Mixture Filtering Approaches to Blind Equalization Based on Estimation of Time-Varying and Multi-Path Channels

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Abstract: **In this paper, we propose a number of blind equalization approaches for time-varying and multi-path channels. The approaches employ cost reference particle filter (CRPF) as the symbol estimator, and additionally employ either least mean squares algorithm, recursive least squares algorithm, or H∞ filter (HF) as a channel estimator such that they are jointly employed for the strategy of "Rao-Blackwellization," or equally called "mixture filtering." The novel feature of the proposed approaches is that the blind equalization is performed based on direct channel estimation with unknown noise statistics of the received signals and channel state system while the channel is not directly estimated in the conventional method, and the noise information if known in similar Kalman mixture filtering approach. Simulation results show that the proposed approaches estimate the transmitted symbols and time-varying channel very effectively, and outperform the previously proposed approach which requires the noise information in its application.**

Index Terms: **Blind equalization, cost reference particle filter (CRPF), frequency selective fading, H∞ (H infinity) filter, least mean squares, mixture filtering, Rayleigh fading, recursive least squares.**

I. INTRODUCTION

GENERALLY, a wireless communication channel is considered as a time-varying and multi-path channel which distorts the transmitted signal, or incurs fading of it. If a wide-band signal is employed in a wireless communication system, the channel is considered as a frequency-selective fading channel when it is safely assumed that the coherence bandwidth of the channel is less than the bandwidth of the transmitted signal: under frequency-selective fading, different frequency components of the signal undergo different gains and phase shifts across the band, and a tapped-delay line model for the received signal describes frequency-selective and slow fading channel which also has significant effect of inter-symbol interference (ISI).

In this paper, we propose approaches that jointly estimate the time-varying channel impulse response (CIR) and transmitted symbols based on the received signal that had gone though frequency-selective and Rayleigh fading, without the aid of a training sequence; besides, the noise statistics of the received signal are not known. The main purpose of the blind equalization is estimating the transmitted symbols regardless of whether the channel is estimated or not. Blind equalization is desirable for some applications, such as multipoint or group communications. The history of the blind equalization may originate from [1] where "a stochastic gradient algorithm" was employed. With the beginning of the work in [1], the blind equalization approaches may be classified into three main categories as follows [2].

Approaches in the first category search for the solution by using the method of "steepest decent," which is a traditional iterative procedure, and has been used to find extremes of nonlinear functions. Due to the difficulty to acquire the correlation of the estimation error and the received signal, usually least mean squares (LMS) algorithm is employed where a one-point sample mean of the estimation error and the received signal replaces the correlation of the error and the received signal. Due to its disadvantage of the slow convergence property, recursive least squares (RLS) algorithm is an alternative which minimizes the "least squares" instead of "mean-square error," and consequently, no statistical information about the received signal and the transmitted input data is required. The reference [3] has been popularly cited in the literature. There are some more papers that proposed the approaches based on "steepest decent" algorithm [1], [4], [5]. A notable feature of the proposed approach in [3] is that any noise statistics are not required to be known nor assumed to be Gaussian, which might be a big advantage in practice. Nonetheless, its well known disadvantage is that the algorithm converges slowly; besides that, it requires a constraint that all elements of the initial equalizing coefficients of the reference taps except for one element should be zero, and the non-zero element must be greater than a threshold that is determined by the true initial channel impulse response for reliable convergence even though the the constraint is a sufficient condition rather than a necessary condition. The approaches in the second category are based on the second-order or higher-order statistics of the received signal. Based on the second-order statistics (autocorrelation) of the received signal, CIR is estimated by using the cyclostationary property of the received signal [6], [7]. The channel also can be estimated explicitly from the forth-order cumulants of the received signal [8], [9]. The tricepstrum equalization algorithm (TEA) based on the forth-order statistics of the received signal was proposed in [10], where the trispectrum is the three dimensional discrete fourier transform of the fourth-order cumulant sequence. Finally, methods in the last category employ the maximum likelihood (ML) ap-

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proaches that are based on the Viterbi algorithm. Contrary to traditional ML equalizers that require the batch of the received signal, and cause the delayed estimation, the per-survival processing (PSP) method jointly estimates the transmitted symbol and channel sequentially from the updated received signal [11], [12]. Among the blind equalization methods in the three categories, maximum likelihood criterion is optimal although it may require large computational complexity, especially when the ISI spans many symbols.

In addition to the methods described above, there are some more contributions to the development of the blind equalizers; specifically, the effort was made with less information [13], such as noise statistics in addition to the blind condition. The proposed methods in [14], [15] are based on unknown statistical information of the input signal; however, the noise statistics of the received signal are needed. In [16], the transmitted symbols are estimated by integrating out the channel parameter and the noise variance of the received signal with the assumption that the noise of the received signal is Gaussian, and this method benchmarks the approach proposed in [17]. Joint channel and transmitted symbol estimation schemes are proposed to tackle the flat [18] and frequency-selective fading channels [19] by employing particle filtering (PF), where a strategy of “Rao-Blackwellization” [20] or equivalently “mixture filtering” scheme is employed by combining a couple of different estimators. PF is adopted for symbol estimation, and the Kalman filtering (KF) or RLS algorithm is applied for channel estimation in [18], [19].

In this paper, we propose a number of mixture filtering approaches to blind equalization where recently developed cost reference particle filter (CRPF) [21], [22], [23] and either H∞ filter (HF) [24], [25], LMS algorithm, or RLS algorithm are jointly employed in order to estimate transmitted symbols and CIR at the same time without using any known preamble symbols. These coupled schemes are similar to that of “particle filtering” and the “KF” as employed in [19]. However, the most salient feature of the proposed approaches is that they do not require the information of the noise statistics of the received signal, input signal, and time varying channel process noise in their applications regardless of if the noise is Gaussian or not. We also directly estimate CIR in this blind equalization where usually inverse CIR is employed for an equalizing filter. Based on the estimated CIR, we infer the transmitted symbols in this blind equalization approaches. Furthermore, even though CRPF works in particle filtering framework, it does not require the intractable computation of *expected posterior density* which is often necessary for methods working in Bayesian framework; specifically, in minimum mean square error criterion. Therefore, CRPF may have an advantage in terms of computational cost in its application compared to standard particle filtering. We model the time-varying multi-path channel by auto-regressive (AR) representation which also reflects Rayleigh fading; especially, the proposed approaches show significantly better performance under the AR(p) model with $p = 1$ (where p denotes the order of AR process) which incurs less computational complexity than higher-order models. Simulation results show that the mixture filtering approaches of “CRPF and LMS algorithm” and “CRPF and RLS algorithm” which do not take advantage of noise information outperform the previously proposed approach [19] that requires the noise information in its application. The mixture approach of CRPF and HF also outperforms the mixture of PF and KF even with unknown noise statistics when the employed number of particles is small and signal to noise ratio (SNR) is low.

The rest of this paper is organized as follows. In Section II, the system model is described where the multi-path time-varying channel is modeled by AR representation which also reflects Rayleigh fading. Next, we introduce HF and CRPF algorithms in addition to the rest of all individual methods that are employed for the mixture filtering approaches in Section III. The proposed mixture filtering approaches are described in detail in Section IV, and then the results of simulations are following where the performance of the proposed approaches are assessed. Finally, we make concluding remarks in the last section.

