User-Centric Resource Allocation with Two-Dimensional Reverse Pricing in Mobile Communication Services

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Abstract: **Reverse pricing has been recognized as an effective tool to handle demand variability and uncertainty in the travel industry (e.g., airlines and hotels). To investigate its viability in mobile communication services, as a benchmark case, we first consider that a single mobile network operator (MNO) adopts (MNO-driven) forward pricing only, taking into account heterogeneous and stochastic user demands. To effectively deal with the drawbacks of forward pricing only, we propose (user-driven) two-dimensional reverse pricing on top of forward pricing and design a ξ-approximate polynomial-time algorithm that can maximize the revenue of the MNO. Through analytical and numerical results, we show that the proposed scheme can achieve "triple-win" solutions: Higher average network capacity utilization, the increase in the average revenue of the MNO, and the increment in the total average payoff of the users. To verify its feasibility in practice, we further implement its real prototype and perform experimental studies. We show that the proposed scheme still creates triple-win solutions in practice. Our findings provide a new outlook on resource allocation, and design guidelines for adopting two-dimensional reverse pricing on top of forward pricing.**

Index Terms: **Heterogeneous and stochastic user demands, mobile communications, resource allocation, revenue management, two-dimensional reverse pricing.**

I. INTRODUCTION

TO cope with the ever-increasing growth in mobile data traffic, recent years have witnessed the changing landscape of pricing as a congestion management tool in mobile communication services [1], [2]. Deviating from a flat-rate pricing scheme where a user is charged with a fixed payment irrespective of resource consumption [3], most mobile network operators (MNOs) currently employ a usage-based pricing scheme where a user is charged proportionally to the amount of resources consumed [4]. However, its critical drawback stems from charging all users the same usage fee at any time, regardless of actual network conditions. This could inevitably lead to a lose-lose situation for both users and MNOs, when facing highly volatile and uncertain demand patterns over different times of the day.

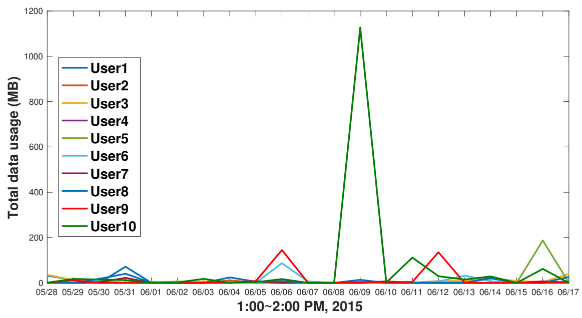

Fig. 1. Data usage patterns for 10 recruited users for a particular hour each day (between 1:00 p.m. to 2:00 p.m.) from May 28 to June 17, in which we only measured 3G and 4G data.

The aim of this paper is how to design a resource allocation scheme that handles unwanted/ unexpected demand fluctuations effectively. Recent measurement studies uncover striking temporal distributions of the mobile data traffic [5]–[8]. In response, researchers from both academia and industry have tailored MNO-driven pricing to the temporal distribution of network demand. From here on, we will use the term "forward pricing" to denote "MNO-driven pricing." In order to reduce peak-to-average ratios of network demand, time-dependent forward pricing have been designed to encourage users to shift a portion of their demand to the off-peak hours with lower prices [9]–[13]. However, a key issue here is to what extent the prediction of user preferences about rearranging it from peak to off-peak hours is accurate. Our measurement, performed over three weeks, shows that even the same user prefers distinctive data usage patterns for a particular hour each day (see Fig. 1). These variability and uncertainty in demand make the adoption of time-dependent pricing particularly challenging. Rather than predicting or overcoming such demand characteristics, how to utilize them is of great practical interest for designing novel pricing mechanisms.

As a complement to forward pricing, in this paper, we propose (user-driven) *two-dimensional reverse pricing* on top of forward pricing for embracing heterogeneous and stochastic user demands wisely, while reducing computational complexity compared to the auction-based pricing schemes [14], [15]. In this

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scheme, a single MNO first asks users to pre-specify their resource demands based on forward pricing. Each user then informs the MNO of his resource demand by maximizing his payoff, the difference between its utility and payment. In return for this self-reported information based on forward pricing, the MNO empowers each user to become a price maker, rather than merely a price taker. The self-reports on resource demands may be biased (either underestimated or overestimated) due to knowledge bias (i.e., lack of knowledge of how much resources the users will consume) or rating bias (i.e., unwillingness to estimate their resource demands). In practice, there may exist some discrepancies between actual (optimal) and self-reported ones. As demonstrated in [16], however, their correlations are moderate to strong. For mathematical tractability, we thus assume that each user is aware of his optimal demand.

Extending our previous work [17] where each user places only the bid price (the per-unit price that he is willing to pay) on a given resource quantity, each user can now submit a two-dimensional bid consisting of the bid price and the corresponding resource quantity. If he wins the bid, then he is allowed to spend his desired resource quantity at his submitted bid price; otherwise, he is allocated his pre-specified resource quantity from the MNO at the forward price per unit resource. Intuitively, the superiority of the proposed scheme is its natural ability to deal with the demand uncertainty. However, the challenge here is two-fold: (i) How to incentivize the users to submit their two-dimensional bids, and (ii) how to determine multiple winners to maximize the revenue of the MNO.

This paper endeavors to design such two-dimensional reverse pricing on top of forward pricing to answer both challenges. As a benchmark case, we first consider that the MNO adopts the forward pricing scheme only, considering the heterogeneous and stochastic network demand. To this end, we use a two-stage Stackelberg game to capture the interaction between the MNO and the users. In Stage I, the MNO announces the price per unit resource to maximize its expected revenue subject to both demand uncertainty and capacity constraint. In Stage II, each user determines his optimal resource demand to maximize the payoff. Based on obtained results, we study how to utilize two-dimensional reverse pricing in conjunction with forward pricing, and quantify the effects of the proposed scheme in terms of the average network capacity utilization, the average revenue of the MNO, and the total average payoff of the users.

The main contributions and results of this paper are summarized in the following.

- *Forward pricing only:* We study the MNO's revenue maximization problem with forward pricing as a chance-constrained programming problem. Here, we consider heterogeneous and stochastic user demands that make our model and analysis different from prior works. For computational tractability, we use a deterministic approximation method and propose a ϵ-optimal approximate linear-time algorithm.
- *Two-dimensional reverse pricing on top of forward pricing:* We study the MNO's revenue maximization problem with two-dimensional reverse pricing on top of forward pricing as a 0/1 knapsack problem. To this end, we propose a ξ-approximate polynomial-time algorithm. We show that the proposed scheme can achieve "triple-win" solutions: Higher average network capacity utilization, the increase in the average revenue of the MNO, and the increment in the total average payoff of the users.
- *Experimental studies:* To verify the feasibility of the proposed scheme, we implement its real prototype and perform experimental studies. We show that the proposed scheme still create triple-win solutions in practice.

The remainder of this paper is organized as follows. We present the system model in Section II. We analyze the revenue maximization of the MNO in Section III. In this section, the MNO employs forward pricing only, whereas in Section IV, the MNO adopts two-dimensional reverse pricing on top of forward pricing. We provide numerical and experimental results to validate the proposed studies in Section V. Finally, we conclude this paper in Section VI.

