Guest Editorial
Fractional-Order Circuits and Systems

In 2008, an article was published [1] that labeled fractional-order continuous-time systems as the "21st century systems." In 2010, another article followed [2] to confirm the importance of this emerging research area and stress on its "multidisciplinary nature." In particular, it is an area where biochemistry, medicine, and electrical engineering overlap giving rise to numerous new potential applications.

Mathematicians have laid the foundations of fractional-calculus more than 200 years ago. Fractional-calculus considers derivatives and integrals of an arbitrary order and therefore classical first-order, second-order, or third-order derivatives and integrals are special cases of a more general situation where a derivative or integral can be for example of order 1/2 or order π. As such, circuits and systems design should not be restricted to the narrow integer-order domain.

Control theorists were early to realize the importance of incorporating fractional-calculus techniques in control systems design. They introduced the general $\mathrm{PI}^{\lambda}D^{\mu}$ controllers instead of traditional proportional-integral-derivative controllers and proposed applications of these controllers for power converters and chaos control, among others (see [2, ref. 25-41]).

Biochemists were also early to adopt and utilize fractional-order dynamics. The impedance spectroscopy measurement technique is widely used to characterize the electrical properties of materials and biological tissues. Biochemists realized long ago that tissue impedances usually scaled with frequency as $(1/\omega)^{\alpha}$ where α is noninteger. The term "constant-phase-element" was introduced as part of the Cole-Cole model; which was found to fit most tissues ranging from fruits and vegetable tissues to the human skull, cancer, and lung diseased tissues as well as the intestine tissue (see [2, 42–57]).

Circuit theorists approached the subject of fractional-order circuits back in the 1960s. In 1964, a paper was published [3] proposing a method to approximate a "fractional capacitor." The concept of "fractional capacitor" was thus born to represent the same element known by biochemists as the "constant phase element." In 1967, another paper [4] was published showing how the "constant-argument immittance" can be approximated. The reason for trying to approximate a fractional-impedance is simply the lack of off-shelf dielectric-material-based fractional capacitors although Westerlund [5] pointed out that dielectric capacitors can only be accurately modeled using fractional derivatives.

It is clear that while the knowledge of fractional calculus and its various techniques is wide spread among biochemists, control theorists, and signal processing researchers, the circuits and systems society is lagging behind, despite some recent publications [6]–[8]. Fractional-order circuits will revolutionize the way we educate future electrical engineers and the applications in biomedicine will be of particular importance [9]. There is no reason why we should only remain modeling and designing systems and circuits in the integer-order subspace.

This special issue offers an opportunity for the circuits and systems community to explore the rapid advances in fractional-order circuits and systems design and their applications in a number of disciplines. A total of 20 papers have been accepted in this special issue and are arranged to cover three main themes: fractional-order circuits and systems, fractional-order biomedical models, and fractional-order control. Below is a summary of the contributions under each of these themes.

FRACTIONAL-ORDER CIRCUITS AND SYSTEMS

As the focus of this special issue is mainly on fractional-order circuits and systems, most contributions are indeed focused on this area.

In the first paper "A Systematic Approach for Implementing Fractional Order Operators and Systems," the authors show how classes of fractional-order systems can be realized in digital hardware using low-cost field programmable gate arrays. Fractional-order derivatives and integrals are realized in fixed-point hardware, with the desired accuracy. Fractional-order transfer functions are also realized by using the fractional-order derivatives and integrals as building blocks. The resultant hardware, while offering a robust low-cost solution, achieves both high speed and high accuracy.

In the next paper "Minimal Realizations for Some Classes of Fractional Order Transfer Functions" the authors attempt to find the minimal realizations for some classes of fractional-order transfer functions which lead to reduced hardware complexity when these transfer functions are implemented using analog circuits. The author of "Closed Form Rational Approximations of Fractional, Analog and Digital Differentiators/Integrators" shows the equivalence between two well-known continued fraction expansions of the fractional-order Laplace operator (s^{α}) and provides closed-form formulae, which greatly reduce the effort of approximating this operator. Along the same direction, the paper "Caputo-Based Fractional Derivative in Fractional Fourier Transform Domain" proposes a novel closed-form analytical expression of the fractional derivative of a signal in the Fourier transform and fractional Fourier transform domains by utilizing the fundamental principles of fractional-order calculus. The application of a low-pass finite impulse response fractional order filter is investigated for an amplitude-modulated signal corrupted with high-frequency chirp noise.