II. SYSTEM MODEL

A widely employed power spectrum density (PSD) model for a mobile radio channel is “Jakes’ model” [26]. The PSD can be obtained by: computing the autocorrelation function of the transfer function of time-varying multi-path channel, and then taking the Fourier transform of the autocorrelation function. Then, the PSD is expressed as follows.

$$S(f) = \begin{cases} \frac{1}{\pi f_m \sqrt{1-\left(\frac{f}{f_m}\right)^2}}, & |f| \le f_m \\ 0, & |f| > f_m \end{cases} \tag{1}$$

where f_m is the maximum Doppler frequency shift that is determined by the mobile terminal speed and the carrier frequency. In reverse, this PSD can be modeled by an AR process [27] which can be driven by an input of white random process through an all-pole model of an infinite impulse response (IIR) filter. Therefore, The dynamic state equation of CIR is expressed by (2) when we assume $(L+1)$ multi-paths of the received signals. In (2)

- k : Sampling time index
- η : Coefficient of the denominator of IIR filter
- p : Order of AR(p) process
- $\boldsymbol{I}$: $(L+1)\times(L+1)$ identity matrix
- $\mathbf{0}_A$: $(L+1)\times(L+1)$ zero matrix
- $\mathbf{0}_\nu$: $(L+1)\times 1$ zero vector
- D : Coefficient of the numerator of IIR filter
- $\mathfrak{h}_k$: The state vector $[\boldsymbol{h}_k\ \boldsymbol{h}_{k-1}\ \cdots\ \boldsymbol{h}_{k-p+1}]^\top$ where $\top$ denotes the transpose.
- $\boldsymbol{h}_k$: The channel impulse response vector at time k and is equal to $[h_{k,0}\ h_{k,1}\ \cdots\ h_{k,L}]^\top$ where each element is a complex value.
- $\boldsymbol{v}_k$: White noise vector, $[v_{k,0}\ v_{k,1}\ \cdots\ v_{k,L}]^\top$ where each element is a complex value, and real and imaginary part have the same variance.

And, the dimension of the matrices are: $[\mathfrak{h}_k] = p\cdot(L+1)\times 1$; $[\boldsymbol{A}] = p\cdot(L+1)\times p\cdot(L+1)$; $[\boldsymbol{\nu}_k] = [\mathfrak{h}_k]$; and η and D are determined according to f_m. Then, we obtain the corresponding transfer function with the input $\boldsymbol{v}_k$ and the output $\mathfrak{h}_k$ as follows.

$$H(z) = \frac{D}{1-\eta_1 z^{-1}-\eta_2 z^{-2}\cdots-\eta_p z^{-p}} \tag{3}$$

$$\underbrace{\begin{bmatrix} \boldsymbol{h}_k \\ \boldsymbol{h}_{k-1} \\ \vdots \\ \boldsymbol{h}_{k-p+1} \end{bmatrix}}_{\mathfrak{h}_k} = \underbrace{\begin{bmatrix} \eta_1 \boldsymbol{I} & \eta_2 \boldsymbol{I} & \eta_3 \boldsymbol{I} & \cdots & \eta_p \boldsymbol{I} \\ \boldsymbol{I} & \boldsymbol{0}_A & \boldsymbol{0}_A & \cdots & \boldsymbol{0}_A \\ \boldsymbol{0}_A & \boldsymbol{I} & \boldsymbol{0}_A & \cdots & \boldsymbol{0}_A \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0}_A & \boldsymbol{0}_A & \boldsymbol{0}_A & \cdots & \boldsymbol{0}_A \end{bmatrix}}_{\boldsymbol{A}} \cdot \underbrace{\begin{bmatrix} \boldsymbol{h}_{k-1} \\ \boldsymbol{h}_{k-2} \\ \vdots \\ \boldsymbol{h}_{k-p} \end{bmatrix}}_{\mathfrak{h}_{k-1}} + D \underbrace{\begin{bmatrix} \boldsymbol{v}_k \\ \boldsymbol{0}_\nu \\ \vdots \\ \boldsymbol{0}_\nu \end{bmatrix}}_{\boldsymbol{\nu}_k} \tag{2}$$

Then, $|H(f)|^2$ will approximate PSD (1) [12]. If each element of the input $\boldsymbol{v}_k$ is complex Gaussian distributed with the same variance, the output envelope will be *Rayleigh* distributed. The time-varying and multi-path channel has been modeled as a frequency-selective fading channel, and the mobile radio channel also reflects Rayleigh fading. We assume that the signaling time and the sampling period have the same value, and they are synchronized as well. Then, the received signal at the receiver after filtering through an ideal low pass filter is expressed as follows:

$$y_k = \boldsymbol{b}_k^\top \mathfrak{h}_k + w_k \tag{4}$$

where transmitted symbols $\boldsymbol{b}_k = [b_k \; b_{k-1} \; \cdots \; b_{k-L} \; \boldsymbol{0}_b \; \cdots \; \boldsymbol{0}_b]^\top$ whose dimension is $p \cdot (L+1) \times 1$; $[\boldsymbol{0}_b] = 1 \times (L+1)$; and w_k is additive noise. The variance of w_k is unknown, and furthermore, it does not have to be Gaussian for equalization by the proposed approaches in this paper. Note that the measurement (4) is independent of the IIR filter order p, but only depends on the channel length $(L+1)$.

III. FILTERING METHODS

In this paper, LMS and RLS algorithms, the KF, HF, PF, and CRPF are employed for a number of mixture filtering approaches. In this section, we explain about CRPF and HF that are not well-known to researchers, before we propose the mixture filtering approaches in the next section; and details of the relatively well-known algorithms of LMS, RLS, KF, and PF are not described in this paper. The methods are explained based on the previously described system model related to the equalization problem under investigation. LMS algorithm, RLS algorithm, the KF, and HF are used for CIR estimation while PF and CRPF are employed for symbol estimation.

A. H_∞ Filtering

HF is employed for CIR estimation, and estimated symbol $\hat{\boldsymbol{b}}$ is provided by CRPF in the mixture filtering algorithm that will be introduced in the following section. Proposed discrete time HF in this paper is originally designed and proposed based on *game theory* approach (specifically, zero-sum game) in several papers, such as [28] and [24], etc. Therefore, the approach is basically more like *game theory* approach rather than conventional HF that is originally employed in *control theory* area. The original HF has not been popularly employed due to its level of mathematical understanding and the necessity of a good modeling of the system. The discrete time HF designed in [24], [28] was motivated by continuous time HF that has been designed in [29], [30], etc. These game theory-based HFs are well-designed for the estimation of parameters that are modeled by the discrete time dynamic state system that includes the state process noise of the dynamic state system. The "cost function" in terms of "game theory," or in other words, "H_∞ norm" in terms of HF is defined, and minimized in game theory-based discrete time HF. In the zero-sum game, the cost function is designed on the basis of the strategy that the probability of maximum expected point-loss is minimized regardless of the strategy of the opponent; therefore, in the game of HF, the filter is a player prepared for the worst strategy that the other player (the nature) can provide, and the goal is providing an uniformly small estimation error for any processes and measurement noises and any initial states. Consequently, we do not have to know the noise statistics of the "dynamic state system" and "measurement" noises for the filtering process. The filtering scheme of HF is similar to that of the KF. However, HF does not require the statistics of the state and measurement noises regardless of if it is Gaussian or not. Moreover, whereas the mean square error is minimized in the KF, the worst case error (or maximized error) is minimized in HF. This is why the KF may also be referred to as H_2 filter. More specifically, the norm or the cost function is defined in HF, and the maximum norm, which is specifically called H_∞ norm, is minimized. Related to the system equations (2) and (4), HF estimates $\mathfrak{h}_k$ with uniformly small errors given arbitrary $\boldsymbol{w}_k$, $\boldsymbol{\nu}_k$, and $\mathfrak{h}_0$. This idea is very similar to the case of the zero-sum game where maximum benefit-loss is minimized. Therefore, the cost function related to the zero-sum game is defined as follows [24] based on the system equations (2) and (4).