II. SYSTEM MODEL

To abstract the interaction between a single MNO and users, and to introduce our two-dimensional reverse pricing smoothly, without loss of generality, we consider a specific system model hereafter. This, however, does not mean that our scheme is only applicable to such specific model. Now let us consider a resource-constrained network with a total amount of available resource Q (e.g., bandwidth, data amount, etc.). The MNO allocates the resource to a set of $\mathcal{N} = \{\infty, \in, \cdots, \mathcal{N}\}$ of users.

We consider a time-slotted system where the resource scheduling horizon is divided into a set $\mathcal{T} \triangleq \{\infty, \cdots, \mathcal{T}\}$ of time slots. In general, users tend to use more data traffic in afternoon and evening, while the reverse is true in morning and night. Note that this diurnal pattern has been demonstrated by a number of studies [5]–[8]. To characterize such temporal demand pattern across users, yet be still simple enough for analytical tractability, we assume that the resource demand of each user is randomly and independently distributed over each time slot, but not identically distributed across different time slots[1]. Accordingly, we here focus on the case of $T = 1$ and the results obtained here can be extended to the general case of $T > 1$. We thus drop the time index T from all the parameters for the analysis.

For a given time slot, let us define s_i as the resource demand of user i. If the demand s_i is satisfied through resource allocation, then each user $i \in \mathcal{N}$ has a following stochastic utility function

$$u_i(\theta_i, \delta_i, s_i) = (\theta_i + \delta_i) \ln(1 + s_i), \tag{1}$$

where θ_i is the (average) deterministic willingness to pay of user i, and δ_i is the stochastic willingness to pay of user i with zero mean. In fact, θ_i is a value that indicates the instinctual state of the user and is difficult to grasp accurately. However, it is possible to statistically deduce the value by performing a sealed-bid auction repeatedly as in the proposed method. Note that the

[1] In the papers [9], [12], the authors deal with the demand shift problem in time domain. They assume that users can delay their demand in the future, and thus the user's traffic consumptions among time slots are dependent. It may reflect the characteristics in reality better. However, based on the experimental results, we confirmed that the user's characteristics are well approximated to the beta distribution even when the dependency is not taken into consideration.

utility function (1) reflects the property of diminishing marginal returns, and has been widely used to model resource allocation in mobile communication services [2], [18].

In this work, we consider two types of pricing schemes.

1. *Forward Pricing only*: The MNO sets the ex-ante unit price p subject to both demand uncertainty and capacity constraint. As the price p usually remains constant for long periods of time (i.e., months or years), the aim of the MNO is to maximize its expected revenue. In this scheme, each user $i \in \mathcal{N}$ acts as a price taker and adjusts his resource demand s_i in accordance with his time varying demand preferences over time. It implies that some residual uncertainty stemming from the actual demand fluctuations may lead to underutilization of network capacity.
2. *Two-Dimensional Reverse Pricing on top of Forward Pricing*: To effectively handle unwanted/unexpected demand fluctuations, the MNO allows each user $i \in \mathcal{N}$ to submit a two-dimensional bid (b_i, q_i) in return for pre-specifying his resource demand s_i based on the forward price p. To be more specific, in this scheme, the MNO acts as an bid-taker and selects multiple users (winners) to maximize its revenue. On the other hand, each user $i \in \mathcal{N}$ acts as a bidder and submits a two-dimensional bid (b_i, q_i) to the MNO for maximizing his payoff as well as reporting his demand s_i based on the forward price p in priori. If user i wins the bid, then he receives the resource allocation q_i at his bid price b_i. Otherwise, user i gets the resource allocation s_i at the price p.

III. BENCHMARK SCENARIO: FORWARD PRICING ONLY

As a benchmark case, we first consider that the MNO employs the forward pricing scheme only. In this case, the MNO decides the ex-ante price per unit resource p to maximize its expected revenue under both demand uncertainty and capacity constraint in Stage I. In Stage II, each user acts as a price taker and adjusts the amount of resources. This study serves as a baseline to quantify the effects of adopting two-dimensional reverse pricing scheme on the average network capacity utilization, the average revenue of the MNO, and the total average payoff of the users. By using backward induction, we start to solve the two-stage game from Stage II to Stage I.

A. Users' Demand in Stage II

If the MNO announces a price p per unit resource in Stage I, the demand function of each user $i \in \mathcal{N}$ in Stage II is derived as the outcome of the following payoff maximization problem

$$\max_{s_i \geq 0} \quad u_i(\theta_i, \delta_i, s_i) - p s_i, \tag{2}$$

which leads to

$$s_i^* = \left(\frac{\theta_i + \delta_i}{p} - 1\right)^+, \quad i \in \mathcal{N}, \tag{3}$$

where $(\cdot)^+ \triangleq \max(\cdot, 0)$.

For analytical tractability, δ_i is assumed to be zero when $\theta_i \leq p$. It means that user i always demands zero resource ($s_i^* = 0$) when the average (or deterministic) willingness to pay of user i is less than or equal to the offered price. When $\theta_i > p$, on the other hand, δ_i is assumed to be an independent bounded random variable such that $\delta_i \in [p - \theta_i, p + \theta_i]$ and $\mathbb{E}[\delta_i] = 0$. We further assume that $\theta_1 > \theta_2 > \cdots > \theta_N$.

Under these assumptions, we can rewrite the user i's demand (3) as

$$s_i^* = \mathbb{1}(\theta_i > p)\left(\frac{\theta_i + \delta_i}{p} - 1\right), \quad i \in \mathcal{N}, \tag{4}$$

where $\mathbb{1}$ is the indicator function: $\mathbb{1}(A) = 1$ is A is true, 0 otherwise.

B. MNO's Forward Pricing in Stage I

Given the users' demand functions (4), the MNO seeks to maximize its expected revenue under both demand uncertainty and capacity constraint. This is obtained by solving the following stochastic optimization problem

$$\mathbf{P1}: \max_{p \geq 0} \quad p\,\mathbb{E}\left[\sum_{i \in \mathcal{N}} \mathbb{1}(\theta_i > p)\left(\frac{\theta_i + \delta_i}{p} - 1\right)\right] \tag{5}$$

$$\text{s.t.} \quad \sum_{i \in \mathcal{N}} \mathbb{1}(\theta_i > p)\left(\frac{\theta_i + \delta_i}{p} - 1\right) \leq Q, i \in \mathcal{N}. \tag{6}$$

Note that problem **P1** is computationally intractable, due to a large number of random variables [19]. To overcome such curse of dimensionality, a worst-case design approach can be used to ensure feasibility against all possible realization of the uncertain demands [17]. However, it may result in an extremely conservative pricing strategy. Perhaps the natural way is to employ the chance constrained method that transforms the inequality constraint (6) to a chance constraint, ensuring that the probability of demand exceeding capacity is below a specified threshold. This motivates us to define a region of an ϵ-optimal price.

Definition 1 (ϵ-optimal Price)*:* We say that a price p is ϵ-optimal if the probability that the aggregate resource demand exceeds the total capacity is less than ϵ.

