Multi-Stage Turbo Equalization for MIMO Systems with Hybrid ARQ

Sangjoon Park and Sooyong Choi

Abstract: **A multi-stage turbo equalization scheme based on the bit-level combining (BLC) is proposed for multiple-input multiple-output (MIMO) systems with hybrid automatic repeat request (HARQ). In the proposed multi-stage turbo equalization scheme, the minimum mean-square-error equalizer at each iteration calculates the extrinsic log-likelihood ratios for the transmitted bits in a subpacket and the subpackets are sequentially replaced at each iteration according to the HARQ rounds of received subpackets. Therefore, a number of iterations are executed for different subpackets received at several HARQ rounds, and the transmitted bits received at the previous HARQ rounds as well as the current HARQ round can be estimated from the combined information up to the current HARQ round. In addition, the proposed multi-stage turbo equalization scheme has the same computational complexity as the conventional bit-level combining based turbo equalization scheme. Simulation results show that the proposed multi-stage turbo equalization scheme outperforms the conventional BLC based turbo equalization scheme for MIMO systems with HARQ.**

Index Terms: **Bit-level combining (BLC), hybrid automatic repeat request (HARQ), minimum mean-square-error (MMSE) equalizer, multiple-input multiple-output (MIMO) systems, turbo equalization.**

I. INTRODUCTION

TURBO equalization schemes based on the minimum mean-square-error (MMSE) criterion are known as the sub-optimal detection schemes for multiple-input multiple-output (MIMO) systems and their error performances are comparable to the optimal detection schemes with a reduced computational complexity [1]–[7]. When hybrid automatic repeat request (HARQ) is employed in MIMO systems for packet transmissions and retransmissions, the combining of previously received signals and retransmitted signals can provide the additional diversity or coding gain [3]–[17]. Therefore, turbo equalization schemes should be modified to consider the combining process for retransmissions in MIMO systems with HARQ.

The combining process for HARQ retransmissions is mainly classified into the symbol-level combining (SLC) and the bit-level combining (BLC) [3]–[13]. In general, SLC based detection schemes outperform BLC based detection schemes for MIMO systems with HARQ. However, SLC based detection schemes do not only require a significantly larger computational complexity than BLC based detection schemes, but they are also applicable only in the cases where there is at least one repeatedly transmitted symbol by retransmissions, e.g., Chase combining based HARQ (CC-HARQ). Meanwhile, BLC based detection schemes can be applied even when there is no repeatedly transmitted symbol by retransmissions, e.g., incremental redundancy based HARQ (IR-HARQ), and they require a smaller computational complexity than SLC based detection schemes. However, the error performance of the BLC based detection schemes is inferior to that of the SLC based detection schemes and the BLC based turbo equalization schemes also show a worse performance than the SLC based turbo equalization schemes [4]–[6], [8]–[13].

Therefore, in order to improve the performance of the BLC based turbo equalization schemes without increasing the computational complexity, we propose a new BLC based turbo equalization scheme for MIMO systems with HARQ. In general, the turbo equalization scheme with the MMSE criterion consists of the MMSE equalizer and the forward error correction (FEC) decoder. Only the extrinsic log-likelihood ratios (LLRs) for the transmitted bits received at the current HARQ round are calculated by the MMSE equalizer in the conventional BLC based turbo equalization scheme [5], [6]. However, in the proposed turbo equalization scheme, the subpacket estimated by the MMSE equalizer is sequentially replaced at each iteration according to the HARQ rounds of received subpackets. Therefore, the extrinsic LLRs for the transmitted bits received at the previous HARQ rounds as well as the current HARQ round are updated at every retransmission in the proposed turbo equalization scheme. Consequently, the proposed turbo equalization scheme can exploit more information from HARQ retransmissions than the conventional BLC based turbo equalization scheme for MIMO systems with HARQ. Since multiple turbo iteration stages are executed for different subpackets received at several HARQ rounds, the proposed turbo equalization scheme is considered as the multi-stage turbo equalization scheme.

This paper is organized as follows. Section II presents the MIMO system model with HARQ. Section III describes the conventional BLC based turbo equalization scheme, and Section IV explains the proposed multi-stage turbo equalization scheme and compares the proposed turbo equalization scheme to the conventional one. Section V shows simulation results, and Section VI makes conclusions.

II. SYSTEM MODEL

Consider a spatially multiplexed MIMO system with N_i transmit and N_o receive antennas. The subpacket generation

Manuscript received December 17, 2014; approved for publication by Shigeaki Ogose, Division II Editor, December 1, 2015.

This work was supported by Institute of BioMed-IT, Energy-IT and Smart-IT Technology (BEST), a Brain Korea 21 plus program, Yonsei University.