$$J = \frac{\sum_{k=0}^{N-1} \| \mathfrak{h}_k - \hat{\mathfrak{h}}_k \|^2_{\boldsymbol{\chi}_k}}{\| \mathfrak{h}_0 - \hat{\mathfrak{h}}_0 \|^2_{\boldsymbol{P}_0^{-1}} + \sum_{k=0}^{N-1} \left(\| \boldsymbol{\nu}_k \|^2_{\boldsymbol{W}_k^{-1}} + \| \boldsymbol{w}_k \|^2_{\boldsymbol{V}_k^{-1}} \right)} \tag{5}$$

where $\boldsymbol{\chi}_k$, $\boldsymbol{P}_0$, $\boldsymbol{W}_k$, and $\boldsymbol{V}_k$ are the weight parameters that are positive definite matrices; and $\| \boldsymbol{\nu}_k \|^2_{\boldsymbol{W}_k^{-1}}$ denotes $\boldsymbol{\nu}_k{}^\top \boldsymbol{W}_k^{-1} \boldsymbol{\nu}_k$. If interested in estimating the second element of $\mathfrak{h}_k$, then $\boldsymbol{\chi}_k(2,2)$ should be large relatively to other elements of $\boldsymbol{\chi}_k$. If it is known that the second element of $\boldsymbol{w}(k)$ is small, then $\boldsymbol{V}_k(2,2)$ is chosen to be small relatively to other elements. Direct minimization of J is not tractable; therefore, the performance bound γ is introduced, and it satisfies $J < \gamma^{-1}$. Then, J' is defined in (6) And, the problem becomes a matter of solving the following minimax problem.

$$\min_{\hat{\mathfrak{h}}_k} \left(\max_{\boldsymbol{w}_k, \boldsymbol{\nu}_k, \mathfrak{h}_0} J' \right) \tag{7}$$

HF reduces to the KF when: $\gamma = \infty$; and the true covariance matrices of the parameters are selected for $\boldsymbol{\chi}_k$, $\boldsymbol{P}_0$, $\boldsymbol{W}_k$, and $\boldsymbol{V}_k$; therefore, the KF does not guarantee any bound for the

$$J' = -\gamma^{-1} \parallel \mathfrak{h}_0 - \hat{\mathfrak{h}}_0 \parallel^2_{\boldsymbol{P}_0^{-1}} + \sum_{k=0}^{N-1} \left[\parallel \mathfrak{h}_k - \hat{\mathfrak{h}}_k \parallel^2_{\boldsymbol{\chi}_k} - \gamma^{-1} \left(\parallel \boldsymbol{\nu}_k \parallel^2_{\boldsymbol{W}_k^{-1}} + \parallel \boldsymbol{w}_k \parallel^2_{\boldsymbol{V}_k^{-1}} \right) \right] \quad (6)$$

cost function from the HF point of view. When the bound γ is selected, it has to satisfy the condition $\gamma \boldsymbol{\chi}_k < \boldsymbol{P}_k^{-1} + \hat{\boldsymbol{b}}_k \boldsymbol{V}_k^{-1} \hat{\boldsymbol{b}}_k^H$ to maintain $\boldsymbol{P}_k > 0$.

B. Cost Reference Particle Filtering

In this paper, CRPF takes on a crucial role for a number of proposed mixture filtering approaches. CRPF plays a role of estimating transmitted symbols while a number of the other algorithms are jointly estimating the channel impulse response. We introduce CRPF algorithm with its theoretical background, specifically, for symbol estimation based on the system model of (2) and (4).

CRPF has a couple of notable advantageous features: its computational cost is not as expensive as that of PF because the computation of the "expected posterior function" is not required as opposed to the cases of approaches which work in Bayesian framework including standard PF; more specifically, in minimum mean square error criterion. In addition, the noise information is not needed in its application [21]. The cost function in CRPF, which is recursive and additive structure, is defined as

$$\mathcal{C}(\boldsymbol{b}_{0:k}^{(i)} | y_{1:k}, \lambda) = \lambda \mathcal{C}(\boldsymbol{b}_{0:k-1}^{(i)} | y_{1:k-1}, \lambda) + \Delta\mathcal{C}(\boldsymbol{b}_k^{(i)} | y_k). \quad (8)$$

Equation (8) is equivalently expressed in a simple form as $\mathcal{C}_k^{(i)} = \lambda \mathcal{C}_{k-1}^{(i)} + \Delta\mathcal{C}_k^{(i)}$ where i indicates the particle index, λ is the forgetting factor [1] $(0 \leq \lambda \leq 1)$ which makes it possible to adaptively change the amount of contributions of past particles in evaluating cost function, and $\Delta\mathcal{C}$ is the "incremental cost function" which indicates the accuracy of the particle $\boldsymbol{b}_k^{(i)}$ for given y_k; the cost increment can be computed by

$$\Delta\mathcal{C}_k^{(i)} = \parallel y_k - \left[\boldsymbol{b}_k^{(i)}\right]^H \mathfrak{h}_k^{(i)} \parallel^q \quad (9)$$

where $q > 0$, H denotes the Hermitian transpose, and $\mathfrak{h}_k^{(i)}$ is assumed to be given by another algorithm in the manner of mixture filtering. The cost function is a measure of "estimate quality" like the weight as in PF. Similarly to PF, the cost-based random measure is represented by a set of *particles* and associated *costs* as given by $\Xi_k = \left\{\boldsymbol{b}_k^{(i)}, \mathcal{C}_k^{(i)}\right\}_{i=1}^M$ where M is the total number of used particles. In addition to the cost function, the "risk function" is defined in CRPF as

$$\mathcal{R}(\boldsymbol{b}_k^{(i)} | y_k) = \Delta\mathcal{C}\left(E\left[\mathfrak{h}_k^{(i)}\right] | y_k\right) = \Delta\mathcal{C}\left(\boldsymbol{A}\mathfrak{h}_{k-1}^{(i)} | y_k\right). \quad (10)$$

A good choice of the risk function is given by

$$\mathcal{R}(\boldsymbol{b}_k^{(i)} | y_k) = \parallel y_k - \left[\boldsymbol{b}_k^{(i)}\right]^H \cdot [\boldsymbol{A}\mathfrak{h}_{k-1}^{(i)}] \parallel^q \quad (11)$$

where $q > 0$, and $\parallel \cdot \parallel$ denotes the norm of the vector. The risk function measures the adequacy of the particle $\boldsymbol{b}_k^{(i)}$ (equivalently corresponding $\mathfrak{h}_{k-1}^{(i)}$) given the observation y_k. Also, the risk function is a prediction of the cost increment $\Delta\mathcal{C}(\boldsymbol{b}_k^{(i)} | y_k)$. Then the "predictive cost function" which includes the risk term is defined as $\mathcal{R}_k^{(i)} = \lambda \mathcal{C}_{k-1}^{(i)} + \mathcal{R}(\boldsymbol{b}_k^{(i)} | y_k)$. Based on above definitions, the sequential algorithm proceeds by recursively repeating the steps of "risk evaluation," "resampling," "particle propagation," and "updating the cost" with time. CRPF has almost the same computational complexity as PF even though CRPF has a few more steps than PF in each iteration; therefore, CRPF has considered to have $O\ (M)$ complexity. More details of CRPF and its applications can be referred to [21], [22], and [31].