Under the chance constrained method, Problem **P1** can be reformulated as follows:

$$\mathbf{P1.1}: \max_{p \geq 0} \quad p\,\mathbb{E}\left[\sum_{i \in \mathcal{N}} \mathbb{1}(\theta_i > p)\left(\frac{\theta_i + \delta_i}{p} - 1\right)\right] \tag{7}$$

$$\text{s.t.} \quad \mathbb{P}\left[\sum_{i \in \mathcal{N}} \mathbb{1}(\theta_i > p)\left(\frac{\theta_i + \delta_i}{p} - 1\right) \geq Q\right] \leq \epsilon, i \in \mathcal{N} \tag{8}$$

where $\epsilon \in (0, 1)$ is a risk tolerance, which is typically small. The parameter ϵ partially explains the MNO's attitude towards quality of service (QoS) and revenue, i.e., risk tolerance. As ϵ grows, the MNO can increase revenue by accommodating more users while taking the greater risk of network congestion.

However, it is still challenging to solve the above chance constrained problem directly. This is because checking the feasible probability of a given price p in (8) would require complicated multivariate numerical integrations. It is therefore desirable to find a good deterministic approximation of the chance constraint (8), while retaining both tractability and ease of practical implementation. Such approximation can be based on Hoeffding's inequality [20].

Let us first consider a special case of problem **P1.1** in which Q is sufficiently large so that all the users are admitted into the network because of lowered price, i.e., $\mathbb{1}(\theta_i > p) = 1, \forall i \in \mathcal{N}$. With the above assumption, problem **P1.1** can be rewritten as

$$\mathbf{P1.2}: \max_{p\geq 0} \quad p\,\mathbb{E}\left[\sum_{i\in\mathcal{N}}\left(\frac{\theta_i+\delta_i}{p}-1\right)\right] \overset{(a)}{=} \sum_{i\in\mathcal{N}}(\theta_i - p) \tag{9}$$

$$\text{s.t.} \quad \mathbb{P}\left[\sum_{i\in\mathcal{N}}\left(\frac{\theta_i+\delta_i}{p}-1\right)\geq Q\right]\leq\epsilon, \tag{10}$$

where (a) follows from the fact that the bounded random variables δ_i, $i \in \mathcal{N}$ are independent with support $[p-\theta_i, p+\theta_i]$. With Hoeffding's inequality, we can derive the lower bound on the MNO's ϵ-optimal optimal price, as summarized in the following lemma.

Lemma 1*:* The ϵ-optimal price for problem **P1.2** is given by

$$p \geq \frac{\sum_{i=1}^N \theta_i + \sqrt{2\ln\frac{1}{\epsilon}\sum_{i=1}^N \theta_i^2}}{Q+N}. \tag{11}$$

Proof: Let $s_i = \frac{\theta_i+\delta_i}{p} - 1 > 0$, $i \in \mathcal{N}$. Applying Hoeffding's inequality yields

$$\begin{aligned}\mathbb{P}\left[\sum_{i\in\mathcal{N}} s_i \geq Q\right] &= \mathbb{P}\left[\sum_{i\in\mathcal{N}} s_i - \mathbb{E}\left[\sum_{i\in\mathcal{N}} s_i\right] \geq Q - \mathbb{E}\left[\sum_{i\in\mathcal{N}} s_i\right]\right] \\ &\overset{(a)}{\leq} \exp\left(-2\frac{\left(Q-\sum_{i\in\mathcal{N}}\left(\frac{\theta_i}{p}-1\right)\right)^2}{\sum_{i\in\mathcal{N}}\left(\frac{2\theta_i}{p}\right)^2}\right) \\ &\leq \epsilon,\end{aligned}$$

where (a) follows from Hoeffding's inequality [20].

Now, a sufficient condition for a price p to be ϵ-optimal is the following.

$$e^{\frac{-2\left(Q+N-\sum_{i\in\mathcal{N}}\frac{\theta_i}{p}\right)^2}{\sum_{i\in\mathcal{N}}\left(\frac{2\theta_i}{p}\right)^2}} \leq \epsilon \leftrightarrow p \geq \frac{\sum_{i\in\mathcal{N}}\theta_i + \sqrt{2\ln\frac{1}{\epsilon}\sum_{i\in\mathcal{N}}\theta_i^2}}{Q+N}. \tag{12}$$

This completes the proof. ■

With Lemma 1, we now have the approximate solution of problem **P1.2**.

Corollary 1*:* The ϵ-optimal approximate solution of problem **P1.2** is given by

$$p^* = \frac{\sum_{i=1}^N \theta_i + \sqrt{2\ln\frac{1}{\epsilon}\sum_{i=1}^N \theta_i^2}}{Q+N}. \tag{13}$$

Proof: Since the objective function (9) is a decreasing function of the price p, taking the minimum value of p in (11) completes the proof. ■

Lemma 2*:* The price in (13) is the ϵ-optimal approximate solution of problem **P1.1** if and only if $Q > \frac{\sum_{i=1}^N \theta_i + \sqrt{2\ln\frac{1}{\epsilon}\sum_{i\in\mathcal{N}}\theta_i^2}}{\theta_N} - N$.

Proof: Suppose that the price in (13) is the ϵ-optimal approximate solution of problem **P1.1**. This means that the indicators for all users should be equal to 1, i.e., $\theta_i > p$, $i \in \mathcal{N}$. Substituting (13) into these inequalities gives $\theta_i > \frac{\sum_{i\in\mathcal{N}} \theta_i + \sqrt{2\ln\frac{1}{\epsilon}\sum_{i\in\mathcal{N}}\theta_i^2}}{Q+N}, \forall i \in \mathcal{N}$. Since $\theta_1 > \theta_2 > \cdots > \theta_N$, $Q > \frac{\sum_{i\in\mathcal{N}} \theta_i + \sqrt{2\ln\frac{1}{\epsilon}\sum_{i\in\mathcal{N}}\theta_i^2}}{\theta_N} - N$, completing the proof of the "if" part.

Next, we prove the "only if" part by contradiction. Suppose not, that is, suppose that the price in (13) is still the ϵ-optimal approximate solution of problem **P1.1** with $Q \leq \frac{\sum_{i=1}^N \theta_i + \sqrt{2\ln\frac{1}{\epsilon}\sum_{i\in\mathcal{N}}\theta_i^2}}{\theta_N} - N$. For notational convenience, let $S_N = \sum_{i\in\mathcal{N}}\theta_i + \sqrt{2\ln\frac{1}{\epsilon}\sum_{i\in\mathcal{N}}\theta_i^2}$. Since $Q \leq \frac{S_N}{\theta_N} - N$, we observe that $p^* \geq \theta_N$, indicating that user N does not subscribe to the MNO.