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Digital object identifier 10.1109/JCN.2016.000049

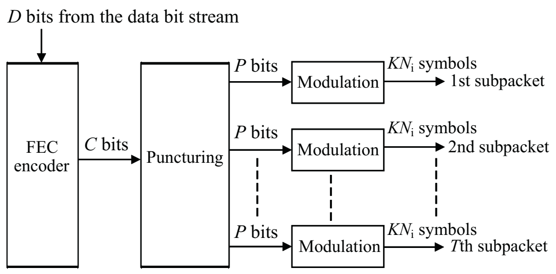

Fig. 1. The subpacket generation process at the transmitter.

process at the transmitter is illustrated in Fig. 1. For a given packet, a data bit sequence of length D is encoded using the FEC encoder to produce a coded bit sequence of length C. For the coded bit sequence, a puncturing algorithm according to the employed HARQ retransmission strategy is performed to select P bits that will be included in the subpacket for the tth HARQ round of the packet (denoted as the tth subpacket in the sequel) from $t = 1$ to $t = T$, where T is the maximum HARQ round allowed by the system. If the packet is not successfully decoded when $t < T$, then the $(t+1)$th subpacket for the packet is transmitted during the next transmission interval. If the packet is successfully decoded or t reaches T, then the packet is terminated and the first subpacket for a new packet containing a new data bit sequence is transmitted during the next transmission interval.

Let $\{\mathbf{x}_{t,1}, \cdots, \mathbf{x}_{t,K}\}$ denote a set of the coded bits selected from the puncturing stage that will be included in the tth subpacket, where $\mathbf{x}_{t,k} = \{\mathbf{x}_{t,k,1}, \cdots, \mathbf{x}_{t,k,N_\text{i}}\}$ for $1 \leq k \leq K$. Each $\mathbf{x}_{t,k,n}$ for $1 \leq n \leq N_\text{i}$ consists of Q coded bits, thereby $KN_\text{i}Q = P$. Each $\mathbf{x}_{t,k}$ for $1 \leq k \leq K$ is modulated as the $N_\text{i} \times 1$ transmit symbol vector $\mathbf{s}_{t,k} = [s_{t,k,1}, \cdots, s_{t,k,N_\text{i}}]^T$, where each $s_{t,k,n}$ for $1 \leq n \leq N_\text{i}$ is modulated from $\mathbf{x}_{t,k,n}$ using the 2^Q-ary constellation set $\mathcal{S}$ that satisfies $\sum_{s\in\mathcal{S}} s = 0$ and $\sum_{s\in\mathcal{S}} |s|^2/2^Q = 1$. Then, $\{\mathbf{s}_{t,1}, \cdots, \mathbf{s}_{t,K}\}$ is the symbol sequence for the tth subpacket, and the input-output relationship of the system for $\mathbf{s}_{t,k}$ from $k = 1$ to $k = K$ can be written as

$$\mathbf{r}_{t,k} = \mathbf{H}_{t,k}\mathbf{s}_{t,k} + \mathbf{n}_{t,k}, \tag{1}$$

where $\mathbf{r}_{t,k}$ is the $N_\text{o} \times 1$ receive signal vector and $\mathbf{H}_{t,k}$ is the $N_\text{o} \times N_\text{i}$ channel matrix for $\mathbf{s}_{t,k}$. Further, $\mathbf{n}_{t,k}$ is the $N_\text{o} \times 1$ noise vector whose elements are i.i.d. complex Gaussian random variables with zero mean and variance σ^2. Therefore, the signal-to-noise ratio (SNR) is defined as $1/\sigma^2$.

To describe the reception procedures of the turbo equalization schemes, the MMSE equalizer and the FEC decoder of the turbo equalization schemes are separately modeled. Since the inner operations of the MMSE equalizer and the FEC decoder in the conventional turbo equalization scheme will not be modified, the detailed descriptions on the inner operations of both modules are omitted in this paper.[1] The MMSE equalizer and the FEC decoder can be respectively modeled using their input-output relationships as

[1]For the detailed descriptions on the inner operations of the modules in the conventional turbo equalization scheme, please refer [1]–[7].

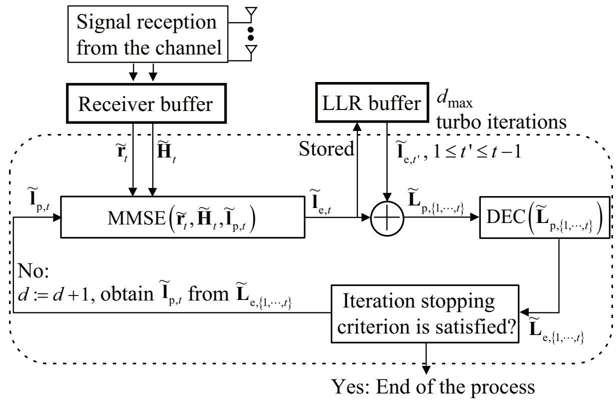

Fig. 2. The reception process of the conventional BLC based turbo equalization scheme for the tth HARQ round of a packet.

$$\widetilde{\mathbf{l}}_{\text{e},t} = \text{MMSE}\left(\widetilde{\mathbf{r}}_t, \widetilde{\mathbf{H}}_t, \widetilde{\mathbf{l}}_{\text{p},t}\right) \tag{2}$$

and

$$\widetilde{\mathbf{L}}_{\text{e},\{1,\cdots,t\}} = \text{DEC}\left(\widetilde{\mathbf{L}}_{\text{p},\{1,\cdots,t\}}\right). \tag{3}$$

