Stochastic Dynamic Analysis for Power Systems Under Uncertain Variability

Ping Ju, *Senior Member, IEEE*, Hongyu Li, *Student Member, IEEE*, Xueping Pan, Chun Gan, *Member, IEEE*, Yong Liu, *Member, IEEE*, and Yilu Liu, *Fellow, IEEE*

***Abstract*—This paper proposes a novel method to analyze impacts of uncertain variability on power system dynamics. There is considerable interest in integrating intermittent renewable energy and plug-in electric vehicles into power systems. Therefore, power systems are in an environment with increasing uncertain variability. In this paper, the uncertain variability is described as a continuous-time stochastic process, and thus, the power system under uncertain variability is modeled by stochastic differential equations. To quantify impacts of uncertain variability on power system dynamics, an intra-region probability index is presented. Based on the stochastic averaging method, an analytical method is proposed to calculate the intra-region probability. Compared to Monte Carlo simulation, the proposed method is many orders of magnitude faster without sacrificing the result accuracy. Furthermore, several insights are given to improve the power system dynamics under uncertain variability.**

***Index Terms*—Renewable energy, electric vehicles, stochastic differential equations, power system dynamics.**

I. Introduction

WITH the ever-growing penetration of intermittent renewable energy and plug-in electric vehicles, uncertainty is increasing in power systems. Impacts of uncertainty on power systems are of concern [1], [2]. Research on power systems under the uncertainty attracts significant attention [3]–[9]. This study focuses on the impacts of uncertain variability on power system dynamics.

The uncertain variability is considered as a stochastic continuous disturbance to the system, which can be described by a stochastic process. Furthermore, the dynamic system under uncertain variability can be modeled by a set of stochastic differential equations (SDEs) [10]–[15]. A systematic and general approach to model stochastic dynamic systems is proposed in [10], by using stochastic differential-algebraic equations. In [11], a multi-machine power system model that captures the uncertain variability from the renewable power generation and plug-in electric vehicles is presented based on SDEs. In [12], the stochastic perturbations in transmission lines and system loads are considered, and then the dynamic system under these perturbations is modeled by a set of SDEs. In [13], the mechanical power input of an asynchronous wind turbine is regarded as a stochastic excitation, and the dynamic system equation under stochastic excitation is formulated based on SDEs. In [14], power systems influenced by stochastic perturbations in load and variable renewable generation are modeled by stochastic differential-algebraic equations. In [15], a stochastic model of the single-machine infinite-bus (SMIB) system is proposed based on SDEs, and then the stochastic stability is analyzed.

Analysis of power system under uncertain variability is strongly related to Monte Carlo simulation. In [16], a probabilistic risk-based approach is presented to assess the rotor angle stability of power systems, by using Monte Carlo simulation. A new quasi-Monte Carlo method is put forward to analyze the dynamic effects of plug-in electric vehicles and wind energy conversion systems in [17], by using probabilistic small signal stability analysis. In [18], a novel framework is proposed to assess the stochastic transient stability, by utilizing Monte Carlo simulation on SDEs. Monte Carlo simulation and the stochastic Lyapunov stability method are combined to assess the transient stability of structure-preserved power systems in [19]. The probability of system failure is estimated in [20], by utilizing importance sampling based Monte Carlo simulation. Monte Carlo simulation has the advantages of good adaptabilities, while the low computational efficiency and unclear physical mechanism often make it undesirable.

Due to the high efficiency without sacrificing the result accuracy, analytical methods with no repeated simulations are attractive [21]–[25]. In [26], the Fokker–Planck's equation is adopted to analyze the evolution of the state's probability density function, and then the probability density function is used to determine the impact of stochastic load perturbations on system stability. In [27], an analytical method for security assessment is proposed for nonlinear power system under stochastic load disturbances. However, the above methods are applied in simple systems, and the applications in large-scale power systems have not been addressed.

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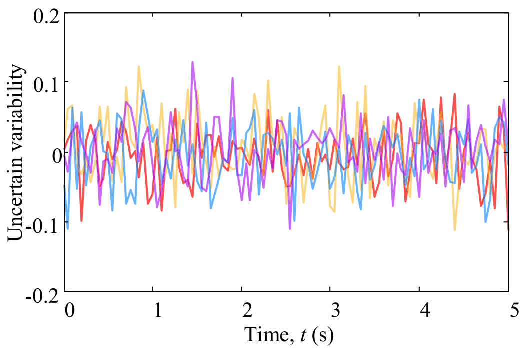

Fig. 1. Four trajectories of uncertain variability.

In recent decades, dynamic analysis of power system under uncertain variability has been deeply considered. However, there are still some issues needing solutions: 1) as the dominant analysis method, Monte Carlo simulation suffers from the low computational efficiency and unclear impact mechanism; 2) the applications of analytical analysis methods encounter difficulties in large-scale power systems, due to the high dimension.

To deal with the issues mentioned above, this paper provides a highly accurate and efficient analysis method to assess the dynamic impacts of uncertain variability on power systems. First, the uncertain variability in a power system is considered as a stochastic continuous disturbance, and then the system under uncertain variability is modeled by SDEs. Second, the intra-region probability is proposed to quantify impacts of uncertain variability on power system dynamics. Finally, the analytical method based on stochastic averaging is provided to calculate the intra-region probability. Compared to Monte Carlo simulation, the proposed method has two significant advantages. First, the proposed method is many orders of magnitude faster than Monte Carlo simulation. Second, the proposed method presents the impact mechanism of uncertain variability on power system dynamics, which cannot be revealed by Monte Carlo simulation.

This paper is organized as follows: Section II introduces power system dynamics under uncertain variability. Section III proposes the novel analytical method for dynamic impacts of uncertain variability on power systems. Section IV gives simulation examples to validate the proposed method. Finally, the conclusions are presented in Section V. Derivations are shown in Appendix.

II. Power System Dynamics Under Uncertain Variability

A. Model of Power System Under Uncertain Variability

With the growing integration of renewable power generation and plug-in electric vehicles, increasing uncertain variability is brought into power systems. In this paper, the uncertain variability is considered as an additive stochastic continuous disturbance for power systems [26], [27]. Fig. 1 shows four trajectories of uncertain variability. Due to the small magnitude, uncertain variability was seldom considered for dynamic analysis in power systems before. However, power systems are operating under more stressed conditions with increasing uncertain variability.