Now, we need to show that p^* still remains the ϵ-optimal approximate solution of the following stochastic optimization problem

$$\max_{p\geq 0} \quad p\,\mathbb{E}\left[\sum_{i\in\mathcal{N}\setminus\mathcal{N}}\mathbb{1}(\theta_i > p)\left(\frac{\theta_i+\delta_i}{p}-1\right)\right] \tag{14}$$

$$\text{s.t.} \quad \mathbb{P}\left[\sum_{i\in\mathcal{N}\setminus\mathcal{N}}\mathbb{1}(\theta_i > p)\left(\frac{\theta_i+\delta_i}{p}-1\right)\geq Q\right]\leq\epsilon,\ i\in\mathcal{N}\setminus\mathcal{N}. \tag{15}$$

Note that the above problem has the same structure with problem **P1.1**. Thus, we assume that Q is sufficiently large so that all the users except user N are admitted into the network, i.e., $\theta_i > p$, $i \in \mathcal{N}\setminus\mathcal{N}$. Following the same steps as the proofs of Lemma 1 gives

$$p \geq \frac{\sum_{i=1}^{N-1} \theta_i + \sqrt{2\ln\frac{1}{\epsilon}\sum_{i=1}^{N-1} \theta_i^2}}{Q+N-1}. \tag{16}$$

Since the objective function (14) is a decreasing function of the price, we should take the minimum value of p in constraint (16), while satisfying $\theta_N \leq p$ and $\theta_i > p$, $i \in \mathcal{N}\setminus\mathcal{N}$. Following the above proof of the "if" part, the ϵ-optimal approximate solution of the above problem is

$$p^\dagger = \begin{cases} \theta_N, & \text{if } \frac{S_{N-1}}{\theta_N} - N + 1 < Q \leq \frac{S_N}{\theta_N} - N, \\ \frac{S_{N-1}}{Q+N-1}, & \text{if } \frac{S_{N-1}}{\theta_{N-1}} - N + 1 < Q \leq \frac{S_{N-1}}{\theta_N} - N + 1. \end{cases} \tag{17}$$

It is observed that the ϵ-optimal approximate solution in (17) is different from the solution in (11). This contradicts with the fact that the price (13) is the ϵ-optimal approximate solution for problem **P1.1**, and thus completes the proof. ■

Algorithm 1 ϵ-optimal approximate algorithm for problem **P1.1**.

- Step 1: Set $K = N$ and sort θ_i for $i = 1, 2, \cdots, K$ in decreasing order, i.e., $\theta_1 > \theta_2 > \cdots > \theta_N$.
- Step 2: Compute $\tilde{Q}_K = \frac{\sum_{i=1}^{K}\theta_i+\sqrt{2\ln\frac{1}{\epsilon}\sum_{i=1}^{K}\theta_i^2}}{\theta_K} - K$.
- Step 3: Compare Q with $\tilde{Q}_K$. If $Q \leq \tilde{Q}_K$, then set $K = K-1$, and go to step 2. Otherwise, go to step 4.
- Step 4: With K, compute $p_K^* = \frac{\sum_{i=1}^{K}\theta_i+\sqrt{2\ln\frac{1}{\epsilon}\sum_{i=1}^{K}\theta_i^2}}{Q+K}$. If $K = N$, then the ϵ-optimal price p^* is p_K^*. Otherwise, p^* is $\max\{p_K^*, \theta_{K+1}\}$.

Using Lemmas 1 and 2, we can find the ϵ-optimal approximate price p^* of problem **P1.1**, which is summarized in the following proposition.

Proposition 1*:* The ϵ-optimal approximate solution for problem **P1.1** is given by

$$p^* = \begin{cases} p_N^*, & \text{if } Q > \tilde{Q}_N, \\ \max\{p_{N-1}^*, \theta_N\}, & \text{if } \tilde{Q}_{N-1} < Q \leq \tilde{Q}_N, \\ \max\{p_{N-2}^*, \theta_{N-1}\}, & \text{if } \tilde{Q}_{N-2} < Q \leq \tilde{Q}_{N-1}, \\ \vdots & \vdots \\ \max\{p_1^*, \theta_2\}, & \text{if } \tilde{Q}_1 < Q \leq \tilde{Q}_2, \end{cases} \tag{18}$$

where $p_K^* = \frac{Z}{Q+K}$ and $\tilde{Q}_K = \frac{Z}{\theta_K} - K$ where $Z = \sum_{i=1}^{K}\theta_i + \sqrt{2\ln\frac{1}{\epsilon}\sum_{i=1}^{K}\theta_i^2}$, $K \in \mathcal{N}$.

Proof: If $Q > \tilde{Q}_N$, the ϵ-optimal approximate price p^* can be obtained from Lemmas 1 and 2. If $\tilde{Q}_{N-1} < Q \leq \tilde{Q}_N$, p^* can be derived from the proof of Lemma 2. For the rest of other intervals of Q, p^* can be achieved by following the similar steps as the proofs of Lemmas 1 and 2, and hence their proofs omitted. ■

Proposition 1 provides a useful result to determine the value of the ϵ-optimal approximate price p^* in practice. For a given capacity Q, there exists a user index threshold $K < N$ satisfying $\tilde{Q}_K < Q \leq \tilde{Q}_{K+1}$ and $\theta_K > p^* \geq \theta_{K+1}$, and the threshold $K = N$ satisfying $Q > \tilde{Q}_N$ and $p^* < \theta_N$. Since $\theta_1 > \theta_2 > \cdots > \theta_N$, it implies that only the set of users with index less than or equal to K will be allocated via forward pricing. By exploiting this property, the values of p^* and K can be simply computed as in Algorithm 1. Note that its compleixity is $\mathcal{O}(|\mathcal{N}|)$.

Proposition 1 offers an important insight into the drawbacks of the forward pricing scheme only. As ϵ decreases, the resource constraint in (8) becomes more tight. To satisfy it, the MNO inevitably charges a higher price to the users. The higher price induces the lower network demand, resulting in the decrease in the total user payoff. These motivate the MNO to utilize two-dimensional reverse pricing on top of forward pricing so that it can achieve "triple-win" solutions.

IV. TWO-DIMENSIONAL REVERSE PRICING ON TOP OF FORWARD PRICING

We now investigate how to design two-dimensional reverse pricing on top of forward pricing to embrace the demand uncertainty wisely. In this scheme, the MNO first asks each user $i \in \mathcal{N}$ to pre-specify his optimal resource demand s_i^* based on the published unit-price p^* in (18). With this information, the MNO announces basic requirements, user-centric scoring functions (in terms of pre-specified resource demands, residual capacity of the network and two-dimensional bids) to the users. Then, each user $i \in \mathcal{N}$ submits a bid of the form (b_i, q_i) as a sealed bid for maximizing his payoff. Based on the obtained bids, the MNO selects the optimal bidders (winners) to maximize its revenue.

A. Basic Rules

A.1 Bid Quantity Requirements

Without loss of generality, we assume that the MNO limits the users' bid quantities, i.e.,

$$0 < s_i^* \leq q_i \leq Q - \sum_{j\in\mathcal{N}\setminus\rangle} s_j^*, i \in \mathcal{N}, \tag{19}$$

where the first inequality in (19) means that only the set of users that pre-specify their optimal (positive) resource demands via forward pricing can place on bids, and the last inequality in (19) implies the maximum allowable residual capacity that the MNO can provide via reverse pricing for each user $i \in \mathcal{N}$.

A.2 Resource Allocation and Payment Rules

Let us denote by (b_i, q_i) the submitted bid by each bidder $i \in \mathcal{N}$ where b_i is the bid price that he is willing to pay for the unit of resource and q_i is the corresponding resource quantity. We define $\boldsymbol{x} \triangleq \{x_i, i \in \mathcal{N}\}$ to be the winner vector, i.e., $x_i = 1$ if user i wins the bid and $x_i = 0$ otherwise. User i thus receives the resource allocation q_i at the submitted bid price b_i when $x_i = 1$. When $x_i = 0$, on the other hand, user i still gets the resource allocation s_i^* at the unit price p^* in (18).

