the aforementioned background, this paper explores the problem of online DR scheduling in isolated MGs. In the studied program, a controller at the operator side receives consumers' connection requests (with preferred power demands, durations and valuations) arriving in an online manner. Constrained by time-varying generation capacity[2] (inflicted by volatile RE sources), the MG operator seeks to determine in real-time binary scheduling decisions that maximize the total valuation of satisfied customers while adhering to the grid codes and operational limits. Given this context, the key contributions of the current article along with its roadmap can be summarized as follows:

- *Descriptive MG Model:* To secure sound operation of MG, the DR problem is defined within an AC OPF framework, incorporating *power flow constraints and voltage tolerances* of the distribution feeder. Grounding on the peculiarities and physical properties of MGs (in a sense to be clarified in Section III), the formulated mixed integer non-convex optimization problem is then decoupled into two subproblems, which are tackled successively. Though computationally parsimonious, this decomposition is not necessarily exact, nevertheless, for the current application and settings the suboptimality gap between the original problem and the transformed variant appears tightly bounded, as revealed through extensive simulation results reported in Section V.
- *Competitive Algorithmic Design:* For the resulting problem, a *competitive randomized online algorithm* is proposed with a definite theoretical guarantee on the quality of solution (Theorem 1 in Section IV). The algorithm, which relies on the primal-dual schema introduced in [37], is myopic in that it controls the demand scheduling process hinging solely on the current and past system state information and hence is readily implementable in practice.
- *Comprehensive Performance Analysis:* We conduct both analytical and empirical evaluation of the featured approach. Specifically, in Section V, the algorithm is applied to two isolated MGs, namely a variant of the CIGRE MV benchmark system [38] and a 4-bus feeder borrowed from the Canadian Benchmark Distribution System (CBDS). These systems epitomize two opposites – the former is a large radial system with multiple nodes and relatively long feeders, while the latter is a simple but quintessential model with just a few nodes and short feeder sections. We validate the efficiency and practicality of the proposed approach against two references: (1) an optimal offline algorithm that possesses a-priori knowledge about all the future inputs; (2) a representative greedy method following first-come-first-serve scheduling policy. Performance of the algorithms is scrutinized extensively under diverse settings and scenarios with respect to customer loads and valuations.

[1] For a more expressive DR model, one might consider discrete and continuous loads in tandem, which would be a logical extension to this work.

[2] An MG, once islanded, is highly likely to run short of power.

II. System Model

Towards defining the online DR problem formally, this section starts by modeling the system and its components, each of which is detailed in the corresponding subsection. It's noteworthy that the employed model is quite generic, devoid of specific assumptions and sophisticated design constructs and can thus be applied in various contexts.

Notational Convention: In the sequel, vectors are denoted in boldface letters with $\mathbf{0}$ $\mathbf{1}$ symbolizing the vectors of all zeros and ones, respectively. For a complex number $\nu \in \mathbb{C}$, we let $|\nu|$ be its magnitude, $\arg(\nu)$ be the phase angle it makes with the real axis, ν^* be its complex conjugate and write $\nu^{\mathrm{R}} \triangleq \mathrm{Re}(\nu)$ for the real and $\nu^{\mathrm{I}} \triangleq \mathrm{Im}(\nu)$ for the imaginary components of ν, respectively.

A. System Overview

By convention, the intended DR topology considers a single aggregator or load serving entity (LSE), which coordinates the scheduling of DR participant demands over the temporal domain $\mathcal{T} \triangleq \{1, \ldots, m\}$. Here, $\mathcal{T}$ is discretized into m equal periods with a duration corresponding to the desired time resolution granularity at which DR decisions are to be produced (e.g., 5 seconds, 0.5 min. or 1 min.).

We envision a scenario with real-time bidirectional communication infrastructure (e.g., a wireless or wired local area network) linking LSE and consumer premises. Each customer is equipped with a smart meter that monitors the power consumption of electrical devices. At the operator side, a controller receives customers' requests with preferred power demands, durations and valuations upon their connection to MG. LSE computes on the fly the power allocation among the arrived requests, which is then transmitted to customers' smart meters. These scheduling decisions once outputted cannot be undone or altered neither by consumer intervention nor by LSE. Moreover, each *request is scheduled non-preemptively*, that is, if started it remains active continuously until completion.

B. Customer Model

Consider a set of customers $\mathcal{N} \triangleq \{1, \ldots, n\}$, including residential and commercial energy consumers, serviced by LSE. Recall, that in the online setting studied here, $\mathcal{N}$ is not known in advance, but is revealed progressively over time as customers continue to arrive. Without loss of generality, the latter process is assumed to occur in a sequential fashion from 1 to n.

A customer $k \in \mathcal{N}$ is associated with *a complex-valued power demand* $S_k \in \mathbb{C}$ and *a preferred scheduling interval* $T_k = [t_1, t_2] \subseteq \mathcal{T}$, which is declared at the beginning of t_1 when

k connects to MG. Here, S_k captures the net apparent power requirements of customer k's loads (all of which are controllable and can be shed in response to supply conditions) over each time instant $t \in T_k$. Let $\theta \triangleq \max_{k,k' \in \mathcal{N}} |\arg(S_k) - \arg(S_{k'})|$ be the maximum difference between the phase angles of any pair of customer power demands. In practice, $\theta < \frac{\pi}{2}$ due to regulations stipulating electric equipment to conform with a maximum allowable power factor. Concretely, θ is usually restricted to lie in the range of $[-36°, 36°]$ (commensurate to a load power factor of $[0.8, 1]$) [39]. We shall assume $S_k^{\mathrm{R}} \geq 0$ (customer k's *active power demand*) and $S_k^{\mathrm{I}} \geq 0$ (customer k's *reactive power demand*) for $\forall\, k \in \mathcal{N}$, which incurs no loss in generality as the power vectors can be rotated into the non-negative quadrant of the plane.