Itō SDEs are usually used to describe power system dynamics under uncertain variability [10]–[15]. In this paper, generators are modeled by second-order generator models, and loads are described with constant impedance models. Then, the stochastic model of a multi-machine power system under uncertain variability can be expressed as a set of Itō SDEs [26]

$$\begin{cases} \mathrm{d}\delta_i = \omega_0 \Delta\omega_i \mathrm{d}t & i = 1, 2, \cdots, n \\ M_i \mathrm{d}\Delta\omega_i = [P_{mi} - G_{ii} {E_i}^2 - \sum_{j=1, j\neq i}^{n} E_i E_j B_{ij} \sin(\delta_i - \delta_j) \\ -D_i \Delta\omega_i] \mathrm{d}t + \sigma_i \mathrm{d}B_i(t) \end{cases} \tag{1}$$

where δ_i, $\Delta\omega_i$, M_i, P_{mi}, E_i and D_i are the rotor angle, rotor speed deviation, inertia coefficient, mechanical power, internal voltage and damping coefficient of ith generator, respectively; ω_0 is the synchronous machine speed, which equals to $2\pi f$ with the nominal system frequency f in Hertz; $G_{ii} E_i^2$ denotes the active power from the loads, because the networks and loads are merged; G_{ii} is the ith diagonal element in conductance matrix; B_{ij} is the off-diagonal element in the ith row and jth column of the susceptance matrix; $\sigma_i \mathrm{d}B_i(t)$ denotes the uncertain variability on ith generator; $B_i(t)$ denotes an independent standard Wiener process; σ_i is the intensity of the uncertain variability on ith generator; n denotes the number of generators; and all the parameters and variables are described by per-unit value except M_i and t.

B. Intra-Region Probability

In a power system under deterministic disturbances (e.g., a three-phase fault), it is known that the impact of a smaller disturbance is weaker. However, in a power system under stochastic disturbances, a small disturbance may have significant impacts [28]. In practice, it is desired that system states are operated in the secure region. For example, system frequency needs to be maintained within certain limits for meet frequency performance criteria [23]. However, when the uncertain variability is introduced, it is a random event that system states are within the region. The probability of operating states being in a region is a proper index to evaluate impacts of uncertain variability on power system dynamics. In this paper, this probability is called as the intra-region probability. It is reasonable that a stable power system with the higher intra-region probability is less impacted by the uncertain variability. The intra-region probability is defined as follows:

$$P(t) = P\{\boldsymbol{X}(t) \in \Omega_B\} \tag{2}$$

where $\boldsymbol{X}(t)$ denotes the operating state at time t, and $\boldsymbol{X}(t) = [\delta_1(t), \cdots, \delta_n(t), \Delta\omega_1(t), \cdots, \Delta\omega_n(t)]$; Ω_B denotes the region that operating states need to be in; and $P\{\boldsymbol{X}(t) \in \Omega_B\}$ denotes the probability of $\boldsymbol{X}(t)$ being within Ω.

C. Calculation Method Based on Energy Function

It is feasible to analytically calculate the intra-region probability (2) in an SMIB system with only two-dimensional states. However, the calculation becomes burdensome in large-scale power systems, due to the high-dimensional operating states

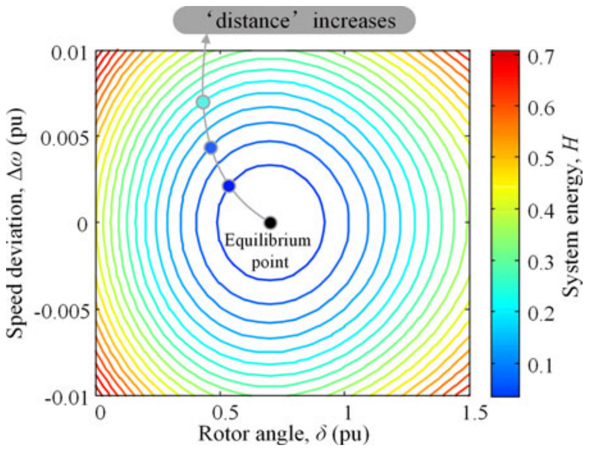

Fig. 2. System energy of an SMIB system in the state space.

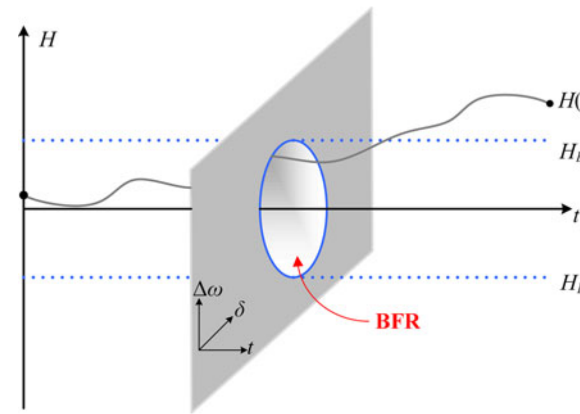

Fig. 3. The relationship between operating states and BFR.

[26]. In this paper, energy function method is adopted to address the problem of high dimension. The energy function of the system (1) can be expressed as follows [29]:

$$H = \frac{1}{2}\sum_{i=1}^{n} M_i\omega_0\Delta\omega_i^2 + \sum_{i=1}^{n} P_i(\delta_{is} - \delta_i) + \left[\sum_{i=1}^{n}\sum_{j=i+1}^{n} E_iE_jB_{ij}\cos(\delta_{is} - \delta_{js}) - \sum_{i=1}^{n}\sum_{j=i+1}^{n} E_iE_jB_{ij}\cos(\delta_i - \delta_j)\right] \tag{3}$$

where δ_{is} and δ_{js} denote the equilibrium rotor angle of the ith and jth generators, respectively.

By the power system energy function, high-dimensional operating states are transformed into system energy values. In Fig. 2, the system energy of an SMIB system is shown in the state space. It is easy to see that the operating state with a higher system energy value is further from the equilibrium point. In other words, the system energy can be regarded as the 'distance' of an operating state from the equilibrium point. Hence, it is reasonable that an operating state is in a predefined region (i.e., the 'distance' of the operating state from equilibrium point is limited) when the system energy value is lower than a preset value. In this paper, the predefined region, in which the system energy is lower than a predefined limit H_B, is called the bound fluctuation region (BFR). In a real application, the energy limit H_B can be set according to the operating requirements.

The relationship between operating states and BFR is illustrated in Fig. 3. The gray trajectory $H(t)$ represents the energy evolution of operating states along with the time. The blue dotted lines H_B represent the energy boundary of BFR. It can be seen that operating states are bounded when they are within the BFR.

Based on the above analysis, the intra-region probability can be expressed by the following form:

$$P(t) = P\{H(t) < H_B\} \tag{4}$$

where $H(t)$ denotes the system energy along with the time, H_B denotes the predefined energy boundary, and $P\{H(t) < H_B\}$ denotes the probability of $H(t)$ being lower than H_B.

