Decentralized Participation of Flexible Demand in Electricity Markets—Part I: Market Mechanism

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***Abstract*—In the deregulated power systems setting, the realization of the significant demand flexibility potential should be coupled with its integration in electricity markets. Centralized market mechanisms raise communication, computational and privacy issues while existing dynamic pricing schemes fail to realize the actual value of demand flexibility. In this two-part paper, a novel day-ahead pool market mechanism is proposed, combining the solution optimality of centralized mechanisms with the decentralized demand participation structure of dynamic pricing schemes and based on Lagrangian relaxation (LR) principles. Part I presents the theoretical background, algorithmic approaches and suitable examples to address challenges associated with the application of the mechanism and provides an implementation framework. Non-convexities in reschedulable demand participants' price response and their impacts on the ability of the basic LR structure to reach feasible market clearing solutions are identified and a simple yet effective LR heuristic method is developed to produce both feasible and high quality solutions by limiting the concentrated shift of reschedulable demand to the same low-priced time periods.**

***Index Terms*—Demand side participation, electricity pool markets, Lagrangian relaxation.**

NOMENCLATURE

t	Time period index ($t = 1, 2, \ldots, 24$).
r	Iteration index ($r = 1, 2, \ldots, R$).
I, N_I	Set and number of demand participants in the market.
J, N_J	Set and number of generation participants in the market.
$\boldsymbol{d_i^r}$	Vector of hourly demand responses $d_{i,t}^r$ of demand participant i at r.
$\boldsymbol{s_j^r}$	Vector of hourly generation responses $s_{j,t}^r$ of generation participant j at r.
$C_j(\cdot)$	Daily cost function of generation participant j.
$\boldsymbol{l_i^d}$	Vector of local constraints of demand participant i.
$\boldsymbol{l_j^s}$	Vector of local constraints of generation participant j.
$\boldsymbol{e^r}$	Vector of hourly demand-supply imbalances e_t^r at r.
$\boldsymbol{\lambda^r}$	Vector of hourly electricity prices λ_t^r at r.

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I. INTRODUCTION

RECENT developments have paved the way for the wide penetration of flexible electrical demand technologies in power systems, exhibiting an ability to reschedule the users' demand requirements in time through the employment of different types of storage [1]. Suitable deployment of such flexibility could induce significant technical, economic and environmental benefits and various strategies for flexible demand integration in system operation have been proposed in the literature and implemented around the world [1], [2]. Before the deregulation of the electricity sector, such strategies were limited to centrally administrated programmes, where the regulated utility exercised direct management of loads or deliberate intervention in the electricity prices faced by the consumers to influence their energy use according to the utility's economic, planning and reliability requirements.

In the liberalized environment however, the realization of the demand flexibility potential needs to be coupled with integration schemes driven by competitive market dynamics and the individual consumers' interests, and enabling the latter to access the price-setting process. Such schemes will close the gap between wholesale and retail electricity market segments, enable a more active demand participation in the market setting and subsequently lead to more efficient and competitive markets [3]. This paradigm change however necessitates suitable modifications in traditional, one-sided market mechanisms which were designed to treat the demand side as a fixed, inflexible forecasted load [3].

Approaches examined in the relevant literature to achieve integration of the demand side in electricity markets can be divided into two categories. The first [4]–[7] revolves around the extension of traditional centralized mechanisms to include two-sided participation. Both generation and demand participants submit their economical and technical characteristics in the form of bids and offers to the market operator and the latter clears the market through the solution of a global optimization problem (usually social welfare maximization). Under the assumption of competitive behavior by the market participants, this approach yields the optimal outcome from the system perspective. Under significant demand side participation however, the communication and computational scalability of centralized mechanisms is at least questionable; transmission of the diverse complex operational constraints and physical parameters of a

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very large number of flexible loads to the central clearinghouse will yield information collection and communication problems, while the vast number of decision variables and constraints associated with such loads will create a massive computational burden to the market operator. Last but not least, centralized mechanisms are likely to raise privacy concerns by the consumers, related to the disclosure of sensitive information, such as habits, preferences and load assets' properties, during the bidding process.

The second approach involves dynamic pricing schemes [1], [2], [8] which constitute the first effort towards decentralized market participation of the demand side. Without revealing their individual properties to a central entity, consumers are exposed to pre-determined time-variable prices reflecting more accurately the actual system costs than the traditional fixed tariffs, and are thus encouraged to activate their flexibility and modify their demand patterns according to the posted prices in order to reduce their payments. However, such schemes fail to realize the actual value of demand flexibility, since the posted price signals are not influenced by demand response close to real-time [4], [8]; without this feedback of demand on prices, it is difficult to determine the most efficient price signals that should be communicated to the consumers and inefficient or even infeasible market outcomes occur. If for example a large number of consumers are exposed to dynamic prices, the resulting shift of their demand towards the lowest-priced periods will create new, possibly significant peaks, the economic implications of which are not encapsulated in the communicated prices.

This two-part paper develops, analyzes and tests a novel pool market mechanism enabling flexible demand participation in electricity markets and combining the solution optimality of centralized mechanisms with the decentralized demand participation structure of dynamic pricing schemes. Based on *Lagrangian relaxation* (LR) principles [9], [10], the proposed mechanism involves a two-level iterative process, consisting of a number of independent local surplus maximization sub-problems—expressing the market participants' price response—coordinated by a global price update algorithm—expressing the market operator's effort to reach an optimal clearing solution; this two-level process is modified by suitable heuristic methods (*LR heuristics*) when it cannot reach a feasible solution due to non-convexities in participants' price-response sub-problems.

