

# Generalized Area Spectral Efficiency of Wireless Ad-hoc Networks over Rayleigh Fading

Jun Zhu, Lei Zhang, Hong-Chuan Yang, and Mazen O. Hasna

**Abstract:** Generalized area spectral efficiency (GASE) was introduced as a metric to quantify the spectral utilization efficiency of wireless transmissions. Unlike other performance metrics, GASE takes into account the spatial property of wireless transmissions. In this paper, we extend the research on GASE from link level to network level. In particular, we consider a wireless ad-hoc network with Poisson distributed nodes. We take into account the co-channel interference and derive the generic closed-form moment generating function (MGF) expression of aggregate interference in such network. With the interference statistics, we calculate the ergodic capacity, affected area, and GASE of the network over Rayleigh fading channels. Furthermore, we analyze the effect of carrier sense multiple access with collision avoidance (CSMA/CA) mechanism on the GASE performance of such network. Finally, we propose a new cognitive paradigm that allows the secondary transmitters that are located outside the primary affected area to transmit. With this paradigm, we can achieve high ergodic capacity while effectively utilizing the space-spectrum resource of primary network. Besides, through mathematical analysis and numerical examples, we show that GASE provides a new perspective on transmission power selection and secondary network optimization.

**Index Terms:** Aggregate interference, ergodic capacity, Poisson point process, Rayleigh fading.

## I. INTRODUCTION

WITH the development of wireless communication technology, a variety of wireless network models are emerging, such as ad hoc network, wireless sensor network, cognitive network, and etc. Those networks are usually confined in a specific geometrical area with dense node distribution. Nodes in the network share the same spectrum bandwidth without centralized coordination, and thus generate interference on each other. Therefore, spatial property and node intensity are essential to the system performance. However, most conventional performance metric for wireless networks focus on the quantification of either spectrum utilization efficiency or link reliability. Seldom do they consider the spatial property of wireless transmission. In [1], generalized area spectral efficiency (GASE) was proposed to evaluate the spectral efficiency as well as spatial utilization efficiency of arbitrary wireless transmissions. GASE

is defined as the ratio of overall effective ergodic capacity of the transmission link over its affected area, where a significant amount of transmission power is observed and parallel transmissions over the same frequency will suffer high interference level. In previous work [1], GASE performance analysis was carried out on link-level transmission scenarios, such as dual-hop relay transmission, three-node cooperative relay transmission and underlay cognitive radio transmission. These works focused on the transmission power optimization from individual node perspective. In this paper, we extend the analysis to network-level scenario. In particular, we take into account mutual co-channel interference among the nodes that are randomly distributed in the network. We also consider the impact of node intensity as well as node coordination schemes on the system GASE performance.

In wireless networks, a collection of nodes share the same spectrum bandwidth to increase the system spectrum utilization efficiency. From conventional point of view, the side effect of this approach is severe co-channel interference, which may deteriorate the system performance. Therefore, knowledge of interference statistics is essential to the performance analysis of wireless networks. In particular, the statistics of co-channel interference in wireless networks are affected by the following essential physical parameters, namely: (1) Spatial distribution of interferers; (2) propagation characteristic of the medium, including path loss, shadowing and fading; (3) spatial region over which the interferers are distributed. Specifically, if no knowledge regarding node locations is available a priori, a typical assumption is that the nodes are distributed according to a homogeneous Poisson point process [2], [3]. Intensive research has been carried out on the application of Poisson point process to wireless networks, including network connectivity and coverage [4]–[7], packet throughput [8], error probability and link capacity in the presence of interferers in a Poisson field [11], [12], [32], [33]. If we further assume individual interference power follows a distance-dependent decaying power law, then the aggregate interference at the receiver can be modeled as shot noise [13] associated with a particular Poisson point process. In [11], it showed that the shot noise interference from a homogeneous Poisson field of interferers distributed over the entire space can be modeled using the symmetric  $\alpha$ -stable distribution [14]. Following this work,  $\alpha$ -stable distribution has been extensively applied to characterizing the interference in wireless networks [15]–[19]. The interference statistics are given by their MGF function. However, due to the complexities of these MGF functions, no closed-form PDF/CDF expression was given except for several special cases, limiting the usage of the interference statistics in the derivation. In this paper, we further develop these results and apply them into the GASE analysis. Specifi-

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cally, we derive the generic closed-form expression of the interference statistics in Poisson distributed network over Rayleigh fading channels. These expressions can be readily applied into the calculation of ergodic capacity and affected area of such networks. Through mathematical analysis and numerical examples, we analyze the effect of node intensity and transmission power on the ergodic capacity, affected area and GASE of wireless networks.

We also consider the impact of node coordination on system GASE performance. First, we consider a more practical wireless network in which carrier sense multiple access with collision avoidance (CSMA/CA) mechanism is employed. In CSMA/CA network, two close nodes are prohibited to transmit simultaneously, and thus ensures a minimum distance between each active transmitter. In this scenario, we introduce Matèrn point process [20], [21] to model the spatial distribution of transmitter-receiver pairs in CSMA/CA networks. Matèrn point process is established from its underlying Poisson point process by dependent thinning operations, where it retains certain points of a Poisson point process in such a way that no pairs of points are at distance less than a threshold. The statistics we derived in Poisson field is still valid by imposing a guard zone around the transmitter. However, special treatment should be exercised on the calculation of the affected area. Through numerical examples, we compare the performance, in terms of ergodic capacity, affected area and GASE, of wireless networks with and without implementing CSMA/CA mechanism.

Finally, we analyze GASE of two-tier cognitive networks. Cognitive radio is used as a promising technology to improve the spectrum utilization by letting the unlicensed (secondary) user share a frequency bandwidth with the licensed (primary) owner under the condition that no harmful interference is caused to the licensee [22]–[24]. Typically, there are three main cognitive radio paradigms [25]: Interweave, overlay and underlay. With interweave paradigm, cognitive users opportunistically exploit the primary radio spectrum only when primary users are detected to be idle [24]. In overlay paradigm, cognitive users help maintain and/or improve primary users' communication while utilizing some spectrum resources for their own communication needs [26]. The underlay paradigm allows cognitive users share the frequency bandwidth of primary users only if the resultant interference power level at the primary receiver is below a given threshold [27], [28]. These paradigms either explore the time-spectrum hole or use sophisticated techniques to protect the primary network.

In this paper, we propose a new paradigm from the space-spectrum perspective. In particular, primary network is first established and defines its primary affected area. Secondary nodes can transmit only when they are outside the primary affected area. This mechanism allows the secondary network fully explore the space-spectrum holes in the primary network, but it also increases the interference level of the entire network. Through numerical examples, we show the effect of secondary cognitive users on the total ergodic capacity, affected area, whereas we use GASE metric to measure the tradeoff between the former and the latter. Part of the contributions have been reported in [29], which considered ergodic capacity (not GASE) for a single transmitter-receiver pair.