III. Stochastic Dynamic Analysis of Power Systems Under Uncertain Variability

In this section, the stochastic dynamic analysis of a power system under uncertain variability is proposed to calculate the intra-region probability (4) analytically. This section is organized as follows: first, a simplified model of the system energy under uncertain variability is presented by stochastic averaging method (SAM); second, an explicit formulation of the solution to the system energy is obtained by mathematical deduction; third, the statistical information of the system energy over time is solved analytically; fourth, the intra-region probability (4) is analytically calculated based on the solved statistical information.

A. Simplified Model of System Energy Under Uncertain Variability

SAM is proposed in [30] to simplify stochastic dynamic systems. By using SAM, one can average rapidly varying quantities to derive the approximate differential equations of slowly varying quantities, which are much simpler than the original system model [30]. In many cases, SAM can significantly simplify the motion equations of the original stochastic system as one-dimensional SDEs.

By utilizing SAM on (1), one can obtain a simplified model of the system energy under uncertain variability, as follows:

$$\mathrm{d}H = \overline{m}(H)\mathrm{d}t + \overline{\sigma}(H)\mathrm{d}B(t) \tag{5}$$

where $\overline{m}(H)$ and $\overline{\sigma}(H)$ are two coefficients and can be derived by following equations (the derivation is shown in Appendix A); and $B(t)$ denotes a standard Wiener process.

$$\begin{cases} \overline{m}(H) = \sum_{i=1}^{n}(\omega_0{\sigma_i}^2/2M_i) - \sum_{i=1}^{n}(2D_i/M_i)/(2n-1)\cdot H \\ \overline{\sigma}^2(H) = \sum_{i=1}^{n}(2\omega_0{\sigma_i}^2/M_i)/(2n-1)\cdot H \end{cases}. \tag{6}$$

Furthermore, (5) can be expressed in the following form

$$\mathrm{d}H = \lambda(\mu - H)dt + \sigma\sqrt{H}dB(t) \tag{7}$$

where

$$\begin{cases} \lambda = \sum_{i=1}^{n} (2D_i/M_i)/(2n-1) \\ \mu = (2n-1)\sum_{i=1}^{n} (\omega_0 {\sigma_i}^2/M_i)/\sum_{i=1}^{n} (4D_i/M_i) \\ \sigma = \sqrt{\sum_{i=1}^{n} (2\omega_0 {\sigma_i}^2/M_i)/(2n-1)} \end{cases}. \tag{8}$$

In stochastic theory, the dynamic equation (7) is called as a mean-reverting square root process (MRSRP) [31], [32]. MRSRP is widely used in finance as the model of volatility, interest rate, and other financial quantities [31]. In the following part, the explicit formulation of the solution to (7) will be offered.

B. Explicit Formulation of the Solution to Mean-Reverting Square Root Process

It is difficult to give the explicit formulation of the solution to MRSRP directly. In this paper, a solvable ideal system is used to obtain the explicit formulation of the solution to MRSRP (7) indirectly. Then, the explicit formulation of the solution to (7) can be expressed as follows (the derivation is shown in Appendix B):

$$H = \sum_{i=1}^{k} \left[\int_0^t \frac{\sigma}{2} e^{\lambda(s-t)/2} \mathrm{d}B_i(s)\right]^2. \tag{9}$$

From (9), it can be seen that the system energy value under uncertain variability becomes a random variable. The statistical information is the key role for a random variable. In the next part, the statistical information of the system energy over time will be solved by the mathematical deduction.

C. Statistical Information of System Energy Under Uncertain Variability

The statistical information of $\int_0^t \frac{\sigma}{2} e^{\lambda(s-t)/2}\mathrm{d}B_i(s)$ is analyzed initially, because $\int_0^t \frac{\sigma}{2} e^{\lambda(s-t)/2}\mathrm{d}B_i(s)$ are key parts of MRSRP's solution (9). $\int_0^t \frac{\sigma}{2} e^{\lambda(s-t)/2}\mathrm{d}B_i(s)$ under different i follow Gaussian distributions with the same statistical information at a certain time t [33]. The mean and variance of $\int_0^t \frac{\sigma}{2} e^{\lambda(s-t)/2}\mathrm{d}B_i(s)$ can be derived as follows:

$$\begin{cases} MEAN = E\left[\int_0^t \frac{\sigma}{2} e^{\lambda(s-t)/2}\mathrm{d}B_i(s)\right] = 0 \\ VAR = \mathrm{var}\left[\int_0^t \frac{\sigma}{2} e^{\lambda(s-t)/2}\mathrm{d}B_i(s)\right] \\ \quad = \int_0^t \frac{\sigma}{2} e^{\lambda(s-t)/2} \frac{\sigma}{2} e^{\lambda(s-t)/2}\mathrm{d}s \\ \quad = \frac{\sigma^2}{4\lambda}(1-e^{-\lambda t}) = \frac{\sum_{i=1}^{n} (\omega_0 {\sigma_i}^2/M_i)}{4\sum_{i=1}^{n} (D_i/M_i)} \\ \quad \times \left[1 - e^{-2t\sum_{i=1}^{n} (D_i/M_i)/(2n-1)}\right] \end{cases} \tag{10}$$

where *MEAN* and *VAR* denote the mean and variance, respectively.

In practice, damping coefficients may be zero in a system. However, one still can obtain the mean and variance based on (10) when all the damping coefficients are zero. As all damping coefficients $D_i \to 0$, the limit of the mean and variance can be deduced as follows:

$$\begin{cases} \lim\limits_{D_i \to 0} MEAN = 0 \\ \lim\limits_{D_i \to 0} VAR = \lim\limits_{\sum_{i=1}^{n} (D_i/M_i) \to 0} VAR \\ = \lim\limits_{\sum_{i=1}^{n} (D_i/M_i) \to 0} \frac{\sum_{i=1}^{n} (\omega_0 {\sigma_i}^2/M_i)}{4\sum_{i=1}^{n} (D_i/M_i)} \\ \quad \left[1 - e^{-2t\sum_{i=1}^{n} (D_i/M_i)/(2n-1)}\right] \\ = \lim\limits_{\sum_{i=1}^{n} (D_i/M_i) \to 0} \frac{t\sum_{i=1}^{n} (\omega_0 {\sigma_i}^2/M_i)}{2(2n-1)} \\ \quad \frac{e^{-2t\sum_{i=1}^{n} (D_i/M_i)/(2n-1)} - 1}{-2t\sum_{i=1}^{n} (D_i/M_i)/(2n-1)} \\ = \frac{t\sum_{i=1}^{n} (\omega_0 {\sigma_i}^2/M_i)}{2(2n-1)} \lim\limits_{\sum_{i=1}^{n} (D_i/M_i) \to 0} \\ \quad \left[\frac{e^{-2t\sum_{i=1}^{n} (D_i/M_i)/(2n-1)} - 1}{-2t\sum_{i=1}^{n} (D_i/M_i)/(2n-1)}\right] \\ = \frac{t\sum_{i=1}^{n} (\omega_0 {\sigma_i}^2/M_i)}{2(2n-1)} \end{cases}. \tag{11}$$

Clearly, the variance of the system energy is determined by the intensity of uncertain variability, inertia coefficient, system damping, and time.