Authors in [11] present a model of a similar LR-based market mechanism, which is however characterized by two significant drawbacks: 1) it covers a single time period and therefore is not able to capture and address issues associated with the inter-temporal behaviour of flexible demand and 2) perfectly convex characteristics—unable to accurately reflect the participants' techno-economic properties—are assumed for both demand and generation participants in order to avoid the above solution infeasibility complications. On the other hand, multi-time period LR-based algorithms have been extensively researched in centralized generation-only scheduling problems [12]–[25]. The motivation for employing LR in these studies is not decentralizing the scheduling process—as the problem is solved by a central entity with perfect knowledge of generators' characteristics—but addressing the computational complexity related to the on-off generators' commitment characteristics, the wide variety of complex constraints characterizing the operation of generating units of different technologies, and the large number of available units in real-scale systems, since LR breaks down the initial problem to a number of smaller, unit-wise sub-problems

In the context of the problem addressed in this paper however, new challenges arise, related to the special characteristics of decentralized flexible demand participation: a) the formulation of flexible demand technologies' price-response sub-problems, taking into account their inter-temporal techno-economic properties, b) the identification of non-convex characteristics in flexible demand's price response and the analysis of their impacts on the ability of the basic LR structure to reach feasible solutions, c) the formulation of suitable LR heuristics achieving near-optimal feasible solutions in face of such non-convexities' implications, and complying with the decentralized demand participation objective and d) the derivation of an efficient price update algorithm to reduce the number of required message exchanges between the market operator and the decentralized demand participants for reaching a satisfactory solution. Approaches to deal with these challenges are proposed and analyzed theoretically and numerically in this two-part paper.

Part I presents the theoretical and mathematical foundations of the proposed market mechanism, provides suitable algorithmic approaches and examples to address application challenges b), c) and d) above and outlines a basic framework for its practical implementation. Part II [26] demonstrates the applicability of the proposed mechanism by considering two reschedulable demand technologies with significant potential—electric vehicles (EV) with flexible charging capability and electric heat pump (EHP) systems accompanied by heat storage for space heating—and presenting suitable case studies validating the properties of the mechanism and illustrating the benefits of these technologies' market participation.

The rest of Part I is organized as follows. The theoretical background and mathematical formulation of the proposed mechanism are detailed in Section II while considerations related to its practical implementation framework are examined in Section III. Section IV identifies non-convexities in the price response of reschedulable demand participants and their impact on solution feasibility of the basic LR structure. Section V develops a novel LR heuristic method achieving high quality feasible solutions in face of such non-convexities. Section VI presents the price update algorithm employed and derives the overall structure of the proposed mechanism. Finally, Section VII concludes this work.

II. Theoretical Background and Mathematical Formulation of Proposed Market Mechanism

The examined market is a pool energy-only market with a day-ahead horizon and hourly resolution (24 commodities, representing the active electrical power at each hour of the next day, are traded simultaneously), with the market clearing objective set as the maximization of the social welfare, and the prices set on a uniform (transmission capacity constraints and losses are not taken into account and thus the prices are the same for all market participants irrespectively of their location in the network) and marginal (the price at hour t is equal to the system marginal cost of serving an additional unit of demand at t) basis. The participants are assumed to behave competitively, acting as price-takers and revealing their actual economic and technical characteristics to the market.

Under a traditional centralized market mechanism, the market operator derives participants' cost/benefit functions and constraints from their respective bids/offers and clears the market by solving the social welfare maximization problem over the considered day-ahead horizon. It is assumed in this paper that demand participants' benefit functions are constant and their preferences are expressed solely in the form of constraints; this assumption is justified by a) the theoretical and practical difficulties of benefit functions' derivation due to the significant uncertainties related to the human valuation of electrical energy and b) the fact that the considered flexible demand technologies (EV and EHP systems with heat storage) involve explicit storage components enabling demand flexibility without affecting the consumers' level of service. Therefore, the market clearing optimization problem is converted from a social welfare maximization problem to an equivalent generation cost minimization problem:

$$\min_{\substack{\boldsymbol{d_i},\forall i\in I\\ \boldsymbol{s_j},\forall j\in J}} f = \min_{\substack{\boldsymbol{d_i},\forall i\in I\\ \boldsymbol{s_j},\forall j\in J}} \left(\sum_{j=1}^{N_J} C_j(\boldsymbol{s_j})\right). \tag{1}$$

Constraints:

$$e_t = \sum_{i=1}^{N_I} d_{i,t} - \sum_{j=1}^{N_J} s_{j,t} = 0\ ,\ \forall t \in [1,24] \tag{2}$$

$$\boldsymbol{l_i^d}\left(\boldsymbol{d_i}\right)\ ,\ \forall i \in I \tag{3}$$

$$\boldsymbol{l_j^s}\left(\boldsymbol{s_j}\right)\ ,\ \forall j \in J. \tag{4}$$

This problem is subject to both system constraints (2), coupling all market participants and expressing the hourly system demand-supply balance, as well as local constraints (3)–(4), each of which is associated with the operational characteristics of an individual participant (e.g., maximum generation capability of a generation participant or battery capacity of a flexible EV participant) and may correspond to an equality or inequality constraint.