In (2), $\widetilde{\mathbf{r}}_t$ and $\widetilde{\mathbf{H}}_t$ are the sets of the receive signal vectors and the channel matrices for the tth subpacket, respectively, $\widetilde{\mathbf{l}}_{\text{p},t}$ is the set of the input *a-priori* LLRs to the MMSE equalizer for the coded bits in the tth subpacket, $\{\mathbf{x}_{t,1}, \cdots, \mathbf{x}_{t,K}\}$, and $\widetilde{\mathbf{l}}_{\text{e},t}$ is the set of the output extrinsic LLRs from the MMSE equalizer for $\{\mathbf{x}_{t,1}, \cdots, \mathbf{x}_{t,K}\}$. In (3), $\widetilde{\mathbf{L}}_{\text{p},\{1,\cdots,t\}}$ is the set of the input *a-priori* LLRs to the FEC decoder for the entire coded bits transmitted up to the tth HARQ round, which is a combination of all $\widetilde{\mathbf{l}}_{\text{e},t'}$ for $1 \leq t' \leq t$, and $\widetilde{\mathbf{L}}_{\text{e},\{1,\cdots,t\}}$ is the set of the output extrinsic LLRs from the FEC decoder for the entire coded bits transmitted up to the tth HARQ round.

III. CONVENTIONAL BIT-LEVEL COMBINING BASED TURBO EQUALIZATION SCHEME

The reception process of the conventional BLC based turbo equalization scheme for MIMO systems with HARQ [5], [6] is illustrated in Fig. 2. Before the first iteration, the input *a-priori* LLRs to the MMSE equalizer for coded bits in the tth subpacket, $\widetilde{\mathbf{l}}_{\text{p},t}$, are all initialized to zero. Then, at the beginning of the dth iteration to detect the packet at its tth HARQ round, the MMSE equalizer is executed to obtain the extrinsic LLRs for coded bits in the tth subpacket as

$$\widetilde{\mathbf{l}}_{\text{e},t} = \text{MMSE}\left(\widetilde{\mathbf{r}}_t, \widetilde{\mathbf{H}}_t, \widetilde{\mathbf{l}}_{\text{p},t}\right). \tag{4}$$

Then, the calculated $\widetilde{\mathbf{l}}_{\text{e},t}$ is combined with $\widetilde{\mathbf{l}}_{\text{e},t'}$ for $1 \leq t' \leq t-1$ to obtain $\widetilde{\mathbf{L}}_{\text{p},\{1,\cdots,t\}}$, and the FEC decoder is executed using $\widetilde{\mathbf{L}}_{\text{p},\{1,\cdots,t\}}$ as

$$\widetilde{\mathbf{L}}_{\text{e},\{1,\cdots,t\}} = \text{DEC}\left(\widetilde{\mathbf{L}}_{\text{p},\{1,\cdots,t\}}\right). \tag{5}$$

Then, $\widetilde{\mathbf{L}}_{\text{e},\{1,\cdots,t\}}$ from the dth iteration is used to obtain $\widetilde{\mathbf{l}}_{\text{p},t}$ which is the input *a-priori* LLRs to the MMSE equalizer at the

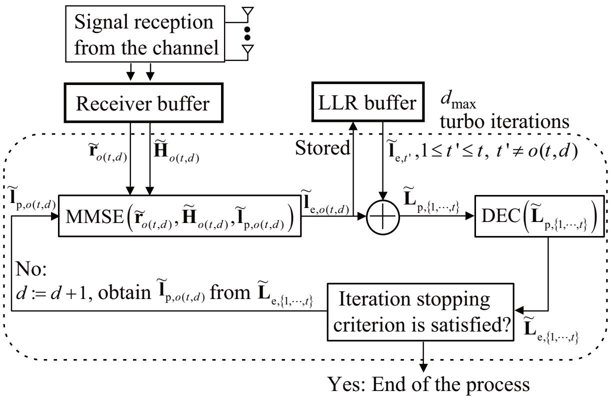

Fig. 3. The reception process of the proposed multi-stage turbo equalization scheme for the tth HARQ round of a packet.

next $(d+1)$th iteration.[2] This process is repeated until an iteration stopping criterion is satisfied, e.g., d reaches the maximum allowed number of turbo iterations, $d_{\max}$.

IV. PROPOSED MULTI-STAGE TURBO EQUALIZATION SCHEME

As described in Section III, in the conventional turbo equalization scheme for the tth HARQ round of a packet, the MMSE equalizer calculates $\widetilde{\mathbf{l}}_{\mathrm{e},t}$ regardless of d. Since $\widetilde{\mathbf{l}}_{\mathrm{e},t}$ is obtained from $\widetilde{\mathbf{l}}_{\mathrm{p},t}$ which was obtained from the combined LLRs up to the tth HARQ round, $\widetilde{\mathbf{L}}_{\mathrm{e},\{1,\cdots,t\}}$, $\widetilde{\mathbf{l}}_{\mathrm{e},t}$ is calculated by using the entire information obtained up to the tth HARQ round. However, each $\widetilde{\mathbf{l}}_{\mathrm{e},t'}$ for $1 \le t' \le t-1$ estimated at the previous t'th HARQ round is not updated at the tth HARQ round. That is, each $\widetilde{\mathbf{l}}_{\mathrm{e},t'}$ for $1 \le t' \le t-1$ is estimated by using the information obtained up to the t'th HARQ round, and the information obtained from the $(t'+1)$th HARQ round to the tth HARQ round is not utilized for the estimation of $\widetilde{\mathbf{l}}_{\mathrm{e},t'}$.

