Extracting Rare Failure Events in Composite System Reliability Evaluation Via Subset Simulation

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***Abstract*—This paper proposes an efficient method for evaluating composite system reliability via subset simulation. The central idea is that a small failure probability can be expressed as a product of larger conditional probabilities, thereby turning the problem of simulating a rare failure event into several conditional simulations of more frequent intermediate failure events. In existing methods, system states are simply assessed in a binary secure/failure manner. To fit into the context of subset simulation, the adequacy of system states is parametrized with a metric based on linear programming, thus allowing for an adaptive choice of intermediate failure events. Samples conditional on these events are generated by Markov chain Monte Carlo simulation. The proposed method requires no prior information before imulation. Different models for renewable energy sources can also be accommodated. Numerical tests show that this method is significantly more efficient than standard Monte Carlo simulation, especially for simulating rare failure events.**

***Index Terms*—Linear programming, Markov chain Monte Carlo, Monte Carlo methods, power system reliability, rare event simulation, risk analysis, subset simulation.**

NOMENCLATURE

Sets and Indices:

i	Index for intermediate failure events and simulation levels.
F_0	Intact sample space.
F_i	Nested subsets of F_0 with threshold b_i.
F	Target failure domain with threshold b.
m	Total number of simulation levels.
i_b	Bus index.
N_b	Total number of buses.
L	Number of branches in the system.
$\mathcal{C}$	Set of system components with cardinality d.
$\mathcal{D}$	Set of discrete-state components.
j	Index for system components.
k	Index for sampled states within a simulation level.
N	Number of samples per level.
l	Index for Markov chains within a simulation level.
N_c	Number of Markov chains per level.
h	Index for sampled states within a Markov chain.
N_s	Number of samples per chain.
N_T	Total number of samples.

Variables in the Optimization Problems:

L_C	Amount of load curtailment for the current state.
G_D	Metric for system deficiency defined by (5).
d_{i_b}	Supplied load at bus i_b.
D	N_b-vector composed of d_{i_b}.
p_{i_b}	Power generation at bus i_b.
P	N_b-vector composed of p_{i_b}.
$d_{i_b}^{\max}$	Load demand at bus i_b.
$D^{\max}$	N_b-vector composed of $d_{i_b}^{\max}$.
$P^{\max}$	N_b-vector of generation capacity.
$P^{\min}$	N_b-vector of minimum output of generators.
Γ	$L \times N_b$ matrix of power transfer distribution factors.
$\overline{F}$	L-vector of maximum line flow.
$\underline{F}$	L-vector of minimum line flow.
β	Load scale factor in (5).
DI	Deficiency Index defined by (8).

Variables in the Evaluation Procedure:

p_F	Failure probability.
Y	General response variable.
p_0	Level probability.
$\boldsymbol{\theta}$	State variable for the target system.

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$\boldsymbol{\theta}_k$	kth sampled state of the system.
θ_j	State variable for the jth system component.
$\theta_{k,j}$	kth sampled state of the jth component.
$\pi(\cdot)$	Joint distribution function of $\boldsymbol{\theta}$.
$\pi_j(\cdot)$	Marginal PDF/PMF of θ_j.
$\Pi_j(\cdot)$	Marginal CDF of θ_j.
U_j	Random variable uniformly distributed on $[0,1]$.
$p_j^*(\cdot\|\cdot)$	Proposal PDF for component j.
ξ_j	Random variable generated by the proposal PDF.
$\tilde{\boldsymbol{\theta}}$	Candidate state of the system.
$\phi(\cdot)$	Standard Gaussian PDF.
$\Phi(\cdot)$	Standard Gaussian CDF.
ω_j	State variable for component j in the Gaussian space.
r_j	Acceptance ratio in the Metropolis algorithm.
$\boldsymbol{\theta}_k^{(0)}$	kth generated sample at level 0.
$\mathrm{DI}_k^{(0)}$	Deficiency Index corresponding to $\boldsymbol{\theta}_k^{(0)}$.
$b_k^{(i)}$	kth sorted Deficiency Index at level i.
$p_k^{(i)}$	Exceedance probability corresponding to $b_k^{(i)}$.
$\boldsymbol{\theta}_{l0}^{(i)}$	Seed for the lth Markov chain at level i.
$\boldsymbol{\theta}_{lh}^{(i)}$	hth state obtained in the lth Markov chain at simulation level i.
$\mathrm{DI}_{lh}^{(i)}$	Deficiency Index corresponding to $\boldsymbol{\theta}_{lh}^{(i)}$.

I. Introduction

Monte Carlo simulation (MCS) is widely used in power system reliability evaluation due to its robustness to the dimension of the problem, capability of handling contingencies of all orders, and flexibility in accommodating various power system models and operation modes [1], [2]. Nevertheless, an intrinsic drawback of this method is the low efficiency in estimating failures with small probabilities. Essentially, information from rare samples that lead to failure is required for estimation of small failure probabilities. On average it requires a formidable number of samples before enough failure events occur, because the number of samples required to achieve a given convergence criterion is inversely proportional to the probability of the failure event. This limitation is further exacerbated by increasingly interconnected power systems with high reliability.

Improving the computational efficiency of composite system reliability evaluation has been a long-standing interest. Assorted variance reduction techniques (VRTs) are available, including stratified sampling, control variates, and antithetic variates [3]. Control variates and antithetic variates reduce variance by constructing a correlated variable that has the same mean value as the original random variable but lower variance [4]. Stratified sampling splits the sample space into several non-overlapping subpopulations called *strata* before simulation to improve the representativeness of sampled states [5], [6]. The efficiency of the above methods relies on an appropriate alteration of the probability distributions of the estimator, which inevitably requires some prior knowledge about failure before simulation. Their performance is thus system-specific.

Importance sampling (IS) [7]–[10] is one of the most popular VRTs applied to the reliability evaluation of power systems. In IS, an importance sampling density (ISD) is chosen to generate samples that lead to failure more frequently so as to gain more information about failure, and the choice of the ISD is crucial to the performance of IS. Recently, a promising scheme for constructing an appropriate ISD in power system engineering based on cross-entropy (CE) methods has appeared in the literature [11]–[13]. In the CE-based MCS method, the state variables representing generation and transmission equipment are properly *distorted* according to a CE-based optimization process. This method has also been successfully applied to systems with renewable energy sources [14]–[16]. In order to get an analytical update rule for the optimization process, the probability distributions of the system state variables to be distorted are confined within the natural exponential family.