According to Section I, the objective of the proposed market mechanism is to achieve the same solution with the centralized mechanism (1)–(4), but without requiring the demand participants to submit their individual properties to the market operator. In these terms, the LR decomposition technique [9], [10] constitutes a suitable mathematical foundation for the developed mechanism. By applying LR on the (primal) problem (1)–(4), the latter is solved indirectly by solving its *Lagrangian dual problem*:

$$\max_{\boldsymbol{\lambda}} \varphi(\boldsymbol{\lambda}) = \max_{\boldsymbol{\lambda}} \min_{\substack{\boldsymbol{d_i},\ \forall i\in I\\ \boldsymbol{s_j},\ \forall j\in J}} L. \tag{5}$$

Inner minimization constraints: (3) and (4)
where φ is the *dual function* of the problem and L is the *Lagrangian function* of the problem, derived by relaxing the system constraints (2) through a vector of Lagrangian multipliers $\boldsymbol{\lambda}$ and adding the relaxation terms to the objective function (1), and expressing an additive combination of the individual participants' economic surpluses:

$$L\left(\boldsymbol{d_1},\ldots,\boldsymbol{d_{N_I}},\boldsymbol{s_1},\ldots,\boldsymbol{s_{N_J}},\boldsymbol{\lambda}\right) = \sum_{j=1}^{N_J} C_j\left(\boldsymbol{s_j}\right) + \boldsymbol{\lambda}^T\left(\sum_{i=1}^{N_I}\boldsymbol{d_i} - \sum_{j=1}^{N_J}\boldsymbol{s_j}\right). \tag{6}$$

By exploiting the additive separability of L with respect to each participant's surplus for a certain value of vector $\boldsymbol{\lambda}$, the dual problem (5) is decomposed into N_I+N_J independent optimization sub-problems—one sub-problem corresponding to each demand participant (7) and each generation participant (8)—coordinated iteratively by a $\boldsymbol{\lambda}$ update process until maximization of φ is achieved. This mathematical decomposition scheme can be interpreted as a two-level iterative market clearing mechanism with the elements of $\boldsymbol{\lambda}$ representing the 24 hourly electricity prices. At the local level, each participant solves independently their individual surplus maximization problem [payment minimization problem (7) for demand participant i—since constant benefit functions have been assumed—and profit maximization problem (8) for generation participant j] for given values of the prices and submit their optimal demand/generation responses to the market operator. At the global level, the latter updates the value of $\boldsymbol{\lambda}$ posted to the participants, according to their responses, in an effort to gradually maximize φ:

$$\min_{\boldsymbol{d_i^r}} (\boldsymbol{\lambda}^r)^T \boldsymbol{d_i^r} \quad \text{Constraints}: \boldsymbol{l_i^d}\left(\boldsymbol{d_i^r}\right) \tag{7}$$

$$\max_{\boldsymbol{s_j^r}} (\boldsymbol{\lambda}^r)^T \boldsymbol{s_j^r} - C_j(\boldsymbol{s_j^r}) \quad \text{Constraints}: \boldsymbol{l_j^s}\left(\boldsymbol{s_j^r}\right). \tag{8}$$

According to the strong duality theory [10], when the examined problem is strictly convex, the solution of the dual problem φ^* is identical to the solution of the primal problem f^* and satisfies the relaxed system constraints of the primal. Furthermore, according to the economic theory [27], this solution constitutes a market (or general) equilibrium since: **Condition a**) each participant maximizes their individual surplus at the prevailing prices (surplus optimum state) and **Condition b**) system constraints are satisfied.

When however the examined problem exhibits non-convexities, f^* is not guaranteed to coincide with φ^* (the positive difference $f^* - \varphi^*$ is referred to as the *inherent duality gap* of the problem) and the latter is not guaranteed to satisfy the relaxed system constraints irrespectively of the price update algorithm employed [10]. In the case that such solution infeasibility occurs, the mechanism needs suitable modifications to achieve solutions that will be both primal-feasible and satisfactory in terms of primal optimality (i.e., as close as possible to f^*).

As discussed in relevant studies [12], [13], [22], [25] such modifications generally proceed along heuristic lines (usually referred to as LR heuristics) since little theory exists to guide the search of near-optimal primal-feasible solutions and different approaches are adopted according to the problem properties. However, a favourable attribute of Lagrangian duality is that a sufficient indication of the optimality of any primal-feasible solution f^f is provided by the properties of the dual problem. According to the weak duality theory [10], the optimal dual φ^* is always a lower bound of the optimal primal f^*. Given that any f^f is an upper bound of f^*, its optimality is indicated by the *relative duality gap* (RDG)(9)[19], [22], [24]. It should be stressed that RDG provides a worst-case indication and not the exact value of the difference $f^f - f^*$ for two reasons: 1) f^* is higher than φ^* if an inherent duality gap exists and 2) due to the properties of the dual problem (Section VI), an approximate value $\hat{\varphi} \leq \varphi^*$ of the maximum dual is determined by the LR-based mechanism:

$$RDG = \frac{f^f - \varphi^*}{\varphi^*}. \tag{9}$$

III. Implementation Framework

Small consumers have neither the expertise nor the motivation (due to the associated discomfort) to negotiate themselves in the market. In order to address this challenge, an agent-mediated participation scheme is envisaged, in line with relevant work in the literature [28]; dedicated software programs—in the sequel indicated as load market agents—are embedded in flexible demand appliances and act as their market representatives by receiving price signals, solving local price response sub-problems (7) (taking into account users' preferences and appliances' operational constraints) and submitting demand responses to the market operator at each iteration of the clearing process. Users' preferences can be determined either explicitly by the users before the start of the clearing process or by a suitable learning model incorporated into agents' intelligence [28]. The agents can also direct the control of the appliances in real time, according to the clearing outcome of the market. Furthermore, suitable two-way communication technologies are required to enable the iterative interaction between the market operator and the load market agents.

However, sending prices and receiving responses from potentially thousands or even millions of agents would be prone to scalability limitations. Therefore, a tree-like organization of demand side participation is required, where flexible load aggregators spread the trial prices determined by the market operator to their consumers' agents and incorporate these agents' individual responses into an aggregate response submitted to the market operator.

In mathematical terms, inflexible demand participants do not need to be treated in a different fashion in the context of the proposed mechanism. In a practical implementation framework however, it is meaningless to send prices and receive responses iteratively from them since their demand does not depend on $\boldsymbol{\lambda}$. Before the start of the clearing process, electricity retailers representing such inflexible demand submit the predicted value of this demand in the day-ahead horizon, which is then included in the computations of the mechanism as a fixed quantity.