To update as many $\widetilde{\mathbf{l}}_{\mathrm{e},t'}$ for $1 \le t' \le t$ as possible at every retransmission, we propose the multi-stage turbo equalization scheme based on the BLC. Fig. 3 illustrates the reception process of the proposed multi-stage turbo equalization scheme for MIMO systems with HARQ. Before the first iteration, the input *a-priori* LLRs to the MMSE equalizer for the coded bits in the tth subpacket, $\widetilde{\mathbf{l}}_{\mathrm{p},t}$, are all initialized to zero. Let $o(t,d)$ denote the HARQ round of the subpacket in which the coded bits are estimated by the MMSE equalizer at the dth iteration during the reception process for the tth HARQ round of a packet. Then, at the beginning of the dth iteration, the MMSE equalizer is executed to obtain the extrinsic LLRs for the coded bits in the $o(t,d)$th subpacket as

$$\widetilde{\mathbf{l}}_{\mathrm{e},o(t,d)} = \mathrm{MMSE}\left(\widetilde{\mathbf{r}}_{o(t,d)}, \widetilde{\mathbf{H}}_{o(t,d)}, \widetilde{\mathbf{l}}_{\mathrm{p},o(t,d)}\right). \tag{6}$$

Then, the calculated $\widetilde{\mathbf{l}}_{\mathrm{e},o(t,d)}$ is combined with $\widetilde{\mathbf{l}}_{\mathrm{e},t'}$ for $1 \le t' \le t$ and $t' \ne o(t,d)$ to obtain $\widetilde{\mathbf{L}}_{\mathrm{p},\{1,\cdots,t\}}$, and the FEC decoder is executed using $\widetilde{\mathbf{L}}_{\mathrm{p},\{1,\cdots,t\}}$ as

$$\widetilde{\mathbf{L}}_{\mathrm{e},\{1,\cdots,t\}} = \mathrm{DEC}\left(\widetilde{\mathbf{L}}_{\mathrm{p},\{1,\cdots,t\}}\right). \tag{7}$$

Then, $\widetilde{\mathbf{L}}_{\mathrm{e},\{1,\cdots,t\}}$ from the dth iteration is used to obtain $\widetilde{\mathbf{l}}_{\mathrm{p},o(t,d+1)}$ which is the input *a-priori* LLRs to the MMSE equalizer at the next $(d+1)$th iteration. Therefore, we can obtain a more accurate $\widetilde{\mathbf{l}}_{\mathrm{e},o(t,d+1)}$. This process is repeated until an iteration stopping criterion is satisfied, e.g., d reaches $d_{\max}$.

Since there are no modifications on both the MMSE equalizer and the FEC decoder, the proposed multi-stage turbo equalization scheme has the identical computational complexity to the conventional turbo equalization scheme for a given $d_{\max}$. In addition, the proposed multi-stage turbo equalization scheme is applicable to both CC-HARQ and IR-HARQ since it is built on the concept of the BLC [3]–[13]. The only additional cost by the proposed multi-stage turbo equalization scheme is the increased receiver buffer size to store $\widetilde{\mathbf{r}}_{t'}$ and $\widetilde{\mathbf{H}}_{t'}$ for $1 \le t' \le t-1$, as shown in Figs. 2 and 3.

The proposed multi-stage turbo equalization scheme can be distinguished from the BLC based turbo equalization schemes in [3], [7] that execute the equalization procedures for all received subpackets before the combining process, which provides an improved performance at the expense of a high complexity. In addition, the proposed multi-stage turbo equalization scheme is also different from the approaches in [14]–[16] that reuse the extrinsic information generated from the decoding procedures of previous HARQ rounds, e.g., reusing $\widetilde{\mathbf{l}}_{\mathrm{p},t'}$ with $t' < t$ at the initialization.

A. Subpacket Order Calculation

As shown in Section III, the performance of the proposed multi-stage turbo equalization scheme is greatly affected by the order of the subpackets estimated by the MMSE equalizer in each iteration, $o(t,d)$. In order to achieve fine error performances, the order can be calculated by tracking the error performance of the coded bits or convergence behavior of each subpacket during the iterative reception process, e.g., analyzing the extrinsic information transfer (EXIT) characteristics [2]. However, such approaches do not only require the heavy computational burden, but they also need to be performed newly for every channel variation and iteration, which is not suitable for practical systems.