As an emerging alternative to MCS, population-based intelligent search (PIS) relies on metaheuristics that have a population of solutions as its core [17]–[19]. Algorithms originally developed as optimization tools are utilized to discover states that have a great contribution to the interested indices. However, problems associated with the convergence process, prevention of revisited states, and memory management in PIS methods still require further research.

In an attempt to solve the above-mentioned limitations, we introduce subset simulation (SS), a simulation-based reliability method originally developed for reliability analysis of structures in civil engineering [20]. Because of its independence of the inherent properties of a system, this method is useful for applications in different areas of science and engineering [21]. Unlike the VRTs that alter the probability distribution of system states, SS utilizes a fundamental concept in probability: conditional probability. A small failure probability can be expressed as a product of larger conditional probabilities of the intermediate events, which effectively converts a rare event simulation problem into a sequence of more frequent ones.

No room is left for intermediate failure events in most of the existing methods, since system states are simply characterized as either success or failure [2]. In well-being analysis [22], success states are split into the healthy and marginal states. Although the marginal state is a plausible candidate for an intermediate failure event, the partition between healthy and marginal state is inflexible. A higher-level granularity of the degree of adequacy is required in order to apply SS. In this paper, we define a new metric for system deficiency considering line flow constraints by constructing a linear program. This metric reflects the degree of adequacy of system states, which makes possible an adaptive choice of intermediate failure events.

Besides the choice of intermediate failure events, efficient generation of samples conditional on these events is required by SS, which is generally a highly nontrivial problem. This however can be done by a class of powerful simulation techniques called Markov chain Monte Carlo (MCMC) methods [3]. We customize the modified Metropolis algorithm [20] by transforming discrete input variables into a standard Gaussian space via the inversion principle in order to accommodate the discrete-state components in power systems.

We propose a framework for composite system reliability evaluation with SS. Since the state space is not altered, this method is robust to the scale and configuration of the target system. It requires no prior knowledge about failure before simulation. Sophisticated models for components of the power system including multilevel load and renewable energy sources are also easily accommodated. Apart from reliability indices such as loss of load probability (LOLP) and expected energy not supplied (EENS), the probability distribution of the degree of system adequacy can be obtained without extra computation, providing useful information for decision-makers in power system planning.

The paper proceeds as follows. Section II introduces Subset Simulation and defines intermediate failure events. The modified Metropolis algorithm is described in Section III. The proposed procedure for reliability evaluation of composite systems is presented in Section IV. Various results are illustrated in Section V, and conclusions follow.

II. Intermediate Failure Events for Composite System Reliability Evaluation

In this section, we define intermediate failure events for composite system reliability evaluation. After a brief introduction to subset simulation and how it utilizes intermediate failure events to simulate small failure events efficiently, we define a metric for system deficiency taking into consideration the security constraints of the transmission network through linear programming. It is proved that this metric is valid for parametrizing the intermediate failure events of power systems. A sequence of intermediate failure events can be chosen by varying this parameter.

A. Overview of Subset Simulation

Subset simulation was originally developed for seismic risk analysis of building structures subjected to stochastic earthquake motions [20], [23], where the problem involved a large number (theoretically infinite) number of random variables arising primarily from the time-domain stochastic description of ground motions. Applications of SS to different disciplines have appeared, e.g., in aerospace engineering [24], [25], fire engineering [26], geotechnical engineering [27], [28], nuclear engineering [29], [30], and meteorology [31]. Performance of subset simulation in a set of benchmark problems is presented in [32] and [33].

Subset simulation is based on the notion of representing a small failure probability p_F as a product of larger conditional probabilities. Consider a sequence of nested subsets of the sample space, beginning with the intact sample space F_0 and ending with the *target* failure domain F:

$$F_0 \supset F_1 \supset \cdots \supset F_{m-1} \supset F_m = F. \tag{1}$$

Failure events reside in subsets $F_1, \cdots, F_{m-1}$ are called *intermediate failure events*. As a result, the failure probability $p_F = P(F)$ can be written as a product of conditional probabilities:

$$\begin{aligned} p_F = P(F_m) &= P\left(\bigcap_{i=1}^{m} F_i\right) \\ &= P(F_1)\prod_{i=1}^{m-1} P(F_{i+1}|F_i). \end{aligned} \tag{2}$$

By choosing appropriate intermediate failure events, although p_F is small, $P(F_1)$ and all conditional probabilities $P(F_i|F_{i-1})$ can be made sufficiently large so that they can be efficiently estimated by simulation. Thus, the problem of simulating rare failure events in the original sample space can be in principle replaced by a sequence of simulations of more frequent events in several conditional probability spaces.

Let $Y(\boldsymbol{\theta})$ be a scalar response variable that depends on the current system state $\boldsymbol{\theta}$. In the actual SS implementation, without much loss of generality, the target failure event is parametrized by $Y(\boldsymbol{\theta})$ exceeding a specified threshold level b, that is, $F = \{\boldsymbol{\theta} \,|\, Y(\boldsymbol{\theta}) > b\}$. The sequence of intermediate failure domains F_i can then be defined as $F_i = \{\boldsymbol{\theta} \,|\, Y(\boldsymbol{\theta}) > b_i\}, i = 1, \ldots, m$, where $b_1 < b_2 < \cdots < b_m = b$ is a sequence of intermediate threshold values.

The choice of intermediate thresholds determines the values of the conditional probabilities $P(F_i|F_{i-1})$, and hence the efficiency of SS. One can use predetermined $\{b_1, \ldots, b_m\}$. But then we will lose control of the conditional probabilities, which leads to suboptimal performance. In fact, too big a conditional probability leads to more simulation levels m, whereas too small a conditional probability leads to more samples required in a single level. It is more efficient to choose the thresholds b_i adaptively, so that the estimated conditional probabilities are equal to a fixed value p_0. This can be easily done using the ordered statistic of the generated samples of Y (see Section IV). Detailed study of the optimal p_0 is presented in [34].