In DR literature, it is customary to model user preferences by a *utility function* [32]. For simplicity, here this is encapsulated in a *utility value* u_k assigned to a customer $k \in \mathcal{N}$ that quantifies the extent of satisfaction obtained (or alternatively, the willingness to pay) when own power demand is satisfied. To be precise, if S_k is fed at each time $t \in T_k$ (i.e., $x_k = 1$), u_k is the perceived utility for customer k, otherwise if shed (i.e., $x_k = 0$) zero utility is imparted.

C. MG Architecture

The employed MG system encompasses a hybrid mix of traditional and RE supplies that could collectively yield a variable (depending on the availability of RE and storage reserves) yet dispatchable capacity. The intermittent RE sources induce time-varying generation, denoted by $C_t \in \mathbb{R}_+$ for a time slot $t \in \mathcal{T}$. Note that $(C_t)_{t \in \mathcal{T}}$ is assumed to be known (or at least estimated) ahead of time. Otherwise, scheduling a current customer's demand non-preemptively without violating the future capacity constraint might spell infeasibility. To incorporate the physical electrical network into the DR problem a model of the distribution network is designed below. We shall confine the scope to networks of radial topology, which are prevalent in practice [40].

A power distribution network can be represented by a connected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ (which is a tree for radial structures), where $\mathcal{V}$ denotes the electric buses and $\mathcal{E}$ symbolizes the branches (lines). The nodes in $\mathcal{V}$ are indexed by $\{0, 1, \ldots, |\mathcal{V}|\}$, where root 0 is reserved for the slack bus which has a fixed voltage and flexible power injection from the collective generation source. Each edge $(i,j) \in \mathcal{E}$ is parameterized by a complex impedance $z_{i,j} \in \mathbb{C}$.

Let $V_i^t \in \mathbb{C}$ denote the voltage at node $i \in \mathcal{V}$ at time $t \in \mathcal{T}$. Define $I_{i,j}^t$ to be the current flowing through line $(i,j) \in \mathcal{E}$ and, with a slight abuse of notation, $\widehat{S}_{i,j}^t \in \mathbb{C}$ to be the transmitted power through that edge at time t. Denote by $v_i^t \triangleq \left|V_i^t\right|^2$ and $\ell_{i,j}^t \triangleq \left|I_{i,j}^t\right|^2$ the squared magnitude of voltage and current, respectively. At each node $i \in \mathcal{V}_+ \triangleq \mathcal{V} \backslash \{0\}$, attached is a set of customers $\mathcal{N}_i$ such that $\mathcal{N} = \cup_{i \in \mathcal{V}_+} N_i$.

A power flow in a steady state is characterized by a set of equations, which for radial networks can be captured through Branch Flow Model (BFM) [41]. Given v_0 and $\{z_{i,j}\}_{(i,j) \in \mathcal{E}}$, the BFM in $\mathcal{G}$ at time $t \in \mathcal{T}$ takes the form

$$\ell_{i,j}^t = \frac{\left|\widehat{S}_{i,j}^t\right|^2}{v_i^t}, \quad \forall\, (i,j) \in \mathcal{E} \tag{1}$$

$$v_j^t = v_i^t + |z_{i,j}|^2 \ell_{i,j}^t - 2\mathrm{Re}(z_{i,j}^* \widehat{S}_{i,j}^t), \quad \forall\, (i,j) \in \mathcal{E} \tag{2}$$

$$\widehat{S}_{i,j}^t = \sum_{h:(j,h) \in \mathcal{E}} \widehat{S}_{j,h}^t + \bar{s}_j^t + z_{i,j} \ell_{i,j}^t, \quad \forall\, (i,j) \in \mathcal{E} \tag{3}$$

where $\bar{s}_j^t \in \mathbb{C}$ is the aggregate load at bus j. Equations (1)–(3), essentially, ensue from the union of Ohm's law, the definition of power flow and its conservation. In the context of DR, $\bar{s}_j^t$ is guided by LSE's decisions on customers' loads connected to bus j at time t. That is, $\bar{s}_j^t = \sum_{k \in N_j : t \in T_k} S_k x_k$, where $x_k \in \{0, 1\}$ is the scheduling decision for customer k. Eqn. (3) can then be reformulated as

$$\widehat{S}_{i,j}^t = \sum_{l:(j,l) \in \mathcal{E}} \widehat{S}_{j,l}^t + \sum_{k \in N_j : t \in T_k} S_k x_k + z_{i,j} \ell_{i,j}^t, \forall\, (i,j) \in \mathcal{E}\,. \tag{4}$$

Aside from BFM equations, the seamless operation of MG is subject to the following spatio-temporal constraints for any $t \in \mathcal{T}$

$$v^{\min} \leq v_i^t \leq v^{\max}\,, \quad \forall\, i \in \mathcal{V}_+ \tag{5}$$

where $v_{\min}, v_{\max} \in \mathbb{R}_+$ are the minimum and maximum permissible squared voltage magnitude at any node, respectively.

While islanded, the MG will operate with a limited apparent power generation and thus at each time step $t \in \mathcal{T}$

$$\left| \sum_{j:(0,j) \in \mathcal{E}} \widehat{S}_{0,j}^t \right| \leq C_t\,. \tag{6}$$

Observe that BFM is non-convex due to the quadratic equality constraints in Eqn.(1) and hence is computationally intractable in general. To convexify the model, we therefore relax them to inequalities for $\forall\, t \in \mathcal{T}$ as follows

$$\ell_{i,j}^t \geq \frac{\left|\widehat{S}_{i,j}^t\right|^2}{v_i^t}\,, \qquad \forall\, (i,j) \in \mathcal{E}\,. \tag{7}$$

Reasoning analogously, this relaxation is adopted in [34], [42] and, as demonstrated in [43], it happens to be exact (i.e., the equality in (7) is achieved) for radial networks under certain mild conditions detailed therein as well as in [44].