A.3 User-Centric Score Functions

Given that each user $i \in \mathcal{N}$ submits his bid (b_i, q_i), the aim of the MNO is to solve the following optimization problem

$$\mathbf{P2}: \max_{\boldsymbol{x}\geq\mathbf{0}} \sum_{i\in\mathcal{N}} x_i\left(b_i q_i - p^* s_i^*\right) + p^* s_i^* \tag{20}$$

$$\text{s.t.} \sum_{i\in\mathcal{N}} x_i\left(q_i - s_i^*\right) + s_i^* \leq Q \tag{21}$$

$$x_i \in \{0, 1\},\ i \in \mathcal{N}, \tag{22}$$

where $\boldsymbol{x} \geq \mathbf{0}$ represents $x_i \geq 0, \forall i \in \mathcal{N}$. We use bold symbols to denote vectors in the sequel. Constraint (21) implies the maximum allowable amount of resources allocated to the users via the proposed scheme. Constraint (22) denotes the admission control decision.

It is observed that the objective function (20) depends on not only each user i's submitted bid (b_i, q_i) via reverse pricing but also the revenue $p^* s_i^*$ via forward pricing. Hence, the MNO wants to maximize the difference between $b_i q_i$ and $p^* s_i^*$ by determining each winner element of the winner vector $\boldsymbol{x}$ subject to the resource constraint (21). To this end, we introduce a user-centric score function which transforms a multi-dimensional bid to a single score.

Definition 2 (User-centric Score Function)*:* In the proposed two-dimensional reverse pricing scheme in conjunction with the forward pricing scheme, the score function announced to each user i is

$$S_i(b_i, q_i) = \frac{b_i q_i - p^* s_i^*}{q_i - s_i^*},\ i \in \mathcal{N}. \tag{23}$$

From (23), it can be seen that S_i increases in $b_i q_i - p^* s_i^*$ but decreases in $q_i - s_i^*$. This implies that the MNO seeks to maximize the additional profit $b_i q_i - p^* s_i^*$ while minimizing the additional resource consumption $q_i - s_i^*$ via reverse pricing for each user i. Note that the maximum values of S_i is p^*. To sum up, S_i may be interpreted as the marginal profit, i.e., the additional profit from selling one extra unit via reverse pricing for each user i.

A.4 Target Score

We assume that the MNO sets a positive and unique score $\tilde{S}$, which is called a target score. It means that each user i should generate the score $S_i = \tilde{S}$ when submitting a bid (b_i, q_i) according to the announced score function (23). The reason is two-fold. First, with a positive target score, the MNO can avoid the revenue loss from utilizing two-dimensional reverse pricing on top of forward pricing. Second, with a unique target score, the MNO does not price-discriminate across all the users and treats them fairly by extracting the same marginal profit from them. It is worth noting that when $\tilde{S}$ is small, in general, more potentially profitable trades are expected to occur, at the cost of the higher marginal profit via reverse pricing. When $\tilde{S}$ is large, on the other hand, the reverse is generally true. Thus, the MNO should set $\tilde{S}$ as a *strategic* variable, taking this trade-off into account. A numerical example of this will be discussed in V.A.

B. MNO's Winner Selection Strategy

We now consider how the MNO determines the winner vector $\boldsymbol{x}$ for problem **P2**. Based on the reported resource quantities based on the announced unit price p^* in (18), we first consider the infeasible case of problem **P2** where $\sum_{i\in\mathcal{N}} s_i^* \geq Q$. Since the unit price p^* is ϵ-optimal, it can happen with the probability of less than ϵ. Thus, we have $x_i = 0$ for all $i \in \mathcal{N}$, degenerating the forward pricing only case (problem **P1.1**). From now on, we will only focus on the feasible case of problem **P2** in which $\sum_{i\in\mathcal{N}} s_i^* < Q$.

Let $\mathcal{R}$ be a subset of the total user set $\{1, 2, \cdots, N\}$, such that $S_i = \tilde{S}$ for $i \in \mathcal{R}$. From (23), we have $b_i q_i - p^* s_i^* = \tilde{S}(q_i - s_i)$ for $i \in \mathcal{R}$. Then, problem **P2** can be simplified as

$$\mathbf{P2.1}: \max_{\boldsymbol{x}\geq \mathbf{0}} \quad \sum_{i\in\mathcal{R}} x_i \tilde{S}\left(q_i - s_i^*\right) \tag{24}$$

$$\text{s.t.} \quad \sum_{i\in\mathcal{R}} x_i \left(q_i - s_i^*\right) \leq Q - \sum_{j\in\mathcal{N}} s_j^* \tag{25}$$

$$x_i \in \{0, 1\},\ i \in \mathcal{R}, \tag{26}$$

where $x_j = 0$ for $j \in \mathcal{N} \setminus \mathcal{R}$.

Note that the above problem is a one-dimensional 0/1 knapsack problem (also known as the sum of subset problem [21]). It is not straightforward to solve because finding an optimal solution to this problem through an exhaustive search requires an exponential complexity in the number of users, i.e., $\mathcal{O}(\in^{|\mathcal{R}|})$. Thus, it is only practical for small values of $|\mathcal{R}|$. It is of much importance to develop efficient algorithms for finding approximate solutions. This leads us to define a ξ-approximate algorithm.

Algorithm 2 ξ-Approximate algorithm for problem **P2.1**.

- Step 1: For a given $0 < \xi \leq \frac{1}{2}$, set $l = \frac{1}{\xi} - 1$. Find a subset of $\mathcal{R}' \subseteq \mathcal{R}$ such that $q_i - s_i^* > \frac{Q - \sum_{k\in\mathcal{N}} s_k^*}{l+1}$ for $i \in \mathcal{R}'$. If not, go to step 4.
- Step 2: Select the subset $\hat{\mathcal{R}} \subseteq \mathcal{R}'$ that maximizes $\sum_{i\in\hat{\mathcal{R}}} q_i - s_i^*$ without exceeding $Q - \sum_{k\in\mathcal{N}} s_k^*$.
- Step 3: If $q_j - s_j^* > Q - \sum_{k\in\mathcal{N}} s_k^* - \sum_{i\in\hat{\mathcal{R}}} q_i - s_i^*$ for all $j \in \mathcal{R} \setminus \hat{\mathcal{R}}$, then $x_i = 1$ for all $i \in \hat{\mathcal{R}}$ and $x_j = 0$ for all $j \in \mathcal{R} \setminus \hat{\mathcal{R}}$. Otherwise, go to step 4.
- Step 4: Find the element $\hat{\jmath} \in \mathcal{R} \setminus \hat{\mathcal{R}}$ such that $\arg\max_{\hat{\jmath}\in\mathcal{R}\setminus\hat{\mathcal{R}}} q_{\hat{\jmath}} - s_{\hat{\jmath}}^*$ without exceeding $Q - \sum_{k\in\mathcal{N}} s_k^* - \sum_{i\in\hat{\mathcal{R}}} q_i - s_i^*$
- Step 5: Set $\hat{\mathcal{R}} = \hat{\mathcal{R}} \cup \{\hat{\rceil}\}$. Go to step 3.