For instance, if the failure probability of a certain composite system $p_F \sim 10^{-4}$, it would be inefficient to simulate by a direct MCS. However, if conditional simulations are conducted and thresholds of intermediate failure events are chosen so that $P(F_1), P(F_{i+1}|F_i) \sim 0.1, i = 1, 2, 3$, the efficiency can be significantly improved.

B. Target Failure Events in Composite Power Systems

For reliability evaluation of generating systems, the response Y can be simply chosen as the difference between system load and the sum of online generation capacity, and b chosen as 0. For composite systems, the target failure events are instead defined as the inability to generate and transport sufficient energy to satisfy the demands at all bulk supply points without violating the system operational constraints [35]. Mathematically, the response $Y(\boldsymbol{\theta})$ is identified with $L_C(\boldsymbol{\theta})$, a nonnegative real number indicating the amount of load curtailment needed in this state. The target failure domain is defined as $F = \{\boldsymbol{\theta} \,|\, L_C(\boldsymbol{\theta}) > 0\}$.

Let us first review the process of determining $L_C(\boldsymbol{\theta})$. For a system state $\boldsymbol{\theta}$, if no outages happen, or security constraints are not violated after a contingency analysis, then $L_C(\boldsymbol{\theta}) = 0$. For those states where violation of security constraints exists, post-contingency remedial actions such as generation re-dispatch and/or load shedding should be conducted. Such actions are usually modeled as a linear program (3) which minimizes load curtailment. The DC modeling of the power system is used in this paper. The calculation of the Γ matrix could be found in [36]:

$$\begin{aligned} \underset{p_{i_b}, d_{i_b}}{\text{Minimize}} \quad & L_C = \sum_{i_b=1}^{N_b} d_{i_b}^{\max} - \sum_{i_b=1}^{N_b} d_{i_b} \\ \text{subject to} \quad & \sum_{i_b=1}^{N_b} p_{i_b} - \sum_{i_b=1}^{N_b} d_{i_b} = 0, \\ & P^{\min} \le P \le P^{\max}, \\ & 0 \le D \le D^{\max}, \\ & \overline{F} \ge \Gamma(P - D) \ge \underline{F}. \end{aligned} \tag{3}$$

C. Parametrizing Intermediate Failure Events in Composite Power Systems

It is impossible to create intermediate events in the context of SS by simply relaxing L_C, because all success states with various degrees of adequacy *collapse* into a single L_C that equals zero. That is, all success states, which constitute the majority of the samples obtained by a direct MCS and contain a variety of generation and transmission contingencies, lead to the same result: $L_C(\boldsymbol{\theta}) = 0$.

Although the marginal state in a well-being analysis [22] is a plausible candidate for an intermediate failure event, the partition between healthy and marginal state is inflexible. Only one intermediate failure event is introduced and this choice of intermediate failure event may be far from optimal.

In order to apply SS, a response variable Y should be chosen so that the samples generated are *driven* by this response to gradually populate towards the target failure region. The response Y should be defined so that:

1) It should be a multivalued metric. Intermediate failure events can thus be created adaptively through varying this metric.
2) Whether the sampled state is a success or a failure can be determined from this metric. That is, assuming that the less the Y, the more secure the system state, there should be a threshold b that $Y(\boldsymbol{\theta}) \le b \Leftrightarrow L_C(\boldsymbol{\theta}) = 0$ and $Y(\boldsymbol{\theta}) > b \Leftrightarrow L_C(\boldsymbol{\theta}) > 0$ for a certain state $\boldsymbol{\theta}$.

D. Constructing a Valid Metric

Constructing a valid metric that satisfies the two requirements is nontrivial. In fact, an intuitive metric for adequacy—the difference between the sum of online generation capacity and the system load—is invalid.

In order to construct such a metric, we modify LP (3) and base the metric on the modified program. The third constraint is first removed, which indicates that the generators can supply an amount of load that is more than actually needed. Then we set $D = \beta D^{\max}$, where β is a scalar representing that the amount of load at all buses are increased or decreased to β times of the original values. Substituting $D = \beta D^{\max}$ into the objective function of (3) yields

$$L_C = \sum_{i_b=1}^{N_b} d_{i_b}^{\max} - \sum_{i_b=1}^{N_b} d_{i_b} = (1-\beta)\sum_{i_b=1}^{N_b} d_{i_b}^{\max} = G_D \tag{4}$$

which is minimized by the optimization and defined as the metric for system deficiency.

Given a sampled state of the composite system, the modified linear program is shown as follows:

$$\begin{aligned} \underset{p_{i_b}, \beta}{\text{minimize}} \quad & G_D = (1-\beta)\sum_{i_b=1}^{N_b} d_{i_b}^{\max} \\ \text{subject to} \quad & \sum_{i_b=1}^{N_b} p_{i_b} - \sum_{i_b=1}^{N_b} \beta d_{i_b}^{\max} = 0, \\ & P^{\min} \le P \le P^{\max}, \\ & \overline{F} \ge \Gamma(P - \beta D^{\max}) \ge \underline{F}. \end{aligned} \tag{5}$$

For this linear program only, we assume that all the bus loads follow the same pattern of variation. This assumption is indispensable in that it ensures $G_D(\boldsymbol{\theta}) \le 0 \Rightarrow L_C(\boldsymbol{\theta}) = 0$, as we will see later. Notably, N_b decision variables of LP (3) that constitute the vector D are replaced by a scalar β, which considerably reduces the scale of this LP.