III. Online DR Scheduling Problem

In this section we define the online DR problem formally along with a measure for assessing the quality of its solutions.

A. Mathematical Formulation

With the system model established in Section II, the online DR scheduling is embodied by the following *quadratically constrained mixed integer programming* problem (hereafter referred to as *complex-demand scheduling problem* or CSP [45]).

$$(\mathrm{CSP}) \quad \max_{\mathbf{x}, \mathbf{v}, \boldsymbol{\ell}, \widehat{\mathbf{S}}} \sum_{k \in \mathcal{N}} u_k x_k$$

$$
\begin{aligned}
\text{subject to}\quad & (2),(4),(5),(6),(7), \qquad \forall t \in \mathcal{T}\\
& x_k \in \{0,1\}, \qquad \forall k \in \mathcal{N}\\
& v_i^t \in \mathbb{R}_+, \qquad \forall i \in \mathcal{V}_+, \forall t \in \mathcal{T}\\
& \ell_{i,j}^t \in \mathbb{R}_+, \widehat{S}_{i,j}^t \in \mathbb{C}, \qquad \forall (i,j) \in \mathcal{E}, \forall t \in \mathcal{T}
\end{aligned}
$$

Here, x_k is a binary decision variable that takes value 1 if and only if the k-th customer's power demand S_k is satisfied for all time slots $t \in T_k$. The crux of solving CSP lies in the said combinatorial structure. This is further complicated by the online setting, wherein the constraint matrix of CSP is unveiled to LSE one column at a time (i.e., upon receiving a customer's demand request) and the corresponding admission/rejection decision x_k, which is irrevocable, has to be computed immediately. To facilitate optimization, a judicious decomposition scheme is devised below.

As MGs are typically small in both scale and size, most power demand can be attributed to customers' loads, and hence, the effect of power lines loss is marginal. Moreover, it is conceivable that retaining demand beneath the generation cap would bound voltage deviations within the acceptable range. Thus, the capacity constraints are likely to be binding most of the time. Following this logic, we define CSP with a generation capacity constraint (CSP_C) as follows.

$$
\begin{aligned}
(\text{CSP}_\text{C}) \quad & \max_{\mathbf{x}} \sum_{k \in \mathcal{N}} u_k x_k \\
\text{subject to} \quad & \left| \sum_{k \in \mathcal{N}: t \in T_k} S_k x_k \right| \leq C_t, \ \ \forall t \in \mathcal{T} \qquad (8)\\
& x_k \in \{0,1\} \ \ \forall k \in \mathcal{N} \qquad (9)
\end{aligned}
$$

CSP_C aims at maximizing the overall net utility of customers arriving online without violating the generation capacity C_t. Admittedly, CSP_C is NP-HARD, since it specializes to the 0-1 *classical Knapsack problem*. In fact, the presence of complex-valued demands engenders substantially more challenging instance, *which is strongly* NP-HARD, as argued in [46].

One computationally efficient means of tackling CSP is to solve CSP_C and then run a standard load flow calculation to check if the solution obtained is feasible for CSP, as detailed in Section IV.

B. Performance Metric

Let the inputs of CSP_C at time t of the arriving customers be $\sigma_t = (u_k, T_k, S_k)_{k \in \mathcal{N}: t \in \mathcal{T}}$. Recall that in an online setting, the decision at the current time t depends solely on the inputs available before or at t, namely, $(\sigma_{t'})_{t' \leq t}$. Given $\sigma = (\sigma_{t'})_{t'=1}^{t}$, let $\mathbb{E}[\text{ALG}[\sigma]]$ be the expected objective value (i.e., $\sum_{k \in \mathcal{N}} u_k x_k$) by a randomized algorithm ALG, and $\text{OPT}(\sigma)$ be that of an offline optimal solution (that knows all future inputs). In *online algorithmic analysis*, the *competitive ratio* is a common performance metric, which typifies the *worst-case* ratio between the expected objective value of an online algorithm and that of an offline optimal solution. Formally, the measure of competitiveness is