This paper focuses on decentralized participation of flexible demand through the proposed mechanism and does not pose a similar requirement on the participation structure of generation participants. Furthermore, as explained in Section V, for the LR heuristic adjustments proposed to be effective, availability of the generation participants' characteristics by the market operator is required. Therefore, the generation participants are assumed to submit a bid with their cost components and constraints to the market operator (as in traditional centralized mechanisms) and their price response sub-problems are solved at each iteration by the market operator based on the bid information.

IV. Reschedulable Demand Participants' Non-Convexities and Impact on Solution Feasibility

Studies associated with the application of LR in generation-only scheduling problems have identified a number of non-convex techno-economic characteristics at the local sub-problems (8) of generation participants. Such non-convexities include not strictly convex objective functions—when generators exhibit linear cost functions [14], [16], [17], [20]—discontinuous objective functions—when generators exhibit binary unit commitment decisions and fixed cost components [20], [21], [23]—non-convex (not-connected) feasible operation domains—when generators exhibit binary unit commitment decisions and minimum stable generation constraints [18], [19], [23]—etc. As explained in these studies, all the above non-convexities create discontinuities in generators' supply functions (defined as the optimal solution of (8) parameterized by $\boldsymbol{\lambda}$) which under certain conditions (associated with the level of system demand, generators' diversity etc.) lead to inability of the basic LR structure to reach a primal feasible solution.

In this section, reschedulable demand participants (RDP) are examined from the same perspective. It can be observed that the objective function of their price response sub-problem (7) (payment minimization) is linear and thus not strictly convex. This non-convexity creates discontinuities in RDP demand functions, as clarified by the following simple example. An RDP i, able to reschedule its demand in time through the employment of some lossless sort of storage, needs to consume E electrical energy in total in the time window $t = 1$ and $t = 2$, with its maximum power demand limit at each hour being $P^{\max}$ with $P^{\max} * 1h > E/2$. It can be easily deduced that its demand functions $d^*_{i,1}(\boldsymbol{\lambda})$ and $d^*_{i,2}(\boldsymbol{\lambda})$ for $t = 1$ and $t = 2$, respectively, are given by

$$\begin{aligned} d^*_{i,1}(\boldsymbol{\lambda}) &= \begin{cases} \min(P^{\max}, \frac{E}{1h}) & \text{if } \lambda_1 < \lambda_2 \\ \max(\frac{E}{1h} - P^{\max}, 0) & \text{if } \lambda_1 > \lambda_2 \end{cases} \\ d^*_{i,2}(\boldsymbol{\lambda}) &= \begin{cases} \max(\frac{E}{1h} - P^{\max}, 0) & \text{if } \lambda_1 < \lambda_2 \\ \min(P^{\max}, \frac{E}{1h}) & \text{if } \lambda_1 > \lambda_2. \end{cases} \end{aligned} \tag{10}$$

If for instance $E = 6$ kWh and $P^{\max} = 5$ kW, then

$$\begin{aligned} d^*_{i,1}(\boldsymbol{\lambda}) &= \begin{cases} 5kW & \text{if } \lambda_1 < \lambda_2 \\ 1kW & \text{if } \lambda_1 > \lambda_2 \end{cases} \\ d^*_{i,2}(\boldsymbol{\lambda}) &= \begin{cases} 1kW & \text{if } \lambda_1 < \lambda_2 \\ 5kW & \text{if } \lambda_1 > \lambda_2. \end{cases} \end{aligned}$$

The RDP demand functions exhibit a discontinuity at the point $\lambda_1 = \lambda_2$. When λ_1 becomes marginally lower than λ_2, the optimal price response of the RDP involves demanding the largest possible proportion of E at $t = 1$ and the smallest possible proportion at $t = 2$ and vice verse. In the extreme case that $\lambda_1 = \lambda_2$ the RDP price response sub-problem becomes singular, as the RDP is indifferent regarding its price response:

irrespectively of the hourly values of its response within its feasible operation domain, its total payments will be the same. It is thus concluded that RDP demand functions exhibit discontinuities due to the combination of their linear payment minimization objective and their demand rescheduling capability. Due to these discontinuities, their optimal price response involves "lumpy" inter-temporal demand shifts between different time periods when the sign of the correlation of the respective prices changes.

The impact of these discontinuities on the solution feasibility of the LR-based clearing mechanism is explored by considering a simplified two time period clearing problem, where the market participants include: 1) a number N of RDP, each of which exhibits the characteristics of the previous example, 2) a number of inflexible demand participants with total hourly demand D_t and 3) a number of generation participants with perfectly convex characteristics, the aggregate hourly supply functions of which are continuous, in the form of (11) (g is a positive scalar):

$$\sum_{j=1}^{N_J} s_{j,t}^{*}(\boldsymbol{\lambda}) = g * \lambda_t, \forall t \in [1, 24]. \tag{11}$$

Assuming that the LR-based clearing process is initialized with $\lambda_1 < \lambda_2$, $\sum_{j=1}^{N_J} s_{j,1} < \sum_{j=1}^{N_J} s_{j,2}$ holds for the generation response due to (11). At the same time all N RDP will strive to increase their demand at $t = 1$ as much as possible due to (10). Assuming $D_1 < D_2$, in the case that

$$N * \min\left(P^{\max}, \frac{E}{1h}\right) + D_1 > N * \max\left(\frac{E}{1h} - P^{\max}, 0\right) + D_2 \tag{12}$$

then $\sum_{i=1}^{N_I} d_{i,1} > \sum_{i=1}^{N_I} d_{i,2}$ holds for the total demand and thus the system constraints $\sum_{i=1}^{N_I} d_{i,1} = \sum_{j=1}^{N_J} s_{j,1}$ and $\sum_{i=1}^{N_I} d_{i,2} = \sum_{j=1}^{N_J} s_{j,2}$ cannot hold simultaneously. If the price update yields $\lambda_1 > \lambda_2$ after some iterations, $\sum_{j=1}^{N_J} s_{j,1} > \sum_{j=1}^{N_J} s_{j,2}$ emerges for the generation response, while the RDP will shift the largest part of their demand to $t = 2$ in a "lumpy" fashion, which—given that $D_1 < D_2$—leads to $\sum_{i=1}^{N_I} d_{i,1} < \sum_{i=1}^{N_I} d_{i,2}$, meaning that the system constraints still cannot be satisfied. The satisfaction of the system constraints cannot be ensured neither in the case where $\lambda_1 = \lambda_2$, since the RDP price response sub-problem becomes singular and the RDP can select any values for their response within their feasible operation domain. In other words, the "lumpy" inter-temporal RDP shifts are sustained through the iterations and do not allow the LR-based process to reach a feasible solution.