In the following, the probability density function of the system energy over time will be deduced based on the mean and variance (10).

The Chi-squared distribution is known as the distribution of a sum of the squares of independent standard Gaussian distribution random variables. The probability density function of a chi-squared distribution with $2n-1$ degrees of freedom is as follows [34]:

$$f(x) = \begin{cases} \frac{(2n-1)2^n n!}{\sqrt{2\pi}(2n)!} x^{n-3/2} e^{-x/2}, & x > 0 \\ 0, & \text{otherwise} \end{cases}. \tag{12}$$

Given that $\int_0^t \frac{\sigma}{2} e^{\lambda(s-t)/2}\mathrm{d}B_i(s)$ with different *i* are independent and follow standard Gaussian distributions, one can deduce the probability density function of the system energy (9) based on (12). The system energy H, scaled down by the variance (i.e., multiplied by $1/VAR$), follows a chi-squared distribution with $2n-1$ degrees of freedom. Substituting (10) into (12), one obtains the probability density function of the system energy (9), as follows:

$$f(H,t) = \begin{cases} \frac{(2n-1)2^n n!}{VAR\sqrt{2\pi}(2n)!} (H/VAR)^{n-3/2} e^{-H/(2VAR)}, & H > 0 \\ 0, & \text{otherwise} \end{cases}. \tag{13}$$

Probability density function (13) is an important property for the system energy under uncertain variability, by which most statistical information of the system energy can be calculated when the system is stable under uncertain variability. In this paper, the intra-region probability is mainly analyzed, which will be presented in the next part.

D. Analytical Solution to Intra-region Probability

The intra-region probability (4) is actually the cumulative distribution function of the system energy H. By integrating (13), one obtains

$$P\{H(t) < H_B\} = \int_{-\infty}^{H_B} f(H,t)\mathrm{d}H$$
$$= \int_0^{H_B} \frac{(2n-1)2^n n!}{VAR\sqrt{2\pi}(2n)!}(H/VAR)^{n-3/2}e^{-H/(2VAR)}\mathrm{d}H. \quad (14)$$

As the time $t \to +\infty$, the limit of the variance can be deduced as follows:

$$\lim_{t\to+\infty} VAR = \lim_{t\to+\infty} \frac{\sum_{i=1}^{n}(\omega_0{\sigma_i}^2/M_i)}{4\sum_{i=1}^{n}(D_i/M_i)}\left[1 - e^{-2t\cdot\sum_{i=1}^{n}(D_i/M_i)/(2n-1)}\right].$$
$$= \frac{\sum_{i=1}^{n}(\omega_0{\sigma_i}^2/M_i)}{4\sum_{i=1}^{n}(D_i/M_i)} \quad (15)$$

Substituting (15) into the intra-region probability (14), one can obtain the limit of the intra-region probability as follows:

$$\lim_{t\to+\infty} P\{H(t) < H_B\}$$
$$= \lim_{t\to+\infty}\int_0^{H_B} \frac{(2n-1)2^n n!}{VAR\sqrt{2\pi}(2n)!}(H/VAR)^{n-3/2} \times e^{-H/(2VAR)}\mathrm{d}H$$
$$= \int_0^{H_B} \frac{(2n-1)2^n n!}{\lim_{t\to+\infty} VAR\sqrt{2\pi}(2n)!}\left(H/\lim_{t\to+\infty} VAR\right)^{n-3/2} \times e^{-H/(2\lim_{t\to+\infty} VAR)}\mathrm{d}H.$$
$$= \frac{4(2n-1)2^n n!\sum_{i=1}^{n}(D_i/M_i)}{\sqrt{2\pi}(2n)!\sum_{i=1}^{n}(\omega_0{\sigma_i}^2/M_i)} \cdot \int_0^{H_B}\left\{\left[4H\sum_{i=1}^{n}(D_i/M_i)\Big/\sum_{i=1}^{n}(\omega_0{\sigma_i}^2/M_i)\right]^{n-\frac{3}{2}} \cdot e^{-2H\sum_{i=1}^{n}(D_i/M_i)/\sum_{i=1}^{n}(\omega_0{\sigma_i}^2/M_i)}\right\}\mathrm{d}H \quad (16)$$

From (16), it can be seen that the intra-region probability converges to a steady-state value.

E. Procedure for Calculating the Intra-Region Probability

The procedure for calculating the intra-region probability by the stochastic dynamic analysis is outlined below and shown in Fig. 4.

Step 1: Obtain the intensity of the uncertain variability in a power system by statistical data.
Step 2: Build the stochastic model of the power system under uncertain variability (1).

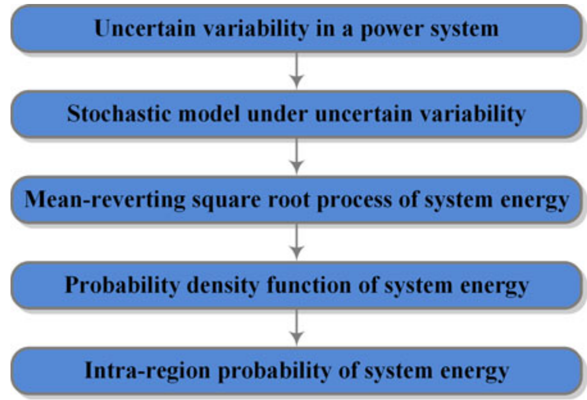

Fig. 4. Procedure for calculating intra-region probability by stochastic dynamic analysis.

Step 3: Deduce the MRSRP of the system energy (5) based on SAM, where the coefficients can be obtained from (6).
Step 4: Calculate probability density function of the system energy (13).
Step 5: Calculate the intra-region probability (14).

In this section, an analytical method based SAM is provided to assess power system dynamics under uncertain variability. Even though the classical model is often used in power system dynamic analysis, more realistic models are desired in real applications. Theoretically, one can utilize SAM, if the power system model can be expressed in a Hamiltonian form [30], [35], [36]. In recent years, much research on modeling a detailed power system in a Hamiltonian form has been carried out [37]–[40].

IV. VALIDATIONS

In this section, case studies are carried out to verify the accuracy and efficiency of the proposed method.

A. Monte Carlo Simulation

In this paper, Monte Carlo simulation is employed to compare with the proposed method. The proper iteration and time step are of great concern in Monte Carlo simulation. One can get more (less) accurate results with a larger (smaller) iteration and smaller (larger) time step, but it takes more (less) time. For most cases, it is difficult to set the iteration and time step directly, so the additional studies to specify the iteration and time step are necessary. In this paper, the time step and iteration number in Monte Carlo simulation are set to 0.001 s and 5000, respectively. The procedure for calculating the intra-region probability by Monte Carlo simulation is outlined below and shown in Fig. 5.