The main contribution of this work can be summarized as follows:

- We further develop the aggregate interference statistics in Poisson distributed network over Rayleigh fading channels. Specifically, we derive the generic closed-form MGF expression of aggregate interference. This result is generic and can be applied both in finite and infinite region.
- We readily apply the closed-form moment generating function (MGF) expression of aggregate interference in the calculation of the ergodic capacity, affected area and GASE of wireless ad hoc network. Besides, we analyze the effect of transmitter coordination on the network performance. We first use Matèrn point process to model the distribution of active transmitters in CSMA/CA network. We find that in sparse network, non CSMA/CA network achieves better ergodic capacity. However, in dense network, CSMA/CA network can ameliorate the increase of aggregate interference level and achieve the same amount of ergodic capacity with fewer transmitters than non CSMA/CA network. We also analyze the common transmission power on the performance of wireless ad hoc network and provide a new perspective on the transmission power optimization.
- We propose a new cognitive radio paradigm to explore space-spectrum resource of wireless network. This paradigm allows the secondary transmitters that are located outside the primary affected area to transmit. We analyze the effect of this paradigm in terms of ergodic capacity, affected area and GASE of two-tier cognitive network. We find that although it deteriorates GASE performance of two-tier cognitive network, secondary transmitters are able to increase ergodic capacity as well as exploit space-spectrum resource of wireless network. However, numerical examples show that the network performance is sensitive to the number of secondary transmitters and their transmission power. Moreover, GASE offers a new perspective on transmitter intensity selection and transmission power optimization.

The remainder of this paper is organized as follows: Section II presents system model and aggregate interference statistics. Section III carries out the analysis on GASE of wireless ad hoc networks with and without implementation of CSMA/CA mechanism. Section IV analyzes GASE of two-tier networks, namely heterogeneous and underlay cognitive network and demonstrates the effect of secondary network on the ergodic capacity and GASE. Section V concludes the work.

## II. SYSTEM MODEL AND INTERFERENCE STATISTICS

### A. System Model

We consider a wireless network in two-dimensional homogeneous space  $\mathbb{R}^2$ . The transmitters in the network are distributed according to Poisson point process  $\Pi = \{r_i\}$  of intensity  $\lambda$  (in the unit of nodes/ $\text{Km}^2$ ), where  $r_i$ 's are the random distances between the transmitters and the origin of the space.<sup>1</sup> We assume all the transmitters are equipped with omni-directional antennas and share the same frequency bandwidth without coordination

<sup>1</sup>The model considered here is more general than Poisson bipolar model [42], in which the distance between transmitter and receiver is fixed.

and restriction. Each transmitter communicates with one and only one receiver. We also assume that each transmitter always has packets to send whenever it gets scheduled. This is applicable in fully saturated network, and the GASE of such network serves as the worst-case lower bound. The case of non-saturated network [30]–[34] will be considered in the future work. For each transmitter-receiver pair, the distance between transmitter and receiver  $d$  is uniformly distributed in the region  $(d_l, d_h)$ , whose PDF is given by

$$f(d) = \frac{2(d - d_l)}{(d_h - d_l)^2}, \quad d_l < d < d_h. \quad (1)$$

The transmitted signal will experience path loss and multipath fading effect. For the sake of clarity, we ignore the shadowing effect. Specifically, the received signal power  $P_r$  at distance  $d$  from a transmitter is given by

$$P_r = \frac{P_t \cdot z}{(d/d_{\text{ref}})^\eta}, \quad (2)$$

where  $P_t$  is the common transmission power of the transmitters,  $\eta$  is the path loss exponent,  $z$  is an independent random variable (RV) that models the multipath fading effect, and  $d_{\text{ref}}$  is the reference distance. Without loss of generality, we set  $d_{\text{ref}} = 1$  m. The term  $1/d^\eta$  indicates the path loss model follows a decaying power law with the distance between transmitter and receiver. For the Rayleigh fading channel model under consideration,  $z$  is an exponential RV with unit mean, i.e.,  $z \sim \mathcal{E}(\infty)$ .

### B. Interference Statistics

With homogeneous assumption, the interference statistics at the reference node  $R_0$  located at the origin of  $\mathbb{R}^2$  represents the statistics of the entire  $\mathbb{R}^2$ . As such, the interference analysis in the following sections is generic and applicable to any points in  $\mathbb{R}^2$ . The aggregate interference experienced by the origin is given, under the assumption of non-coherent addition of interference power, by

$$I = \sum_{i \in \Pi} \bar{\gamma}_i \cdot z_i, \quad (3)$$

where  $\bar{\gamma}_i = P_t/r_i^\eta$  and  $z_i$  is the fading power gain for the  $i$ th transmitter. Note that the individual interference signal power is assumed to follow the decaying power loss law. In addition, the random locations of the transmitting nodes are distributed in  $\mathbb{R}^2$  according to Poisson point process  $\Pi = \{r_i\}$  of intensity  $\lambda$ . Therefore, the aggregate interference power  $I$  can be modeled as shot noise [13]. It follows that the MGF<sup>2</sup> of the aggregate interference power  $I$  from the area of  $r_l \leq r_i \leq r_h$  is given by [8]

$$\Phi_I(s) = \exp \left\{ -\pi\lambda \Psi_I(s) \right\}, \quad (4)$$

<sup>2</sup>The moment generating function is defined as  $\Phi_I(s) = \mathbb{E}[e^{-sI}]$ .

where  $\Psi_I(s)$  is given by

$$\Psi_I(s) = \underbrace{r_h^2 \mathbb{E}_Z [1 - e^{-\bar{\gamma}_h z s}] + (P_t s)^{\frac{2}{\eta}} \mathbb{E}_Z \left[ z^{\frac{2}{\eta}} \Gamma \left( 1 - \frac{2}{\eta}, \bar{\gamma}_h z s \right) \right]}_{\Psi_h(s)} - \underbrace{\left\{ r_l^2 \mathbb{E}_Z [1 - e^{-\bar{\gamma}_l z s}] + (P_t s)^{\frac{2}{\eta}} \mathbb{E}_Z \left[ z^{\frac{2}{\eta}} \Gamma \left( 1 - \frac{2}{\eta}, \bar{\gamma}_l z s \right) \right] \right\}}_{\Psi_l(s)}, \quad (5)$$

where  $\Gamma(\alpha, x) = \int_x^\infty e^{-t} t^{\alpha-1} dt$  is the incomplete gamma function defined in [38, 8.350]. Over Rayleigh fading channels,  $z$  is an exponential RV with unit mean. It can be shown that

$$\mathbb{E}_Z [1 - e^{-\bar{\gamma}_h z s}] = \frac{\bar{\gamma}_h s}{1 + \bar{\gamma}_h s}. \quad (6)$$