Therefore, in this subsection, we propose a criterion that determines $o(t,d)$ at a significantly low complexity for the proposed multi-stage turbo equalization scheme. The goal of the criterion is to update as many $\widetilde{\mathbf{l}}_{\mathrm{e},t'}$ for $1 \le t' \le t$ as possible by using the entire information obtained up to the tth HARQ round. For example, consider the case when $t = 2$ and the iterative reception process is not started yet. Then, before the first iteration, the numbers of updates for $\widetilde{\mathbf{l}}_{\mathrm{e},1}$ and $\widetilde{\mathbf{l}}_{\mathrm{e},2}$ during the reception process at the second HARQ round are equal to 0, and the numbers of updates for $\widetilde{\mathbf{l}}_{\mathrm{e},1}$ and $\widetilde{\mathbf{l}}_{\mathrm{e},2}$ during the reception process at the first HARQ round are respectively $d_{\max}$ and 0. Although the numbers of updates for both $\widetilde{\mathbf{l}}_{\mathrm{e},1}$ and $\widetilde{\mathbf{l}}_{\mathrm{e},2}$ at the second HARQ round are equal to zero, $\widetilde{\mathbf{l}}_{\mathrm{e},1}$ was already updated

[2]The detailed procedures for obtaining $\widetilde{\mathbf{l}}_{\mathrm{p},t}$ from $\widetilde{\mathbf{L}}_{\mathrm{e},\{1,\cdots,t\}}$ as well as the combining procedures for obtaining $\widetilde{\mathbf{L}}_{\mathrm{p},\{1,\cdots,t\}}$ from $\widetilde{\mathbf{l}}_{\mathrm{e},t'}$ for $1 \le t' \le t$ can be different according to the puncturing rule and the modulation type.

Table 1. The calculation process of $o(t,d)$.

Step 0)	Set $t_c := t$.
Step 1)	Generate a set $\mathbf{T}_{t_c} = \{t^* \mid t^* = \arg\min_{1\le t'\le t_c} f(t', t_c, d-1)\}$.
Step 2)	If $\lvert\mathbf{T}_{t_c}\rvert = 1$, set $o(t,d) = t^* (\in \mathbf{T}_{t_c})$ and stop the process. Otherwise, if $\lvert\mathbf{T}_{t_c}\rvert > 1$, set $t_c := t_c - 1$ and go to Step 3).
Step 3)	Generate a set $\mathbf{T}_{t_c} = \{t^* \mid t^* = \arg\min_{t'\in\mathbf{T}_{t_c+1}} f(t', t_c, d_{\max})\}$.
Step 4)	If $\lvert\mathbf{T}_{t_c}\rvert = 1$, set $o(t,d) = t^* (\in \mathbf{T}_{t_c})$ and stop the process. Otherwise, if $\lvert\mathbf{T}_{t_c}\rvert > 1$, set $t_c := t_c - 1$ and go back to Step 3).

Table 2. Numbers of updates for $\widetilde{\mathbf{l}}_{e,t}$ by the MMSE equalizer when $T=3$.

	Conventional	Proposed
$\widetilde{\mathbf{l}}_{e,1} \leftarrow \widetilde{\mathbf{l}}_{p,1} \leftarrow \widetilde{\mathbf{L}}_{e,\{1\}}$ $(t=1)$	$d_{\max}$	$d_{\max}$
$\widetilde{\mathbf{l}}_{e,1} \leftarrow \widetilde{\mathbf{l}}_{p,1} \leftarrow \widetilde{\mathbf{L}}_{e,\{1,2\}}$ $(t=2)$	0	$\lfloor d_{\max}/2 \rfloor$
$\widetilde{\mathbf{l}}_{e,2} \leftarrow \widetilde{\mathbf{l}}_{p,2} \leftarrow \widetilde{\mathbf{L}}_{e,\{1,2\}}$ $(t=2)$	$d_{\max}$	$\lceil d_{\max}/2 \rceil$
$\widetilde{\mathbf{l}}_{e,1} \leftarrow \widetilde{\mathbf{l}}_{p,1} \leftarrow \widetilde{\mathbf{L}}_{e,\{1,2,3\}}$ $(t=3)$	0	$\lfloor d_{\max}/3 \rfloor$
$\widetilde{\mathbf{l}}_{e,2} \leftarrow \widetilde{\mathbf{l}}_{p,2} \leftarrow \widetilde{\mathbf{L}}_{e,\{1,2,3\}}$ $(t=3)$	0	$[d_{\max}/3]$
$\widetilde{\mathbf{l}}_{e,3} \leftarrow \widetilde{\mathbf{l}}_{p,3} \leftarrow \widetilde{\mathbf{L}}_{e,\{1,2,3\}}$ $(t=3)$	$d_{\max}$	$\lceil d_{\max}/3 \rceil$

at the first HARQ round. Therefore, $o(2,1)$ should be set to 2 to update $\widetilde{\mathbf{l}}_{e,2}$ at the first iteration. Consequently, after the first iteration, the numbers of updates for $\widetilde{\mathbf{l}}_{e,1}$ and $\widetilde{\mathbf{l}}_{e,2}$ at the second HARQ round are now 0 and 1, respectively. Further, since $\widetilde{\mathbf{l}}_{e,2}$ experienced more updates than $\widetilde{\mathbf{l}}_{e,1}$ at the second HARQ round, $o(2,2)$ should be set to 1 to update $\widetilde{\mathbf{l}}_{e,1}$ at the second iteration, and so on.