Step 1: Obtain the intensity of the uncertain variability in a power system by statistical data.
Step 2: Build the stochastic nonlinear model of the power system under uncertain variability (1).
Step 3: Simulate a single system states trajectory by the stochastic nonlinear model (1).
Step 4: Calculate a single system energy trajectory based on the simulated system states trajectory.

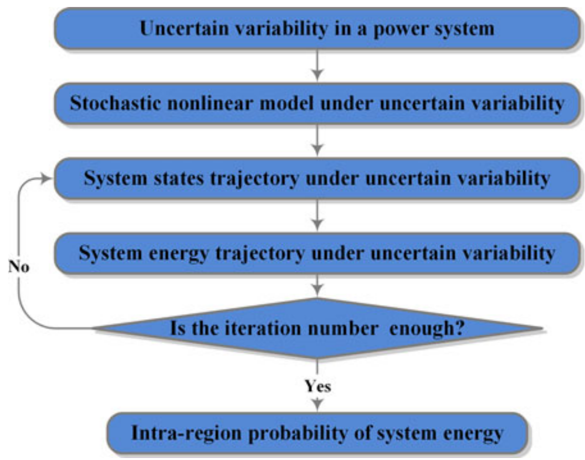

Fig. 5. Procedure for calculating intra-region probability by Monte Carlo simulation.

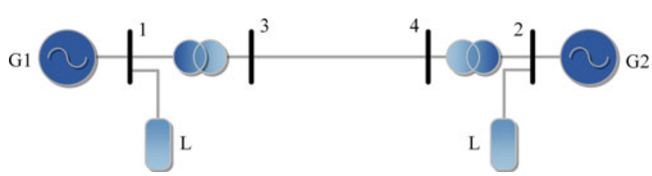

Fig. 6. The 2-machine power system.

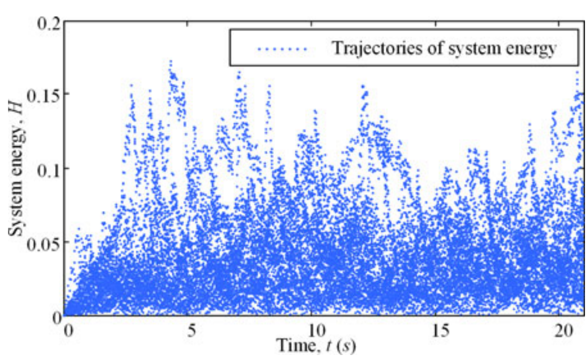

Fig. 7. 20 trajectories of system energy in the 2-machine power system under uncertain variability.

Step 5: Execute *Step 3* and *Step 4* repeatedly, if the iteration number is not enough; go to *Step 6*, if the iteration number is enough.

Step 6: Calculate the probability of the system energy being less than H_B, according to (4).

B. 2-Machine Power System

A 2-machine power system is adopted as the first simulation system, as shown in Fig. 6. The intensity of uncertain variability is defined as σ_1 and σ_2 on Bus 1 and Bus 2, respectively.

In Monte Carlo simulation, each one trial provides the behavior of a single trajectory, so multiple trials are needed to obtain statistical information of the stochastic system. Fig. 7 shows 20 trajectories of the system energy in the 2-machine power system under uncertain variability. Due to the introduction of uncertain variability, the trajectories of the system energy are randomly fluctuant, from which it is hard to obtain insights directly. By statistical analysis of the multiple trials, one obtains the histogram of the system energy, as shown in Fig. 8.

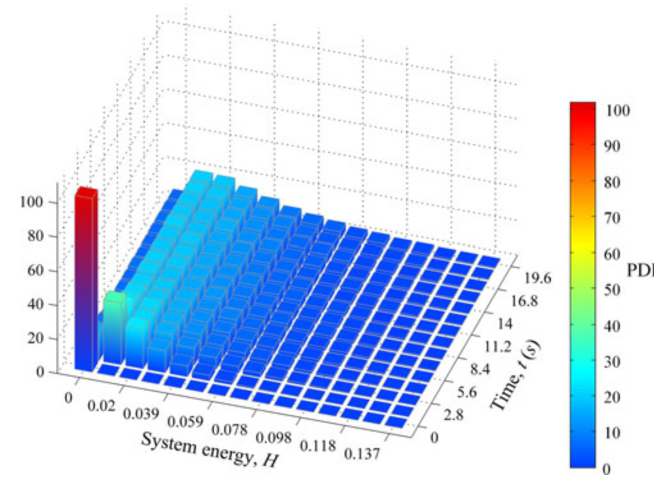

Fig. 8. Probability density function of system energy from Monte Carlo simulation.

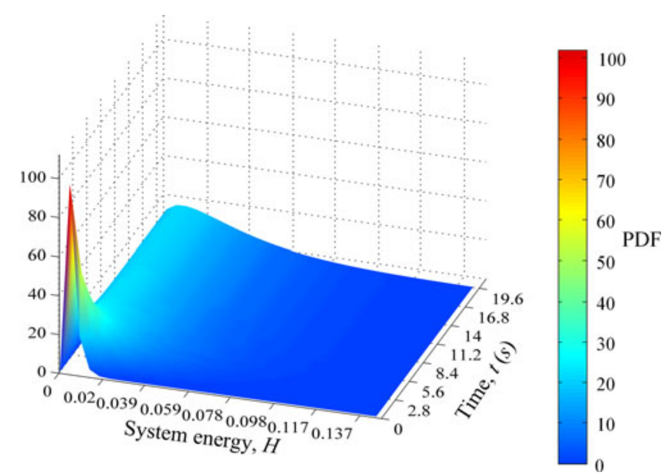

Fig. 9. Probability density function of system energy from the proposed method.

Usually, the histogram of the system energy by Monte Carlo simulation can be regarded as a decent approximation of the probability density function. However, the burdensome computation makes Monte Carlo simulation undesirable. Moreover, due to the unclear mechanism of impacts, Monte Carlo simulation is like a black box. It is desired to develop an analytical method, which can provide the statistical information of the stochastic system directly. The proposed method in Section III provides such a method. Fig. 9 shows the probability density function of the system energy by (12). It can be seen that the probability density function from the proposed method agrees well with that from Monte Carlo simulation.