Each algorithm has different requirement for information of noise and equation parameters in its application. In this paper, the measurement equation (4) can be specified once the channel length $(L + 1)$ is determined; nonetheless, the measurement noise information can not be determined without assuming it to be known. Even though the parameters of the dynamic state equation (2) is determined according to the order of $\mathrm{AR}(p)$ model and Doppler shift, a method such as LMS algorithm does not require the parameters such as $\boldsymbol{A}$, η, and D; moreover, LMS algorithm is not concerned about if the state noise or input noise $\boldsymbol{v}$ is Gaussian or not, and its algorithm is the simplest among the considered approaches. Similar requirement regarding the noise and parameter information is needed for RLS algorithm although it has more steps than LMS algorithm. CRPF and HF are another approaches that do not require the noise information although they need the state equation parameters such as $\boldsymbol{A}$, η, and D; CRPF does not need D. Both LMS and RLS algorithms do not need any of the required information described above for CRPF and HF that is because CRPF and HF are designed for the problems which are described by the dynamic state system, and usually the parameters of the state equation are assumed to be known in their applications. On the contrary, PF and the KF need all information of the state equation parameters and noise information; moreover, in the application of the KF, we need to assume that the noises of the state equation and observation to be the Gaussian distributions. In the following section, we propose a number of mixture blind equalization approaches, and describe the details about them.

IV. MIXTURE FILTERING APPROACHES

In this section, we describe a number of newly proposed mixture filtering approaches. PF and the KF are jointly employed in [19], and we abbreviate the name of this approach as "PF-KF" from here on. Whereas much more information of the state equation parameters and the noise information (furthermore, it has to be Gaussian) is needed in the application of PF-KF, much less information is needed in applications of the proposed approaches in this paper. Three approaches proposed here are enumerated as "CRPF and LMS algorithm (CRPF-LMS)," "CRPF and RLS algorithm (CRPF-RLS)," and "CRPF and HF (CRPF-HF)," respectively. They jointly estimate the transmitted symbols and time-varying multi-path channel in the manner of mix-

[1] At the same time, λ avoids attributing an excessive weight to old observations when a long series of data are available [21].

ture filtering scheme [20]. The contributions of this paper is proposing these joint estimation approaches to blind equalization with less-information provided. These equalizers have severe depth of blindness in their applications.

A. Mixture Algorithm of CRPF and LMS

CRPF and LMS jointly estimate the transmitted symbols and CIR, respectively, in the algorithm. Among the considered mixture approaches, CRPF-LMS has the lowest computational complexity since LMS algorithm requires the minimum number of steps. As long as the performance of the approach is satisfied, this approach might be the most efficient in terms of computational complexity. A value between 0 and 2 is usually selected as the step size ϵ of LMS algorithm. It may improve the performance of the method if an appropriate value of ϵ is carefully chosen in the application. Along with the mixture of CRPF-RLS, this approach requires the least information of the problem, and almost none is required for its application once the Doppler shift is provided. We describe the detailed steps of the algorithm as follows:

I) **Initialization**: (`for` $i = 1, \cdots, M$) Assign the initial cost $\mathcal{C}_0^{(i)} = 0$, initialize the initial state $\mathfrak{h}_0^{(i)}$, and select the step size ϵ for LMS algorithm.

II) **Recursive update**: `for` $k = 1, \cdots, K$ (the total time steps or symbols)

1. Generate (`for` $i = 1, \cdots, M$) $\boldsymbol{b}_k^{(i)}$ from uniformly distributed symbols.
2. Compute (`for` $i = 1, \cdots, M$)
   a) The predictive cost $\mathcal{R}_k^{(i)} = \lambda \mathcal{C}_{k-1}^{(i)} + \| y_k - \left[\boldsymbol{b}_k^{(i)}\right]^H \cdot [\boldsymbol{A} \cdot \mathfrak{h}_{k-1}^{(i)}] \|^q$ for $q > 0$.
   b) Probability mass function (PMF), $\tilde{\pi}_k^{(i)} \propto \mu_1(\mathcal{R}_k^{(i)}) = \frac{1}{\left(\mathcal{R}_k^{(i)} - \min\left\{\mathcal{R}_k^{(i)}\right\}_{i=1}^M + \kappa\right)^\beta}$, where $\kappa, \beta > 0$: small value of κ is to ensure the denominator is not zero.
3. Take the selection step, or resampling $\breve{\Xi}_{k-1} = \left\{\breve{\mathfrak{h}}_{k-1}^{(i)}, \breve{\mathcal{C}}_{k-1}^{(i)}, \breve{\boldsymbol{b}}_k^{(i)}\right\}_{i=1}^M$ according to $\tilde{\pi}_k^{(i)}$ where "˘" denotes the resampled version.
4. Update (`for` $i = 1, \cdots, M$) $\mathfrak{h}_k^{(i)}$ from $\breve{\mathfrak{h}}_{k-1}^{(i)}$ by LMS algorithm.
   a) $e_k^{(i)} = y_k - \left[\breve{\boldsymbol{b}}_k^{(i)}\right]^\top \breve{\mathfrak{h}}_{k-1}^{(i)}$. b) $\mathfrak{h}_k^{(i)} = \breve{\mathfrak{h}}_{k-1}^{(i)} + \epsilon e_k^{(i)} \left[\breve{\boldsymbol{b}}_k^{(i)}\right]^*$ where $*$ denotes complex conjugate.
5. Compute the cost (`for` $i = 1, \cdots, M$), $\mathcal{C}_k^{(i)} = \lambda \breve{\mathcal{C}}_{k-1}^{(i)} + \| y_k - \left[\tilde{\boldsymbol{b}}_k^{(i)}\right]^H \mathfrak{h}_k^{(i)} \|^q$, and normalized PMF, $\pi_k^{(i)} \sim \mu_2\left(\mathcal{C}_k^{(i)}\right) = \frac{1}{\left(\mathcal{C}_k^{(i)} - \min\left\{\mathcal{C}_k^{(i)}\right\}_{i=1}^M + \kappa\right)^\beta}$.
6. Compute the MAP estimate of the symbols and MMSE of CIR.
   a) Estimation of $\boldsymbol{b}_{1:k}$ = $\hat{\boldsymbol{b}}_{1:k} = \arg\max_{\boldsymbol{b}_{1:k}} \left[\sum_{i=1}^M \delta\left(\boldsymbol{b}_{1:k} - \tilde{\boldsymbol{b}}_{1:k}^{(i)}\right) \pi_k^{(i)}\right]$ where, $\delta(\cdot)$ denotes the Kronecker-delta function
   b) Estimation of $\mathfrak{h}_k = \hat{\mathfrak{h}}_k = \mathfrak{h}_k^{\text{mean}} = \sum_{i=1}^M \pi_k^{(i)} \mathfrak{h}_k^{(i)}$.

At the step of II)-4, usually new particles are generated by using the Gaussian propagation density with the current particle as the mean and a properly selected variance; and usually estimated parameter of interest is in the "state" rather than in the measurement equation. The generated particle is the transmitted symbol in this paper; nonetheless, each symbol particle is associated with a specific CIR particle. Therefore, CRPF does not require the initialization and computation of the variance for the particle propagation density in this paper. The initialization of the variance is crucial part in the original application of CRPF for satisfactory performance of the algorithm.

B. Mixture Algorithm of CRPF and RLS

In the mixture algorithm of CRPF and RLS method, RLS method replaces the role of LMS method in CRPF-LMS. RLS algorithm is computationally more complex than LMS algorithm while it is less complex than the KF and HF. The feature of CRPF-RLS is very similar to that of CRPF-LMS except the first has slightly higher computational complexity than the second; therefore, if both approaches have similar performances, then it might be more efficient to employ CRPF-LMS rather than CRPF-RLS in terms of computational complexity. Additionally, CRPF-RLS is less tractable than CRPF-LMS because we have to select another coefficient while CRPF-LMS requires the decision of only one coefficient in the algorithm. A value between 0 and 1 is selected for the forgetting factor Λ, and Δ is a small positive value. These two coefficients may need to be chosen carefully if better performance is required. The details of the algorithm are described as follows:

I) **Initialization**: (`for` $i = 1, \cdots, M$) Assign the initial cost $\mathcal{C}_0^{(i)} = 0$, select the forgetting factor Λ and the value Δ to initialize $\mathfrak{P}_k^{(i)}$, then compute $\mathfrak{P}_0^{(i)} = \Delta^{-1}\boldsymbol{I}$ where $\boldsymbol{I}$ is the $p(L+1) \times p(L+1)$ identity matrix, and initialize the initial state $\mathfrak{h}_0^{(i)}$.

