**TEEE Control Systems** welcomes suggestions for books to be reviewed in this column. Please contact either Scott R. Ploen, Hong Yue, or Thomas Schön, associate editors for book reviews.



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# CONTROLLER DESIGN FOR DISTRIBUTED PARAMETER SYSTEMS

by KIRSTEN A. MORRIS

Reviewed by Ralph C. Smith®

When first studying control theory, it is common to initially focus on linear systems that have a finite, and often small, number of states whose dynamics depend only on time. Examples include vibrating systems or circuits modeled by ordinary differential equations. These are often termed *lumped*-

parameter systems, and the theoretical basis for their control and estimation rely on results from linear algebra. In contrast, many engineering systems have spatially varying dynamics and are modeled by partial differential equations (PDEs) with solutions that depend on a continuum of state values. These are often called distributed parameter systems. Whereas the goals for controller design in the infinitedimensional case are similar to those for finite-dimensional systems (establishing stability, developing feedback control for disturbance rejection and tracking, performing state estimation, and performing input-output controller design), their analysis is significantly more difficult. The reasons include the reliance on functional analysis and infinite-dimensional systems theory, and the necessity to establish criteria so that control algorithms based on model approximations are effective for the intended infinitedimensional devices.

This book provides a self-contained introduction to control and estimator design for distributed parameter

Digital Object Identifier 10.1109/MCS.2022.3216693 Date of current version: 9 January 2023 systems. Although there exist previous texts on this topic, there are several features that make this text very timely and unique. For example, carefully chosen examples are given to motivate the analysis and illustrate differences from finite-dimensional theory. The accessibility of the text is further enhanced by the focus on linear models with bounded control and observation operators, and on PDEs rather than delay-differential equations. To convey the active and evolving nature of the field, the author concludes each chapter with a self-contained section summarizing the primary contents for the chapter and related advanced topics, along with a comprehensive (yet concise) set of references. This self-contained text provides a valuable resource for readers interested in both the fundamental principles and implementation techniques for controller design for distributed parameter systems.

#### **CONTENTS**

The first chapter summarizes three classes of PDE models used throughout the text. The diffusion equation (modeling heat flow) is covered first, where it is shown that analytical solutions exist for certain boundary conditions. Various forms of the 1D wave equation are then used to illustrate the effects of boundary conditions, damping, and properties of hyperbolic equations. The final model quantifies beam displacements for various forces and boundary conditions. This model has a fourth-order spatial derivative and includes both internal and viscous damping. These models are also augmented by brief illustrations of related physical phenomena.

Chapter 2 summarizes the infinite-dimensional systems theory employed throughout the text. Prerequisite material on functional analysis is given in the appendix. The compilation of the background theory required for understanding the control and observation of PDEs into a single chapter as well as an appendix constitute a significant strength of the book. This is achieved, in part, through the excellent use of examples to motivate infinite-dimensional topics. To illustrate, a solution of the heat equation is used to generalize the matrix exponential to motivate semigroups and their generators on Hilbert spaces, which in turn are often motivated by energy principles. For readers familiar with finite-dimensional linear systems theory, the concepts of controllability, observability, and input-output maps will be familiar. However, the details are significantly more subtle and nuanced for infinite-dimensional systems. For example, one must delineate between exact and approximate controllability as these concepts are equivalent for finite-dimensional systems. Overall, the coverage of topics is remarkably complete, and omitted topics are noted with references in the final section. These include theory for unbounded input–output operators [1] and port-Hamiltonian theory for infinite-dimensional systems [2].

Chapter 3 focuses on stability properties of distributed parameter systems. As with controllability and observability, stability concepts for infinite-dimensional systems are significantly more complex than for finite-dimensional models. This is due in part to the property that the spectrum of an infinite-dimensional operator contains components other than eigenvalues, which can affect stability. This yields concepts such as asymptotic and exponential stability, which are equivalent for lumped-parameter systems. As illustrated by examples, the stability of PDEs can be affected by boundary conditions and the nature of damping in the model. It is noted at the end of the chapter that omitted topics including the theory for systems with unbounded control inputs can be found in [1], whereas [3] and [4] provide details regarding the use of backstepping for stabilization using boundary controls.

Chapter 4, on optimal linear-quadratic controller design, has similar objectives to the finite-dimensional case: construction of feedback gains that minimize finite- and infinite-time cost functionals, formulation of control laws in terms of Riccati solutions, and determination of optimal actuator locations. However, as with system stability, the infinite-dimensional models introduce several issues that must be addressed. For example, it is illustrated using a modal approximation of the undamped beam equation that feedback gains do not converge, and the model cannot be stabilized using a finite number of control variables. This motivates the development of criteria that guarantee systems are uniformly stabilizable and detectable. A variation of the beam model is later used to demonstrate the role of damping when establishing the convergence of actuator locations. Finally, the section on numerical linear algebra presents techniques that are important when solving the matrix algebraic Riccati equation (ARE) to construct feedback control gains. References are then given, including [5] (which covers finite-dimensional results for optimal linearquadratic control) and [6] and [7] (which provide additional theory for infinite-dimensional systems).