Therefore, $o(t,d)$ can be calculated by the sequential comparing procedures of the numbers of updates for $\widetilde{\mathbf{l}}_{e,t'}$ $(1 \le t' \le t)$ by the MMSE equalizer during the reception process at each HARQ round, as described in Table 1. $f(t', t_c, d)$ in Table 1 indicates the number of updates for $\widetilde{\mathbf{l}}_{e,t'}$ by the MMSE equalizer until the dth iteration at the t_cth HARQ round. At Step 1), the numbers of updates for $\widetilde{\mathbf{l}}_{e,t'}$ $(1 \le t' \le t)$ by the MMSE equalizer until the $(d-1)$th iteration at the tth HARQ round are calculated, and the HARQ rounds of the received subpackets with the minimum number of updates are selected. If there are more than one HARQ rounds satisfying the above condition, among the selected HARQ rounds from Step 1), the numbers of updates for $\widetilde{\mathbf{l}}_{e,t'}$ $(t' \in \mathbf{T}_t)$ at the previous $(t-1)$th HARQ round are calculated, and the HARQ rounds of the received subpackets with the minimum number of updates are selected. This process is repeated until only one HARQ round is remained and selected as $o(t,d)$. Therefore, $o(t,d)$ can be pre-determined before the iterative reception process at the tth HARQ round.[3]

The conventional turbo equalization scheme can be considered as the special case of the proposed multi-stage turbo equalization scheme with $o(t,d) = t$ regardless of t and d. By updating more $\widetilde{\mathbf{l}}_{e,t'}$ for $1 \le t' \le t$ with new information provided by retransmissions, the proposed multi-stage turbo equalization scheme can achieve a better performance than the conventional turbo equalization scheme for MIMO systems with HARQ. Table 2 shows the numbers of updates for $\widetilde{\mathbf{l}}_{e,t}$ by the MMSE equalizer in the conventional and proposed turbo equalization schemes when $T=3$, where $\lfloor\cdot\rfloor$, $\lceil\cdot\rceil$, and $[\,\cdot\,]$ in Table 2 denote the floor, ceiling, and nearest integer functions, respectively. Since $o(t,d) = t$ regardless of t and d in the conventional turbo equalization scheme, each $\widetilde{\mathbf{l}}_{e,t'}$ for $1 \le t' \le t$ is updated from $\widetilde{\mathbf{l}}_{p,t'}$ which was obtained from $\widetilde{\mathbf{L}}_{e,\{1,\cdots,t'\}}$ and no other updates are performed. Meanwhile, in the proposed multi-stage turbo equalization scheme, every $\widetilde{\mathbf{l}}_{e,t'}$ for $1 \le t' \le t$ is updated from $\widetilde{\mathbf{l}}_{p,t}$ which was obtained from $\widetilde{\mathbf{L}}_{e,\{1,\cdots,t\}}$. Although the number of updates for $\widetilde{\mathbf{l}}_{e,t}$ by the MMSE equalizer in the proposed multi-stage turbo equalization scheme is smaller than that in the conventional turbo equalization scheme, the proposed multi-stage turbo equalization scheme can update every $\widetilde{\mathbf{l}}_{e,t'}$ for $1 \le t' \le t$ using the entire information obtained up to the tth HARQ round if $d_{\max} \ge t$. Therefore, the proposed multi-stage turbo equalization scheme can exploit more information from HARQ retransmissions than the conventional turbo equalization scheme as $d_{\max}$ increases.

V. SIMULATION RESULTS

In this section, the performances of the BLC based turbo equalization schemes are numerically compared. In addition to the conventional BLC based turbo equalization scheme and the proposed multi-stage turbo equalization scheme, the performance of the experimentally scheduled turbo equalization scheme is also evaluated as a control group. In each iteration of the experimentally scheduled turbo equalization scheme, the subpacket order $o(t,d)$ is decided to $t^*(1 \le t^* \le t)$ if the number of bits errors after the FEC decoding procedures at the dth iteration is minimized when the t^*th subpacket is utilized as the input for the MMSE equalizer at the dth iteration. That is, $o(t,d)$ in the experimentally scheduled turbo equalization scheme can be decided after the numerical simulations for all t' with $1 \le t' \le t$ at each d.[4]

A rate-1/2 low-density parity-check (LDPC) code in [17] with $D = 576$ and $C =$1,152 is considered as the mother code. $N_i = 4$, $N_o = 2$, and quadrature phase shift keying (QPSK) modulation with $Q = 2$ is used. IR-HARQ is considered with $T = 3$. A punctured codeword by the rate-compatible puncturing algorithm in [18] with $P = 768$ is modulated as the symbol sequence with $K = 96$ and transmitted as the subpacket for each HARQ round. A symbol interleaver is applied to the symbol sequence, where the interleaving pattern is fixed over one subpacket and varies from the previous subpacket to the next subpacket [12]. A time-varying frequency-flat Rayleigh fading

[3]Although the calculation of $o(t,d)$ proposed in this paper can be done by a much simpler procedure, e.g., $o(t,d) = t - [(d-1)/t]$ with the nearest integer function $[\cdot]$ for any t and d, we described the calculation of $o(t,d)$ as in Table 1.

[4]Furthermore, the experimentally scheduled turbo equalization scheme requires the perfect knowledge about the transmitted bits at the receiver before the end of the iterative reception process, which is impossible for practical systems.