We will next show that G_D is a valid metric. From the formulation of optimization (5) we can see that G_D satisfies the first condition listed in Section II-C. For the second condition, we claim that for a certain sampled state $\boldsymbol{\theta}$ of the system, if $L_C(\boldsymbol{\theta})$ is determined by optimization (3) and $G_D(\boldsymbol{\theta})$ is obtained by (5), then the following equations hold:

$$G_D(\boldsymbol{\theta}) \le 0 \Leftrightarrow L_C(\boldsymbol{\theta}) = 0 \tag{6}$$

$$G_D(\boldsymbol{\theta}) > 0 \Leftrightarrow L_C(\boldsymbol{\theta}) > 0. \tag{7}$$

To prove the above equations, we first show that (6) holds. Assume that $G_D(\boldsymbol{\theta}) \le 0$. $\{\hat{\beta} = 0, \hat{p}_{i_b} = 0\}$ is always a feasible solution to (5) (both generation and load are set to zero): Such a feasible solution leads to an objective function $\widehat{G}_D = \sum_{i_b=1}^{N_b} d_{i_b}^{\max} > 0$. Since (5) is a linear program, its feasible region is a convex polyhedron, and its objective function is a real-valued affine function defined on this polyhedron. Hence, there exists a feasible solution $\{\tilde{p}_{i_b}, \tilde{\beta}\}$ with objective function $\widetilde{G}_D$ that satisfies $\widehat{G}_D > \widetilde{G}_D = 0 \ge G_D(\boldsymbol{\theta})$. This implies that $\tilde{\beta} = 1$. Then, $\{\tilde{p}_{i_b}, \tilde{d}_{i_b}\}$ where $\tilde{d}_{i_b} = \tilde{\beta} d_{i_b}^{\max} = d_{i_b}^{\max}$ must be a feasible solution to (3) because all the constraints are satisfied. Substituting $\tilde{d}_{i_b} = d_{i_b}^{\max}$ into the objective function of (3) yields $\widetilde{L}_C = 0$. Since load curtailment is a nonnegative value, $\{\tilde{p}_{i_b}, \tilde{d}_{i_b}\}$ is not only a feasible, but also an optimal solution to (3), and thus $L_C(\boldsymbol{\theta}) = 0$.

Next, suppose $L_C(\boldsymbol{\theta}) = 0$. Denote an optimal solution to optimization (3) by $\{p'_{i_b}, d'_{i_b}\}$. In this case $d'_{i_b} = d_{i_b}^{\max}$ must hold true for $i_b = 1, \ldots, N_b$. Therefore, $\{p'_{i_b}, \beta' = 1\}$ must be a feasible solution to (5) with objective function $G'_D = 0$, because we can again verify that all the constraints are satisfied. Since linear program (5) tries to minimize $G_D(\boldsymbol{\theta})$, we have $G_D(\boldsymbol{\theta}) \le G'_D = 0$.

For (7), since $L_C(\boldsymbol{\theta})$ is nonnegative, (6) implies (7).

E. Generating Intermediate Failure Events With a Deficiency Index

Through the above metric, various degrees of adequacy corresponding to different success states are revealed by system deficiency G_D. One problem is that when load shedding occurs, although (7) holds, $G_D(\boldsymbol{\theta})$ is not necessarily equal to $L_C(\boldsymbol{\theta})$. The amount of $L_C(\boldsymbol{\theta})$ with regard to each sampled state is of interest to power system planners. It is also related to energy-related reliability indices such as EENS. We therefore propose a Deficiency Index (DI) that equals to the amount of load curtailment when the system fails. This index appropriately reveals the degree of adequacy of a system state:

$$\mathrm{DI}(\boldsymbol{\theta}) = \begin{cases} G_D(\boldsymbol{\theta}), & G_D(\boldsymbol{\theta}) \leq 0, \\ L_C(\boldsymbol{\theta}), & G_D(\boldsymbol{\theta}) > 0. \end{cases} \tag{8}$$

When assessing the reliability of the composite power system, we define the response $Y(\boldsymbol{\theta}) = \mathrm{DI}(\boldsymbol{\theta})$. After a certain state is sampled, the LP (5) is first solved to determine the value of G_D. If $G_D > 0$, then this state is a failure, and LP (3) needs to be solved to determine the amount of load curtailment.

All possible states of the power system can be parametrized with the parameter DI so that the sequence of intermediate failure events $\{F_i : i = 1, \ldots, m-1\}$ can be chosen by varying the parameter. That is, intermediate failure domains can be defined as

$$F_i = \{\boldsymbol{\theta} \,|\, \mathrm{DI}(\boldsymbol{\theta}) > b_i\} \tag{9}$$

where $b_1 < \cdots < b_m = 0$ is a sequence of intermediate threshold values.

III. Efficient Generation of Conditional Samples

Besides the determination of intermediate failure events, one needs to generate samples conditional on these events. Here we customize the Metropolis algorithm used in the conditional simulations to accommodate the discrete-state components in composite power systems.

Let the set of system components be $\mathcal{C}$, with index j and cardinality d. The components represented by discrete variables make up a set $\mathcal{D}$, and $\mathcal{D} \subseteq \mathcal{C}$. The vector $\boldsymbol{\theta}_k = [\theta_{k,1}, \ldots, \theta_{k,d}]$ indicates the k-th sampled state of system. State variables $\theta_{k,j}$ can be either continuous or discrete variables with marginal probability distribution/mass functions $\pi_j(\cdot)$. Since no limitation is imposed on the probability distribution of the state variables, $\theta_{k,j}$ could be a discrete variable following Bernoulli distribution which represents a two-state generator or transmission line, a discrete variable obeying categorical distribution that models multilevel load, or a continuous variable following Beta distribution which represents solar irradiation. Their joint distribution function is represented by $\pi(\boldsymbol{\theta})$.

In the conditional simulations, one needs to obtain samples $\boldsymbol{\theta}_k \sim \pi(\cdot|F_i)$. Although one can use a direct MCS approach to obtain samples that lie in failure region F_i, it is inefficient to do so. Here we describe the algorithm for efficiently generating samples according to the conditional distribution $\pi(\cdot|F_i)$, which will be used in the SS algorithm (see Section IV). The algorithm is a modified version of the original Metropolis algorithm so that it works even when the number of random variables is large without suffering from the curse of dimension [20], [37]. The significance of Metropolis algorithm to power system reliability evaluation is that we can simulate a sample of the power system having the conditional distribution $\pi(\cdot|F_i)$, then the next state of the Markov chain will also be distributed as $\pi(\cdot|F_i)$.

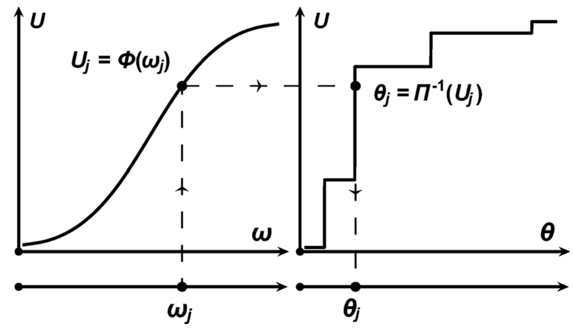

Fig. 1. Illustration of the proposed transformation.