Applying [38, 6.455.1], we can further show that

$$\begin{aligned} & \mathbb{E}_z \left[ z^{\frac{2}{\eta}} \Gamma \left( 1 - \frac{2}{\eta}, \bar{\gamma}_h z s \right) \right] \\ &= \frac{(\bar{\gamma}_h s)^{1 - \frac{2}{\eta}}}{(1 + \frac{2}{\eta})(1 + \bar{\gamma}_h s)^2} F \left( 1, 2; 2 + \frac{2}{\eta}; \frac{1}{1 + \bar{\gamma}_h s} \right), \end{aligned} \quad (7)$$

where  $F(\mu_1, \mu_2; \nu; t)$  is the Gauss hypergeometric function defined in [38, 9.111]. Substituting (6), (7) into (5) and applying [38, 9.137.4],  $\Psi_I(s)$  can be simplified to

$$\begin{aligned} \Psi_I(s) &= \frac{\bar{\gamma}_h r_h^2 s}{1 + \bar{\gamma}_h s} F \left( 1, 1; 1 + \frac{2}{\eta}; \frac{1}{1 + \bar{\gamma}_h s} \right) \\ &\quad - \frac{\bar{\gamma}_l r_l^2 s}{1 + \bar{\gamma}_l s} F \left( 1, 1; 1 + \frac{2}{\eta}; \frac{1}{1 + \bar{\gamma}_l s} \right). \end{aligned} \quad (8)$$

Finally, after substituting (8) into (4), we can obtain the MGF of the aggregate interference power  $I$  over Rayleigh fading channels, as

$$\begin{aligned} \Phi_I(s) &= \exp \left\{ -\pi\lambda \left[ \frac{\bar{\gamma}_h r_h^2 s}{1 + \bar{\gamma}_h s} F \left( 1, 1; 1 + \frac{2}{\eta}; \frac{1}{1 + \bar{\gamma}_h s} \right) \right. \right. \\ &\quad \left. \left. - \frac{\bar{\gamma}_l r_l^2 s}{1 + \bar{\gamma}_l s} F \left( 1, 1; 1 + \frac{2}{\eta}; \frac{1}{1 + \bar{\gamma}_l s} \right) \right] \right\}. \end{aligned} \quad (9)$$

From (5), we can also derive MGF of the aggregate interference for several special cases as follows.

#### B.1 Infinite Space

When  $r_h \rightarrow \infty$ , the transmitters are distributed in an infinite space. It can be shown that  $\Psi_h^\infty(s) = k(sP_t)^{2/\eta}$ , where  $k = (2\pi/\eta)/\sin(2\pi/\eta)$ . MGF of the aggregate interference in infinite area is given by

$$\begin{aligned} & \Phi_\infty(s) \\ &= \exp \left\{ -\pi\lambda \left[ k(sP_t)^{\frac{2}{\eta}} - \frac{\bar{\gamma}_l r_l^2 s}{1 + \bar{\gamma}_l s} F \left( 1, 1; 1 + \frac{2}{\eta}; \frac{1}{1 + \bar{\gamma}_l s} \right) \right] \right\}. \end{aligned} \quad (10)$$

## B.2 Continuous Space

When  $r_l = 0$ , the space is continuous without singular point at the origin. It can be obtained that  $\Psi_l^0(s) = 0$ , and MGF of aggregate interference in continuous area is given by

$$\Phi_0(s) = \exp \left\{ -\pi\lambda \frac{\bar{\gamma}_h r_h^2 s}{1 + \bar{\gamma}_h s} F\left(1, 1; 1 + \frac{2}{\eta}; \frac{1}{1 + \bar{\gamma}_h s}\right) \right\}. \quad (11)$$

## B.3 Continuous Infinite Space

For nodes distributed in continuous infinite space, its aggregate interference MGF function is given by

$$\Phi_{(0,\infty)}(s) = \exp \left\{ -\pi\lambda k(sP_t)^{\frac{2}{\eta}} \right\}. \quad (12)$$

The PDF of the aggregate interference  $I$ ,  $f_I(x)$ , can be derived by applying the inverse Laplace transform on  $\Phi_I(s)$ , i.e.,  $f_I(x) = \mathcal{L}^{-1}\{\Phi_I(s)\}$ . Due to the complexity of  $\Phi_I(s)$ , no generic closed-form expression is known for  $f_I(x)$ . However, for special case ( $r_l = 0, r_h \rightarrow \infty, \eta = 4$ ), its PDF can be derived from (12), and is given by

$$f_I(x) = \frac{\lambda}{4} \left( \frac{\pi P_t}{x} \right)^{\frac{3}{2}} \exp \left\{ -\frac{P_t \pi^4 \lambda^2}{16x} \right\}, \quad (13)$$

which is equivalent to [8, eq. 11]. Correspondingly, its CDF expression,  $F_I(x)$ , is given by

$$F_I(x) = \operatorname{erfc} \left( \frac{\sqrt{P_t} \lambda \pi^2}{4\sqrt{x}} \right). \quad (14)$$

Note that although the aforementioned analytical results are general enough to apply to both finite and infinite region, we limit ourselves to the case with  $r_l = 0, r_h = \infty$  in the rest of the paper, which is considered to be the most realistic scenario.

## III. GASE ANALYSIS FOR WIRELESS AD HOC NETWORKS

GASE is defined as the ratio of overall effective ergodic capacity of the transmission link over the affected area, where a significant amount of transmission power is observed and parallel transmissions over the same frequency will suffer high interference level. In this paper, we extend this definition from link-level to network-level analysis. Specifically, we analyze GASE performance of wireless ad hoc network in Poisson field over Rayleigh fading channels. We first derive the total ergodic capacity and affected area of wireless ad hoc network by applying the statistics of aggregate interference obtained in the above section. Then GASE of such network is given by the ergodic capacity over the affected area. This network-level GASE analysis not only considers the transmission power of individual node in the network, but also the node intensity and co-channel interference among them. Furthermore, we analyze the impact of node coordination on system GASE performance. In particular, we employ CSMA/CA mechanism in wireless ad hoc network. Through numerical examples, we compare the system performance of wireless network with and without CSMA/CA in terms of ergodic capacity, affected area and GASE.

## A. Ergodic Capacity Analysis

Ergodic capacity is an appropriate measure if delays can be afforded and coding over many indecent channel realizations (i.e., over many coherence blocks) is possible. The total ergodic capacity of wireless network is given by

$$C_{\text{total}} = \sum_{i=1}^{\lambda\Omega} C_i(d_i), \quad (15)$$

where  $\lambda$  is the intensity of the transmitters (in the unit of nodes/Km<sup>2</sup>),  $\Omega$  is the total area of  $\mathbb{R}^2$ ,  $d_i \in [d_l, d_h]$  is the distance between the  $i$ th transmitter-receiver pair, and  $C_i(d_i)$  is the ergodic capacity of individual transmitter-receiver pair. It follows that the ergodic capacity  $C_i(d_i)$  can be calculated by averaging the instantaneous capacity,  $C = \log_2(1 + \Gamma_i)$ , over the distribution of the received signal-to-interference-plus-noise ratio (SINR),  $\Gamma_i$ , as

$$C_i(d_i) = \int_0^\infty \log_2(1 + \Gamma_i) dF_{\Gamma_i}(\gamma), \quad (16)$$