Furthermore, one can calculate the intra-region probability based on the probability density function, which is proposed as

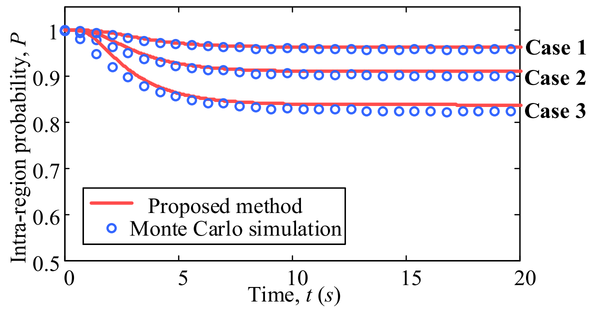

Fig. 10. Intra-region probability under different intensities of uncertain variability (Case 1: $\sigma_1 = \sigma_2 = 0.035$; Case 2: $\sigma_1 = \sigma_2 = 0.040$; Case 3: $\sigma_1 = \sigma_2 = 0.045$).

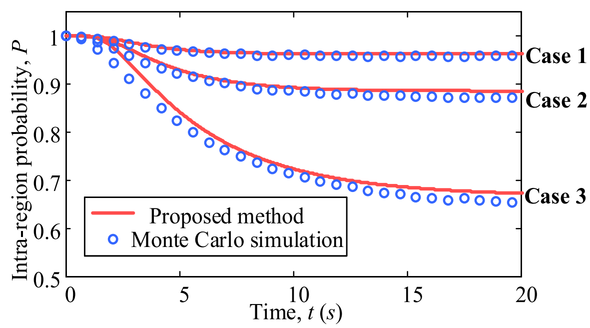

Fig. 11. Intra-region probability under different damping coefficients (Case 1: $D_1 = D_2 = 4$; Case 2: $D_1 = D_2 = 7$; Case 3: $D_1 = D_2 = 10$).

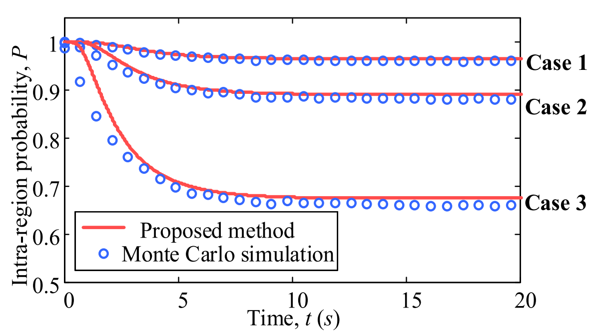

Fig. 12. Intra-region probability under different energy boundaries (Case 1: $H_B = 0.10$; Case 2: $H_B = 0.07$; Case 3: $H_B = 0.04$).

a reasonable index to assess impacts of uncertain variability on power system dynamics. In this simulation case, the intra-region probability is investigated by the proposed analytical method and Monte Carlo simulation. Intra-region probability is simulated in several cases with different intensities of uncertain variability, damping coefficients, and energy boundaries, as shown in Figs. 10–12. Clearly, all analytical results are much close to Monte Carlo simulation results, which demonstrates the high accuracy of the proposed method. In addition, the intra-region probability increases with increasing of the damping coefficient, the decreasing of uncertain variability's intensity, and increasing of energy boundary. It is suggested that the operations on improving system damping, mitigating uncertain variability and improving system tolerance of uncertain variability would be helpful for power system dynamics under uncertain variability.

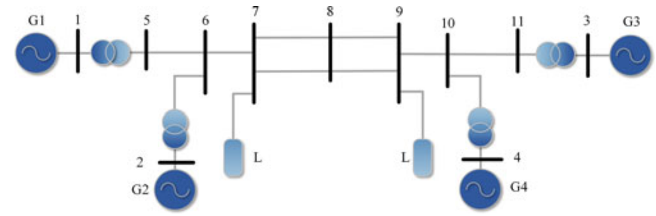

Fig. 13. Kundur's 4-machine 2-area system.

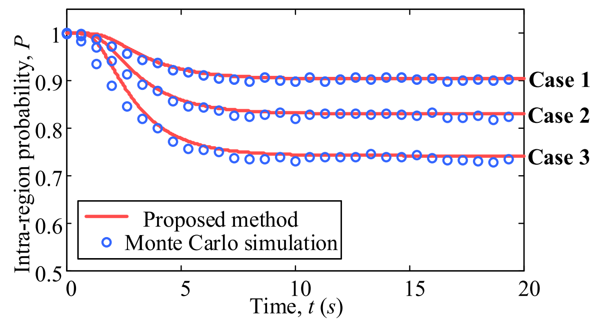

Fig. 14. Intra-region probability under different intensities of uncertain variability (Case 1: $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = 0.060$; Case 2: $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = 0.065$; Case 3: $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = 0.070$).

From Figs. 10–12, it can also be found that the intra-region probability converges to a certain value when $t \rightarrow +\infty$. This certain value is also known as the "steady-state solution" [13], [26], [41]. In other words, the dynamic behavior of power system under uncertain variability seems like "steady-state" after a long period. This phenomenon indicates that the system energy under uncertain variability has a stationary distribution, which is of great interest for study. By the proposed analytical method, it is easy to calculate the "steady-state solution" of the intra-region probability, as shown in (15).

C. Kundur's 4-Machine 2-Area System

Kundur's 4-machine 2-area system is adopted as the second simulation system to show the scalability of the proposed method. The system is shown in Fig. 13, and the parameters can be found in [29]. The loads are modeled by constant impedance models. Then, the system is reduced to a 4-bus system by eliminating all the load buses. The uncertain variability is considered as the stochastic mechanical power inputs of generators [13]. Finally, the stochastic model of this power system can be expressed as (1).

By the proposed method and Monte Carlo simulation, the intra-region probability from several cases with different intensities of uncertain variability, damping coefficients, and energy boundaries is simulated, as shown in Figs. 14–16. Clearly, the results from the proposed method are still close to those from Monte Carlo simulation, which verifies the effectiveness of the proposed method in a multi-machine power system. Furthermore, the same phenomenon that presents the system energy under uncertain variability has a stationary distribution can be observed again.

TABLE I
ACCURACY AND EFFICIENCY COMPARISONS OF THE PROPOSED METHOD AND MONTE CARLO SIMULATION

Number of generators	Monte Carlo simulation		Proposed method	
	Results	Computational time (s)	Results	Computational time (s)
4	0.8980	37.7	0.9020	0.7
10	0.8982	173.3	0.9092	0.7
16	0.8913	438.1	0.8823	0.7
50	0.8599	5045.3	0.8719	0.7

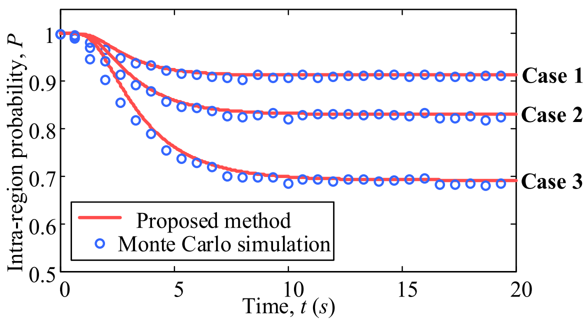

Fig. 15. Intra-region probability under different damping coefficients (Case 1: $D_1 = D_2 = D_3 = D_4 = 7.5$; Case 2: $D_1 = D_2 = D_3 = D_4 = 6.3$; Case 3: $D_1 = D_2 = D_3 = D_4 = 5.1$).