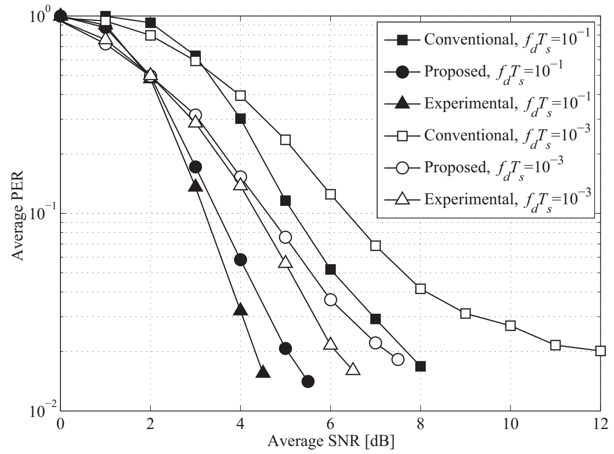

Fig. 4. Average PERs of the BLC based turbo equalization schemes when $d_{\max} = 8$ and $t = 2$.

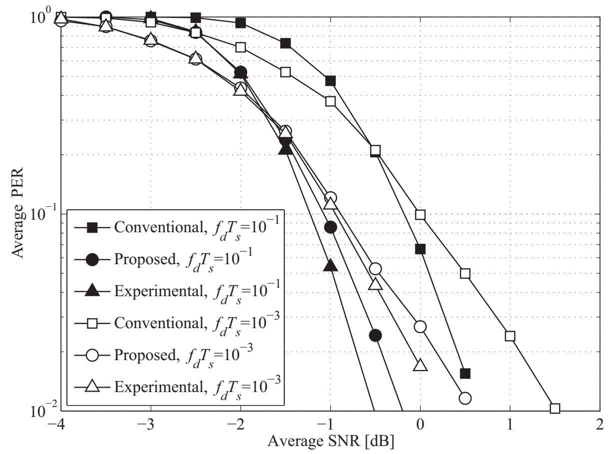

Fig. 5. Average PERs of the BLC based turbo equalization schemes when $d_{\max} = 8$ and $t = 3$.

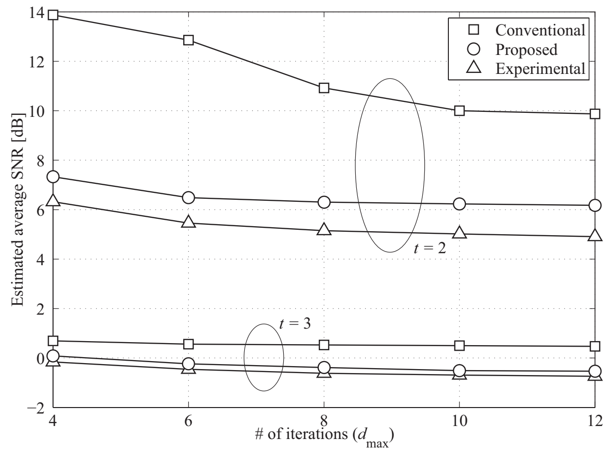

Fig. 6. Estimated average SNRs to achieve a PER of $2 \cdot 10^{-2}$ for the BLC based turbo equalization schemes according to $d_{\max}$ when $f_dT_s = 10^{-2}$.

channel with a normalized Doppler frequency f_dT_s is considered with the perfect channel estimation at the receiver. The standard belief propagation decoding algorithm [19] is used for the FEC decoder, and the number of total FEC decoding iterations is fixed to 120 regardless of $d_{\max}$.

The average packet error rates (PERs) of the turbo equalization schemes at the tth HARQ round when $d_{\max} = 8$ are compared in Figs. 4 and 5 for $t = 2$ and $t = 3$, respectively. It is shown from Figs. 4 and 5 that the proposed multi-stage turbo equalization scheme outperforms the conventional turbo equalization scheme regardless of t and f_dT_s. In addition, the SNR gains of the proposed turbo equalization scheme over the conventional turbo equalization scheme are more significant when $f_dT_s = 10^{-3}$ than when $f_dT_s = 10^{-1}$. In particular, when $t = 2$, the SNR gains at a PER of $2 \cdot 10^{-2}$ are about 0.85 dB and 0.95 dB for $f_dT_s = 10^{-1}$ and 10^{-3}, respectively. These gains are significantly increased when $t = 2$, which are about 2.65 dB and 4.8 dB for $f_dT_s = 10^{-1}$ and 10^{-3}, respectively. Since the number of the updates for $\widetilde{\mathbf{l}}_{e,t'}$ $(1 \leq t' < t)$ by the MMSE equalizer with a given $d_{\max}$ increases as t decreases in the proposed multi-stage turbo equalization scheme, the performance improvement is more significant when $t = 2$ than when $t = 3$. Meanwhile, the experimentally scheduled turbo equalization scheme achieves the lowest average PERs regardless of t and f_dT_s, and the proposed multi-stage turbo equalization scheme shows the slightly worse average PERs than the experimentally scheduled turbo equalization scheme in the high SNR region. However, the performance gap between the proposed multi-stage turbo equalization scheme and the experimentally scheduled turbo equalization scheme is negligible compared to that between the proposed multi-stage turbo equalization scheme and the conventional turbo equalization scheme.