where  $\Gamma_i = \frac{\mathbf{P}_i}{\mathbf{I} + \mathbf{N}}$ ,  $\mathbf{P}_i$  is the received signal power,  $\mathbf{I}$  is the aggregate interference power, and  $\mathbf{N}$  is the noise power,  $F_{\Gamma_i}(\gamma)$  is the CDF of  $\Gamma_i$ . Over Rayleigh fading channels, the received signal power  $\mathbf{P}_i$  follows exponential distribution with average received signal power determined by the path loss, i.e.,  $\mathbf{P}_i \sim \mathcal{E}(d_i^\eta/P_t)$ . It can be shown that the CDF of the SINR at the receiver is given by

$$F_{\Gamma_i}(\gamma) = P \left\{ \frac{\mathbf{P}_i}{\mathbf{I} + \mathbf{N}} < \gamma \right\} = 1 - \exp \left( -\frac{N d_i^\eta}{P_t} \gamma \right) \Phi_I \left( \frac{d_i^\eta}{P_t} \gamma \right), \quad (17)$$

where  $\Phi_I(\cdot)$  is the MGF of the aggregate interference power given by (9). Substitute (17) into (16) and make some manipulations, the ergodic capacity of individual transmitter-receiver pair can be written as

$$C_i(d_i) = \frac{1}{\ln 2} \int_0^\infty \frac{e^{-\frac{N d_i^\eta}{P_t} \gamma}}{1 + \gamma} \Phi_I \left( \frac{d_i^\eta}{P_t} \gamma \right) d\gamma. \quad (18)$$

When  $\lambda\Omega$  is sufficiently large,  $C_{\text{total}}$  can be approximately calculated as

$$C_{\text{total}} = \lambda\Omega \int_{d_l}^{d_h} C(d) f(d) dd, \quad (19)$$

where  $f(d)$  is the PDF of the distance  $d$  between a pair of transmitter-receiver given by (1). Substitute (1) and (18) into (19), we can arrive at

$$C_{\text{total}} = \kappa \int_{d_l}^{d_h} \int_0^\infty \lambda e^{-\frac{N d^\eta}{P_t} \gamma} \Phi_I \left( \frac{d^\eta}{P_t} \gamma \right) \frac{d - d_l}{1 + \gamma} d\gamma dd, \quad (20)$$

where  $\kappa = \frac{2\Omega}{\ln 2 \cdot (d_h - d_l)^2}$ . For continuous infinite space, substituting (12) into (20), we can arrive at

$$C_{(0,\infty)} = \kappa \int_{d_l}^{d_h} \int_0^\infty \underbrace{\lambda \exp(-\pi\lambda k d^2 \gamma^{\frac{2}{\eta}})}_{\mathfrak{F}_I} \underbrace{\exp(-N d^\eta \gamma / P_t)}_{\mathfrak{F}_N} (d - d_l) / (1 + \gamma) d\gamma dd, \quad (21)$$

where  $\mathfrak{F}_N = \exp(-Nd^n\gamma/P_t)$  presents the effect of transmission and noise power on ergodic capacity, and  $\mathfrak{F}_I = \exp(-\pi\lambda_k d^2\gamma^{\frac{2}{n}})$  presents the effect of aggregate interference on ergodic capacity. Note that when  $N/P_t \rightarrow 0$ ,  $\mathfrak{F}_N \rightarrow 1$ .

### B. GASE Result

The affected area is defined as the area where the aggregate interference power is greater than a threshold value  $I_{th}$ . Mathematically speaking, the affected area can be calculated by the following area integral

$$A_{\text{aff}} = \oint_{\mathbb{R}^2} \Pr\{I > I_{th}\} d\mathbf{S}, \quad (22)$$

where  $\mathbf{S}$  is the area integral variable. As we assume the interference statistics is identical on the homogeneous Euclidean plane  $\mathbb{R}^2$ , the affected area can be written as

$$A_{\text{aff}} = \left(1 - F_I(I_{th})\right)\Omega, \quad (23)$$

where  $F_I(I_{th})$  is the CDF of the aggregate interference. Finally, with (19) and (23), the overall GASE is given by

$$\eta_{\text{GASE}} = \frac{C_{\text{total}}}{A_{\text{aff}}} = \frac{\lambda \int_{d_i}^{d_h} C(d)f(d) dd}{1 - F_I(I_{th})}. \quad (24)$$

### C. Effect of CSMA/CA

The above section considered the transmitters in wireless ad hoc networks distributed according to Poisson point process, which implies that the transmitters' locations are independent with each other. However, this strong assumption is not valid in most practical wireless ad hoc network. Medium access control (MAC) protocol ensures that two close transmitters cannot transmit simultaneously by implementing the CSMA/CA mechanism. Before establishing a successful connection with the target receiver, the transmitter broadcasts a request-to-send (RTS) signal with power  $P_{\text{RTS}}$ . Other transmitters that receive the RTS signaling will postpone their transmission. If only considering the path loss effect, the transmitter defines a guard zone with radius  $R_{\text{RTS}}$ , proportional to  $P_{\text{RTS}}$ . As such, the distance between two active transmitters should be larger than  $R_{\text{RTS}}$ . Poisson point process does not take this constraint into account and leads to inaccuracy in the distribution of active transmitters in wireless ad hoc network that implements CSMA/CA mechanism. Alternatively, we introduce Matèrn point process [20], [21] to model the spatial distribution of active transmitters in CSMA/CA network.

Matèrn point process can be obtained by thinning an underlying Poisson point process. Specifically, we consider a collection of potential transmitters  $\{X_i\}_{i=1,\dots,K}$  independently and uniformly distributed in  $\mathbb{R}^2$ , where  $K$  is an RV describing the total potential transmitters in  $\mathbb{R}^2$  and follows a discrete Poisson Law. The  $K$  potential transmitters constitute the underlying Poisson point process  $\Pi_0$  with intensity  $\lambda_0$ . To build Matèrn point process  $\Theta(K)$ , the transmitters  $X_1$  is first selected into the active transmitters set  $\mathcal{X}$ . At the  $i$ th step, the transmitters  $X_i$  is selected into  $\mathcal{X}$  if and only if none of the previous  $i - 1$  transmitters lies in a circle centered at  $X_i$  with radius  $R_{\text{RTS}}$ . The procedure stops

when all the  $K$  transmitters have been considered. As such, the transmitters  $\{X_j\}_{j=1,\dots,N(K)}$  constitute a Matèrn point process  $\Theta(K)$ , where  $N(K)$  is a RV describing the number of active transmitters selected into  $\Theta(K)$  from the total  $K$  potential transmitters in  $\Pi_0$ . Without considering the boundary effect, the active transmitter intensity of Matèrn point process  $\lambda_m$  can be calculated as

$$\lambda_m = \frac{1 - e^{-\lambda\pi R_{\text{RTS}}^2}}{\pi R_{\text{RTS}}^2}. \quad (25)$$

We can follow the same procedure as previous sections to determine the statistics of aggregate interference as well as the ergodic capacity by substituting  $\lambda$  with  $\lambda_m$ .