II) **Recursive update**: `for` $k = 1, \cdots, K$ (the total time steps or symbols)

1.–3. Identical with the steps as in CRPF-LMS.

4. Update (`for` $i = 1, \cdots, M$) $\mathfrak{h}_k^{(i)}$ from $\breve{\mathfrak{h}}_{k-1}^{(i)}$ by RLS algorithm.
   a) $\boldsymbol{z}_k^{(i)} = \mathfrak{P}_{k-1}^{(i)} \left[\breve{\boldsymbol{b}}_k^{(i)}\right]^*$
   b) $\boldsymbol{g}_k^{(i)} = \boldsymbol{z}_k^{(i)} / \left(\Lambda + \left[\breve{\boldsymbol{b}}_k^{(i)}\right]^H \boldsymbol{z}_k^{(i)}\right)$
   c) $\alpha_k^{(i)} = y_k - \left[\breve{\boldsymbol{b}}_k^{(i)}\right]^H \breve{\mathfrak{h}}_{k-1}^{(i)}$
   d) $\mathfrak{h}_k^{(i)} = \breve{\mathfrak{h}}_{k-1}^{(i)} + \alpha_k^{(i)} \boldsymbol{g}_k^{(i)}$.
   e) $\mathfrak{P}_k^{(i)} = \Lambda^{-1}\left\{\mathfrak{P}_{k-1}^{(i)} - \boldsymbol{g}_k^{(i)} \left[\boldsymbol{z}_k^{(i)}\right]^H\right\}$

5.–6. Identical with the steps as in CRPF-LMS.

C. Mixture Algorithm of PF and the Kalman Filter

The KF is used for CIR estimation when it is jointly employed with PF for equalization [19]. As opposed to the cases of LMS and RLS, the KF requires much more information such as $\boldsymbol{A}$, D, the covariance of the state noise ν, and the variance of the measurement noise w. Besides, the KF assumes both noises are Gaussian for satisfactory performance. The feature of PF is simulation-based approximation technique, and consequently, it works very well for the problems that are modeled by nonlinear functions of the state, and moreover, the noise does not need to be Gaussian; nonetheless, the information of noises $\boldsymbol{\nu}$ and w in (2) and (4) has to be known in its application. The generic PF with two kinds of proposal densities (optimal density and prior density) were employed in [19]; the generic PF with prior proposal density is identical with sequential importance resampling (SIR) PF if the resampling process is performed every time step. We consider SIR PF in this paper, and the details of the algorithm can be referred to [19]. This approach requires the most amount of information in its application; furthermore, the noises of the equations are assumed to be Gaussian distributed due to the application of the Kalman algorithm. The algorithm also has the highest computational complexity; therefore, significant performance superiority might be required for this algorithm at the expense of high computational complexity.

D. Mixture Algorithm of CRPF and H_∞ Filter

Both CRPF and HF do not need the noise information of the state and the measurement equations; and, both algorithms do need the parameter information of the state equation. Therefore, this mixture seems to be very similar to PF-KF approach except that it does not need the noise information regardless of if it is Gaussian or not. Detailed steps of the proposed mixture filtering approach are described as follows:

I) **Initialization**: (for $i = 1, \cdots, M$) Initialize the bound γ and weighting parameters for HF, assign the initial cost $\mathcal{C}_0^{(i)} = 0$, and nitialize the initial state $\mathfrak{h}_0^{(i)}$.

II) **Recursive time update**: for $k = 1, \cdots, K$

1.–3. Identical with the steps as in CRPF-LMS.

4. Compute a bank (M) of HF steps,

a) $\boldsymbol{S}_k^{(i)} = \left\{\boldsymbol{I} - \gamma\boldsymbol{\chi}_k\boldsymbol{P}_{k-1}^{(i)} + \left[\tilde{\boldsymbol{b}}_k^{(i)}\right]^H \boldsymbol{V}_k^{-1}\tilde{\boldsymbol{b}}_k^{(i)}\boldsymbol{P}_{k-1}^{(i)}\right\}^{-1}$

b) $\boldsymbol{H}_k^{(i)} = \boldsymbol{A}\boldsymbol{P}_{k-1}^{(i)}\boldsymbol{S}_k^{(i)}\left[\tilde{\boldsymbol{b}}_k^{(i)}\right]^\top \boldsymbol{V}_k^{-1}$.

c) $\mathfrak{h}_k^{(i)} = \boldsymbol{A}\tilde{\mathfrak{h}}_{k-1}^{(i)} + \boldsymbol{H}_k^{(i)}\left[\boldsymbol{y}_k - \left[\tilde{\boldsymbol{b}}_k^{(i)}\right]^\top \tilde{\mathfrak{h}}_{k-1}^{(i)}\right]$.

d) $\boldsymbol{P}_k^{(i)} = \boldsymbol{A}\boldsymbol{P}_{k-1}^{(i)}\boldsymbol{S}_k^{(i)}\boldsymbol{A}_k^H + \boldsymbol{W}_k$.

5.–6. Identical with the steps as in CRPF-LMS.

V. SIMULATIONS

In this section, we assess the performance of the proposed mixture filtering approaches via computer simulations. In simulations, we vary the channel length and/or the order of AR model to investigate the performance of the approaches under various scenarios; besides, we also look into the effectiveness to the performance when we employ various numbers of particles. Bit error rate (BER) performance is evaluated by using the differential quadrature phase shift keying (DQPSK) modulation scheme. More specifications of the simulation parameters are summarized in Table 1. We consider the order of AR model p up to 3; then the parameters in the state equation (2) changes according to p. If $p = 1$, $\eta_1 = 0.9850$, and $D = 0.015$. If $p = 2$, $\eta_1 = -0.5017$, $\eta_2 = -0.9944$, and $D = 2.4961$. If $p = 3$, $\eta_1 = 0.4833$, $\eta_2 = -0.5002$, $\eta_3 = 0.9795$, and $D = -0.0374$, respectively. The coefficient μ of LMS algorithm in CRPF-LMS is selected carefully depending on channel models. The coefficients Λ and Δ of RLS algorithm in CRPF-RLS has to be carefully selected as well. We tried to run the simulations with various coefficients of LMS and RLS algorithms for each channel model, and selected the one which generated the best performance. On the other hand, once the parameters of HF and CRPF are properly selected for a channel model, it generally works well for the rest of channel models as well; therefore, we apply the same set of parameters for HF and CRPF regardless of the channel model. The coefficients of the LMS and RLS algorithms in their mixture filtering approaches are selected as follows:

- When $L = 1$ and $p = 1$: $\mu = 0.6$ for LMS; $\Lambda = 0.8$ and $\Delta = 0.8$ for RLS.
- When $L = 2$ and $p = 1$: $\mu = 0.4$ for LMS; $\Lambda = 0.8$ and $\Delta = 0.8$ for RLS.
- When $L = 3$ and $p = 1$: $\mu = 0.1$ for LMS; $\Lambda = 1$ and $\Delta = 0.8$ for RLS.
- When $L = 4$ and $p = 1$: $\mu = 0.1$ for LMS; $\Lambda = 1$ and $\Delta = 0.8$ for RLS.

