Nanomagnetics

Skyrmion State Stability in Magnetic Nanodots With Perpendicular Anisotropy

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Abstract—Stability of skyrmion magnetization configuration in a circular magnetic nanodot with an uniaxial anisotropy perpendicular to the dot plane is calculated. It is shown that the skyrmion state can have magnetic energy lower than the vortex and perpendicular single domain state in a finite range of magnetic anisotropy fields at room temperature even in the case of absence of the Dzyaloshinskii–Moriya exchange interaction and external magnetic field.

Index Terms—Nanomagnetics, magnetic anisotropy, nanodot, skyrmion, vortex.

I. INTRODUCTION

It is well known that in small ferromagnetic particles with sizes smaller than the exchange length (~10 nm), the exchange energy dominates, which results in almost homogeneous magnetization configurations (single-domain states). Increasing the particle sizes the magnetostatic and anisotropy energies play more essential role and lead to stabilization of inhomogeneous magnetization configurations (domain structures) [Hubert 1998]. In the case of soft magnetic materials, the simplest example is a magnetic vortex [Metlov 2002, Scholz 2003], which was extensively investigated during the past 15 years. Other magnetization configurations such as skyrmion-like bubble domains can be stabilized in magnetic films [Thiele 1970] and magnetic dots [Moutafis 2006, 2007] if a perpendicular magnetic anisotropy and/or perpendicular magnetic field are present.

Magnetic skyrmions have recently attracted much attention of researchers due to their small sizes about of 1–10 nm assuming applications to ultrahigh density magnetic recording and easy manipulation using spin-polarized currents [Fert 2013, Sampaio 2013]. The skyrmions were observed in noncentrosymmetric weak ferromagnets with B20 structure such as MnSi, FeCoSi, etc., and insulating weak ferromagnets Cu_2OSeO_3 [Fert 2013]. It was established and widely accepted that the Dzyaloshinskii–Moriya (DM) exchange interaction plays a decisive role in the stabilization of such skyrmions [Rössler 2006, Rohart 2013, Sampaio 2013, Du 2013]. However, these materials have low Curie temperatures, well below room temperature, and need an external magnetic field to stabilize the skyrmions that is inconvenient for their applications in spintronic devices. Therefore, the question arises whether the skyrmion state can be stabilized in strong ferromagnets like Fe, Co, and Ni without the presence of the DM interaction and bias magnetic field at room temperature. Stabilization of the skyrmion state in magnetic dots, stripes, and other patterned nanostructures is of special interest.

It was shown by Thiele [1970] that the bubble skyrmion state without DM interaction in infinite films with perpendicular anisotropy can be stabilized by nonzero perpendicular magnetic field. Following the work by Skyrme [1962], it was shown [Abanov 1998, Kirakosyan 2006] that skyrmions in thin magnetic films can be stabilized by introducing a high-order isotropic exchange interaction proportional to $(\nabla^2\mathbf{M})^2$ that is beyond the traditional micromagnetic approach, where only the dominating terms $(\nabla M_\alpha)^2$ ($\mathbf{M}(\mathbf{r})$ is the sample magnetization, $\alpha = x, y, z$) are accounted for. Then, Ezawa [2010] found using a trial function that a single skyrmion state (bubble skyrmion with a large radius of about 1 μm) can be stabilized in a magnetic film with perpendicular anisotropy by a magnetic field. There is no quantitative difference between magnetization configurations of the large-radius bubble skyrmions and small-radius DM or exchange skyrmions. Both of them are stable in some part of parameter space and bear a topological charge. That is, the DM interaction is not mandatory to stabilize skyrmions, but it is required to select particular skyrmion chirality [Kiselev 2011].

Recently, Kiselev [2011] calculated the stability of small-radius skyrmion in an infinite film as a function of DM interaction strength and perpendicular magnetic field. Meanwhile, it was calculated that the skyrmions can be stabilized in ultrathin Co (0.4 nm) layers in contact with metals having strong spin-orbit coupling like Pt if the interfacial DM interaction is strong enough [Sampaio 2013, Rohart 2013]. If the DM interaction is absent, Johnson [2012] showed that skyrmion in finite isotropic ferromagnetic particles can be stabilized by a bias field. Sun [2013] simulated artificial stable skyrmions in patterned layered magnetic films composed of vortex state dots and a sublayer with perpendicular anisotropy. The idea by Sun *et al.* then was experimentally proved by Li [2014].

In this letter, we demonstrate that the skyrmions can be stabilized in a nanodot by perpendicular anisotropy in absence of the external magnetic field and DM interaction. Such skyrmions can be considered as specific bubble domains adopted for finite in-plane dot sizes. We distinguish the bubble-skyrmions stabilized in dots from common bubble domains in magnetic films having perpendicular anisotropy [Hubert 1998] because there are neither well-defined domains nor domain walls in dots; magnetic nonuniformity occupies all the dot volume. The common bubble domain radius is essentially larger than the domain wall width. In the case of small dot radius, there is no difference between the bubble-skyrmions and DM skyrmions.