The affected area of wireless ad hoc network that implements CSMA/CA mechanism is given by

$$A_{\text{CSMA/CA}} = \bigcup_{X_i \in \Theta} \mathcal{B}_{X_i} + \oint_{\mathbb{R}^2 \setminus \bigcup_{X_i \in \Theta} \mathcal{B}_{X_i}} \Pr\{I > I_{th}\} d\mathbf{S}. \quad (26)$$

The first part,  $\bigcup_{X_i \in \Theta} \mathcal{B}_{X_i}$ , represents the union area of circles  $\mathcal{B}_{X_i}$ 's centered at the active transmitting nodes,  $X_i$ 's, with radius  $R_{\text{RTS}}$ . According to [21], it equals to

$$\begin{aligned} \bigcup_{X_i \in \Theta} \mathcal{B}_{X_i} &= (1 - e^{-\lambda\pi R_{\text{RTS}}^2})\Omega - \frac{\Omega}{\pi R_{\text{RTS}}^2} \int_{R_{\text{RTS}}}^{2R_{\text{RTS}}} \nu(\mathcal{B}_o \cap \mathcal{B}_y) \\ &\left( \frac{1 - e^{-\lambda\nu(\mathcal{B}_o \cup \mathcal{B}_y)}}{\nu(\mathcal{B}_o \cup \mathcal{B}_y)} - \frac{e^{-\lambda\pi R_{\text{RTS}}^2} - e^{-\lambda\nu(\mathcal{B}_o \cap \mathcal{B}_y)}}{\nu(\mathcal{B}_o \cup \mathcal{B}_y) - \pi R_{\text{RTS}}^2} \right) 2\pi y dy \\ &+ K(\lambda, R_{\text{RTS}}), \end{aligned} \quad (27)$$

where

$$\begin{cases} \nu(\mathcal{B}_o \cap \mathcal{B}_y) = 2R_{\text{RTS}}^2 \cos^{-1}\left(\frac{y}{2R_{\text{RTS}}}\right) - \frac{1}{2}y\sqrt{4R_{\text{RTS}}^2 - y^2}, \\ \nu(\mathcal{B}_o \cup \mathcal{B}_y) = 2\pi R_{\text{RTS}}^2 - \nu(\mathcal{B}_o \cap \mathcal{B}_y), \end{cases} \quad (28)$$

and  $K(\lambda, R_{\text{RTS}})$  is negligible for numerical evaluation [21, Proposition 2].

The second part represents the area outside  $\bigcup_{X_i \in \Theta} \mathcal{B}_{X_i}$  and the aggregate interference power of which is greater than  $I_{th}$ . Under homogeneous assumption, it can be written as

$$\oint_{\mathbb{R}^2 \setminus \bigcup_{X_i \in \Theta} \mathcal{B}_{X_i}} \Pr\{I > I_{th}\} d\mathbf{S} = \left( \Omega - \bigcup_{X_i \in \Theta} \mathcal{B}_{X_i} \right) \left( 1 - F_I(I_{th}) \right). \quad (29)$$

Substituting (29) into (26), we can arrive at the affected area of CSMA/CA network, as

$$A_{\text{CSMA/CA}} = \bigcup_{X_i \in \Theta} \mathcal{B}_{X_i} \cdot F_I(I_{th}) + \left(1 - F_I(I_{th})\right)\Omega. \quad (30)$$

Finally, GASE of the wireless network with CSMA/CA is given by

$$\eta_{\text{GASE}} = \frac{\lambda_m \Omega C}{\bigcup_{X_i \in \Theta} \mathcal{B}_{X_i} \cdot F_I(I_{th}) + \left(1 - F_I(I_{th})\right)\Omega}. \quad (31)$$

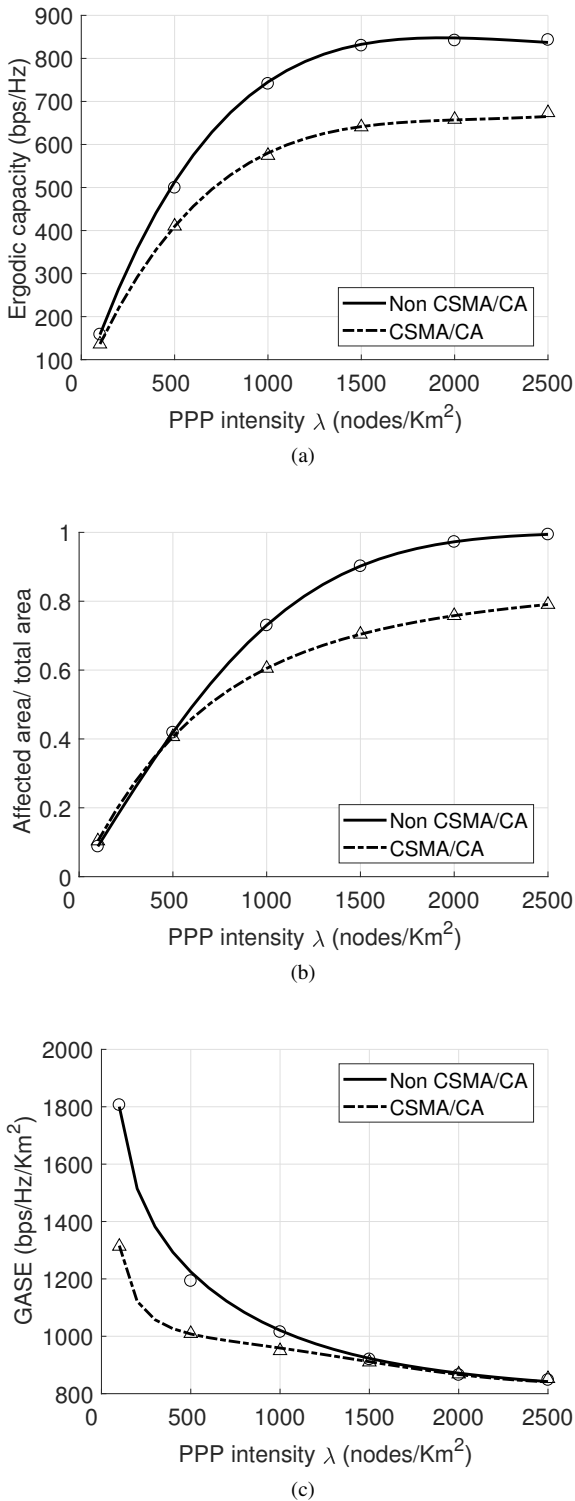


Fig. 1.  $P_t = 10$  dBm,  $\eta = 4$ ,  $I_{th} = -20$  dBm,  $R_{RTS} = 40$  m,  $\Omega = 1$  Km<sup>2</sup>,  $d_l = 1$  m,  $d_h = 20$  m,  $r_l = 0$ ,  $r_h = \infty$ : (a) Ergodic capacity, (b) affected area, and (c) GASE.

#### D. Numerical Examples

We consider a wireless ad hoc network in continuous infinite area of  $\Omega = 1000 \times 1000$  m<sup>2</sup>. The average number of transmitters is given by  $\lambda\Omega$ . The receivers are assumed to be uniformly distributed in a annulus of radius  $d_l = 1$  m and  $d_h = 20$  m

centered at the transmitter. As each transmitter communicates with only one receiver, there are totally  $\lambda\Omega$  transmitter-receiver pairs for concurrent communications. The simulation results are shown as discrete dots, which match well with the analytical results. In Fig. 1, we plot the ergodic capacity, affected ratio as well as GASE of wireless ad hoc network as function of the Poisson point process intensity  $\lambda$ .