Let $p_j^*(\xi|\theta)$, the *proposal PDF*, be a univariate PDF for ξ with symmetry property $p_j^*(\xi|\theta) = p_j^*(\theta|\xi)$. Generate a Markov chain of samples $\{\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \ldots\}$ from a given sample $\boldsymbol{\theta}_1$ by computing $\boldsymbol{\theta}_{k+1}$ from $\boldsymbol{\theta}_k$ as follows:

1) *Generate a candidate state* $\tilde{\boldsymbol{\theta}}$: Simulate ξ_j from the proposal PDFs $p_j^*(\xi_j|\theta_j)$ for each component $j = 1, \ldots, d$, and compute the ratio $r_j = \pi(\xi_j)/\pi(\theta_{k,j})$. Set $\tilde{\theta}_j = \xi_j$ with probability $\min\{1, r_j\}$, and $\tilde{\theta}_j = \theta_{k,j}$ otherwise.
2) *Accept/reject* $\tilde{\boldsymbol{\theta}}$: If $\tilde{\boldsymbol{\theta}} \in F_i$, accept it as the next sample, i.e., $\boldsymbol{\theta}_{k+1} = \tilde{\boldsymbol{\theta}}$. Otherwise reject it and $\boldsymbol{\theta}_{k+1} = \boldsymbol{\theta}_k$.

The above algorithm is suitable for components that are represented by continuous variables. However, many components of composite power systems are usually represented by discrete-state models. In view of this, we first transform the discrete variables into the standard Gaussian space according to the inversion principle. Let $\Pi_j(\cdot)$ be the marginal CDF of θ_j, then $U_j = \Pi_j(\theta_j)$ is uniformly distributed on $[0, 1]$. If we further perform a transformation by

$$\omega_j = \Phi^{-1}(U_j) = \Phi^{-1}(\Pi_j(\theta_j)) \tag{10}$$

where $\Phi(\cdot)$ is the standard Gaussian CDF, then ω_j will be standard Gaussian. Transformation from ω_j to θ_j is done by the inverse of (10), which is illustrated in Fig. 1.

With the above transformation, the modified Metropolis algorithm proceeds as follows:

1) Obtain $\omega_{k,j}$ for all $j \in \mathcal{D}$.
2) For components represented by continuous variables, proceed as before. For $j \in \mathcal{D}$, simulate ξ_j from a continuous proposal PDF, and compute the ratio $r_j = \phi(\xi_j)/\phi(\omega_{k,j})$, where $\phi(\cdot)$ is the PDF of standard Gaussian distribution. Set $\tilde{\omega}_j = \xi_j$ with probability $\min\{1, r_j\}$, and $\tilde{\omega}_j = \omega_{k,j}$ otherwise.
3) Get $\tilde{\boldsymbol{\theta}}$ by transforming $\tilde{\omega}_j$, $j \in \mathcal{D}$ back into the discrete space.
4) Accept/reject $\tilde{\boldsymbol{\theta}}$ as before.

The Metropolis algorithm is thus adapted for discrete-state components. Good candidates for a proposal PDF in a continuous space, for example the uniform and Gaussian PDF centered at the current sample, can be readily adopted in simulating discrete-state systems.

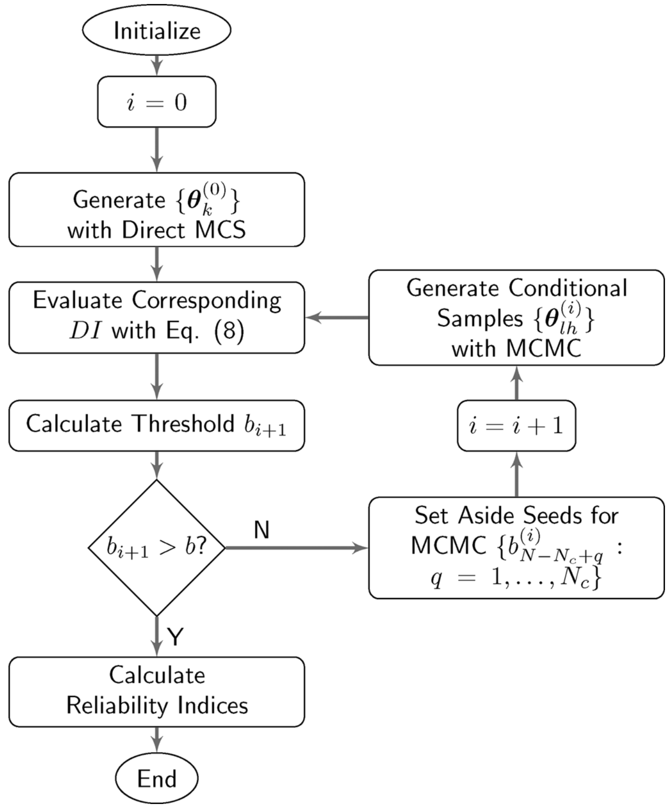

Fig. 2. Brief flowchart for the proposed evaluation procedure.

IV. Proposed Evaluation Procedure

In this section, we first illustrate the procedure for reliability evaluation of composite systems via subset simulation. A brief flowchart is also shown in Fig. 2. Then, the calculation of reliability indices as well as some additional aspects is described. The readers are also encouraged to refer to [21] for a comprehensive view of the subset simulation method.

A. Illustration of the Evaluation Procedure

Input: level probability p_0, and the number of samples per level N. Then the number of Markov chains $N_c = p_0 N$, and number of samples per chain $N_s = p_0^{-1}$. N and p_0 should be chosen that N_c and N_s are integers. The number of simulation levels m will be determined in this procedure. Notice that $N = N_c N_s$.