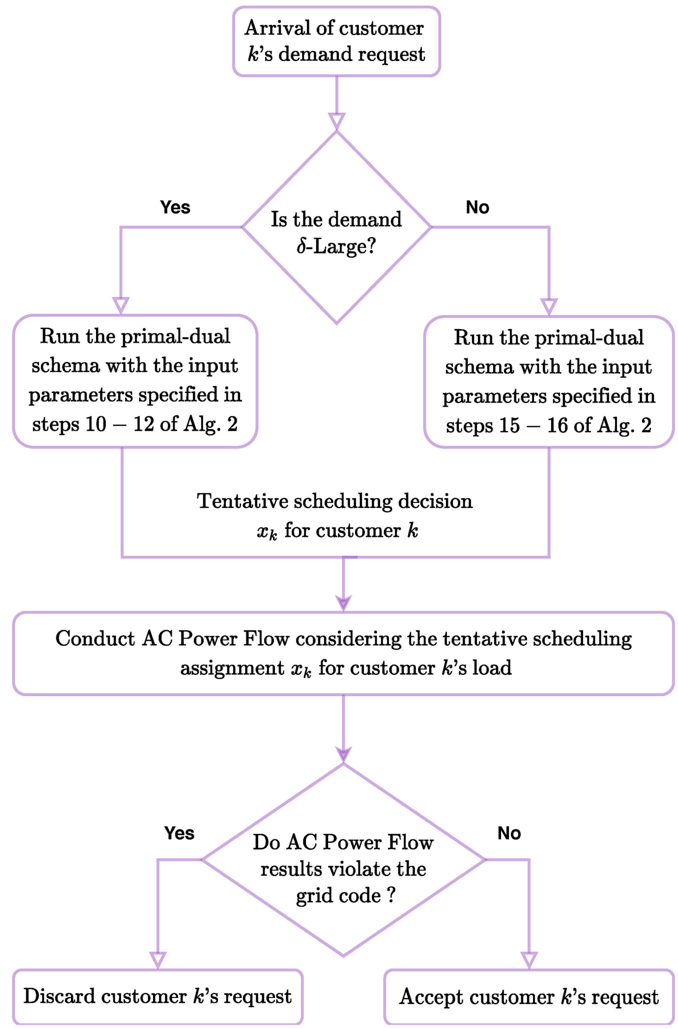

Fig. 1. Synoptic block-diagram of the developed online DR approach.

defined as

$$
\texttt{CR}(\text{ALG}) \triangleq \min_{\sigma} \frac{\mathbb{E}[\text{ALG}[\sigma]]}{\text{OPT}(\sigma)}. \qquad (10)
$$

If $\texttt{CR}(\text{ALG}) = c$ (note that $c <= 1$), ALG guarantees at least c fraction of the optimal offline objective value under any input instance σ.

IV. Competitive Online Algorithm

This section designs an efficient randomized online algorithm (`Online`), formalized in Alg. 1 and illustrated in Fig. 1, that solves the CSP problem in real-time without any foresight into forthcoming demand requests. The key theoretical contribution culminates in Theorem 1.

At the core of `Online` is the proposed algorithm $\texttt{Online}_\text{C}$, elucidated in Alg. 2, which attains the competitive ratio quantified in Theorem 1. As depicted in Fig. 1, given a DR scheduling decision determined by $\texttt{Online}_\text{C}$ (step 3 in Alg. 1), `Online` performs an additional load flow analysis to verify its feasibility for CSP (step 4 in Alg. 1). $\texttt{Online}_\text{C}$, which is extended from the

Algorithm 1: $\texttt{Online}[k, u_k, T_k, S_k]$.

Global Initialization: $\mathbf{x} \leftarrow \mathbf{0}, \mathbf{x}' \leftarrow \mathbf{0}$

Execution upon the k-th customer:

$x'_k \leftarrow \texttt{Online}_\text{C}[k, u_k, T_k, S_k]$
if $\nexists\, (v_i^t, \ell_{i,j}^t, \widehat{S}_{i,j}^t)_{t\in\mathcal{T},(i,j)\in\mathcal{E}}$ satisfying Constr. (2), (4), (5) and (7) with $\mathbf{x} = \mathbf{x}'$ **then**
  $x'_k \leftarrow 0$
$x_k \leftarrow x'_k$
return x_k

algorithm presented in [47] for the Unsplittable Flow problem, as a subroutine invokes a primal-dual (PD) schema similar to that introduced in [37] for *online fractional packing problem* (FPP). The adapted PD schema is stated in Alg. 3.

The basic idea is that $\texttt{Online}_\text{C}$ relates the quadratically constrained CSP_C to a linearly constrained packing problem by Lemma 3 (in the supplementary materials). This allows leveraging the framework of [37] to obtain a close-to-optimal fractional solution $\widehat{\mathbf{x}}$. To convert $\widehat{\mathbf{x}}$ into an integral solution $\mathbf{x}$ without sacrificing much in the objective value, we avail of a rounding technique known as *randomized rounding with correction* [47]. For this to work, the demands are categorized (based on a fixed threshold) into two subsets: $\mathcal{I}_\text{L}$ designated for those with *large* magnitude relative to the generation capacity and $\mathcal{I}_\text{S}$ for the remaining ones. The PD schema is then invoked in parallel on the two sub-problems of CSP_C defined by these subsets to compute fractional solutions $(\widehat{\mathbf{x}}, \widehat{\mathbf{y}})$ and $(\widetilde{\mathbf{x}}, \widetilde{\mathbf{y}})$ for the small and large demands, respectively. Rounding these to an integral solution $\mathbf{x}$ probabilistically concludes the execution. For a more elaborate description of Algs. 2 and 3 we refer the reader to Sec. B and C in the supplementary materials.

The scant information concerning the future inputs limits the performance of an online algorithm severely as compared to an offline approach possessing global knowledge. In this context, contending against such an omnivalent benchmark is a daunting task. Indeed, as shown in [48], in the absence of assumptions it is impossible to devise an online algorithm with non-trivial competitive ratio. This necessitates the introduction of the assumptions listed hereunder.

- The largest demand of a customer is at most the smallest capacity over all time slots, namely,

$$\max_{k\in\mathcal{N}} \left|S_k\right| \le C^{\min} \triangleq \min_{t\in\mathcal{T}} C_t\,.$$

  This is known in the literature as *the no bottleneck assumption* (NBA) [47].
- There exist positive $a^{\max}$, $a^{\min}$, $u^{\max}$, $u^{\min}$ and $T^{\max}$ known *a priori* such that

$$a^{\min} \le \frac{|S_k|}{u_k} \le a^{\max},\ u^{\min} \le u_k \le u^{\max}$$

and $|T_k| \le T_{\max}$ for $\forall\, k \in \mathcal{N}$.

Note that the NBA assumption naturally holds in power systems, since individual demands are typically much smaller than generation capacity.

Algorithm 2: $\texttt{Online}_\text{C}[k, u_k, T_k, S_k]$.