Fig. 1(a) shows that there exists a maximal value of ergodic capacity with respect to the intensity  $\lambda$ , which implies the ergodic capacity does not always increase with  $\lambda$ . As  $\lambda$  increases, the average number of transmitters increases correspondingly. The increasing number of transmitters has two effect on the network. For one thing, it increases the level of network aggregate interference. For another thing, more transmitter-receiver pair means more capacity is taken into account of the total ergodic capacity of wireless ad hoc network. In sparse network, i.e.,  $\lambda$  is small, the benefit on capacity incurred by increasing  $\lambda$  is more significant than the negative effect incurred by the increasing interference level. Therefore, the ergodic capacity is an increasing function of  $\lambda$  in sparse network. On the contrary, in dense network, the interference effect dominates and thus the ergodic capacity decreases with respect to  $\lambda$ . Fig. 1(a) also shows that for small value of  $\lambda$ , the ergodic capacity in CSMA/CA network is smaller than that in non CSMA/CA network, which implies that prohibiting close transmitters from simultaneous transmitting decreases the overall system ergodic capacity. However, when  $\lambda$  goes large, without CSMA/CA mechanism, the ergodic capacity decreases dramatically after achieving a maximal value, while the network with CSMA/CA mechanism decreases slightly. This is due to in non CSMA/CA network, the transmitters can be added into the network without restriction. In dense network, too many transmitters will greatly increase the interference level and thus decrease the total ergodic capacity of the network. However, CSMA/CA mechanism prevents excessive transmitters to be activated in dense network. Therefore, when the network is saturated, no more transmitters are allowed to transmit and thus the ergodic capacity only slightly decreases after achieving the maximal value. This phenomenon implies that CSMA/CA mechanism effectively ameliorates the increase of aggregate interference level in dense network. Note that in CSMA/CA network, we use Matérn point process to model the distribution of active transmitters. As Matérn point process is a thinning progress of the Poisson point process, the active transmitters in CSMA/CA network is no greater than its underlying non CSMA/CA network. This means in dense wireless ad hoc network, CSMA/CA network requires fewer transmitters to achieve the same amount of ergodic capacity of non CSMA/CA network.

Fig. 1(b) shows that the affected area is an increasing function of  $\lambda$ . Meanwhile, the affected area of CSMA/CA network increases slower than that of non CSMA/CA network. This is due to for the same value of  $\lambda$ , CSMA/CA network has fewer active transmitters than that of non CSMA/CA network. Finally, Fig. 1(c) shows that in sparse network, non CSMA/CA network enjoys better GASE performance than that of non CSMA/CA network. However, as the number of simultaneous transmitters increases, the latter network outperforms the former one. Meanwhile, GASE of CSMA/CA and non CSMA/CA network are

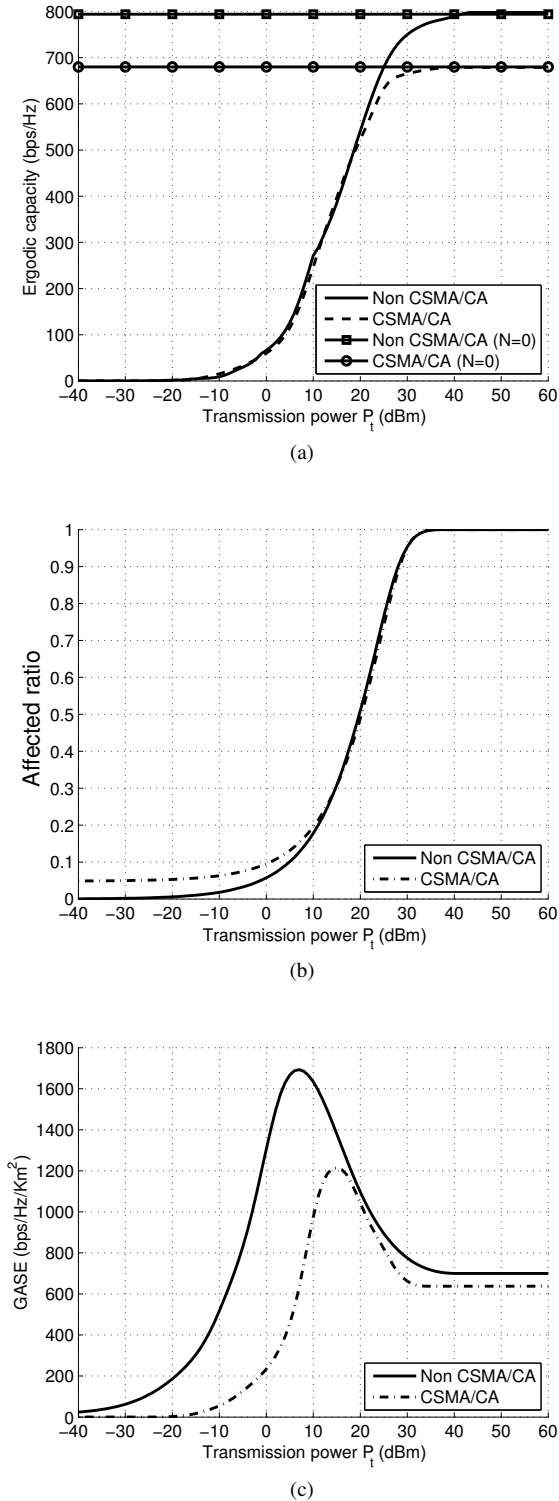


Fig. 2.  $\eta = 4$ ,  $N = -40$  dBm,  $I_{th} = -20$  dBm,  $R_{RTS} = 40$  m,  $\Omega = 1$  Km<sup>2</sup>,  $d_l = 1$  m,  $d_h = 20$  m,  $r_l = 0$ ,  $r_h = \infty$ ,  $\lambda_p = 200$  nodes/Km<sup>2</sup>: (a) Ergodic capacity, (b) affected area, and (c) GASE.

both monotonically decreasing function of  $\lambda$ . This is due to GASE not only considers the negative effect of co-channel interference incurred by simultaneous transmission, but also takes into account the spatial effect of wireless transmission in terms of affected area.

In Fig. 2, we analyze the effect of common transmission power  $P_t$  on network performance. From (21), we can see that if  $P_t \gg N$ , i.e., the common transmission power is sufficiently larger than the noise power  $N$ , then  $\mathfrak{F}_N = \lim_{N/P_t \rightarrow 0} \exp(-Nd^n\gamma/P_t) \rightarrow 1$ . Under this circumstance, the network ergodic capacity is function of transmitter intensity  $\lambda$  and path loss exponent  $\eta$ , irrelevant to individual transmission power  $P_t$ , which means even we continue increase  $P_t$ , we cannot achieve higher network ergodic capacity. This observation can be justified by Fig. 2(a). It shows that the ergodic capacity of wireless ad hoc network is an increasing function of  $P_t$ . However, if  $P_t \gg N$ , the ergodic capacity converges to a constant value, which equals to the value calculated from (21) with  $\mathfrak{F}_N = 1$ .

