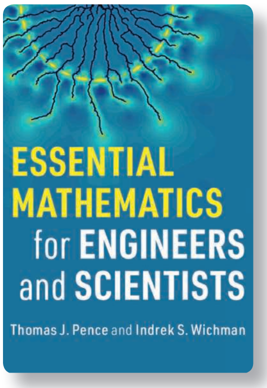
IEEE Control Systems welcomes suggestions for books to be reviewed in this column. Please contact either Scott R. Ploen, Hong Yue, or Thomas Schön, associate editors for book reviews.



Cambridge University Press, 2020,
ISBN: 978-1-10842-544-5,
745 pages, US$120.

ESSENTIAL MATHEMATICS FOR ENGINEERS AND SCIENTISTS

BY THOMAS J. PENCE and INDREK S. WICHMAN

Reviewed by Joel A. Stroch

This book falls within the category of what has been labeled "advanced mathematics for scientists and engineers" and is written for first-year graduate students of engineering and physical science. It is assumed that the reader has gone through the standard undergraduate courses in calculus, linear algebra, ordinary differential equations, and partial differential equations. Unlike other popular texts [1], [2], this book is written at a more advanced level and avoids repetition of several topics that have been covered at the undergraduate level (for example, ordinary differential equations, Laplace transforms, vector calculus, basic linear algebra, Fourier analysis, and elementary boundary value problems). It is clear that the authors have gone to great lengths in selecting which topics to include, and the pedagogical presentation is excellent. The example problems and end-of-chapter exercises are well chosen and help motivate the formal mathematical framework. A unique feature of the text is revisiting topics from different viewpoints, which fosters the beauty and power of the mathematical theory. For example, the Fredholm alternative is first considered in the context of linear algebra and then later in the context of ordinary differential equations. In the preface, the authors grapple with the problem of how the ubiquitous use of computers and software impacts the presentation of topics in advanced mathematics courses. This is an extremely critical issue, and I cannot agree more with their statement: "Although computers *have* altered the way we do our mathematics, and how we construct solutions, and how we consider the parts of a given problem in relation to the whole, it was a change in form, not content." The text is divided into three parts: Linear Algebra, Complex Variables, and Ordinary and Partial Differential Equations. The authors dispel the notion that these three topics are isolated from one another by showing mutual connections in both theory and applications.

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The book is organized into three parts. Part 1 consists of four chapters covering finite-dimensional vector spaces, linear transformations, systems of linear equations, and eigenvalue problems. The introduction of the singular value decomposition in the context of a linear transformation from R^n to R^m is well motivated, and the authors provide a natural connection to the pseudoinverse. I was surprised that there was no mention of ill-conditioning in the normal equations, which is significant in numerical computation. Also absent are the Rayleigh quotient [3] and the more elegant characterization of the eigenvalues of a Hermitian matrix contained in the Courant–Fischer theorem [4]. It should be noted, however, that the text does discuss the Rayleigh quotient in the context of approximating eigenvalues of partial differential equations in Section 9.4.

Part 2 is a self-contained module on complex variables. It consists of eight chapters covering integration in the complex plane, analytic functions, mappings (including the Schwarz–Christoffel transformation), contour integration, series expansions, branch points, and analytic continuation. There are also sections devoted to potential flow and the Laplace and Fourier transforms. I was positively impressed that the authors were careful to strike just the right balance between mathematical rigor and the target audience. Too often, authors of applied math textbooks make statements that are not precise and many times simply incorrect. On page 254, after establishing that satisfaction of the Cauchy–Riemann equations is a necessary condition for a function to be analytic at a point, the authors are careful to mention that an additional condition—continuity of the partial derivatives—is required for this condition to be sufficient. Although the proof is not given (which, in my opinion, is fine), at least it is clearly stated so that the student is aware of it.

Part 3 is devoted to partial differential equations and consists of five chapters covering separation of variables, eigenvalues and eigenfunctions, the Rayleigh quotient, linear ordinary differential equations in the complex plane, and the Green's function. Analysis of the Green's function is extensive, including bounded and unbounded domains, complex contour integration, eigenfunction expansions, and applications to the Poisson equation. Inevitably, any presentation of the Green's function requires transitioning to the realm of generalized functions and the theory of distributions. This presents a difficult challenge to any author, and decisions need to be made with respect to the depth and rigor to which topics are addressed. There is no ideal answer to what is the best approach, and the authors have wisely adopted a heuristic introduction motivated by physical applications. It is regrettable that there is no coverage of the D'Alembert solution to the wave equation nor the method of characteristics. In addition, methods for solving first-order partial differential equations are not covered [5].

SUMMARY

Overall, this text is one of the best textbooks available for introducing students of engineering and the physical sciences to advanced methods of applied mathematics. The material is presented in a cogent manner with numerous insightful examples and end-of-chapter exercises that both reinforce the theory and show applications to areas not covered in the text. After mastering the material in this book, the reader will be equipped to explore more advanced topics such as functional analysis, perturbation methods, and asymptotic expansions.

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REVIEWER INFORMATION

Joel Storch (joel.storch@csun.edu) started his career as an applied mathematician at Brookhaven National Laboratory and Goddard Institute for Space Studies, where he specialized in numerical methods for partial differential equations. In addition, he has more than 30 years of experience in the aerospace industry and was a senior member of the technical staff at the Charles Stark Draper Laboratory, NASA Jet Propulsion Laboratory, The Aerospace Corporation, and Boeing, where he worked on various programs in the areas of spacecraft dynamics and control. Currently, he serves as an adjunct professor in the Department of Mechanical Engineering at California State University, Northridge, Los Angeles, California, 91330, USA, where he performs research in the mechanical properties of carbon nanotubes. His publications have primarily been in the areas of numerical analysis, multibody dynamics, and vibrations.

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