It is noted that Δ of RLS is consistent regardless of the channel model. We have simulated for higher-order AR models. The simulation results showed that CRPF-LMS and CRPF-RLS failed to track time-varying CIR, and consequently failed to detect the transmitted symbols properly. This is because LMS and RLS algorithms do not require the parameters information of the state equation while the KF and HF need and use them. Even for CRPF-HF and PF-KF, their performance is clearly degraded in accordance with the complexity of the channel model and the results may not show satisfactory performance with increased order p.

The total number of sampled received signal is 1000. The number of employed particles is selected within the range $\{10, 20, 50, 100, 200\}$ one by one. The initial true CIR $\boldsymbol{h}_0$ is generated from complex Gaussian of "$\mathcal{CN}(0,1)$" for each signal ray depending on the value L. The initial, given estimate of CIR $\hat{\boldsymbol{h}}_0$ is generated randomly in the same manner as the true initial value is generated (the identical initial particles are generated as $\boldsymbol{h}_0^{(i)} = \hat{\boldsymbol{h}}_0, i = 1, \cdots, M$). The noise variance of w_k is selected according to the predetermined SNR of the measurement at every sampling instant. The parameters for CRPF are: $q = 1$; $\lambda = 0.95$; $\kappa = 0.1$; and $\beta = 2$. The parameters for HF are: $\gamma = 0.001$; $V = 1$; $\boldsymbol{P}_0 = 0.5\boldsymbol{I}$; $\boldsymbol{W}_k = \boldsymbol{W} = \boldsymbol{I}$; $\boldsymbol{\chi}_k = \boldsymbol{I}$. The performance is assessed over 20,000 simulated runs. Then the simulation results are following.

Although we can obtain considerably improved BER performance by employing forward error correction (FEC) and interleaving scheme in the algorithms, we focus on the performance

Table 1. Simulation scenario.

Channel model	Auto-regressive model
Modulation scheme	Differential quadrature phase shift keying
Speed of receiver terminal	100 km/h (about 62 miles/h)
Symbol rate	218 kBd
Carrier frequency	5.82 GHz
f_D (Doppler shift)	539 Hz

of considered algorithms in terms of estimation accuracy rather than error correcting scheme by channel coding.

A. Simulation Results

The BER performance results are compared in Figs. 1−12 when $p = 1$ and $L = 1, \cdots, 4$.

CRPF-HF shows similar performance to that of PF-KF under the specific condition that the employed number of particles is small with low SNR as shown in Figs. 2, 4, 6, and 8; nonetheless, CRPF-HF is outperformed by PF-KF if SNR is increased in that situation. Therefore, CRPF-HF is more robust against the unfavorable situation that the employed number of particles is small and SNR is low even with unknown noise information. However, this is not true anymore under the situation that the channel is modeled by a higher order AR representation, and PF-KF outperforms CRPF-HF even when the number of particles is small and SNR is low although we did not included the result of it due to limited space.

For channel models with $p = 1$, PF-KF shows high performance with a significantly increased number of particles and low SNR. The performance margin of CRPF-HF with an increased number of particles is not significant compared to that of PF-KF with an increased number of particles when SNR is low. The performance margin of CRPF-LMS and CRPF-RLS with an increased number of particles is similarly not significant when SNR is low compared to that of PF-KF; nonetheless, it shows different story when SNR is higher, and CRPF-LMS and CRPF-RLS outperform PF-KF with high SNR and large number of employed particles as shown in Fig. 1, Fig. 3, Fig. 5, and Fig. 7.

In the results of Figs. 1−8, we find that CRPF-LMS filtering shows the worst performance with low SNR regardless of the number of particles employed; the performance of CRPF-RLS is slightly better than that of CRPF-LMS, but still worse than the other two appraoches when SNR is low as shown in from Fig. 1 to Fig. 8. With a large number of employed particles and high SNR, CRPF-LMS and CRPF-RLS outperforms the other methods, and the performance of CRPF-LMS is slightly better than that of CRPF-RLS; nonetheless, if SNR keeps on increasing, the degree of performance improvement of these two methods are gradually mitigated and somewhat converging whereas the performance of PF-KF keeps on improving.

In Figs. 9−12, it shows BER performance of each approach for all range of L together. The results show that the performance of all methods is degraded with the increased L, which makes the inter symbols interference more significant; especially, CRPF-LMS and CRPF-RLS show prominently good performance with $L = 1$ as shown in Figs. 11−12. CRPF-HF and PF-KF also show the best performance when $L = 1$; however, the gap between the performance with $L = 1$ and that with $L = 2$ is not as much as those of CRPF-LMS and CRPF-RLS. Therefore, it might be a great advantage to employ CRPF-LMS and CRPF-RLS, especially CRPF-LMS, under the channel model with $L = 1$ and $p = 1$.

In the cases for higher order of AR channel models, PF-KF outperforms CRPF-HF, and the increased number of particles

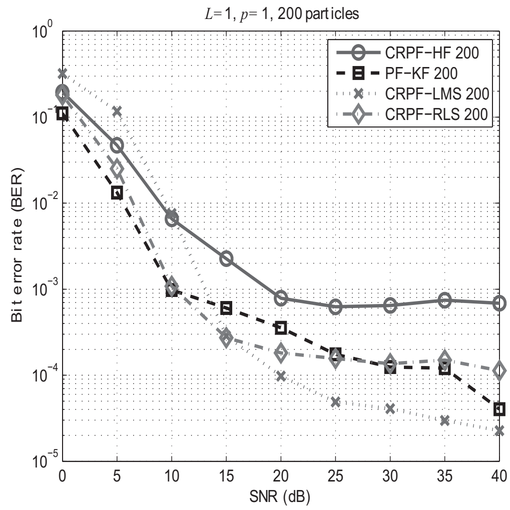

Fig. 1. BER comparison of approaches with 200 particles when $L = 1$ (2 rays) and $p = 1$.

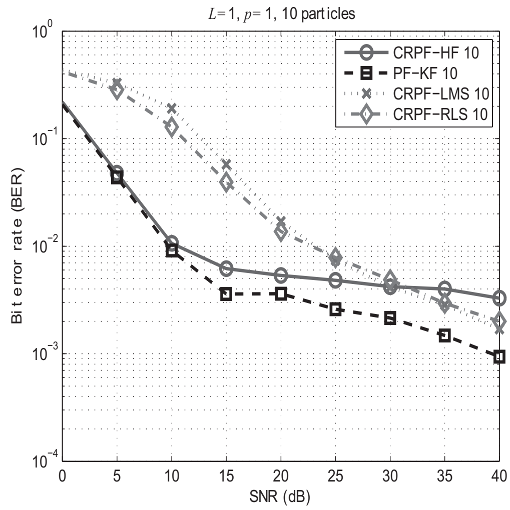

Fig. 2. BER comparison of approaches with 10 particles when $L = 1$ (2 rays) and $p = 1$.

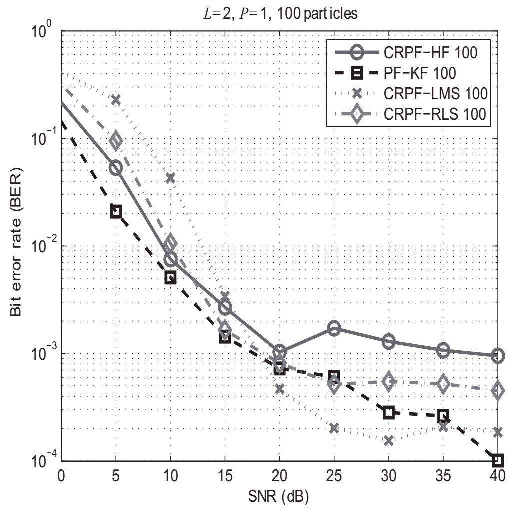

Fig. 3. BER comparison of approaches with 100 particles when $L = 2$ (3 rays) and $p = 1$.