The affected area of CSMA/CA network is calculated by (23), as  $A_{aff} = (1 - F_I(I_{th})) \cdot \Omega$ . For the special case  $\eta = 4$ ,  $F_I(I_{th}) = \text{erfc}\left(\frac{\lambda\pi^2}{4} \sqrt{\frac{P_t}{I_{th}}}\right)$ , which is function of transmitter intensity  $\lambda$ , common transmission power  $P_t$  and aggregate interference threshold  $I_{th}$ . If  $P_t \gg I_{th}$ ,  $F_I(I_{th}) = 1$ , then the affected area  $A_{aff} = \Omega$ , which means all  $\mathbb{R}^2$  is affected. Fig. 2(b) justifies this observation. With Figs. 2(a) and 2(b), we conclude that too large  $P_t$  saturates the network distribution area without help in increasing the system ergodic capacity. On the other hand, too small  $P_t$  leads to small ergodic capacity and insufficient utilization of the network space-spectrum resource.

Fig. 2(c) shows an maximal GASE value with respect to common transmission power  $P_t$ . The maximal value exists because ergodic capacity increases faster than affected area when transmit power is small (less than 10 dBm). By considering ergodic capacity and affected area together, GASE measures the relationship between  $P_t$ ,  $N$ , and  $I_{th}$  with one generic performance metric, and provides a new perspective on the transmission power optimization.

#### IV. GASE ANALYSIS FOR TWO-TIER COGNITIVE NETWORK

In previous section, we investigated the GASE performance of wireless ad hoc networks with and without implementing CSMA/CA mechanism. From the analysis and numerical examples, we found that as the number of transmitter increases, ergodic capacity does not necessarily increase correspondingly, but the affected area does. Besides, the overall GASE performance of wireless ad hoc network is a decreasing function of transmitter intensity  $\lambda$ . From this perspective, we cannot fully utilize the space-spectrum resource and achieve high ergodic capacity at the same time. In this section, we utilize secondary cognitive networks to exploit the space-spectrum potential of CSMA/CA network. We also examine the impact of secondary cognitive network on overall system GASE.

We consider a two-tier cognitive network distributed in continuous infinite space  $\mathbb{R}^2$ . In particular, the primary network is the CSMA/CA network described in Section III.C. The transmitter intensity in primary network is  $\lambda_p$ , and the common transmission power is  $P_p$ . These primary transmitters define the primary affected area  $A_p$ , which is given by (30). The MGF of aggregate interference generated by primary network is given

by

$$\Phi_p(s) = \exp \left\{ -\pi \lambda_p k (s P_p)^{\frac{2}{\eta}} \right\}. \quad (32)$$

The secondary network is distributed in  $\mathbb{R}^2$  according to Poisson point process  $\Pi_{ps}$  with  $\lambda_{ps}$ , independent from the primary network. However, only those secondary transmitters that locate outside the primary affected area  $A_p$  can transmit with power  $P_s$ . The active secondary transmitters constitute a new Poisson point process with intensity  $\lambda_s = (1 - A_p/\Omega)\lambda_{ps}$ . The MGF of aggregate interference generated by active secondary transmitters is given by

$$\Phi_s(s) = \exp \left\{ -\pi \lambda_s k (s P_s)^{\frac{2}{\eta}} \right\}. \quad (33)$$

The total interference of the two-tier cognitive network  $I_c$  is summation of the interference generated by both primary and secondary network, i.e.,  $I_c = I_p + I_s$ . As we assume that primary and secondary network are independently distributed, the MGF of total interference of two-tier cognitive network is given by

$$\Phi_c(s) = \Phi_p(s) \cdot \Phi_s(s). \quad (34)$$

Substitute (32) and (33) into (34), we can arrive at

$$\Phi_c(s) = \exp \left\{ -\pi k \left( \lambda_p P_p^{\frac{2}{\eta}} + \lambda_s P_s^{\frac{2}{\eta}} \right) s^{\frac{2}{\eta}} \right\}. \quad (35)$$

The total ergodic capacity of two-tier cognitive network is given by

$$C_c = \kappa \int_{d_l}^{d_h} \int_0^{\infty} \left\{ \lambda_p e^{-\frac{N d_l^\eta}{P_p} \gamma} \cdot \Phi_c \left( \frac{d^\eta}{P_p} \gamma \right) + \lambda_s e^{-\frac{N d_l^\eta}{P_s} \gamma} \cdot \Phi_c \left( \frac{d^\eta}{P_s} \gamma \right) \right\} \frac{d - d_l}{1 + \gamma} d\gamma dd. \quad (36)$$

The affected area of two-tier cognitive network is given by

$$A_{\text{cog}} = \bigcup_{X_i \in \Theta} \mathcal{B}_{X_i} \cdot F_{I_c}(I_{\text{th}}) + \left( 1 - F_{I_c}(I_{\text{th}}) \right) \Omega. \quad (37)$$

In Fig. 3, we plot the ergodic capacity and affected ratio of cognitive and heterogeneous network as function of the secondary intensity  $\lambda_s$ . As comparison, we also include these values in primary network without secondary network. Fig. 3(a) shows that adding secondary nodes into the existing primary network degrades the ergodic capacity of the primary network. This degradation in heterogeneous network is more severe than that in cognitive network. However, the secondary network can significantly improve the ergodic capacity of wireless network. With small number of secondary user, heterogeneous network shows slightly better ergodic capacity than cognitive network. While excessive secondary users not only deteriorate the ergodic capacity of primary network but also that of the total network, especially in heterogeneous network. This is due to in heterogeneous network, excessive secondary users significantly increase the aggregate interference level and thus degrade the system performance. In underlay cognitive network, the primary network is protected by imposing a maximal tolerable interference power on the primary users, which prohibits excessive simultaneous transmitting secondary nodes. Under this circumstance, when the number of secondary transmitter reaches