A new method for the realization of integer-order approximations of 2-D fractional order filters having frequency-planar beam shaped pass-bands is proposed by the authors in "Approximate Realization of Fractional-Order 2-D IIR Frequency-Planar Filters." The resulting integer-order filters are

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shown to accurately approximate 2-D fractional-order frequency-planar beam shapes both magnitude in and phase. In “Fractional Order Butterworth filter: Active and Passive Realizations,” a general procedure to obtain Butterworth filter specifications in the fractional-order domain is proposed. The necessary and sufficient condition for achieving a fractional-order Butterworth filter with a specific cut-off frequency is derived and several passive and active filters are experimentally validated. This paper is clearly geared towards circuit design so much as the following one “Design and Analysis of CMOS Terahertz Integrated Circuits by a Fractional-Order RLGC Transmission Line Model” is. In this particularly important contribution, a fractional-order model is proposed for the design and analysis of CMOS-based THz integrated circuits that utilize transmission lines, such as standing-wave oscillators, with the accuracy of the model verified up to 110 GHz. On the other far-end of the spectrum, where very low frequency signals are applied, fractional-order models show another striking success with their ability to accurately reproduce the behavior of super-capacitors; also known as ultra-capacitors. This is detailed in the particularly important paper entitled “Measurement of Super-Capacitor Fractional-Order Model Parameters From Voltage Excited Step Response” where the authors show how to construct fractional-order models of super-capacitors from their voltage excited step responses. Both low and high capacity super-capacitors rated up to 3000 F are investigated. Super-capacitors are of great importance for their applications in power electronic circuits of hybrid and renewable energy sources.

The paper “Resonance and Quality Factor of Fractional-Order circuits” introduces general maps for transient and frequency responses of resonance networks showing the damping parameters, resonance and 3 dB frequencies, as well as the quality factor for all possible resonance cases. In the following paper “Analog Circuit Implementation of Fractional Order Damped Sine and Cosine Functions” analog circuit implementations are proposed for two fundamental linear fractional-order systems whose impulse responses are the fractional-order damped sine and cosine functions. The next paper “Analog Modeling of Fractional Switched Order Derivative Using Different Switching Schemes” presents comparison of two different switching schemes of variable order derivatives. Results of analog modeling are presented using approximations of integrators of orders 0.5 and 0.25. Finally, the paper “On FIR Filter Approximation of Fractional-Order Differentiators and Integrators” introduces and compares three different FIR filter structures that approximate fractional-order differentiators and integrators with reduced complexity.

FRACTIONAL-ORDER BIO-MEDICAL MODELS

Modeling of biological tissues is crucial for the design of bio-medical circuits for various applications. Indeed, this is also true for many other applications in the agriculture and food industries. In this special issue, three papers are geared towards the applications of fractional-order circuits in biology and medicine. A comprehensive survey is conducted by the author of “A Survey of Fractional-Order Circuit Models for Biology and Biomedicine” who presents all fractional-order circuit models widely used in biomedicine and biology to fit experimentally collected impedance data. An overview of the different methods used to extract the impedance parameters from collected datasets is presented and applications of fractional-order circuit models for human tissues, plant physiology, respiratory systems, and tissue–electrode interfaces are compared to highlight the accuracy of these different models.

The next paper in this section with the title “Emerging Tools in Engineering: Fractional Order Ladder Impedance Models for Respiratory and Neural Systems” focuses on preserving the natural rules governing the intrinsic fractal (recurrent) structure of some biological tissues through fractional-order impedance models. A generally valid ladder network model allows preserving the anatomy of such tissues.

Finally, the paper “Fractional Order Generalization of Anomalous Diffusion as a Multidimensional Extension of the Transmission Line Equation” proposes a new fractional order generalization of the diffusion equation to describe the anisotropy of anomalous diffusion which is often observed in brain tissues using magnetic resonance imaging (MRI). The model was used to analyze diffusion weighted MRI data acquired from a normal human brain using a clinical MRI scanner.

FRACTIONAL-ORDER CONTROL

In the paper “Observer-Based Approach for Fractional-Order Chaotic Synchronization and Secure Communication” the fractional-order direct Lyapunov theorem is used to derive the closed-loop asymptotic stability conditions. The proposed approach is then applied to secure communications using chaotic modulation. In “Reduced Order Approximation of MIMO Fractional Order Systems” a new two-stage method for reduced integer order approximation of fractional MIMO systems is proposed. The reduced order system is constructed by selecting the dominant poles from an intermediate high integer order model. The input and output matrices are found by matching approximate time moments and Markov parameters of the final reduced order model and the original system.

In the paper “Using Self-Synchronization Error Dynamics Formulation Based Controller for Maximum Photovoltaic Power Tracking in Micro-Grid Systems” a self-synchronization error dynamics based controller for maximum photovoltaic power tracking is proposed. A fractional-order model to express the dynamic behavior of incremental conductance and to adjust the terminal voltage to the maximum power point is proposed. The proposed model reduces the tracking time and improves the conversion efficiency.

The association of fractional calculus and symbolic dynamics is explored in the paper entitled “Symbolic Fractional Dynamics.” Results reveal the strong relationship between the geometrical fractal dimension and one particular definition of the fractional derivative. Finally the paper “Study on Control Input Energy Efficiency of Factional Order Control Systems” studies fractional-order control systems from the view point of control input energy efficiency. The divergent terms of the control input energy function of fractional-order control systems are obtained and shown to have a significant role in the amount of the energy injected to a plant by the controller.

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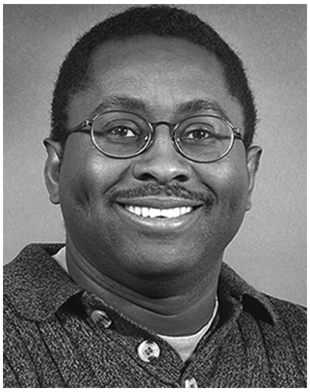

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