Global Initialization:

$\widehat{\mathbf{x}} \leftarrow \mathbf{0}, \widetilde{\mathbf{x}} \leftarrow \mathbf{0}, \mathbf{x} \leftarrow \mathbf{0}$; $\mathcal{I}_\text{S} \leftarrow \emptyset, \mathcal{I}_\text{L} \leftarrow \emptyset$; $C' \leftarrow (C_t)_{t\in\mathcal{T}}$
$\widehat{\mathbf{y}} \leftarrow \mathbf{0}, \widetilde{\mathbf{y}} \leftarrow \mathbf{0}$; $a^{\max} \leftarrow \max\limits_{j\in\mathcal{N}} \left\{ \dfrac{\left|S_j\right|}{u_j} : |S_j| \neq 0 \right\}$
$a^{\min} \leftarrow \min\limits_{j\in\mathcal{N}} \left\{ \dfrac{\left|S_j\right|}{u_j} : |S_j| \neq 0 \right\}$; $s \leftarrow 0$; $l \leftarrow 0$
$u^{\min} \leftarrow \min\limits_{j\in\mathcal{N}}\{u_j : u_j \neq 0\}$; $u^{\max} \leftarrow \max\limits_{j\in\mathcal{N}}\{u_j\}$;
$T^{\max} \leftarrow \max_{j\in\mathcal{N}}\{\left|T_j\right|\}$; $\alpha \leftarrow 0.138$; $\delta \leftarrow 0.333$
$r_\text{S} \leftarrow 2\log\left(1 + \dfrac{(T^{\max}+1)a^{\max}}{a^{\min}}\right)$
$r_\text{L} \leftarrow 2\log\left(1 + \dfrac{T^{\max}u^{\max}}{u^{\min}}\right)$
Choose $\tau \in \{0, 1\}$ at random

Execution upon the k-th customer:

if $\left|S_k\right| \le \delta \min_{t\in T_k}\{C_t\}$ **then**
  $\mathcal{I}_\text{S} \leftarrow \mathcal{I}_\text{S} \cup \{k\}$ ▷ *δ-Small demands*
  $s \leftarrow s+1$; $\mathcal{T} \leftarrow \mathcal{T} \cup \{|\mathcal{T}|+1\}$; $\widehat{T}_s \leftarrow T_k \cup \{|\mathcal{T}|\}$
  $C_{|\mathcal{T}|} \leftarrow u_k$; $a_{s,t} \leftarrow \dfrac{\left|S_k\right|}{u_k}\ \forall t \in \widehat{T}_s$; $a_{s,|\mathcal{T}|} \leftarrow 1$
  $\widehat{x}_s, \widehat{\mathbf{y}} \leftarrow \texttt{PD}[s, (C_t)_{t\in\mathcal{T}}, (a_{i,t}, \widehat{T}_i)_{i\in\{1,\ldots,s\},j\in\mathcal{T}}, \widehat{\mathbf{x}}, \widehat{\mathbf{y}}]$
else
  $\mathcal{I}_\text{L} \leftarrow \mathcal{I}_\text{L} \cup \{k\}$ ▷ *δ-Large demands*
  $l \leftarrow l+1$; $\widetilde{T}_l \leftarrow T_k$; $\widetilde{a}_{l,t} = \frac{1}{u_k}\ \forall t \in \widetilde{T}_l$
  $\widetilde{x}_l, \widetilde{\mathbf{y}} \leftarrow \texttt{PD}[l, \mathbf{1}, (\widetilde{a}_{i,t}, \widetilde{T}_i)_{i\in\{1,\ldots,l\},j\in\mathcal{T}}, \widetilde{\mathbf{x}}, \widetilde{\mathbf{y}}]$
  ▷ *Randomized rounding and correction*
if $k \in \mathcal{I}_L$ **then**
  $x_k \leftarrow \begin{cases} 1, & \text{with probability } \alpha\tau\frac{\widetilde{x}_l}{u_k r_\text{L}} \\ 0, & \text{with probability } 1-\tau\alpha\frac{\widetilde{x}_l}{u_k r_\text{L}} \end{cases}$
else
  $x_k \leftarrow \begin{cases} 1, & \text{with probability } (1-\tau)\frac{\widehat{x}_s}{2u_k r_\text{S}} \\ 0, & \text{with probability } 1-(1-\tau)\frac{\widehat{x}_s}{2u_k r_\text{S}} \end{cases}$
if $\left|\sum_{j\in\{1,\ldots,k\},t\in T_j} S_j x_j\right| > C'_t$ for some $t \in \mathcal{T}$ **then**
  $x_k \leftarrow 0$
else
  if $x_k = 1$ **then**
    $C'_t \leftarrow C'_t - \left|S_k\right|\ \forall\, t \in T_k$
return x_k

We next show analytically that $\texttt{Online}_\text{C}$ is a competitive online algorithm for CSP_C problem.

Theorem 1: Under the NBA assumption, algorithm $\texttt{Online}_\text{C}$ produces a feasible solution to CSP_C with a competitive ratio of

$$\texttt{CR}\left(\texttt{Online}_\text{C}\right) = \Omega\left(\frac{\cos\frac{\theta}{2}}{\log\left(1 + T^{\max}\frac{a^{\max}}{a^{\min}}\right)}\right).$$

Remark: It is noteworthy that, to the best of our knowledge, there are no known results in the literature concerning the lower bound of the competitive ratio of the online problem CSP_C. From the arguments in [37], however, it holds that $\mathcal{O}(\log(\frac{a^{\max}}{a^{\min}}))$ is

Algorithm 3: $\texttt{PD}[k, (\bar{C}_t)_{t\in\mathcal{T}}, (a_{i,t}, \bar{T}_i)_{i\in\{1,\dots,k\},j\in\mathcal{T}}, \mathbf{x}, \mathbf{y}]$.