Definition 3 (ξ-Approximate Algorithm)*:* We say that for any $0 < \xi \leq \frac{1}{2}$, an algorithm for problem **P2.1** is ξ-approximate if it satisfies the following two conditions:

1. $\frac{F^* - \hat{F}}{F^*} \leq \xi$, where F^* is the optimal objective value of problem **P2.1** and $\hat{F}$ is an approximate objective value given by the algorithm.
2. The algorithm is of polynomial complexity $\mathcal{O}(|\mathcal{R}|^{\updownarrow})$ for $l \geq 1$.

Proposition 2*:* The proposed Algorithm 2 is an ξ-approximate algorithm that has complexity $\mathcal{O}(|\mathcal{R}|^{\updownarrow})$ for $\xi = \frac{1}{1+l}$ and $l \geq 1$.

Proof: Suppose that the approximate optimal solution $\hat{\boldsymbol{x}}$ and its value $\hat{F}$ of problem **P2.1** are obtained by Algorithm 2. From Steps 1 of Algorithm 2, the set of $\hat{\mathcal{R}} = \{\rangle \in \mathcal{R} : \hat{\S}_\rangle = \infty\}$ of winners can be divided into two disjoint sets, i.e.,

$$\hat{\mathcal{R}} = \hat{\mathcal{R}}_\infty \cup \hat{\mathcal{R}}_\in, \tag{27}$$

where $\hat{\mathcal{R}}_\infty = \{\rangle \in \hat{\mathcal{R}} | \amalg_\rangle - \int_\rangle^* > \frac{\mathcal{Q} - \sum_{\parallel\in\mathcal{N}} \int_\parallel^*}{\updownarrow + \infty}\}$, and $\hat{\mathcal{R}}_\in = \hat{\mathcal{R}} \setminus \hat{\mathcal{R}}_\infty$. From (20), the approximate objective value $\hat{F}$ can be expressed as

$$\hat{F} = \sum_{i\in\hat{\mathcal{R}}} \tilde{S}\left(q_i - s_i^*\right) + \sum_{k\in\mathcal{N}} p^* s_k^*. \tag{28}$$

Let $\boldsymbol{x}^*$ and F^* be the optimal solution and its value of problem **P2.1**. Similarly, the set of $\mathcal{R}^* = \{\rangle \in \mathcal{R} : \S_\rangle^* = \infty\}$ of optimal winners can be also partitioned into

$$\mathcal{R}^* = \mathcal{R}^*{}_\infty \cup \mathcal{R}^*{}_\in, \tag{29}$$

where $\mathcal{R}^*{}_\infty = \{\rangle \in \mathcal{R}^* | \amalg_\rangle - \int_\rangle^* > \frac{\mathcal{Q} - \sum_{\parallel\in\mathcal{N}} \int_\parallel^*}{\updownarrow + \infty}\}$, and $\mathcal{R}^*{}_\in = \mathcal{R}^* \setminus \mathcal{R}^*{}_\infty$. From (20), the optimal objective value F^* can be written as

$$F^* = \sum_{j\in\mathcal{R}^*} \tilde{S}\left(q_j - s_j^*\right) + \sum_{k\in\mathcal{N}} p^* s_k^*. \tag{30}$$

Now, we prove that $\hat{F} = F^*$ or $\frac{F^* - \hat{F}}{F^*} \leq \xi$ for $\xi = \frac{1}{1+l}$ and $l \geq 1$. To this end, first suppose that $\sum_{i\in\hat{\mathcal{R}}_\in} \left(q_i - s_i^*\right) \geq$

$\sum_{j\in\mathcal{R}^*_\in}\left(q_j - s_j^*\right)$. From Step 2 of Algorithm 2, we know that $\sum_{i\in\hat{\mathcal{R}}_\infty}\left(q_i - s_i^*\right) \geq \sum_{j\in\mathcal{R}^*_\infty}\left(q_j - s_j^*\right)$. Then, it follows $\hat{F} - F^* \geq 0$. Since F^* is the optimal objective value of problem **P2.1**, we thus have $\hat{F} = F^*$.

Next, suppose that $\sum_{i\in\hat{\mathcal{R}}_\in}(q_i - s_i^*) < \sum_{j\in\mathcal{R}^*_\in}\left(q_j - s_j^*\right)$. It implies that some $j \in \mathcal{R}^*{}_\in$ are removed from $\hat{\mathcal{R}}_\in$. From Step 4 of Algorithm 2, we know that $q_j - s_j^* > Q - \sum_{k\in\mathcal{N}} s_k^* - \sum_{i\in\hat{\mathcal{R}}}\left(q_i - s_i^*\right)$. Then, we have

$$\sum_{i\in\hat{\mathcal{R}}}(q_i - s_i^*) > Q - \sum_{k\in\mathcal{N}} s_k^* - (q_j - s_j^*)$$
$$\overset{(a)}{\geq} \left(\frac{l}{l+1}\right)\left(Q - \sum_{k\in\mathcal{N}} s_k^*\right)$$
$$\overset{(b)}{\geq} \left(\frac{l}{l+1}\right)\sum_{j\in\mathcal{R}^*}(q_j - s_j^*), \tag{31}$$

where (a) follows from Step 4 of Algorithm 2, and (b) comes from (25). Now, check $\frac{F^*-\hat{F}}{F^*} \leq \xi$ for $\xi = \frac{1}{1+l}$ and $l \geq 1$, i.e.,

$$\frac{F^* - \hat{F}}{F^*} = \frac{\sum_{j\in\mathcal{R}^*}(q_j - s_j^*) - \sum_{i\in\hat{\mathcal{R}}}(q_i - s_i^*)}{\sum_{k\in\mathcal{N}} p^* s_k^* + \sum_{j\in\mathcal{R}^*}\left(q_j - s_j^*\right)}$$
$$\leq \frac{\sum_{j\in\mathcal{R}^*}(q_j - s_j^*) - \sum_{i\in\hat{\mathcal{R}}}\left(q_i - s_i^*\right)}{\sum_{j\in\mathcal{R}^*}(q_j - s_j^*)} \leq \frac{1}{1+l}, \tag{32}$$

where the last (32) follows from (31). This completes the proof. ■

C. Users' Bidding Strategies

Given the announced target score $\tilde{S}$ and users' score functions (23), each user $i \in \mathcal{N}$ seeks to maximize the following optimization problem

$$\mathbf{P3}: \max_{b_i, q_i} \; u_i(\theta_i, \delta_i, q_i) - b_i q_i \tag{33}$$
$$\text{s.t.} \quad S_i(b_i, q_i) = \tilde{S}, \tag{34}$$
$$u_i(\theta_i, \delta_i, q_i) - b_i q_i \geq u_i(\theta_i, \delta_i, s_i^*) - p^* s_i^*, \tag{35}$$
$$0 \leq b_i \leq p^*, s_i^* < q_i \leq q_i^{\max}. \tag{36}$$

Note that constraint (35) represents the *individual rationality*, meaning that the bidders never get worse by bidding since they can always get the non-negative payoffs (i.e.,$u_i(\theta_i, \delta_i, s_i^*) - p^* s_i, i \in \mathcal{N}$) via forward pricing. The following proposition provides their optimal bidding strategies.