Simulation Level 0 (Direct Monte Carlo)

1) Generate samples of system states $\{\boldsymbol{\theta}_k^{(0)} : k = 1,\ldots,N\}$ using direct MCS. Evaluate all sampled states and calculate their corresponding $\{DI_k^{(0)}\}$ with (8).
2) Sort $\{DI_k^{(0)} : k = 1,\ldots,N\}$ in ascending order to give a new permutation of DI, denoted by $\{b_k^{(0)} : k = 1,\ldots,N\}$. The value of $b_k^{(0)}$ gives the estimate of b corresponding to the exceedance probability $p_k^{(0)} = P(\mathrm{DI} > b)$ where
$$p_k^{(0)} = \frac{N-k}{N}, \quad k = 1,\ldots,N. \tag{11}$$
3) Plot $\{(p_k^{(0)}, b_k^{(0)}) : k = 1,\ldots,N - N_c\}$ to give the complementary cumulative distribution function (CCDF) of DI with exceedance probability ranging from $(1-N^{-1})$ to p_0. The regime for probabilities below p_0 shall be further estimated by higher levels of simulation.
4) Set $b_1 = b_{N-N_c}^{(0)}$ to be the threshold for the next intermediate failure event. The intermediate failure events are thereby adaptively chosen. Let $\{\boldsymbol{\theta}_{l0}^{(1)} : l = 1,\ldots,N_c\}$ be the N_c samples of system state corresponding to $\{b_{N-N_c+q}^{(0)} : q = 1,\ldots,N_c\}$. These samples are used as the seeds for the N_c Markov chains that generate samples conditional on $F_1 = \{\boldsymbol{\theta} \,|\, \mathrm{DI}(\boldsymbol{\theta}) > b_1\}$ at simulation Level 1.

Simulation Level $i = 1,\ldots,m-1$ (MCMC)

1) Generate N_s conditional samples $\{\boldsymbol{\theta}_{lh}^{(i)} : h = 1,\ldots,N_s\}$ from each seed $\boldsymbol{\theta}_{l0}^{(i)} (l = 1,\ldots,N_c)$ with the modified Metropolis algorithm described in Section III. This gives N_c Markov chains, each with N_s samples. The total number of samples at Level i still equals to $N = N_c N_s$. The seeds are discarded after use in order to reduce the correlation between samples at different simulation levels, which is slightly different from the original SS algorithm described in [20]. Evaluate these system samples and calculate the corresponding $\{\mathrm{DI}_{lh}^{(i)} : l = 1,\ldots,N_c; h = 1,\ldots,N_s\}$.
2) Sort all $\mathrm{DI}_{lh}^{(i)}$ in ascending order to give the list $\{b_k^{(i)} : k = 1,\ldots,N\}$, whose corresponding exceedance probabilities are
$$p_k^{(i)} = p_0^i \frac{N-k}{N}, \quad k = 1,\ldots,N. \tag{12}$$
3) Plot $\{(p_k^{(i)}, b_k^{(i)}) : k = 1,\ldots,N - N_c\}$ to give the CCDF of DI with exceedance probability ranging from $p_0^i(1 - N^{-1})$ to p_0^{i+1}.
4) Set $b_{i+1} = b_{N-N_c}^{(i)}$.
   a) If $b_{i+1} > b = 0$, the failure region is reached. Plot the $\{(p_k^{(i)}, b_k^{(i)}) : k = N - N_c + 1,\ldots,N\}$ to cover the probability range below p_0^{i+1}, since no further simulation level will be carried out to obtain a better estimate of this probability regime. Set $m = i + 1$, and the simulation ends.
   b) Else, let $\{\boldsymbol{\theta}_{l0}^{(i+1)} : l = 1,\ldots,N_c\}$ be the N_c samples of system state corresponding to $\{b_{N-N_c+q}^{(i)} : q = 1,\ldots,N_c\}$. These are used as the seeds for the N_c Markov chains that generate samples conditional on $F_{i+1} = \{\boldsymbol{\theta} \,|\, \mathrm{DI}(\boldsymbol{\theta}) > b_{i+1}\}$ at simulation Level $i+1$. Set $i = i + 1$ and enter the next simulation level.

B. Calculation of the Reliability Indices

In the proposed evaluation procedure, the conditional sampling stops once the target failure region is reached. Therefore all plotted failure states are obtained at simulation level $m - 1$. Though correlated, these failure states are conditional on the same intermediate failure event F_{m-1}. This allows us to estimate some of the reliability indices using conditional expectation.

The reliability index LOLP can be easily obtained through the CCDF curve:
$$\mathrm{LOLP} = P(\mathrm{DI} > 0). \tag{13}$$

The calculation of energy-related reliability indices involves the determination of the expected value of L_C. Such a expected value itself is one of the reliability indices, EPNS (expected power not served):

$$\begin{aligned}\text{EPNS} &= E[L_C] \\ &= E[\text{DI}|\text{DI} > 0] \times \text{LOLP}. \end{aligned} \quad (14)$$

Nonsequential MCS can also provide unbiased estimates for the loss of load frequency (LOLF). We use the unbiased estimator given in [38] to determine this index:

$$\text{LOLF} = E[\Delta\lambda|\text{DI} > 0] \times \text{LOLP} \quad (15)$$

where $\Delta\lambda$ is the sum of the transition rates between the current failure state $\boldsymbol{\theta}$ and all the success states that can be reached from $\boldsymbol{\theta}$ in one transition.

Other reliability indices (EENS, LOLD, etc.) can be easily calculated based on the above three estimates. Note that one might enter another level of simulation after the failure region is reached. As a result, the failure states sampled would be conditional on different intermediate failure events. In this case, the sampled failure states need to be weighted according to the intermediate failure event they are conditional on. To this end, the sample partitioning method [21] can be used.

C. Additional Aspects

1) Number of Samples Per Level N: In the direct MCS, since the samples are independent from each other, the coefficient of variation (c.o.v.) is estimated by $\sqrt{(1-\tilde{p}_F)/(N_T\tilde{p}_F)}$ where N_T is the total number of samples and $\tilde{p}_F$ is the estimate for the failure probability. The number of samples needed to achieve a certain level of c.o.v. can also be determined by this equation.

In subset simulation, the number of samples per level N is an input that controls the accuracy of the estimated indices. The samples (except for those generated in simulation level 0) in SS are generated by Markov chain Monte Carlo and are thus correlated. They generally give less information compared to the situation when they were independent. Therefore, the correlation among the MCMC samples must be accounted when estimating the variance of the samples. For verification purposes, the c.o.v. of the reliability indices is statistically assessed by 50 repeated independent runs of the subset simulation in Section V of this paper.