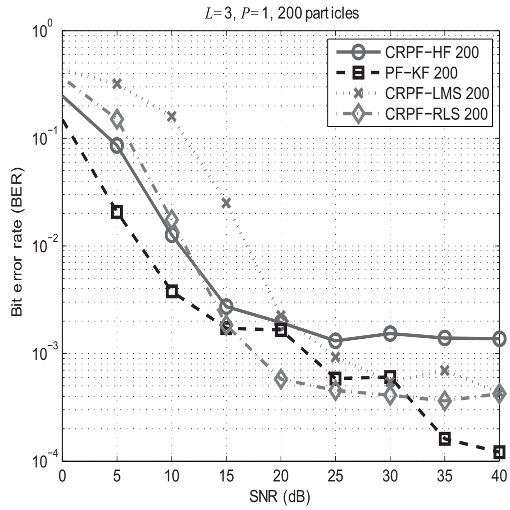

Fig. 5. BER comparison of approaches with 200 particles when $L = 3$ (4 rays) and $p = 1$.

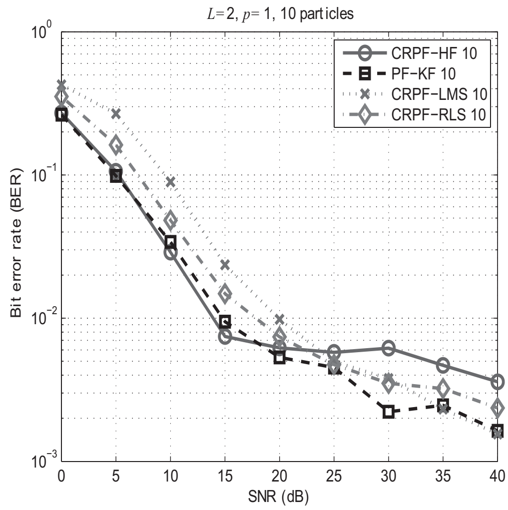

Fig. 4. BER comparison of approaches with 10 particles when $L = 2$ (3 rays) and $p = 1$.

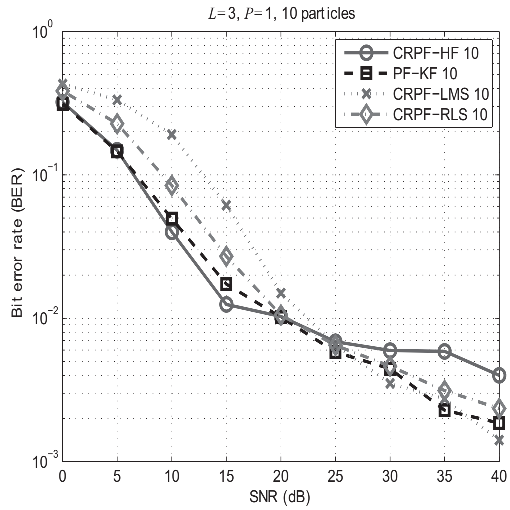

Fig. 6. BER comparison of approaches with 10 particles when $L = 3$ (4 rays) and $p = 1$.

does not make significant differences for CRPF-HF approach whereas the performance is improved for PF-KF with increased number of particles. The other two approaches, i.e., CRPF-LMS and CRPF-RLS do not show considerably good performance because LMS and RLS algorithms are not able to track the channel variations in the mixture filtering algorithms due to the lack of the information of the state equation parameters.

In summary, it is inspiring that CRPF-HF shows outperforming result with small number of employed particles and low SNR, and CRPF-LMS and CRPF-RLS show outperforming results, especially CRPF-LMS shows the best performance with large number of particles and SNR above 15 dB under the AR channel representation with the order $p = 1$ regardless of the number of rays. The proposed approaches in addition to the previously proposed Kalman mixture are summarized in Table 2. The computational complexity (CC) of mixture filtering approaches where a particle filtering is employed can be approximated by big O notation, i.e. "$\mathcal{O}(M)$" where "M" is the number of particles. Therefore all the proposed approaches and the previously proposed Kalman mixture approach has approximately $\mathcal{O}(M)$ because all of them employ particle filtering approach in the algorithms. The disadvantage of the particle filtering approaches is its relatively high computational complexity. Particularly, the mixture filtering approaches have approximately m times higher CC than that of conventional maximum likelihood equalization approaches.

We showed the result as a function of the maximum Doppler shift when $p = 1$ with two-ray channels as shown in Fig. 13. Furthermore, the BER result with the block coding as the function of the maximum Doppler shift was provided in Fig. 14 where the code word length is seven adn the message length is three, respectively.

Table 2. Blind equalization methods where CC stands for computational complexity.

	Noise statistics	Property	Common property
CR-RLS	Unknown	Suboptimal with $p = 1$	With channel estimation
CR-LMS	Unknown	Optimal with $p = 1$	Without training symbols
CR-HF	Unknown	Optimal with low SNR and small numbers of particles	CC, $\mathcal{O}(M)$
PF-KF	Known	Suboptimal with $p = 1$	

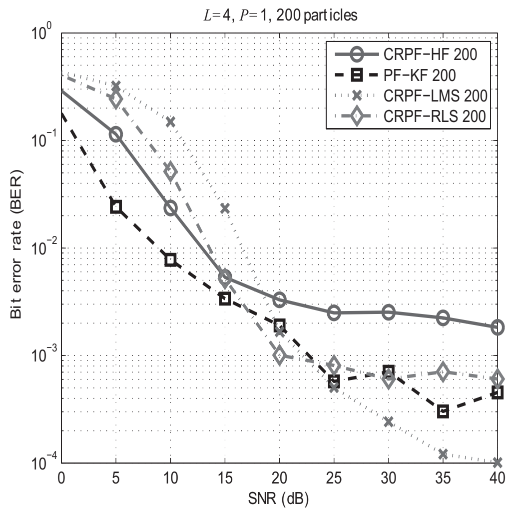

Fig. 7. BER comparison of approaches with 200 particles when $L = 4$ (5 rays) and $p = 1$.

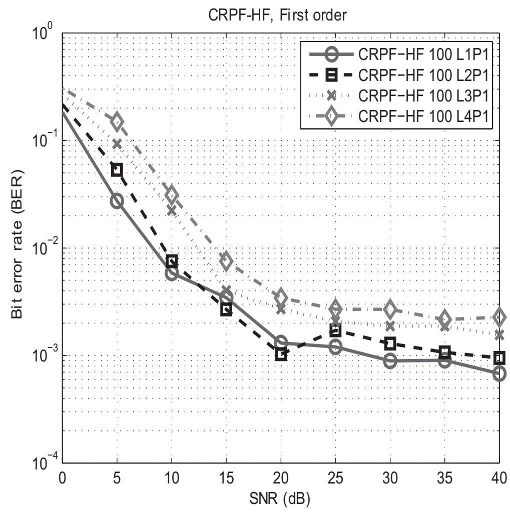

Fig. 9. BER of CRPF-HF filtering with 100 particles when $p = 1$ with various L.

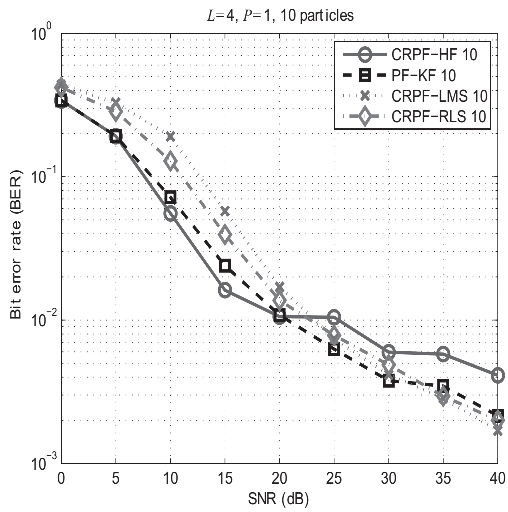

Fig. 8. BER comparison of approaches with 10 particles when $L = 4$ (5 rays) and $p = 1$.