Proposition 3: For each user $i \in \mathcal{N}$, the optimal bidding strategy is given by

$$(b_i^*, q_i^*) = \begin{cases} \left(\frac{(q_i^* - s_i^*)\tilde{S} + p^* s_i^*}{q_i^*}, \min\left\{\frac{\theta_i + \delta_i}{\tilde{S}} - 1, q_i^{\max}\right\}\right), & \text{if constraints (34)–(36) are met,} \\ (0, 0), & \text{otherwise.} \end{cases}$$

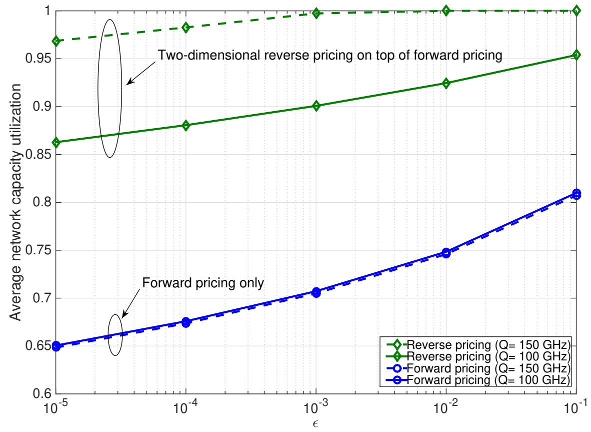

Fig. 2. Average network capacity utilization as a function of ϵ.

Proof: From (34), it follows that $b_i q_i = \tilde{S}\left(q_i - s_i^*\right) + p^* s_i^*$, for $i \in \mathcal{K}$. Substituting it into (33) gives

$$u_i(\theta_i, \delta_i, q_i) - \tilde{S}\left(q_i - s_i^*\right) - p^* s_i^*. \tag{37}$$

It is observed that the rewritten objective function in (37) is a concave function over q_i. Since all the constraints in (34)–(36) are affine, problem **P3** is a convex optimization problem. Exploiting the first order necessary condition yields

$$q_i^* = \min\left\{\frac{\theta_i + \delta_i}{\tilde{S}} - 1, q_i^{\max}\right\}, \; b_i^* = \frac{(q_i^* - s_i^*)\tilde{S} + p^* s_i^*}{q_i^*}, \tag{38}$$

where the "min" operator stems from the constraint (36) and b_i^* follows from (34). Putting (38) into (35) and checking whether (35) holds conclude the proof. ■

V. NUMERICAL AND EXPERIMENTAL RESULTS

A. Numerical Results

We first provide numerical examples to study several key properties of two-dimensional reverse pricing on top of forward pricing. Consider a 100-user in the network. For simplicity, the average willingness to pay of users are chosen as $\theta_i = i$, for $i \in \{1, 2, \cdots, 100\}$. The stochastic willingness to pay of users follows the scaled beta distribution on $[p^* - \theta_i, p^* + \theta_i]$ with shape parameters $[\theta_i - p^*, \theta_i + p^*]$. The total amount of available resource and target score are set as $Q = 100$ GHz and $\tilde{S} = 0.6p^*$ unless specified otherwise. For Algorithm 2, l is set to be 2. Each graph represents an average over 100,000 independent realizations.

Fig. 2 shows the average network capacity utilization (i.e., the average ratio of the total amount of allocated resource to the total amount of available resource) as a function of ϵ. We observe that the effect of reverse pricing is greater when the total resource of the network is larger. With forward pricing only, the average network capacity utilization decreases as ϵ decreases. As shown in Fig. 3, this stems from the fact that the MNO inevitably charges a higher price to the users as ϵ decreases.

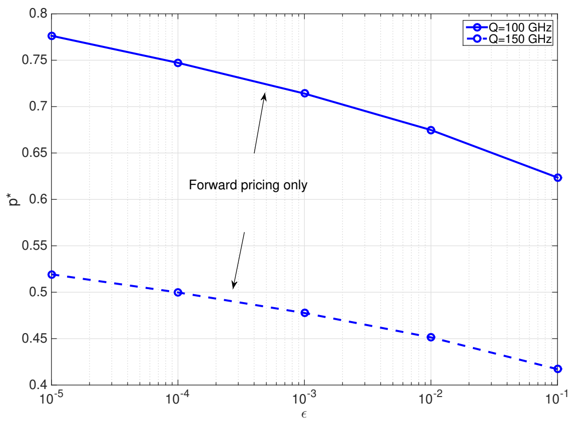

Fig. 3. ϵ-optimal approximate forward price as a function of ϵ.

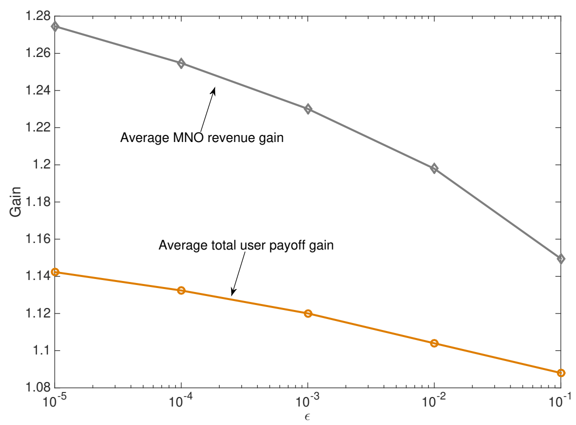

Fig. 4. Average MNO revenue and average total user payoff gains as a function of ϵ.

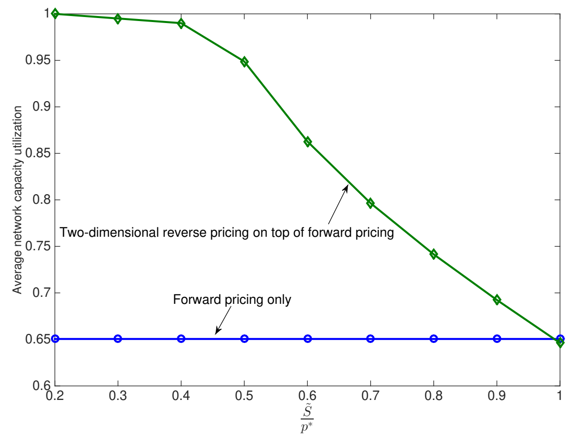

Fig. 5. Average network capacity utilization as a function of $\frac{\tilde{S}}{p^*}$. We assume $\epsilon = 10^{-5}$.

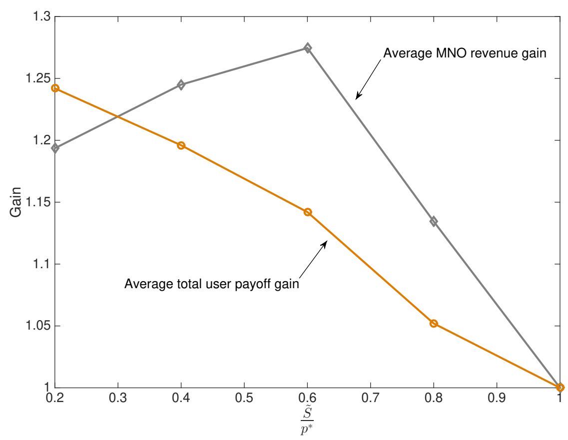

Fig. 6. Average MNO revenue and average total user payoff gains as a function of $\frac{\tilde{S}}{p^*}$. We assume $\epsilon = 10^{-5}$.