1) Example V.1: Based on the above simplified problem, an example with $E = 6$ kWh, $P^{\max} = 5$ kW, $N = 10\,000$, $D_1 = 10$ MW, $D_2 = 20$ MW, $g = 0.5$ MW^2/£ and the sub-gradient method (Section VI-A) employed for the price update is examined. As illustrated in Fig. 1 (black line), the LR-based process not only cannot reach the primal optimal solution of $\sum_{i=1}^{N_I} d_{i,1} = \sum_{i=1}^{N_I} d_{i,2} = 45$ MW but also cannot reach a solution satisfying the system constraints (the norm of demand-supply imbalances $|\boldsymbol{e}|$ does not reach zero). Even if the market operator knew in advance the primal optimal solution and posted the corresponding prices $\lambda_1 = \lambda_2 = 90$£/MWh to the RDP, the price response of the latter would become singular.

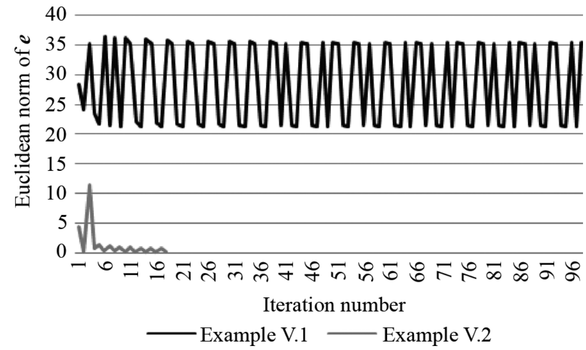

Fig. 1. Ability of the LR-based process to reach a feasible solution in the examined examples.

It can be observed that the outcome of the above simplified case depends on the validity of (12), which in turn depends on the relative size of the "lumpy" RDP rescheduling effect with respect to the temporal variation of inflexible demand i.e., the size of $N * \min(P^{\max}, E/1h) - N * \max(E/1h - P^{\max}, 0)$ with respect to the size of $D_2 - D_1$. If the number and maximum power limit of the RDP are low enough such that the sign of (12) is inversed, both $\sum_{j=1}^{N_J} s_{j,1} < \sum_{j=1}^{N_J} s_{j,2}$ and $\sum_{i=1}^{N_I} d_{i,1} < \sum_{i=1}^{N_I} d_{i,2}$ hold for $\lambda_1 < \lambda_2$; given the continuity of the supply functions (11), this means that prices $\lambda_1 < \lambda_2$ for which the system constraints are satisfied exist.

2) Example V.2: The maximum power limit of the RDP is reduced to $P^{\max} = 3.3$ kW while the rest of the parameters maintain the same values as in Example V.1. As illustrated in Fig. 1 (grey line), the LR-based process reaches a solution $\sum_{i=1}^{N_I} d_{i,1} = 43$ MW and $\sum_{i=1}^{N_I} d_{i,2} = 47$ MW satisfying the system constraints (within the tolerance limit $|\boldsymbol{e}| < 0.01$ MW) after 18 iterations, which is also a primal optimal solution and a market equilibrium.

Furthermore, if the operational diversity of the RDP—e.g., with respect to their consumption time windows—is enhanced, the RDP shifts are more evenly distributed across the different periods and their discontinuities may be evened out without radically affecting the correlation between the total demands at different time periods. It is thus concluded that the effect of the "lumpy" inter-temporal RDP shifts on solution feasibility of the LR-based mechanism depends on the relative number, maximum power limit and heterogeneity of the RDP with respect to the temporal variation of inflexible demand.

V. LR Heuristics for High Quality Feasible Solutions

A. Generation Scheduling With Unaffected RDP Response

Studies associated with the application of LR in generation-only scheduling problems have proposed suitable methods to achieve high quality primal feasible solutions when generators' non-convexities do not allow the basic LR structure to reach a feasible solution [12], [13], [16], [17], [19], [22], [24], [25]. As widely recognized in this literature, such methods proceed along heuristic lines—and are thus usually referred to as LR heuristics—since little theory exists to guide the search of such solutions and different approaches are adopted according to the problem's specific properties and especially the considered non-convex generation characteristics. Despite the significant diversity of the proposed methods in the above studies, all of them share a common characteristic: they modify the LR mechanism by introducing some form of heuristic direct adjustment of some generators' commitment/dispatch values by the central entity solving the problem, in contrast with the basic LR structure involving solely indirect interaction with the generators through the Lagrangian multipliers. Since in the context of the above studies the generation scheduling problem is solved by a central entity with perfect knowledge of generators' characteristics, such direct adjustment is not subject to information availability limitations.

In the context of the examined problem, suitable LR heuristics need to be developed in face of the RDP non-convexities identified in Section IV. The distinctive feature of such LR heuristics with respect to the respective methods in the above studies is that they cannot assume availability of all participants'—specifically the demand participants'—characteristics by the market operator due to the need to comply with the decentralized demand participation objective.