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We consider a cylindrical dot with radius R and thickness L. We assume that L is about of the material exchange length, several tens of nanometers (10–50 nm). That allows neglecting the magnetization dependence on the thickness coordinate z and averaging relevant physical quantities over z. We assume existence of a uniaxial magnetic anisotropy with the easy axis directed along z and consider a competition of the magnetic energies of three distinct magnetization states: perpendicular single domain (SD), vortex (V), and skyrmion (Sk). Then, we determine the critical values of the anisotropy fields and dot sizes for transitions between the SD, V, and Sk states by comparing the energies of these stable (metastable) states and choosing a state with the minimal energy.

II. THEORETICAL MODEL

A. Magnetic Energy Contributions

Inhomogeneous magnetization configurations are described by the magnetization vector field $\mathbf{m}(\mathbf{r}) = \mathbf{M}(\mathbf{r})/M_s$. We introduce the magnetic energy density $w(\mathbf{m}) = A(\partial\mathbf{m}/\partial x_\alpha)^2 + w_m + w_a$, where $w_m = -M_s\mathbf{m}\cdot\mathbf{H}_m/2$ is the magnetostatic energy density, $w_a = -K(\mathbf{m}\cdot\mathbf{n})^2$ is the uniaxial anisotropy energy density, A is the exchange stiffness, M_s is the saturation magnetization, $\mathbf{H}_m[\mathbf{m}(\mathbf{r})]$ is the nonlocal magnetostatic field, and $x_\alpha = x, y, z$. The total dimensionless magnetic energy is a functional of the magnetization configuration $\mathbf{m}(\mathbf{r})$

$$E[\mathbf{m}(\mathbf{r})] = \frac{1}{M_s^2 V}\int_V d^3\mathbf{r}\, w(\mathbf{m}(\mathbf{r})) \tag{1}$$

where integration is conducted over the dot volume $V = \pi R^2 L$.

The equilibrium magnetization configurations are obtained from the integro-differential Euler–Lagrange equations

$$\frac{\delta w}{\delta\mathbf{m}} = \frac{\partial w}{\partial\mathbf{m}} - \frac{\partial}{\partial x_\alpha}\frac{\partial w}{\partial(\partial\mathbf{m}/\partial x_\alpha)} = 0. \tag{2}$$

We use the cylindrical coordinates $\mathbf{r}(\rho,\varphi,z)$ and parameterize the magnetization components via the spherical angles Θ, Φ of $\mathbf{m}$ as $m_\rho + im_\varphi = \sin\Theta\exp(i(\Phi-\varphi))$, $m_z = \cos\Theta$. To minimize the bulk and side surface magnetostatic energy of the curling-like V, Sk magnetization configurations with a chirality $C = \pm 1$, we use the equation $\Phi(\varphi) = \varphi + C\pi/2$ corresponding to $\nabla\cdot\mathbf{m} = 0$, and $m_\rho = 0$. The V and Sk energy does not depend on C in absence of the DM interaction. We also assume that the function $\Theta = \Theta(\rho)$ is axially symmetric. Using these assumptions, we can essentially simplify the magnetostatic term $H_m^\alpha(\mathbf{r}) = M_s\int d^3\mathbf{r}'\Gamma_{\alpha\beta}(\mathbf{r},\mathbf{r}')m_\beta(\mathbf{r}')$ in (2), where $\Gamma_{\alpha\beta}(\mathbf{r},\mathbf{r}') = -\partial^2/\partial x_\alpha\partial x'_\beta|\mathbf{r}-\mathbf{r}'|^{-1}$ is the magnetostatic kernel. Accounting the magnetostatic energy for infinite films by Kiselev [2011] following the bubble-domain theory [Tu 1971] assumes using some identities which are not applicable for a dot of finite radius and more rigorous approach is needed. There is only one nonzero component of $\Gamma_{\alpha\beta}(\mathbf{r},\mathbf{r}')$ averaged over z, z', $G_{zz}(\rho,\rho') = L^{-1}\int dz dz'\Gamma_{zz}(\mathbf{r},\mathbf{r}')$, corresponding to the magnetic energy of the dot face ($z = 0, L$) magnetic charges $\mathbf{m}\cdot\mathbf{n} = m_z$. The magnetostatic field in the form

$$h_m(\rho) = H_m^z(\rho)/M_s = \int d\rho'\rho' g_{zz}(\rho,\rho')m_z(\rho')$$

$$g_{zz}(\rho,\rho') = -(4\pi/L)\int_0^\infty dk\left(1-e^{-kL}\right)J_0(k\rho)J_0(k\rho')$$

leads to the magnetostatic energy ($r = \rho/R$, $\beta = L/R$)

$$E_m[\mathbf{m}] = \frac{4\pi}{\beta}\int_0^\infty dx\left[1-e^{-\beta x}\right]I^2(x),\ I(x) = \int_0^1 drr J_0(xr)m_z(r). \tag{3}$$

The anisotropy energy is $E_a = h_a\int drr\sin^2\Theta$, where $h_a = 2K/M_s^2$ is the reduced anisotropy field ($K > 0$ is the anisotropy constant) and $Q = h_a/4\pi$ is a quality factor. The exchange energy E_{ex} is accounted in the standard form. The total magnetic energy is $E = E_{ex} + E_a + E_m$:

$$E[\Theta(r)] = \int_0^1 drr\left\{\left(\frac{L_e}{R}\right)^2\left[\left(\frac{d\Theta}{dr}\right)^2 + \frac{\sin^2\Theta}{r^2}\right] + h_a\sin^2\Theta\right\} + E_m[\Theta(r)]$$

where $L_e = \sqrt{2A}/M_s$ is the exchange length.