Chapter 5 focuses on the development of control theory and algorithms that handle disturbances due to process noise, signal noise, and/or modeling errors. As for lumped-parameter models, the results utilize  $\rm H_2$  relations for fixed disturbances and  $\rm H_{\infty}$  relations for the more common case of unknown disturbances. In the latter case, the computation of feedback control laws are complicated by the fact that the quadratic term in the ARE is often not negative semidefinite, and solutions generally must be computed in an iterative manner. In addition to supporting the theory, the examples, including diffusion on an

irregular 2D domain, illustrate that the placement of actuators in complex geometries is often counterintuitive. This reiterates the critical role of the theory to guide design. The extensions to unbounded control and observation operators are not covered, and the reader is referred to [8] for additional information.

The role of state estimation, addressed in Chapter 6, is critical for control design of distributed parameter systems as states generally cannot be measured everywhere. Building on results from the previous chapter, an  $H_2$  estimator is developed to minimize the error for a single fixed disturbance. It is also demonstrated that  $H_\infty$  estimators bound the error over all disturbances and hence are more robust. Convergence theory for approximating optimal sensor locations is also discussed and builds on the duality between control and estimation. Algorithms and theory from Chapter 4 are then utilized to determine optimal actuator placement. As in previous chapters, examples serve a key role for illustrating the performance of methods and issues to be avoided. The reader is referred to [5] and [9] for a full account of the finite-dimensional case.

The material from Chapters 4–6 is synthesized in Chapter 7 to provide theory and approximation algorithms for controller design based on output observations. Here, one optimizes both actuator and sensor locations when computing feedback control signals. The initial discussion focuses on dissipativity results to establish stability based on the input–output behavior of systems. An  $\rm H_2$  control design is developed by combining the optimal  $\rm H_2$  estimate with the optimal  $\rm H_2$ -state feedback. This permits decoupling of the estimator and control designs, although sensors and actuators are coupled in the cost. In contrast, it is shown that estimation and control are coupled for  $\rm H_{\infty}$  designs. Attributes of the  $\rm H_{\infty}$  design are illustrated for the Kelvin–Voigt beam model with a distributed disturbance.

#### **SUMMARY**

This book provides a mathematically rigorous framework for control and estimator design for distributed parameter systems. To make the material accessible to a wide range of readers, the author rigorously states results without proof and employs a suite of examples to illustrate the theory and demonstrate differences from finite-dimensional theory. To provide the framework required for stability analysis, linear-quadratic control design, disturbance rejection, state estimation, and input-output control design, the text summarizes the required infinite-dimensional systems theory and functional analysis in a chapter and an appendix. To highlight the central role of examples, several of the functional analysis results are illustrated using an example from sampling theory, which should be familiar to many readers. Through its combination of theory, approximation techniques, and examples, the text provides a valuable resource for both instructors and readers seeking a self-contained introduction to control and estimation for distributed parameter systems.

REVIEWER INFORMATION

Ralph C. Smith (rsmith@ncsu.edu) joined the faculty of North Carolina State University, Raleigh, NC 27695 USA in 1998, where he is presently a distinguished university professor of mathematics. He is coauthor of the research monograph Smart Material Structures: Modeling, Estimation and Control and author of the books Smart Material Systems: Model Development and Uncertainty Quantification: Theory, Implementation, and Applications. He is on the editorial boards of the Journal of Intelligent Material Systems and Structures, International Journal for Uncertainty Quantification, and the SIAM/ASA Journal on Uncertainty Quantification. He is the recipient of the 2016 ASME Adaptive Structures and Material Systems Prize and the SPIE 2017 Smart Structures and Materials Lifetime Achievement Award, and he was named a Society for Industrial and Applied Mathematics Fellow in 2018. His research interests include mathematical modeling and control of smart material systems, Bayesian model calibration, sensitivity analysis, and uncertainty quantification for physical and biological systems.

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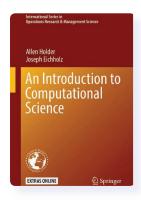
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### **Book Announcements**



Springer, 2019, ISBN: 978-3-030-15677-0, 486 pages, US\$119.99.

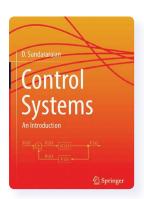
## AN INTRODUCTION TO COMPUTATIONAL SCIENCE

by A. HOLDER and J. EICHHOLZ

This textbook provides an introduction to the growing interdisciplinary field of computational science. The intended audience is the undergraduate who has completed introductory coursework in mathematics and computer science. Students gain computational acuity by developing their own numerical routines, and by practic-

ing with numerical methods as they solve computational models. The text is written in two parts. Part one provides a succinct introduction to the primary algorithms on which a further study of computational science rests. The material is organized so that the transition to computational science from coursework in calculus, differential equations, and linear algebra is natural. Beyond the mathematical and computational content of part one, students gain proficiency with elemental programming constructs and visualization, which

are presented using Matlab. The focus of part two is modeling, wherein students build computational models, compute solutions, and report their findings. The models intersect numerous areas of science and engineering to demonstrate the pervasive role played by computational science.



Springer, 2022, ISBN: 978-3-030-98444-1, 323 pages, US\$79.99.

### CONTROL SYSTEMS: AN INTRODUCTION

by D. SUNDARARAJAN

This textbook is designed for an introductory, one-semester course in control systems for undergraduates and graduates in various engineering departments. It is written in a concise and straightforward manner that attempts to make it easier for students to learn this important subject. The treatment emphasizes the physical

simulation of systems using Matlab. Every new concept is explained with figures and examples to foster a deeper understanding of the theory.

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