Correspondence

Addendum to "Modeling MOSFET Drain Current Non-Gaussian Distribution With Power-Normal Probability Density Function"

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In our paper "Modeling MOSFET Drain Current Non-Gaussian Distribution with Power-Normal Probability Density Function" [1] published in IEEE ELECTRON DEVICE LETTERS, Feburary 2014, the analytic results are derived for power Gaussian distribution (PGD) with integer n only. It appears to be a limitation in practical application, where one may need non-integer n for a more accurate model, e.g., $n = 1.5$. This addendum is to show that our analytic results, without any format change, are also applicable to non-integer n. Therefore, the integer limitation can be stripped off from our model, which in fact is capable of providing continuous coverage.

We redefine the scope of transformation as follows:

$$y(x) = K\left[\left(\frac{x}{\eta}\right)^{\frac{1}{n}} - 1\right] \tag{1}$$

where n is real that satisfies $n \geq 1$. Then, with Taylor expansion, $\mathrm{E}[X]$ is given by

$$\begin{aligned}\mathrm{E}[X] &= \int_0^\infty x\phi(y(x))\frac{dy(x)}{dx}dx \\ &\approx \eta\int_{-\infty}^{\infty}\left[\Sigma_{i=0}^{\infty}C_n^i\left(\frac{y}{K}\right)^i\right]\phi(y)dy \end{aligned} \tag{2}$$

where C_n^i is the generalized binomial coefficient defined as

$$C_n^i = \prod_{k=1}^{i}\frac{n-k+1}{k} \tag{3}$$

Similarly, $\mathrm{E}[X^2]$ is calculated as

$$\mathrm{E}[X^2] \approx \eta^2\int_{-\infty}^{\infty}\left[\Sigma_{i=0}^{\infty}C_{2n}^i\left(\frac{y}{K}\right)^i\right]\phi(y)dy \tag{4}$$

To obtain an explicit and simple solution for K, we only keep the low order terms up to $(y/K)^4$ in both $\mathrm{E}[X^2]$ and $(\mathrm{E}[X])^2$. This is a reasonable approximation when n is small, e.g., $n < \sim K/2$.

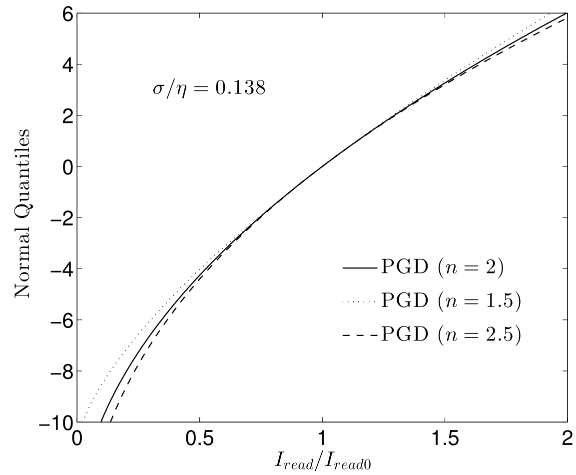

Fig. 1. The normal probability plot for a comparison of PGD with $n = 1.5$, $n = 2$, and $n = 2.5$. We choose $\sigma/\eta = 0.138$, which is used in Fig. 3 of [1], based on σ and η extracted from silicon data.

The approximation leads to the following result

$$\sigma^2 \approx \eta^2 \cdot \left[\frac{C_{2n}^2 - 2C_n^2}{K^2} + \frac{3C_{2n}^4 - \left(C_n^2\right)^2 - 6C_n^4}{K^4}\right] \tag{5}$$

By solving Eq. (5), K can be approximately expressed as a function of median η and variance σ^2 of distribution:

$$K \approx \sqrt{\frac{(n-1)(3n-5)}{\sqrt{\frac{\sigma^2}{\eta^2}\frac{2(n-1)(3n-5)}{n^2} + 1} - 1}} \tag{6}$$

which is exactly same as Eq. (6) in [1] in terms of format. Fig. 1 shows a comparison of PGD with $n = 1.5$, $n = 2$, and $n = 2.5$, given the same σ/η ratio.

REFERENCES

[1] B. Yu *et al.*, "Modeling MOSFET drain current non-Gaussian distribution with power-normal probability density function," *IEEE Electron Device Lett.*, vol. 35, no. 2, pp. 154–156, Feb. 2014.

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