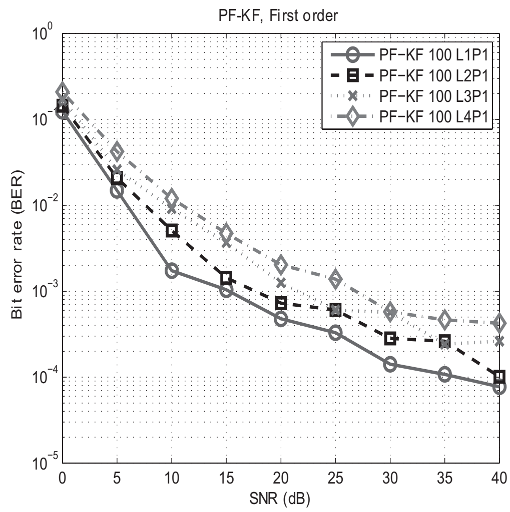

Fig. 10. BER of CRPF-KF filtering with 100 particles when $p = 1$ with various L.

VI. SUMMARY AND CONCLUSIONS

The lower the channel length and the order of AR model are, the better the performance of the proposed approaches is; lower computational complexity is required for lower L and p as well. CRPF-LMS and CRPF-RLS fail to track the time-varying CIR, and consequently also fail to detect the transmitted symbols for the AR channel model with 2 or higher although they outperform the other approaches for the AR (1) model, and CRPF-RLS showed the best performance therein. This is because CRPF-LMS and CRPF-RLS do not require the parameters information of the state equation that are mainly determined by AR process order and the maximum Doppler shift whereas CRPF-HF and PF-KF require the state parameter information. Another notable result is that CRPF-HF outperforms the other approaches when

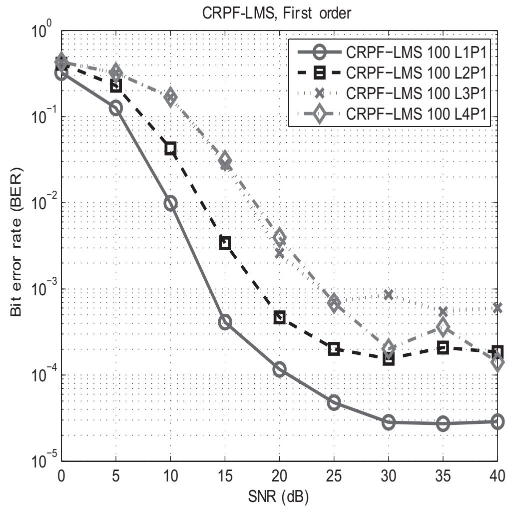

Fig. 11. BER of CRPF-LMS filtering with 100 particles when $p = 1$ with various L.

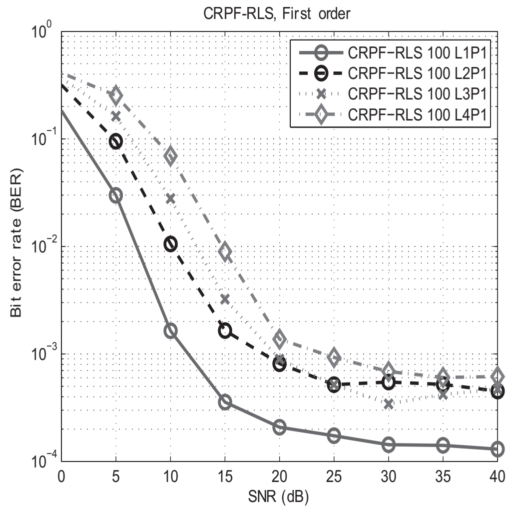

Fig. 12. BER of CRPF-RLS filtering with 100 particles when $p = 1$ with various L.

the employed number of particles is small and SNR is low; especially, it outperforms PF-KF due to the reason that CRPF does not approximate the posterior density by particles. More number of particles may be able to approximate the posterior density more closely for standard PF, but this is not the case for CRPF. Consequently, the effect of increasing the number of particles is weaker in CRPF than that of PF, and also a small number of employed particles does not necessarily mean that most of particles have low quality in CRPF. Although we can obtain significantly improved BER performance by employing forward error correction (FEC, i.e., channel coding) and interleaving in the algorithms, we focused on the performance of considered algorithms in terms of estimation accuracy rather than error correcting scheme. We leave this work as near future work by which we can obtain significantly improved equalizing performance under more practical scenarios (e.g., under low SNR).

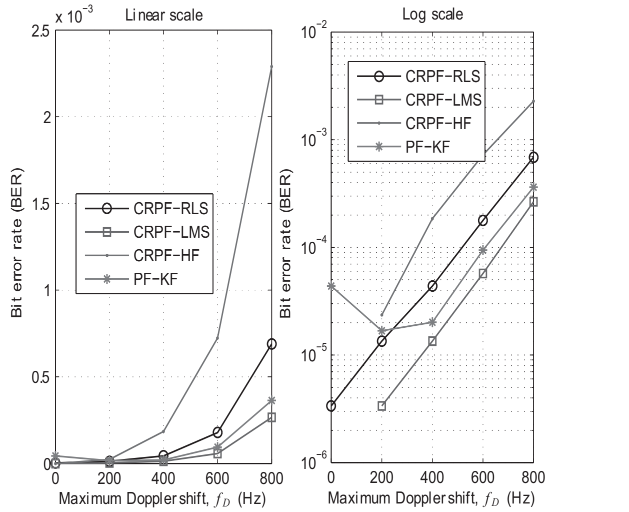

Fig. 13. BER of approaches when $p = 1$, two rays, with 300 particles as the function of maximum Doppler shift (f_D). The corresponding speeds are approximately 37 m/s, 74 m/s, 111 m/s, and 148 m/s, respectively, for 0 Hz, 200 Hz, 400 Hz, 600 Hz, and 800 Hz. The signal to noise ratio was maintained as 40 dB in this 3000 simulations.

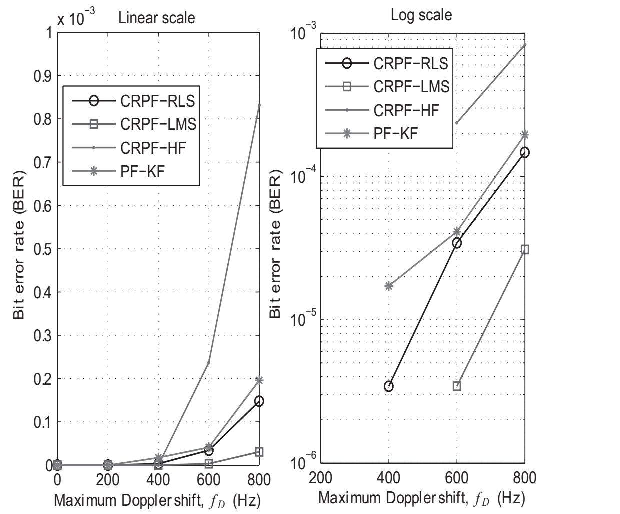

Fig. 14. BER of approaches with block coding when $p = 1$, two rays, with 300 particles as the function of maximum Doppler shift (f_D). The corresponding speeds are approximately 37 m/s, 74 m/s, 111 m/s, and 148 m/s, respectively, for 0 Hz, 200 Hz, 400 Hz, 600 Hz, and 800 Hz. Cyclic block codes were employed where the code word length is 7 and the message length is 3, respectively. The signal to noise ratio was maintained as 40 dB in this 3000 simulations.

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