If one has an acceptable coefficient of variation in mind, the following formula originally from [20] can be used to estimate the total number of samples N_T needed to achieve a desired c.o.v. of δ associated with the estimate $\tilde{p}_F$, given that a rough estimation of the failure probability p_F is known a priori:

$$N_T = mN \approx \left(\frac{\log p_F}{\log p_0}\right)^r \frac{(1+\gamma)(1-p_0)}{p_0\delta^2} \quad (16)$$

where $2 \leq r \leq 3$ depends on the actual correlation of the level probabilities, and γ represents a factor accounting for the correlation among the MCMC samples. Typical values such as $r = 2.5$ and $\gamma = 3$ might be used when estimating N_T. If p_F is not known a priori, some tentative runs of simulation could be conducted. In the numerical tests presented in this paper, for comparison, the number of samples per level is chosen after some tentative runs of simulation so that the c.o.v. of the obtained reliability indices are below a given level.

TABLE I
RELIABILITY INDICES FOR IEEE-RTS-79

	Subset Simulation	Direct MCS
LOLP	1.16×10^{-3}	1.14×10^{-3}
EENS (MWh/yr)	1131.4	1121.7
# of Samples	90 000	341 000
CPU Time (s)	698.7	1982.3

2) Choice of the Proposal PDFs: It is concluded in [20] that the efficiency of Subset Simulation is insensitive to the *type* of the proposal PDFs. On the other hand, the *spread* (or the variance) of the proposal PDFs affects the deviation of the candidate state from the current state, and controls the efficiency of the Markov chain in populating the failure region. The optimal choice of the proposal PDF involves a trade-off between acceptance rate and correlation of the MCMC samples, because small spread tends to increase the correlation between samples due to their proximity, and excessively large spread may reduce the acceptance [20]. A sensitivity study on the spread of the proposal PDF on efficiency is presented in [34]. In this paper, a Gaussian PDF centered at the current sample with unit standard deviation is chosen as the proposal PDF, which is found to give satisfactory performance.

V. RESULTS

Numerical tests are conducted on the original IEEE-RTS-79, IEEE-RTS-79 integrated with wind energy, IEEE-RTS-96, as well as a real-life power system. Circuits and conventional generators are represented by two-state models. Numerical tests are performed on a MATLAB platform with a PC consisting of a 2.2-GHz processor and 4-GB RAM. Linear programs are solved by GUROBI 5.5.

A. IEEE-RTS-79

IEEE-RTS-79 [39] includes 24 buses, 38 circuits, and 32 generators, with a total generation capacity of 3405 MW. The annual system peak load is 2850 MW. The original hourly load curve is used and represented by a non-aggregated model. Although it has been assumed that all the bus loads follow the same pattern as the system load, this assumption is not mandatory for the proposed method.

The level probability p_0 is set at 0.1. The number of samples per level N is set at 30 000 so that after three levels of simulations are conducted, coefficient of variation (c.o.v.) reaches below 5% for the reliability indices. This is also used as the convergence criterion for a direct MCS. Reliability indices estimated by a single run of these two methods are shown in Table I.

In Fig. 3, three independent runs of SS are conducted. For comparison, results obtained by the direct MCS are also plotted. The CCDF of load curtailment, which is of interest to power system planners and crucial in estimation of energy-related reliability indices, is simply the part of the CCDF of DI where $\text{DI} > 0$. Also, the results obtained by SS agree well with those of the direct MCS, except for the low probability region where

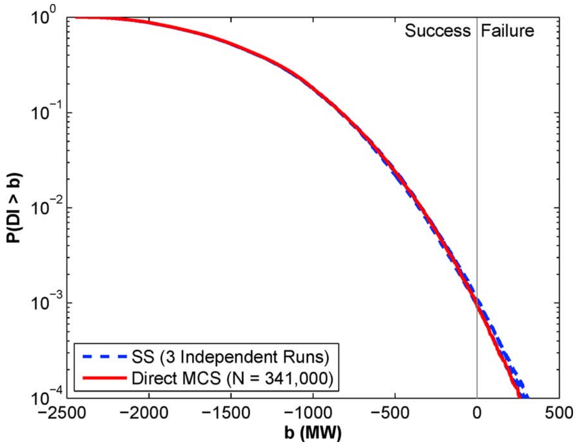

Fig. 3. CCDF of DI obtained from three independent runs of subset simulation and a single run of direct MCS.

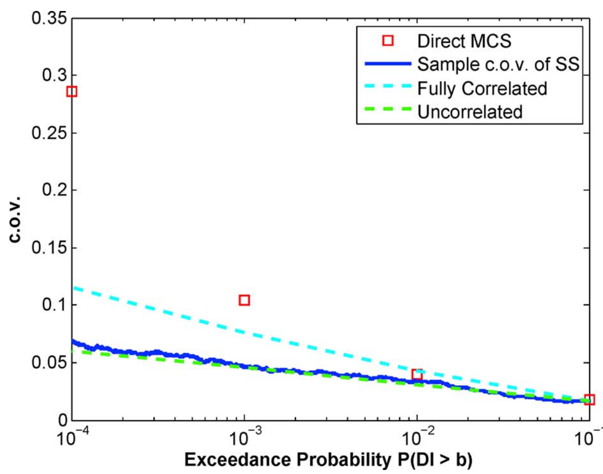

Fig. 4. Coefficient of variation of the exceedance probability estimates.

the error of the MCS is obvious due to its inefficiency in simulating rare events. Theoretically, the estimates of exceedance probabilities obtained by SS are asymptotically unbiased [20].

To investigate the empirical variation of exceedance probability estimates, the sample c.o.v. of the exceedance probability estimates over 50 independent runs of SS are calculated and shown by the solid line in Fig. 4. An additional level of simulation is conducted to examine how c.o.v. changes with decreasing exceedance probability. The dashed lines indicate the approximations of c.o.v. of the exceedance probability estimates obtained by SS assuming that the conditional probability estimators are uncorrelated or fully correlated. Here the sample c.o.v. is close to the approximation where samples in different levels of simulations are assumed uncorrelated, indicating that the loss of efficiency due to correlation between samples in different simulation levels is modest. Detailed description of the statistical properties of the estimators, as well as how SS reduces variance, are found in [20].