In these terms, a simple conceivable LR heuristic for reaching a primal feasible solution after iteration r of the LR mechanism involves direct optimal scheduling of the commitment states and outputs of the generation participants by the market operator to satisfy the total demand, as determined by the sum of inflexible system demand and the optimal price response of RDP at iteration r.

However, the feasibility and optimality of this approach are questionable. If the relative size of the inter-temporal demand shifts of RDP is large, the total demand at iteration r will contain new significant peaks at the periods t with the lowest λ_t^r. Due to the "lumpiness" of these shifts, the resulting peaks are not smoothened out through the iterations but are just shifted in time. In the case that these peaks are higher than the total available generation, a feasible solution cannot be reached by the above generation scheduling problem. Even if such infeasibility does not occur, an optimality issue emerges. Given that the hourly marginal cost of the generation side increases with the size of the hourly demand, a peaky demand profile yields a feasible solution with much higher total generation costs with respect to the optimal level.

1) Example VI.1: For the same problem examined in Example V.1, the above approach can only yield the solutions a) $\sum_{i=1}^{N_I} d_{i,1} = 60$ MW and $\sum_{i=1}^{N_I} d_{i,2} = 30$ MW (after iterations with $\lambda_1^r < \lambda_2^r$) with a total cost of £4500 and b) $\sum_{i=1}^{N_I} d_{i,1} = 20$ MW and $\sum_{i=1}^{N_I} d_{i,2} = 70$ MW (after iterations with $\lambda_1^r > \lambda_2^r$) with a total cost of £5300, both significantly worse than the primal optimal solution $\sum_{i=1}^{N_I} d_{i,1} = \sum_{i=1}^{N_I} d_{i,2} = 45$ MW with a total cost of £4050.

B. Perturbation of RDP Response

Therefore, in order to produce better feasible solutions, the peaks created by the RDP shifts should be limited through suitable perturbation of the RDP price response. Such perturbation however should be carried out without information of the RDP individual characteristics due to the reasons explained above.

According to Section IV, the size of these shifts is limited when the diversity of RDP price response is enhanced. Two conceivable approaches to achieve such diversification are:

— Exposing the RDP population to differentiated trial prices. This can be achieved by splitting the RDP population into different groups and posting differentiated $\boldsymbol{\lambda}^r$ to each group.

— Artificially modifying some parameters of the RDP. This can be achieved by splitting the RDP population into different groups and introducing differentiated input parameters to the load market agents representing the RDP of each group e.g., by adding variable terms with diversified parameters in the objective function of their price response sub-problem.

However, the process of identifying suitable diversified price signals or diversified agents' parameters and suitably allocating the RDP population to different groups in order to achieve a non-peaky aggregate RDP response is highly intractable, especially when taking into account that the individual RDP properties are unknown. Furthermore, a fairness preservation issue emerges [18]. If the response of two RDP with identical characteristics is differentiated by the above diversification process, their market clearing dispatch and subsequently their payments will be different, implying that they are not treated in an equitable fashion.

According to Section IV, the size of the RDP shifts is also limited when their maximum power limit $P^{\max}$ is reduced. Due to the unavailability of information regarding the individual RDP properties, the suitable reduction of $P^{\max}$ through the transmission of absolute maximum power limits to the RDP in order to achieve a non-peaky total demand profile, while ensuring the feasibility of their sub-problems and the preservation of fairness, is a highly intractable task.

An alternative approach envisaged by the authors to suitably limit the maximum RDP power demand is the application of a uniform (same for all RDP) set Ω of relative maximum demand limits ω to the RDP:

$$\Omega = \{\omega_k, k = 1, 2, .., K | \omega_k \in (0, 1] | 1 \in \Omega\} \tag{13}$$

where ω represents the maximum allowable power demand limit $d_{i,t}^{\max}$ of the RDP as a fraction of their respective technically feasible limit $P_{i,t}^{\max}$ according to (14), which is added as a constraint in the RDP price response sub-problem:

$$d_{i,t}^{\max} = \omega * P_{i,t}^{\max}, \forall t \in [1, 24]. \tag{14}$$

This set may be posted to the agents representing the RDP by the market operator before the initiation of the LR-based clearing process or be embedded in the knowledge of the agents. The latter solve their price response sub-problem for each of the values ω_k of the set Ω. If a value ω_k is so restrictive for an RDP that does not allow the satisfaction of its local constraints (e.g., the RDP cannot obtain its total daily energy requirements) its optimal price response for ω_k is set equal to the optimal price response corresponding to the immediately higher value of the set. Since the value $\omega_k = 1$ is always an element of Ω(13), the feasibility of the price response sub-problem is guaranteed (given that the technically feasible maximum limit ensures this feasibility).

The market operator then calculates K solutions of the primal problem by solving the generation scheduling problem for the total demand determined by the sum of inflexible system demand and the optimal price response of RDP for each of the values ω_k. The set Ω includes multiple values in order to heuristically search for a suitable value of the relative maximum demand limit ω resulting in a non-peaky system demand profile and subsequently in a feasible and high quality solution of the market clearing problem.

In general, large values of ω (small restrictions on the RDP maximum demand) may not sufficiently limit the RDP capability to shift their demand towards low-priced periods and thus may still allow large inter-temporal demand shifts and lead to a demand profile with peaks created by the RDP response. On the other hand, small values of ω (large restrictions on the RDP maximum demand) may limit excessively the available flexibility of RDP to shift their demand towards low-priced periods and thus preclude them from smoothing the peaks and filling the off-peak valleys of the inflexible demand profile.