On the other hand, with two-dimensional reverse pricing on top of forward pricing, the average network capacity utilization is still very high even with lower ϵ.

Fig. 4 shows average MNO revenue and average total user payoff gains (i.e., the average ratios of the revenue and the total user payoff of reverse pricing on top of forward pricing to those of forward pricing only, respectively) as a function of ϵ. We observe that both gains increase as ϵ decreases. Perhaps counter-intuitively, it is worth noting that we identify some cases where the average MNO revenue gain is larger than the average total user payoff gain and such difference becomes more distinct as ϵ decreases. This can be explained as follows. As shown in Fig. 3, p^* increases as ϵ decreases, resulting in the decreases in both the average total user payoff and the average MNO revenue. For a given fixed target score $\tilde{S} = 0.6p^*$, however, the MNO can extract the higher marginal profit from multiple winners via two-dimensional reverse pricing as ϵ decreases. On the other hand, the average network utilization is still very close to 1 even when $\epsilon = 10^{-5}$. Thus, the average MNO revenue gain is larger than the average total user payoff gain.

Fig. 5 shows the average network capacity utilization as a function of $\tilde{S}/p^*$. We observe that the difference between two pricing schemes decreases as $\tilde{S}/p^*$ increases. This is due to the fact that the users are less incentivized to submit their two-dimensional bids with higher $\tilde{S}/p^*$.

Fig. 6 shows average MNO revenue and average total user payoff gains, respectively, as a function of $\tilde{S}/p^*$. Intuitively, we observe that the average total user payoff gain decreases $\tilde{S}/p^*$ increases. On the other hand, there is the optimal value of $\tilde{S}/p^*$ for maximizing the average MNO revenue gain. As $\tilde{S}/p^*$ increases, the MNO can extract the higher marginal profit from the winners at the cost of the decrease in the number of winners via two-dimensional reverse pricing. Thus, the MNO should set $\tilde{S}/p^*$ as a a strategic variable, taking this trade-off into account.

B. Experimental Results

Next, we present experimental studies to verify the feasibility of the proposed scheme in practice. To this end, we architected and prototyped the fully functional proposed pricing system via Android application and Amazon web service (AWS) service as shown in Fig. 7. For the experimental studies, we recruited 50 users as our trial participants and all of them were subscribers of SK Telecom (SKT) that is currently South Korea's largest MNO. Then, we conducted one month experiment by giving them the

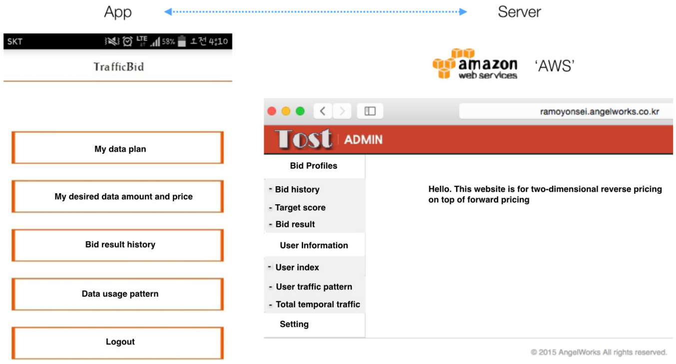

Fig. 7. The implemented proposed scheme app, called "Trafficbid." Users can plan their data amount and corresponding prices automatically or manually, identify their bid result histories, and check their data usage patterns. On the other hand, the MNO can manage users? bid profiles, check their information, and modify a target score over different time periods.

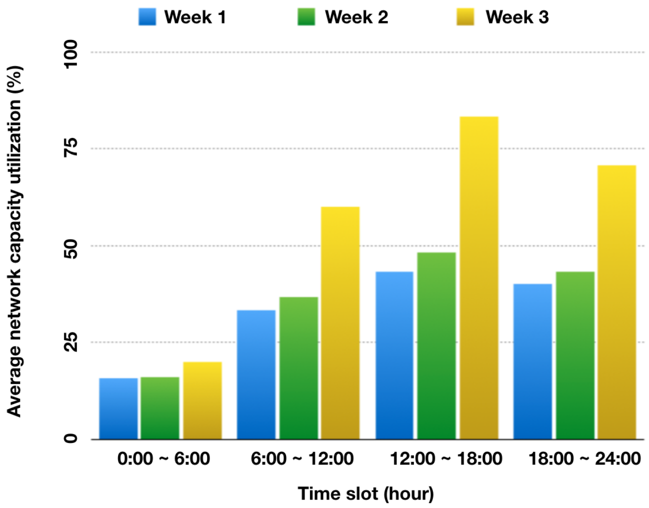

Fig. 8. The implemented proposed scheme app, called "Trafficbid." Users can plan their data amount and corresponding prices automatically or manually, identify their bid result histories, and check their data usage patterns. On the other hand, the MNO can manage users' bid profiles, check their information, and modify a target score over different time periods.

manual about the data usage amount per each application and enabling them to know how much data they would consume. After one month, we then allowed them to submit their two-dimensional bid as well as their data amount based on currently employed forward pricing scheme for each given period (i.e., 6 hours), and conducted three week experiments as follows.

- *1 week:* Participants used the currently employed forward pricing scheme (no two-dimensional reverse pricing employed).
- *2 week:* Participants used the proposed scheme without any specific target score.
- *3 week:* Participants used the proposed scheme with the target score $\tilde{S} = 0.6p^*$.

Fig. 8 shows the average network capacity utilization of all participants per each daily quarter. An interesting observation is that there is no distinct difference of the average network capacity utilization between in week 1 and week 2. If there is no target score requirement, the participants want to user their data amount based on the forward pricing scheme with cheaper bid prices. On the other hand, the average network capacity utilization in week 3 has increased 80% more than that in week 1. It indicates that the participants try to use more data amount with their own bid price in order to satisfy the target score (i.e., $\tilde{S} = 0.6p^*$ in our experiment).

Table 1. Average MNO revenue over three weeks.

	Week 1	Week 2	Week 3
Average MNO revenue	100%	87%	121%

Table 1 shows the average revenue of the MNO over three weeks. Note that the baseline is the MNO's average revenue in week 1. Since the participants tend to submit cheaper bid prices while keeping their current data usage patterns, the MNO's average revenue in week 2 has decreased 13% less than that in week 1. On the other hand, the MNO's average revenue in week 3 has increased 21% than that in week 1. Due to the target score requirement, the MNO can extract more revenue from adopting two-dimensional reverse pricing on top of forward pricing.

VI. CONCLUDING REMARKS

In this paper, we study the revenue-maximizing problem for a single MNO, considering heterogeneous and stochastic user demands. As a benchmark case, we analyze the drawbacks of forward pricing only. To effectively handle them, we propose two-dimensional reverse pricing on top of forward pricing and design a ξ-approximate polynomial-time algorithm. Compared to forward pricing only, we show that the proposed pricing scheme can achieve "triple-win" solutions: Higher average network capacity utilization; an increase in the total average payoff of the users; and an increment in the the average revenue of the MNO. To verify its feasibility in practice, we further implement its real prototype and perform experimental studies. We show that the proposed scheme still creates triple-win solutions as well. Our findings provide a new outlook on resource allocation, and design guidelines for adopting two-dimensional reverse pricing on top of forward pricing.

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