1: $\bar{a}^{\max} \leftarrow \max\limits_{j\in\{1,\dots,k\},t\in\bar{T}_j}\{a_{j,t}\}$; $\bar{T}^{\max} \leftarrow \max_{1\le j\le k}\{|\bar{T}_j|\}$
2: **while** $\sum_{t\in\bar{T}_k} y_t a_{k,t} < 1$ **do**
3: Increase x_k
4: **for** $t \in \bar{T}_k$ **do**
5: $b \leftarrow e^{(2\bar{C}_t)^{-1}\sum_{j\in\{1,\dots,k\},t\in\bar{T}_j} a_{j,t}x_j}$
6: $y_t \leftarrow \max\left\{y_t, \dfrac{b-1}{\bar{T}^{\max}\bar{a}^{\max}}\right\}$
7: **return** x_k, $\mathbf{y}$

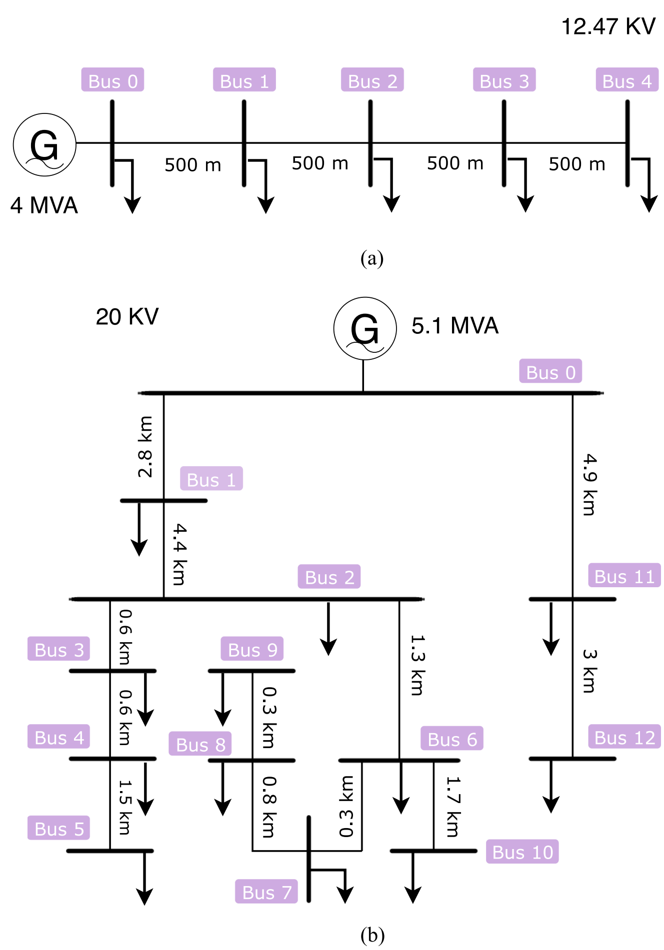

Fig. 2. One-line diagrams of employed MG test systems: (a) A 4-bus feeder taken from CBDS. (b) A slightly modified version of the CIGRE MV benchmark system.

indeed the best possible competitive ratio that could be achieved by any online algorithm for the natural linear programming relaxation of CSP_C with a solitary constraint.

Due to space scarcity, the proof of Theorem 1 is deferred to the supplementary materials.

V. Empirical Evaluation

To complement the analytic result in Theorem 1, this section validates the proposed algorithm `Online` numerically on two isolated MGs, depicted in Fig. 2, simulated with the Pandapower package [49] available within the Python distribution. The objective value obtained by `Online` when applied to CSP is contrasted against that of a common greedy method `FCFS` (first-come-first-serve), which schedules the loads in the arrival order whenever feasible (i.e., if constraints (2), (4), (5), (6), (7) are not violated). The *optimal offline solutions* computed by the numerical solver CPLEX, denoted by `OFL`, serve as a baseline for comparison. We first lay out the configurations and scenarios under study, then in Section V-B discuss the results of the comparative analysis.

A. Simulation Setup and Scenarios

Performance of the candidate DR scheduling algorithms is evaluated on two sample MGs: a modified version of the CIGRE MV benchmark system and a 4-bus feeder borrowed from CBDS. The former, pictured in Fig. 2, is a 20 kV network based on a German MV distribution system [38]. In the adapted version here, a connection to the external grid has been replaced by a collective generation unit with a cumulative capacity of 5.1 MVA. The line parameters and system data are provided in Fig. 2 as well as in Table I in the Appendix. As for the latter, the feeder is a portion of a 12.47 kV radial system, practically deployed in Canada, whose particulars can be consulted in [50]. For both MGs, the time-varying generation capacity $(C_t)_{t\in\mathcal{T}}$ is sampled according to a Bernoulli process.

In the adopted simulation setup, which reflects the model specified in Section II, DR participants arrival follows a Poisson distribution and the scheduling decisions are outputted at a granularity of 1 minute. Each customer $k \in \mathcal{N}$ is assigned a power demand S_k (including both active and reactive power), defined over a certain duration T_k drawn at random from a uniform distribution on $\mathcal{T}$, and a utility u_k that are generated according to a probability preference model. In particular, the following are scenarios for the case studies undertaken.

i) *Customer set*:
   a) *Commercial* (**C**): DR participants are commercial consumers with medium-to-large power demands ranging from $S^{\min} = 100$ KVA to $S^{\max} = 1$ MVA.
   b) *Mixed* (**M**): The customer set comprises a mix of commercial and residential consumers. The latter have power demands ranging from $S^{\min} = 2000$ VA up to $S^{\max} = 20$ KVA and constitute no more than 80% of all customers chosen at random.

ii) *Utility-demand correlation*:
   a) *Quadratic* (**Q**): The utility of a customer is a quadratic function of the power demand and duration in the form of $u_k(|S_k|, |T_k|) = \gamma \cdot (|S_k||T_k|)^2 + \eta \cdot |S_k||T_k| + \psi$, where $\gamma > 0, \eta, \psi \geq 0$ are preset constants.
   b) *Random* (**R**): Independent of the power demand, each customer's utility is generated randomly from $[S^{\min}, S^{\max}]$, according to the demand ranges indicated above.