A suitable value of ω achieves an effective trade-off between the avoidance of demand peaks' creation and the flattening of the inflexible demand profile by the RDP response, leading to an as-flat-as-possible total demand profile. Based on the relevant discussion in Section IV, it can be deduced that the most suitable value of ω will depend on the correlation between the characteristics of the RDP population (number, technically feasible maximum power limit and diversity) and the temporal variation of inflexible demand. For a certain inflexible demand profile, a larger number and technically feasible maximum power limit and a poorer diversity of the RDP population will result in a smaller value of the most suitable ω, as a larger restriction needs to be set on each RDP in order to reach a higher-quality solution.

The number K of the elements of Ω should be suitably determined, as a large number expands the space of the heuristic search for a high quality solution but at the same time increases the volume of the response data transmitted by the RDP to the market operator, the number of price response sub-problems solved by the RDP and the number of generation scheduling problems solved by the market operator.

1) Example VI.2: For the same problem examined in Example VI.1 the proposed approach is applied with $\Omega = \{1, 0.9, 0.8, 0.7, 0.6\}$ (values below 0.6 are not examined since they do not allow the satisfaction of the total energy requirements of the RDP). The hourly system demands and total generation cost corresponding to the solutions determined after an iteration with $\lambda_1^r < \lambda_2^r$ for each value of ω are presented in Table I. The proposed approach has managed not only to improve the obtained feasible solution with respect to the method of Section V-A. (which corresponds to the case of unaffected RDP response or equivalently $\omega = 1$) but also to reach the primal optimal solution of the problem for $\omega = 0.7$.

2) Example VI.3: The number of RDP in the previous example is now reduced to $N = 3000$. The respective solutions of the proposed approach are presented in Table II. As the number of RDP has been reduced, a smaller restriction needs to be set on each RDP in order to reach a higher-quality solution and therefore the most suitable value of ω increases to $\omega = 0.9$. In this example, the primal optimal solution of £1152 would emerge for $\omega = 0.9333$ and thus cannot be reached with the selected granularity of Ω. As explained above, a larger granularity of Ω would approach closer the optimal solution, in the expense of more significant communication and computational requirements.

TABLE I
SOLUTIONS OF EXAMPLE VI.2

ω	**System demand $t = 1$ [MW]**	**System demand $t = 2$ [MW]**	**Generation costs [£]**
1	60	30	4500
0.9	55	35	4250
0.8	50	40	4100
0.7	**45**	**45**	**4050**
0.6	40	50	4100

TABLE II
SOLUTIONS OF EXAMPLE VI.3

ω	**System demand $t = 1$ [MW]**	**System demand $t = 2$ [MW]**	**Generation costs [£]**
1	25	23	1154
0.9	**23.5**	**24.5**	**1152.5**
0.8	22	26	1160
0.7	20.5	27.5	1176.5
0.6	19	29	1202

VI. LAGRANGIAN MULTIPLIERS UPDATE ALGORITHM AND OVERALL STRUCTURE OF PROPOSED MECHANISM

A. Lagrangian Multipliers' Update Algorithm

As discussed in Section II, the Lagrangian multipliers' update process is translated mathematically to a solution procedure of the dual function φ maximization. If the basic LR structure can reach a feasible solution, the dual maximum φ^* coincides with the primal optimum; if not, the value of φ provides a worst-case indication (9) of the optimality of primal feasible solutions and the accuracy of this indication is enhanced as φ approaches φ^*. Therefore, the market operator should in any case employ effective multipliers' update algorithms to reach a (near) maximum value of φ. Furthermore, these algorithms should be efficient in reaching this value in relatively few iterations since this will not only reduce the total time required for reaching the clearing solution but will also reduce the required communication costs of the mechanism, since the number of iterations corresponds to the number of message exchanges between the market operator and the decentralized RDP.

Based on the definition of $\varphi(5)$ and the Lagrangian function $L(6)$, it is deduced that φ constitutes an additive combination of the individual participants' demand and supply functions. Since these functions exhibit discontinuities due to the RDP and generation participants' non-convex characteristics (Section IV), φ is non-differentiable at certain points. Therefore, instead of conventional methods for smooth optimization, optimization techniques for non-differentiable functions should be employed for the multipliers' update process, a requirement also reflected in the literature associated with the application of LR in generation-only scheduling problems [12], [22].

Among such techniques, sub-gradient methods [12], [13], [15], [16], [18], [24], [25] are preferred in the majority of the relevant studies due to their conceptual simplicity and the lower required computational time per iteration. These methods update each multiplier proportionally to the respective relaxed system constraint violation at the latest iteration. In the context of this paper, a sub-gradient method leads to the update algorithm (15) where β^r denotes the step size at iteration r. Although sub-gradient methods—as every non-differentiable optimization method—are not ascending methods (meaning that an increase in the value of φ after every iteration is not guaranteed), the distance between the latest values of $\boldsymbol{\lambda}$ and the respective values $\boldsymbol{\lambda}^*$ at φ^* gradually decreases and their convergence after a finite number of iterations is guaranteed, if the step size satisfies the conditions $\lim_{r\to\infty} \beta^r = 0$ and $\sum_{r=1}^{\infty} \beta^r \to \infty$[13]:

$$\lambda_t^{r+1} = \lambda_t^r + \beta^r * e_t^r, \quad \forall t \in [1, 24]. \tag{15}$$

Since sub-gradient methods proceed towards φ in a slow and oscillatory fashion [13], [22], [24], suitable techniques are required for its step size adjustment at each iteration as well as its initialization, since values of the initial multipliers $\boldsymbol{\lambda}^1$ close to the respective values at φ^* can reduce the number of iterations required to reach φ^*. In this work, the step size update rule presented in [12], [18], and [24] has demonstrated satisfactory performance. The effect of the initialization quality on the required number of iterations is examined in Part II [26].

B. Overall Structure of Proposed Mechanism

Crucial challenges related to the derivation of the overall structure of the LR-based clearing mechanism are associated with the primal-dual interaction between the developed LR heuristic method achieving high quality primal feasible solutions (Section V-B) and the multipliers' update algorithm maximizing the dual function (Section VI-A).