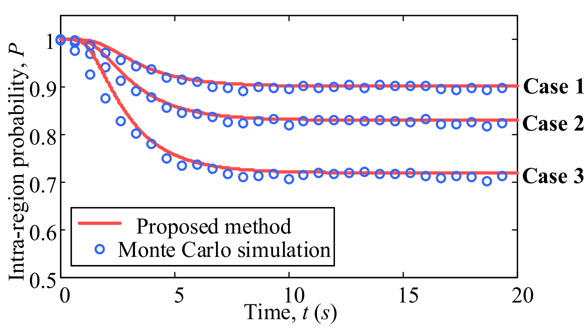

Fig. 16. Intra-region probability under different energy boundaries (Case 1: $H_B = 0.76$; Case 2: $H_B = 0.65$; Case 3: $H_B = 0.54$).

D. Large-Scale Power System

To demonstrate the accuracy of the proposed analytical method in large-scale power systems, several power systems with a different number of generators are simulated (i.e., the Kundur's 4-machine 2-area system, the New England 10-Generator 39-Bus System, the IEEE 16-generator 68-bus power system, and the IEEE 50-generator system). In the Monte Carlo simulation, the simulation time is set to 20 s to obtain the "steady-state solution" of the intra-region probability. Based on the proposed method and Monte Carlo simulation, the accuracy and efficiency comparisons are shown in Table I. It can be seen that the analytical results are close to the Monte Carlo simulation results, which demonstrated the scalability of the proposed method in large-scale power systems. In practice, the computational efficiency is also important. In a power system with 50 generators, it takes 0.7 s by using the proposed method, while 5045.3 s by Monte Carlo simulation. Clearly, the proposed method is many orders of magnitude faster than Monte Carlo simulation.

V. CONCLUSION

With the integration of intermittent renewable energy and plug-in electric vehicles, uncertain variability in power systems attracts significant attention. This paper provides an efficient analysis method for assessing the dynamic impacts of uncertain variability on stable power systems. The main contributions of this paper are given as follows:

1) Intra-region probability is presented as an assessment index for the dynamic impacts of uncertain variability.
2) A novel analytical method is proposed for assessing impacts of uncertain variability on power system dynamics, which can be used in large-scale power systems.
3) It is found that the system energy under uncertain variability has a stationary distribution when the system is stable. Furthermore, the stationary distribution is proved as a multiple of a chi-squared distribution.
4) Insights of increasing system damping, mitigating uncertain variability, and increasing system tolerance of uncertain variability are given to improve power system dynamic characteristics under uncertain variability.

Although the research on power system dynamics under uncertain variability is challenging theoretically and practically, we believe that the proposed stochastic dynamic analysis will serve well in this regard.

APPENDIX A

A. Linearization of Stochastic Model and Energy Function

In practice, the magnitude of uncertain variability is relatively small, so the linearization is reasonable. When the system (1) is stable, the linearized stochastic model can be derived at its equilibrium point, as follows:

$$\begin{cases} d\Delta\delta_i = \omega_0 \Delta\omega_i dt \\ \qquad i = 1, 2, \cdots, n-1 \\ M_i d\Delta\omega_i = -\sum_{j=1, j\neq i}^{n} C_{ij}(\Delta\delta_i - \Delta\delta_j)dt - D_i\Delta\omega_i \\ \qquad\qquad + \sigma_i dB_i(t) \\ \qquad i = 1, 2, \cdots, n \end{cases} \tag{A-1}$$

where $\Delta\delta_i = \delta_i - \delta_n$ and $\Delta\delta_n = 0$; δ_n is the rotor angle of nth generator, which is set as the reference node; and $C_{ij} = E_i E_j B_{ij} \cos(\delta_{is} - \delta_{js})$.

The linearized energy function can be expressed as follows:

$$H = \frac{1}{2}\sum_{i=1}^{n} M_i\omega_0 \Delta\omega_i{}^2 + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=i+1}^{n} C_{ij}(\Delta\delta_i - \Delta\delta_j)^2. \tag{A-2}$$

B. Derivation of the Stochastic Averaging Equation

It can be seen that (A-2) is a quadratic form. By linear transformation, one can transform the linearized energy (A-2) in the standard quadratic form, as follows:

$$H = \boldsymbol{Y}^T\boldsymbol{Y} = y_1{}^2 + \cdots y_i{}^2 \cdots + y_{2n-1}{}^2. \tag{A-3}$$

where $\boldsymbol{Y} = (y_1, \cdots, y_i, \cdots, y_{2n-1})^T$, and $\boldsymbol{Y} = \boldsymbol{C}X$; $\boldsymbol{X} = (\Delta\delta_1, \cdots, \Delta\delta_{n-1}, \Delta\omega_1, \cdots, \Delta\omega_n)^T, \boldsymbol{C}$ is a non-singular matrix with $(2n-1)$th-order.

By the same linear transformation $\boldsymbol{C}$, the system (A-1) can be transformed into the following form:

$$d\boldsymbol{Y} = \boldsymbol{A}\boldsymbol{Y}dt + \boldsymbol{G}d\boldsymbol{B}(t). \tag{A-4}$$

By Itō's lemma [33], one obtains the following Itō equation from (A-3) and (A-4).

$$\begin{aligned} dH &= (\partial H/\partial \boldsymbol{Y})^T d\boldsymbol{Y} + (1/2)tr(\boldsymbol{G}\boldsymbol{G}^T\partial^2 H/\partial \boldsymbol{Y}^2)dt \\ &= \sum_{i=1}^{n}\left(\frac{-2D_i}{M_i}y_{n-1+i}{}^2 + \frac{\sigma_i{}^2\omega_0}{2M_i}\right)dt \\ &\quad + \sqrt{\sum_{i=1}^{n}\left(\frac{2\omega_0\sigma_i{}^2}{M_i}y_{n-1+i}{}^2\right)}dB(t). \end{aligned} \tag{A-5}$$