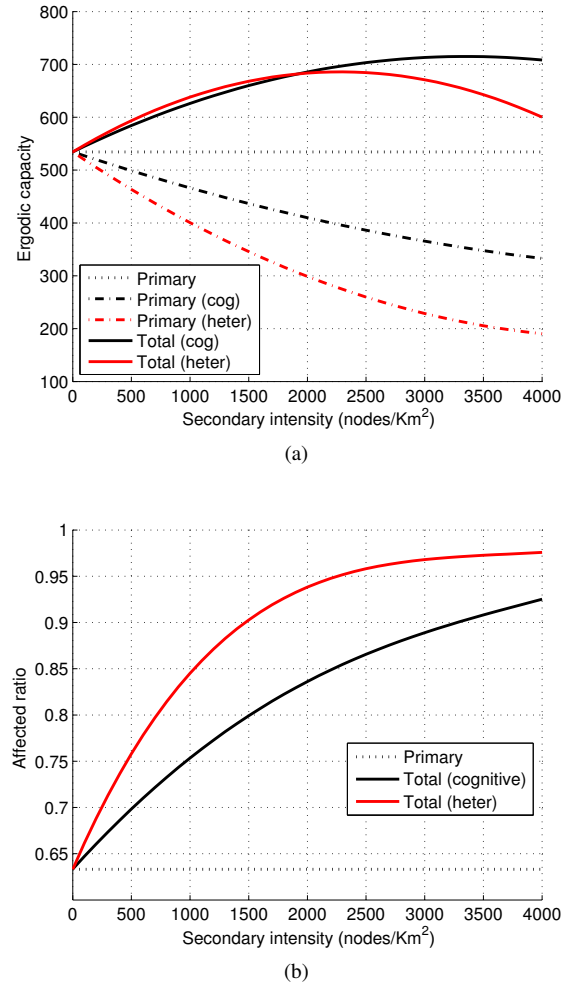


Fig. 3.  $P_t = 10$  dBm,  $P_s = 5$  dBm,  $\eta = 4$ ,  $I_{\text{th}} = -40$  dBm,  $\rho = 20$  m,  $\Omega = 1$  Km<sup>2</sup>,  $r_l = 0$ ,  $r_h = \infty$ ,  $\lambda_p = 1000$  nodes/Km<sup>2</sup>,  $d_l = 5$  m,  $d_h = 20$  m: (a) Ergodic capacity and (b) affected area.

a certain value, even we continue adding secondary nodes into the network, most of them have little opportunity to be selected active. As such, the ergodic capacity of cognitive network converges to a constant value. Fig. 3(b) shows that the area in heterogeneous network is affected heavily than that in cognitive network. Meanwhile, the former increases fast than the latter with respect to the secondary intensity  $\lambda_s$ .

In Fig. 4, we plot the ergodic capacity, affected ratio as well as GASE as function of the secondary transmission power  $P_s$  over the noise power  $N$ . For practical reason, we assume  $P_s$  is not large than  $P_t$ . Fig. 4(a) shows that the ergodic capacity of cognitive and heterogeneous network are all monotonically increasing function of  $P_s$  in the region under consideration. Meanwhile, increasing  $P_s$  also degrades the ergodic capacity of the primary network. In Fig. 4(b), the affected ratios of cognitive and heterogeneous network dramatically increase with respect to  $P_s$ . In Fig. 4(c), the GASE curves present a minimal value as we increase the value of  $P_s$ . The minimal value exists because ergodic capacity increases slower than affected area when secondary transmission power is not large enough. Note that for the  $P_s$  region under consideration, both the cognitive and heterogeneous network show worse GASE performance than the



V. CONCLUSION

In this paper, we analyzed GASE of wireless network in Poisson field over Rayleigh fading channels. We derived the generic closed-form MGF expression of aggregate interference of the wireless network. We then applied the statistics into the calculation of ergodic capacity, affected area and GASE of wireless ad hoc network in Poisson field over Rayleigh fading channels. We also analyzed the effect of CSMA/CA mechanism on network performance. Through mathematical analysis and numerical examples, we found that in sparse scenario, non CSMA/CA network shows better performance than CSMA/CA network; however, in dense scenario, CSMA/CA network can ameliorate the increase of aggregate interference, and achieve same amount of ergodic capacity with fewer transmitters. Finally, we proposed a new cognitive paradigm, which allows the secondary transmitters that are located outside the primary affected area to transmit. Numerical examples show that the number of secondary transmitters and their transmission power are essential to the network performance in terms of ergodic capacity and affected area. Meanwhile, we found that GASE provides a new perspective on transmission power selection and secondary network optimization.

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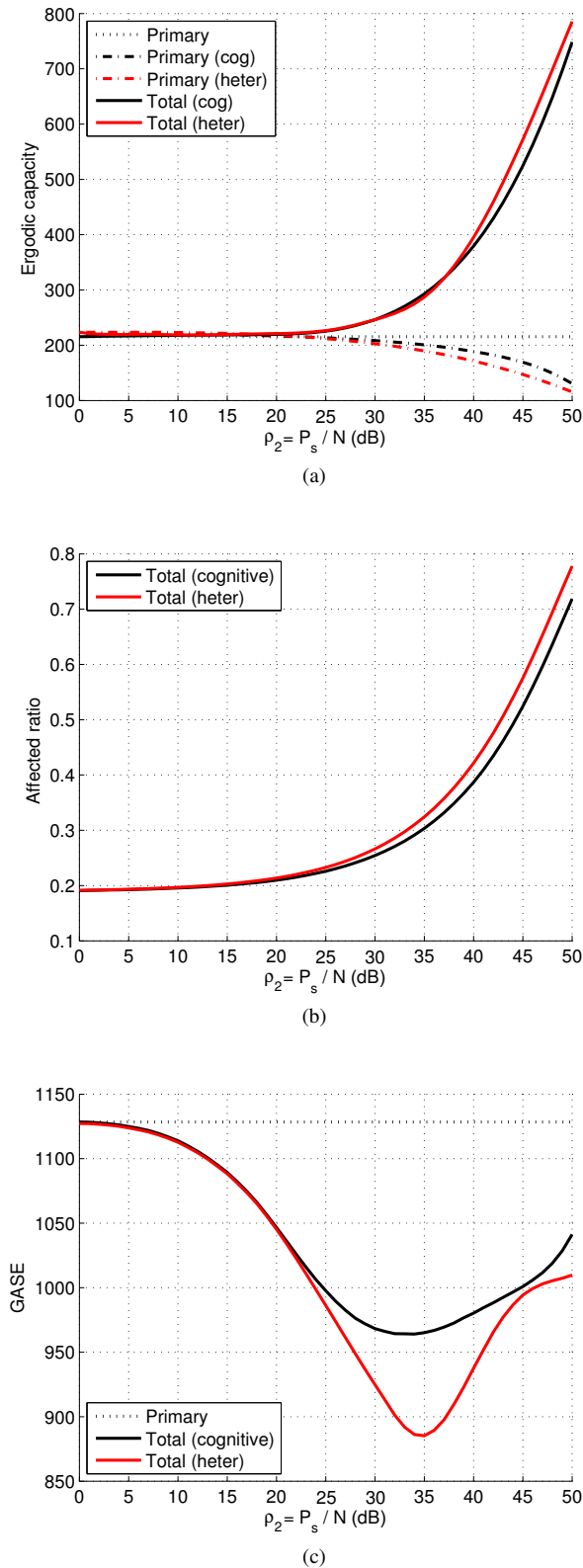


Fig. 4.  $P_t = 10$  dBm,  $\eta = 4$ ,  $I_{th} = -40$  dBm,  $\rho = 20$  m,  $\Omega = 1$  Km<sup>2</sup>,  $\lambda_p = 200$  nodes/Km<sup>2</sup>,  $r_l = 0$ ,  $r_h = \infty$ ,  $\lambda_s = 1000$  nodes/Km<sup>2</sup>,  $d_l = 5$  m,  $d_h = 20$  m: (a) Ergodic capacity, (b) affected area, and (c) GASE.

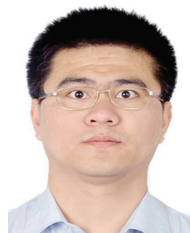
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