



## Iterative Learning Control: An Optimization Paradigm

by DAVID H. OWENS

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**W**ith the recent development of artificial intelligence, learning control becomes a promising direction in the field of control. The combination of artificial intelligence and control technology creates new research opportunities in many areas, such as robotics, manufacturing, and transportation.

The research topics related to learning control include adaptive control, neural-network control, and reinforcement learning, to name a few. Iterative learning control (ILC) is also a category of learning control, and it executes the same control task repeatedly during a finite time horizon. Such a control task widely exists in real world, such as machines in the mining industry, robots in an assembly line, and even a basketball player practicing shots.

To state it more formally, ILC aims to control the system input  $u(t)$  to make the system output  $y(t)$  track a reference signal  $r(t)$ ,  $t \in [0, T]$ . After time  $T$ , the system state will return to the initial state at time zero, and the control task is repeated again. Different from the classical control technology, such as a proportional-integral-derivative (PID) controller, ILC can use the error observations in the previous trials (or iterations) and update the control actions for the next trial. Therefore, ILC can achieve high-precision tracking without lags in transient tracking that always exist in a PID controller. The learning feature of ILC indicates that it has the essence of trial and error, and its key task is how to extract useful information from the repeated error signals to improve the control performance. ILC has no special requirement on the system dynamic structure, and it is applicable to a system whether the system is linear or nonlinear, deterministic, or random.

The basic idea of ILC can be found in a publication in 1978 [7]. However, ILC attracted wide attention after a series of articles in 1984 [4], [5] and has been extensively studied during the past three decades. For more details, interested readers can refer the books [1], [2], [6], and [9] and review articles [3] and [8]. From the task description of ILC in the preceding paragraph, it can be observed that it has close relation to the optimization theory and filtering theory. This monograph by David H. Owens summarizes his rich research on ILC from

the paradigm of optimization. Owens is with the Department of Automatic Control and Systems Engineering at the University of Sheffield, United Kingdom. He has long-term academic research and practice with ILC and introduces the optimization paradigm to the ILC community in the form of “norm-optimal iterative learning control (NOILC).” NOILC aims to minimize not only the tracking errors but also other metrics in a norm form, such as the control energy consumed or disturbance-related quantities. It is similar to quadratic-optimal control but with advantages derived from repeated observations on trials. Different from other books on ILC, this monograph provides a unique perspective of optimization for the ILC problem, and it may motivate new topics in ILC, such as data-driven optimization or control.

The first four chapters introduce the concepts and mathematical background knowledge for ILC. Chapter 5 gives a formal formulation for the ILC problem. Chapters 6 and 7 present two straightforward approaches to study ILC; one is the inverse model algorithm, and the other is the gradient algorithm. Chapter 8 introduces research about combining the inverse and gradient algorithms together. The next five chapters are the key content of this monograph, where the NOILC approach is extensively discussed. Finally, Chapter 14 introduces parameter-optimal iterative control, in which the control is parameterized.

Chapter 1 presents a brief introduction of the concept of ILC, and the difference between repetitive and iterative control is discussed. Some industrial examples, including automated ploughing, automated coal cutting, and metal rolling, are introduced to provide an intuitive concept for the audience. Chapter 2 introduces the necessary mathematics for the ILC problem. Mathematical knowledge about matrix theory, quadratic optimization, Banach spaces, and Hilbert spaces is provided. These provide the mathematical basis for the following chapters about the state-space model and quadratic-optimal control for the ILC problem. Chapter 3 presents state-space models in control theory. Classical control-theory content is also introduced, including the Laplace transformation, transfer function, and system frequency response. The quadratic-optimal control for linear systems is also introduced in this chapter, where the classical Riccati equation is discussed. Chapter 4 presents matrix models to formulate a discrete-time dynamic system, which is widely used in control theory. The control problem is formulated as matrix equations in discrete time, typically like  $x(t+1) = Ax(t) + Bu(t)$  and  $y(t) = Cx(t) + Du(t)$ . The related algebra of matrix and frequency-domain analysis are also given.

Chapter 5, “Iterative Learning Control: A Formulation,” presents a mathematical formulation for ILC problems in a formal manner. The control objective is to let  $\lim_{k \rightarrow \infty} \|r_k - y_k\| = 0$ , where  $\|\cdot\|$  represents a norm and the subscript  $k$  indicates the quantity in the  $k$ th trial. That is, ILC has to satisfy a *perfect tracking* requirement. Some

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conditions for convergence, such as the eigenvalue theory, are also introduced in this chapter. Chapter 6, "Control Using Inverse Model Algorithms," presents a natural method to study ILC. When the system transfer function has a closed-form inverse, ILC can be directly studied using the algebra analysis of matrix inverse and other matrix operations.

Chapter 7, "Monotonicity and Gradient Algorithms," presents another algorithm that uses the gradient of an objective function with respect to the control variables. Gradient-based optimization is widely used in the operations research community. This idea is also naturally applicable to the ILC problem, where the objective function can include not only the tracking error but also other metrics, such as the control energy consumed. Chapter 8, "Combined Inverse and Gradient Based Design," presents an approach that combines the inverse and gradient algorithms. Such a combination has potential benefits to improve the robustness and efficiency of control algorithms.

Chapter 9, "Norm Optimal Iterative Learning Control," presents an optimization framework to formulate the ILC problem. The control problem is reformulated as an optimization problem in which the norm of tracking error and other metrics are used as the optimization objective. It creates a new perspective to study ILC. The monotonicity analysis can be further studied, which guarantees the convergence of the optimization algorithm. Such NOILC formulation is the fundamental idea of this monograph, and the following chapters discuss different aspects of NOILC.

Chapter 10, "NOILC: Natural Extensions," further explains how an iterative control problem can be formulated as an NOILC problem. Some examples are used to clarify such explanations, including the intermediate-point problem and the multitask problem. Chapter 11, "Iteration and Auxiliary Optimization," introduces auxiliary variables to optimize not only tracking errors but also other objectives. The interpretation of auxiliary variables varies in different practical problems.

Chapter 12, "Iteration as Successive Projection," presents ILC as a successive projection problem, which provides a clear and simple way to study ILC in Hilbert spaces. The issue of control constraints is also discussed here. Chapter 13, "Acceleration and Successive Projection," discusses how to accelerate the convergence of NOILC algorithms. Successive projection is used to do convergence acceleration.

Chapter 14, "Parameter Optimal Iterative Control," discusses a special ILC problem where the control law is parameterized. With the optimization paradigm, such ILC problems can be viewed as a parameter-optimization problem with norm objective. The cases of single-dimension or multiple-dimension parameters are both studied in this chapter.

This book presents a comprehensive study of ILC from the optimization paradigm, more specifically, the NOILC optimization paradigm. The organization is clear, and the necessary fundamentals are self-contained. The mathematical analysis is rigorous, and the algorithms are detail complete. This book is suitable for academic researchers rather than engineering practitioners. It can be used as a textbook for graduate students who are interested in ILC. It also gives rich motivations for researchers to study ILC from the optimization perspective. New research topics may spark following the idea of viewing ILC as an optimization problem rather than a control problem.

## REVIEWER INFORMATION

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