In Fig. 6, the estimated average SNRs to achieve a PER of $2 \cdot 10^{-2}$ for the turbo equalization schemes according to $d_{\max}$ are derived when $f_dT_s = 10^{-2}$. This figure shows that the proposed multi-stage turbo equalization scheme outperforms the conventional turbo equalization scheme regardless of t and $d_{\max}$. Similar to the results in Figs. 4 and 5, the SNR gains of the proposed multi-stage turbo equalization scheme over the conventional turbo equalization scheme are more significant when $t = 2$ than when $t = 3$. In addition, it is shown that the SNR gains of the experimentally scheduled turbo equalization scheme over the proposed multi-stage turbo equalization scheme are similar for all $d_{\max}$. In other words, regardless of the number of turbo iterations, the proposed multi-stage turbo equalization scheme that updates as many $\widetilde{\mathbf{l}}_{e,t'}$ for $1 \leq t' \leq t$ as possible provides comparable error performances to the experimentally scheduled turbo equalization scheme. Furthermore, for the subpacket order calculation, the proposed multi-stage turbo equalization requires a simple calculation procedure with a significantly low complexity, while the experimentally scheduled turbo equalization scheme requires the equalization procedures for all received subpackets at each iteration.

Fig. 7 compares the average numbers of retransmissions per packet for the turbo equalization schemes when $f_dT_s = 10^{-2}$ and $d_{\max} = 4$. As shown in Fig. 7, the conventional turbo equalization scheme requires more numbers of retransmissions per packet than the proposed multi-stage turbo equalization scheme and the experimentally scheduled turbo equalization scheme in the entire SNR region. Specifically, when the average SNR is 4

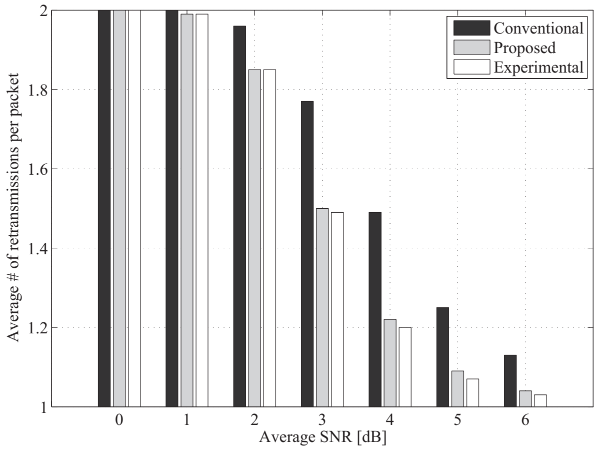

Fig. 7. Average numbers of retransmissions per packet for the BLC based turbo equalization schemes when $f_d T_s = 10^{-2}$ and $d_{\max} = 4$.

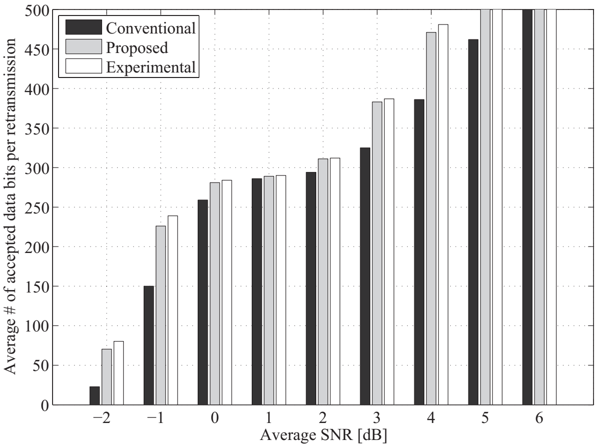

Fig. 8. Average numbers of successfully received data bits per retransmission for the BLC based turbo equalization schemes when $f_d T_s = 10^{-2}$ and $d_{\max} = 4$.

dB, the conventional turbo equalization scheme requires around 1.22 and 1.25 times greater numbers of retransmissions than the proposed multi-stage turbo equalization scheme and the experimentally scheduled turbo equalization scheme, respectively. Meanwhile, the numbers of retransmissions for the proposed multi-stage turbo equalization are nearly identical to those for the experimentally scheduled turbo equalization scheme regardless of the average SNR.

Finally, in Fig. 8, the average numbers of successfully received data bits per retransmission are compared for the turbo equalization schemes when $f_d T_s = 10^{-2}$ and $d_{\max} = 4$. Similar to the previous simulation results, the proposed multi-stage turbo equalization scheme outperforms the conventional turbo equalization scheme and achieves similar performances to the experimentally scheduled turbo equalization scheme regardless of the average SNR.

VI. CONCLUSION

In this paper, the multi-stage turbo equalization scheme is proposed for MIMO systems with HARQ. Since the proposed multi-stage turbo equalization scheme is built on the concept of the BLC, the proposed multi-stage turbo equalization scheme is applicable to both CC-HARQ and IR-HARQ. Further, the proposed multi-stage turbo equalization scheme has the identical computational complexity to the conventional BLC based turbo equalization scheme. Therefore, the proposed multi-stage turbo equalization scheme can be considered as an effective suboptimal receiver for MIMO systems with HARQ. From the simulation results, it is verified that the proposed multi-stage turbo equalization scheme outperforms the conventional BLC based turbo equalization scheme in terms of error performance.

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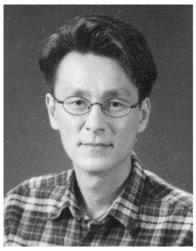

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