Also the c.o.v. of MCS estimates at a probability corresponding to a particular threshold for intermediate failure events using the same total number of samples as in SS (i.e., in this case, $N = 30\,000, 60\,000, 90\,000, 120\,000$ for $P = 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}$, respectively) are plotted in Fig. 4.

TABLE II
RELIABILITY INDICES FOR THE MODIFIED IEEE-RTS-79

	Subset Simulation	Direct MCS
LOLP	8.74×10^{-4}	8.71×10^{-4}
EENS (MWh/year)	869.7	843.5
# of Samples	114 000	472 000
CPU Time (s)	1017.8	2942.0

Notice that the values of c.o.v. for SS and MCS are identical at probability 0.1, since a direct MCS is conducted in Level 0 of SS. While the c.o.v. for a direct MCS grows exponentially with the logarithm of decreasing exceedance probability, such growth is only moderate (visually linear) for SS. Therefore, the smaller the target failure probability, the more efficient is SS over a direct MCS.

The probability distribution of the degree of system adequacy is obtained (Fig. 3). Nevertheless, the probability distributions of reliability indices as described in [40] are indeed restricted to full chronological (sequential) MCS methods. In the next subsection, we show the flexibility and simplicity of SS in accommodating different models for renewable energy sources.

B. Modified IEEE-RTS-79

In order to examine the robustness of SS to different models for power system components, IEEE-RTS-79 is modified by integrating two 160-MW wind farms located at Bus 19 and one 240-MW wind farm located at Bus 1. All these wind farms are connected to the original network through a single transmission line whose unavailability is ignored. While the identification of probabilistic models that accurately capture the intermittency and non-stationarity of wind power is of critical importance, this is not the focus of our paper. We make a simplifying assumption that the wind speeds at the three wind farms follow a two-parameter Weibull distribution. This family of distribution has been shown to give a good fit to observed wind speed data [41]. Parameters of the Weibull distribution are calculated by fitting two-year hourly wind speed data observed on Hainan Island, China. The power output of the wind farm is then determined through a quadratic power curve of the wind turbines. Parameters of the simulation, except for the number of samples per level, are set at the same values as the previous test.

As illustrated in Section III, the only modification to the proposed method for integrating the wind farms is three additional state variables that follow Weibull distribution. Similarly, one can use state variables that follow other types of probability distribution to incorporate a different type of renewable energy source, or use state variables that follow categorical distribution to incorporate any component of the power system that is represented by a multi-state outage table.

Reliability indices of the Modified IEEE-RTS-79 are presented in Table II.

C. IEEE-RTS-96

IEEE-RTS-96 [42] includes 73 buses, 120 circuits and 32 generators, with a total installed capacity of 10 215 MW. The optional DC link connecting two subsystems as indicated in [42] is omitted. The annual system peak load is 8550 MW, and the

TABLE III
RELIABILITY INDICES FOR IEEE-RTS-96

	Subset Simulation	Direct MCS
LOLP	2.08×10^{-5}	2.21×10^{-5}
EENS (MWh/year)	26.44	28.26
# of Samples	895 000	22 977 000
CPU Time (s)	15231.8	276722.2

TABLE IV
RELIABILITY INDICES FOR THE REAL-LIFE SYSTEM

	Subset Simulation	Direct MCS
LOLP	6.95×10^{-4}	7.01×10^{-4}
EENS (MWh/year)	1339.7	1365.3
# of Samples	138 000	612 000
CPU Time (s)	7382.4	31598.3

original hourly load curve is used. We set the number of samples per level to be 179 000 in order to ensure that the c.o.v. of the reliability indices are below 5%. The other parameters remain the same as previous tests. Five levels of simulation are conducted in SS.

The failure events in IEEE-RTS-96 are even rarer than the above systems. The inefficiency of the direct MCS in simulating small failure probabilities is clearly shown in Table III. As for Subset Simulation, the speed-up ratio in terms of CPU time increases to 18.16, which again validates that the smaller the target failure probability, the more efficient is SS over a direct MCS.

D. Real-Life Power System in Northwestern China

To validate the scalability of the proposed method, tests are also performed on a real-life power system in Northwestern China, which contains 773 buses, 279 generating units, and 1036 circuits. The total installed generating capacity is 28.8 GW. System load is set to be constant at 22.4 GW. Parameters of the simulation, except for the number of samples per level, are set at the same values as previous tests.

Reliability indices in Table IV demonstrate that the proposed method achieve satisfactory performance for a large-scale real-life system.

VI. CONCLUSION

This study proposes a framework for composite system reliability evaluation with subset simulation. The inefficiency of Monte Carlo simulation in simulating rare failure events is overcome by breaking the problem into estimating a sequence of conditional probabilities. Computational burden is reduced by extracting the subset of system states with significant contribution to reliability indices.

The degree of adequacy of system states is first parametrized through a metric based on a linear program that takes network security constraints into consideration. The Metropolis algorithm is then customized to account for the discrete-state components in power systems. The proposed method successfully replicates the MCS results while considerably reduces the computational effort.

Every simulation algorithm is involved with a tradeoff between efficiency and robustness. The direct MCS provides best robustness yet poor efficiency when simulating rare events. A major difference of subset simulation from other VRTs is that it utilizes the fundamental notion of conditional probabilities, instead of altering the probability distribution of system states. Though the proposed method might not be as efficient as some algorithms well-designed for certain target systems, it possesses advantages in terms of robustness. No prior knowledge of system failures is required before simulation. Preparation processes, such as constructing an optimal distortion of the sample space in VRTs, or training the learning system in machine-learning-based classification techniques, are thus spared. Also, sophisticated models for power system components can be easily incorporated.

A nonsequential simulation of the composite system is conducted in this study. Future work could focus on the application of the proposed method to the chronological (sequential) simulation of power systems, as well as customized procedures targeting at area and bus indices. The conditional samples generated in subset simulation can also be further investigated to reveal more information of the cause and consequence of failure events in composite systems.

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