By utilizing SAM on (A-5) [30], the system energy H weakly converges to a first-order diffusion process as follows:

$$\mathrm{d}H = \overline{m}(H)\mathrm{d}t + \overline{\sigma}(H)\mathrm{d}B(t). \tag{A-6}$$

where

$$\begin{cases} \overline{m}(H) = \frac{1}{S}\int_\Omega \sum\limits_{i=1}^{n}\left(\frac{-2D_i}{M_i}y_{n-1+i}{}^2 + \frac{\sigma_i{}^2\omega_0}{2M_i}\right)ds \\ \qquad = \sum\limits_{i=1}^{n}\left(\frac{\sigma_i{}^2\omega_0}{2M_i}\right) - \sum\limits_{i=1}^{n}\left(\frac{2D_i}{M_i}\cdot\frac{\int_\Omega y_{n-1+i}{}^2 ds}{\int_\Omega 1ds}\right) \\ \overline{\sigma}^2(H) = \frac{1}{S}\int_\Omega \sum\limits_{i=1}^{n}\left(\frac{2\omega_0\sigma_i{}^2}{M_i}y_{n-1+i}{}^2\right) \\ ds = \sum\limits_{i=1}^{n}\left(\frac{2\omega_0\sigma_i{}^2}{M_i}\cdot\frac{\int_\Omega y_{n-1+i}{}^2 ds}{\int_\Omega 1ds}\right) \\ S = \int_\Omega 1ds \\ \Omega = \{(y_1, \cdots, y_i, \cdots, y_{2n-1}) | \, y_1{}^2 + \cdots y_i{}^2 \cdots + y_{2n-1}{}^2 = H\} \end{cases}. \tag{A-7}$$

C. Symmetry of First Type Surface Integral

By using the symmetry of the first type surface integral [42], one has

$$\int_\Omega y_1{}^2 ds = \cdots = \int_\Omega y_i{}^2 ds = \cdots = \int_\Omega y_{2n-1}{}^2 ds. \tag{A-8}$$

Based on (A-3) and (A-8), one can obtain the following equation.

$$\sum_{i=1}^{2n-1}\int_\Omega y_i{}^2 ds = \int_\Omega\left(\sum_{i=1}^{2n-1} y_i{}^2\right)ds = \int_\Omega H ds = H\int_\Omega 1ds. \tag{A-9}$$

Combining (A-9) and (A-10), one obtains

$$\frac{\int_\Omega y_i{}^2 ds}{\int_\Omega 1ds} = \frac{H}{(2n-1)}. \tag{A-10}$$

By incorporating (A-10) into (A-7), the following equations can be deduced

$$\begin{cases} \overline{m}(H) = \sum_{i=1}^{n}(\omega_0\sigma_i{}^2/2M_i) - \sum_{i=1}^{n}(2D_i/M_i)/(2n-1)\cdot H \\ \overline{\sigma}^2(H) = \sum_{i=1}^{n}(2\omega_0\sigma_i{}^2/M_i)/(2n-1)\cdot H \end{cases}. \tag{A-11}$$

Appendix B

To obtain the explicit formulation of the solution to MRSRP (7), an ideal dynamic system under uncertain variability is constructed as:

$$\begin{cases} dx_1(t) = -\frac{\lambda}{2}x_1(t)dt + \frac{\sigma}{2}dB_1(t) \\ dx_2(t) = -\frac{\lambda}{2}x_2(t)dt + \frac{\sigma}{2}dB_2(t) \\ \ldots \\ dx_k(t) = -\frac{\lambda}{2}x_k(t)dt + \frac{\sigma}{2}dB_k(t) \end{cases} \tag{B-1}$$

where k is the order of the ideal system, which equals to $2n-1$; $x_1(t)$, $x_2(t)$, $\cdots$, and $x_n(t)$ denote the system states of the ideal system; and $x_1(0) = x_2(0) = \cdots = x_k(0) = 0$.

The energy function of the ideal system (B-1) can be defined as follows:

$$H = \sum\nolimits_{i=1}^{k} x_i{}^2(t). \tag{B-2}$$

By Itō's lemma [33], one obtains the following Itō equation from (B-1) and (B-2).

$$\begin{aligned} dH &= \sum_{i=1}^{k}\frac{\partial H}{\partial x_i}dx_i + \frac{1}{2}tr\left(\frac{\sigma}{2}\frac{\sigma}{2}\frac{\partial H}{\partial x_i\partial x_j}\right)dt \\ &= \sum_{i=1}^{k} x_i\left[-\lambda x_i(t)dt + \sigma dB_i(t)\right] + 0.25k\sigma^2 dt \\ &= 0.25k\sigma^2 dt - \lambda\sum_{i=1}^{k} x_i{}^2(t)dt + \sigma\sum_{i=1}^{k}\left[x_i(t)dB_i(t)\right] \\ &= 0.25k\sigma^2 dt - \lambda\sum_{i=1}^{k} x_i{}^2(t)dt + \sigma\sqrt{\sum\nolimits_{i=1}^{k} x_i{}^2(t)}dB(t) \\ &= 0.25k\sigma^2 dt - \lambda H dt + \sigma\sqrt{H}dB(t) \\ &= \lambda(0.25k\sigma^2/\lambda - H)dt + \sigma\sqrt{H}dB(t) \\ &= \lambda(\mu - H)dt + \sigma\sqrt{H}dB(t) \end{aligned} \tag{B-3}$$

where $\frac{1}{2}tr(\frac{\sigma}{2}\frac{\sigma}{2}\frac{\partial H}{\partial x_i\partial x_j})dt$ denotes the Wong-Zakai correction part [43]; $\frac{\partial H}{\partial x_i\partial x_j}$ denotes the Hessian matrix of H [44]; and 'tr'

denotes the trace of a matrix (i.e., the summation of all diagonal elements) [45].

It can be seen that the simplified model (B-3) of the ideal system (B-1) is also an MRSRP and the same as (7). Furthermore, one can obtain the explicit formulation of the solution to (7) based on the analytical solution to (B-3). The analytical solution to the ideal system (B-1) is solved as follows:

$$\begin{cases} x_1(t) = \int_0^t \frac{1}{2}\sigma e^{\lambda(s-t)/2} \mathrm{d}B_1(s) \\ x_2(t) = \int_0^t \frac{1}{2}\sigma e^{\lambda(s-t)/2} \mathrm{d}B_2(s) \\ \ldots \\ x_k(t) = \int_0^t \frac{1}{2}\sigma e^{\lambda(s-t)/2} \mathrm{d}B_k(s) \end{cases}. \tag{B-4}$$

Substituting (B-4) into (B-2), the explicit formulation of the solution to (B-3) can be obtained by

$$H = \sum_{i=1}^{k} \left[\int_0^t \frac{\sigma}{2} e^{\lambda(s-t)/2} \mathrm{d}B_i(s) \right]^2. \tag{B-5}$$

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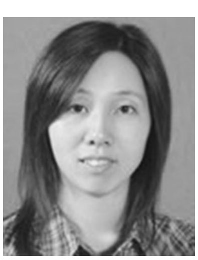

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