A first emerging issue is associated with the evaluation of φ and the relaxed constraints' violations $\boldsymbol{e}$ at iteration r. As discussed in [15], if the dual problem is modified by the employed LR heuristics, the value of φ does not constitute anymore a lower bound of f^* and thus cannot indicate the optimality of the primal feasible solutions determined by the LR heuristics. Therefore, the value φ and $\boldsymbol{e}$ at iteration r should be evaluated based on RDP responses corresponding to $\omega_k = 1$ (the value not modifying the properties of the RDP price response sub-problems) and the generation participants' outputs corresponding to their sub-problems' solution for $\boldsymbol{\lambda}^r$ rather than their respective outputs at the feasible solutions determined by the LR heuristics.

The second issue is associated with the order of LR heuristics and multipliers' update execution and the termination criteria of the iterative mechanism. Two relevant approaches are reported in the literature. In the first [12], [16], [17], [25] the LR heuristics are applied and primal feasible solutions are determined only after the dual function is (approximately) maximized, while in the second [13], [19], [22], [24] the LR heuristics are carried out in parallel with the dual maximization, implying in the context of this paper that at each iteration r, the RDP sub-problems are solved for $\boldsymbol{\lambda}^r$ and each element of Ω, and K primal solutions are determined.

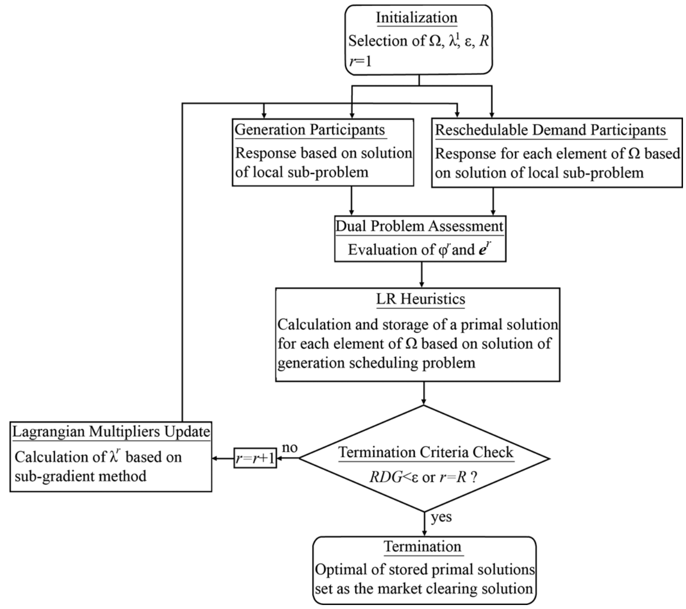

Fig. 2. Overall structure of proposed market clearing mechanism.

The latter approach is adopted in this paper for the following reasons: 1) termination criteria of the dual maximization based on changes in the values of φ, $\boldsymbol{e}$, or $\boldsymbol{\lambda}$ between successive iterations are not easy to derive since the non-differentiable optimization methods employed for the dual maximization are not ascending and their convergence towards φ^* does not always proceed smoothly, 2) the adoption of the parallel approach enables the calculation of the RDG (9) corresponding to the best primal feasible solution calculated so far, and thus the formation of a suitable termination criterion of the iterative mechanism: the mechanism terminates after iteration r if the RDG between the minimum primal solution $\hat{f}$ and the maximum value $\hat{\varphi}$ of the dual function in the r previous iterations is lower than a pre-determined tolerance value ε and 3) even if suitable termination criteria of the dual maximization could be derived, there is no mathematical guarantee that the RDP responses to $\boldsymbol{\lambda}^*$ will yield a better primal feasible solution than the respective responses to a different vector $\boldsymbol{\lambda}^r$ corresponding to iteration r; in other words, the adoption of the parallel approach expands the space of the heuristic search for a high quality primal feasible solution [22].

In practical terms, a very large number of iterations may be required in some cases in order to satisfy the termination criterion described at point 2) above. In order to avoid this, an additional termination criterion is envisaged, which constitutes of terminating the mechanism when a pre-determined maximum number of iterations R is reached. When both of these termination criteria are employed, the market operator can determine a desired trade-off between the certainty on the optimality of the clearing solution and the maximum number of message exchanges with the RDP, by suitably selecting the values of parameters ε and R, respectively.

The overall structure of the proposed market clearing mechanism, derived according to the above discussion, is presented in Fig. 2. After the termination of the clearing mechanism, the optimal of the determined primal feasible solutions is selected as the market clearing one and the RDP are dispatched according to this solution by posting them the corresponding values of $\boldsymbol{\lambda}$ and ω.

VII. Conclusions

A novel pool market mechanism is proposed in this two-part paper in order to achieve decentralized, market-based realization of the demand flexibility potential. Based on LR principles, this mechanism overcomes the problems of previous approaches by combining the solution optimality of centralized mechanisms with the decentralized demand participation structure of dynamic pricing schemes.

Part I presents the theoretical and mathematical foundations of the mechanism and outlines an implementation framework. Non-convex characteristics in reschedulable demand's price response and their impact on the ability of the basic LR structure to reach feasible solutions are identified and analyzed. In face of such non-convexities' implications, a simple yet effective LR heuristic method for producing high quality feasible solutions is developed, complying with the decentralized demand participation objective and limiting the creation of new peaks by the concentrated shift of reschedulable demand to the same periods by imposing a suitable relative maximum demand limit on the latter.

Part II [26] demonstrates the applicability of the proposed mechanism by considering two reschedulable demand technologies with significant potential, electric vehicles with flexible charging capability and electric heat pump systems accompanied by heat storage, formulating their price response sub-problems and presenting suitable case studies validating the properties of the mechanism and illustrating the benefits of these technologies' market participation.

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