In what follows, we shall refer to the case studies by the corresponding acronyms. For instance, the case study named QM stands for the one with quadratic utility-demand correlation and mixed customers.

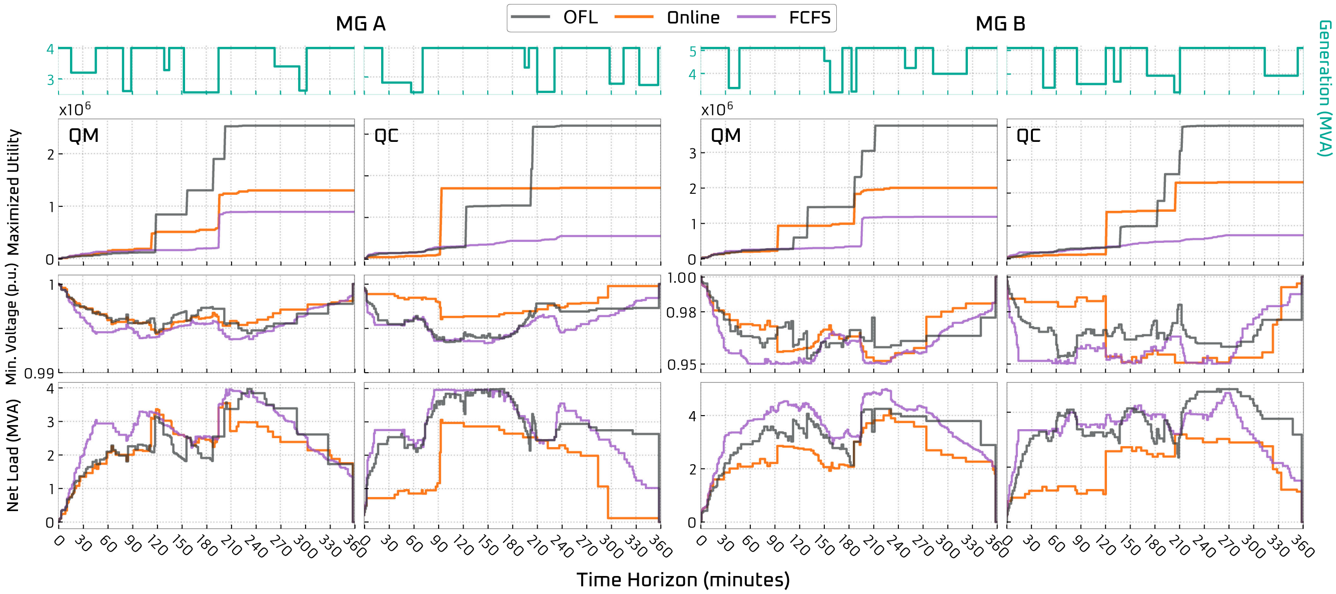

Fig. 3. Performance comparison of `Online`, `FCFS`, and `OFL` on the employed MG test systems considering case studies QC and QM. The lower two subplot rows illustrate the MGs' evolving system dynamics over time for all three approaches.

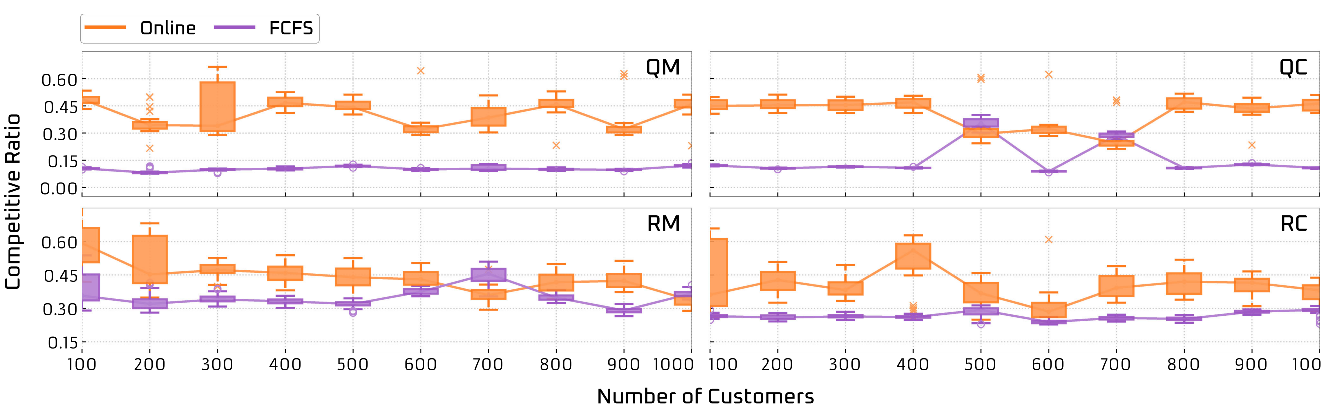

Fig. 4. Box plot of competitive ratios of `Online` and `FCFS` for CSP against the number of customers arriving online on MG A (outliers are pictured as points outside the boxes).

B. Case Studies

As an illustrative example, the candidate algorithms are first contrasted in a single-shot simulation for case studies QM and QC on each of the two MGs, which results appear in Fig. 3. Concretely, the figure depicts the objective values reached by `Online`, `OFL` and `FCFS` for CSP problem along with the observed minimum voltage (across all the nodes) and net load profiles observed on MGs when implementing their DR scheduling decisions. Each case study considers 500 customers arriving online within a time span of 360 minutes.

As evidenced by Fig. 3, `Online` approached `OFL` achieving around 50% of `OFL`'s utility in both case studies while maintaining voltage magnitudes within the IEEE standard 1547 limits (i.e., $>= 0.95$ p.u. and $<= 1.05$ p.u.), whereas `FCFS` drifted away nearing the 15% mark. Admittedly, it is straightforward to verify that `FCFS` might yield arbitrarily worse utility than `OFL`. Consider the following engineered scenario. Let the input instance of CSP be composed of 2 consumers, customer I with a utility of 1 and a power demand of 1 MVA for a duration of 350 minutes as well as customer II with a utility of 1000 and a power demand of 3.5 MVA for a duration of 150 minutes. Suppose I's request arrives at time step 1 on MG A, while II's at instant 2. Then, `FCFS` will greedily admit the former (provided power flow and operational constraints are satisfied) and reject II due to the capacity bottleneck of 4 MVA, effectively attaining a competitive ratio of $\frac{1}{1000}$.

For a more exhaustive evaluation, we next investigate the performance of `Online` and `FCFS` in a larger scale simulation on MG A with up to 1000 consumers. Here, the algorithms were invoked 50 times for each of the n number of customers, n varying between 100 to 1000 in steps of 100, considering random changes in the inputs of CSP. For each step, random perturbations were effected on the generation profile of MG. The results are summarized in Fig. 4, which portrays the competitive ratios obtained by `Online` and `FCFS` at 95% confidence interval. On the whole, `Online` prevailed in maximizing the net customer utility with a competitive ratio averaging to around 0.45, far outstripping FCFS in all but few case studies. In QC and RM, however, FCFS exhibited comparable or slightly superior performance to Online for the customer sets of cardinalities 500 and 700. This owes chiefly to the probabilistic nature of $\texttt{Online}_\mathrm{C}$, which may reject otherwise satisfiable demand request by vicissitude. For practical applications, one can tweak the parameters α and τ of $\texttt{Online}_\mathrm{C}$ to attenuate such occurrences.

Overall, it transpired from the simulations' results that the tentative scheduling decisions returned by $\texttt{Online}_\mathrm{C}$ (i.e., step prior to carrying out AC power flow) satisfied the power flow and operational constraints (2), (4), (5) and (7) of the distribution network in most cases. Specifically, no violation was observed for MG A, effectively rendering the load flow calculations in Online (i.e., steps 4 and 5) redundant. As for MG B, several such were instances were detected, yet, the induced loss in utility (from rejecting the violating requests) accounted for no more than 5% of the total objective value. In consequence, the competitive ratio stated in Theorem 1 for $\texttt{Online}_\mathrm{C}$ *was essentially delegated onto* CSP problem, hence the prominent empirical record of Online (the lowest observed competitive ratio thereof across all the case studies was about 0.21). The implications of these findings are twofold: 1) They underline the salience of incorporating power flow calculations within the design of online DR strategies 2) They call for greater commitment towards devising *competitive online DR scheduling algorithms*. For future work, one can extend the proposed online algorithm to a distributed setting (such that DR scheduling decisions are computed at user end) as well as examine the suboptimality gap of $\texttt{Online}_\mathrm{C}$ on other practical islanded MG test systems (possibly including electric vehicle users with divisible power consumption levels).

VI. Conclusion

The MG paradigm is bolstered by the elevated needs for leveraging emerging communication technologies and evolving load types. While entailing tangible benefits and opportunities, it introduces intricacies, which can be inimical to the grid stability. In response to these, the extant DR methods require a drastic switch from traditionally offline to a *real-time (i.e., online) domain of operation. Although online decision problems have been well-studied in the literature, a unique challenge arising in MGs is the presence of non-linear constraints, a departure from the traditional settings. Against this backdrop,* the present work proposes *a competitive randomized online algorithm* to achieve real-time DR scheduling in isolated MGs. We formulate the complex-demand DR as an AC OPF problem with consideration of the loads' active and reactive power requirements. To verify the effectiveness of the proposed approach, numerical simulations are provided on two practical MG systems, comparing it against the benchmark greedy algorithm FCFS. The results demonstrate a significant difference in performance qualities (in the *order of 2 times* on average), with the devised online algorithm featuring prominently. According to the findings, the proposed method on average *achieved about 45%* of the objective value of the offline optimal solution.

Appendix

TABLE I
Line Parameters of the Modified CIGRE MV System (MG B)

Node From	Node To	Resistance [Ω/km]	Reactance [Ω/km]	Capacitance [nF/km]
0	1	0.501	0.716	151.175
1	2	0.501	0.716	151.175
2	3	0.501	0.716	151.175
3	4	0.501	0.716	151.175
4	5	0.501	0.716	151.175
2	6	0.501	0.716	151.175
6	7	0.501	0.716	151.175
7	8	0.501	0.716	151.175
8	9	0.501	0.716	151.175
6	10	0.501	0.716	151.175
0	11	0.51	0.366	10.0968
11	12	0.51	0.366	10.0968

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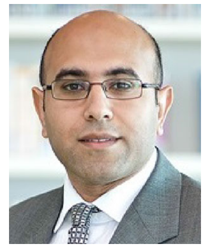

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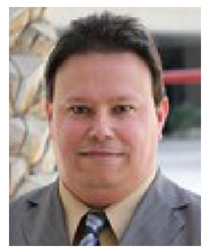

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