B. Equilibrium Magnetization Direction

Using (2), we can write the equation for the equilibrium magnetization angle $\Theta(\rho)$ in the form

$$L_e^2\nabla^2\Theta(\rho) = \frac{1}{2}\left(h_a + L_e^2/\rho^2\right)\sin 2\Theta(\rho) + h_m(\rho)\sin\Theta(\rho). \tag{4}$$

The nonlinear integro-differential equation (4) is complicated and can be solved only numerically. Instead of its numerical solution, we use the method of a reasonable trial function to directly minimize the total energy $E[\Theta]$. Several trial functions for the angle $\Theta(r)$ were used so far: $\cos\Theta(r) = \sin(kr)/kr$ [Guslienko 2004], $\cos\Theta(r) = 1/\cosh(kr)$ [Moutafis 2006], $\cos\Theta(r) = 1 - 2\exp\left(-k^2r^2\right)$ [Finazzi 2013], $\Theta(r) = kr$ [Beg 2013]. The total energy was minimized with respect to parameter k, which has sense of an inverse skyrmion radius. We use the skyrmion ansatz

$$\cos\Theta(r) = \left(c^2 - r^2\right)/\left(c^2 + r^2\right) \tag{5}$$

for $r \le 1$ within the dot, where $c = R_c/R$ is the reduced skyrmion radius defined by the equation $m_z(c) = 0$. This ansatz is a solution of the nonlinear sigma model [Abanov 1998]. The energy $E(c)$ is calculated explicitly as a function of the Sk radius, and the dot magnetic (L_e, h_a) and geometrical parameters (L, R). The perpendicular SD state ($\Theta = 0$) energy corresponds to $1/c \to 0$, and the vortex (half-skyrmion) energy is calculated using the ansatz by Usov [1993]. We use the exchange length value $L_e = 18$ nm.

C. Skyrmion Topological Charge

The skyrmion topological charge S in thin films $S = (1/4\pi)\int d^2\rho\mathbf{m}\cdot(\partial\mathbf{m}/\partial x \times \partial\mathbf{m}/\partial y)$ is proportional to degree of

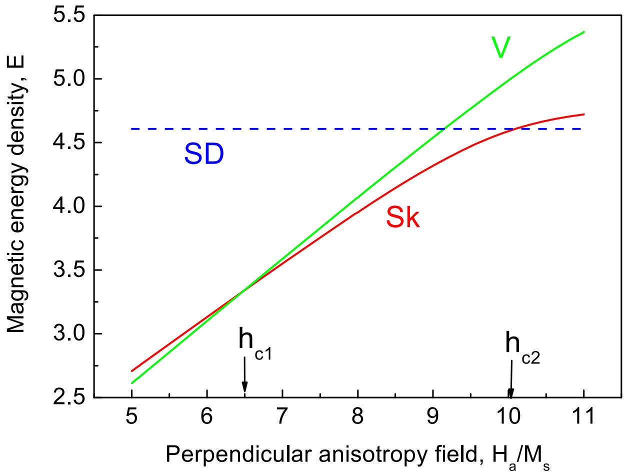


Fig. 1. Magnetic energies of the vortex (V), skyrmion (Sk) and single domain (SD) magnetization states versus anisotropy field. $R = 100$ nm, $\beta = L/R = 0.3$. Arrows show the critical fields of skyrmion stability.

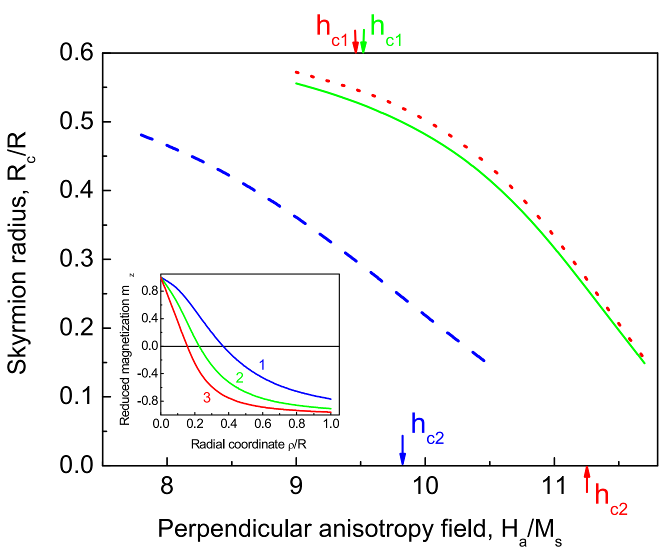
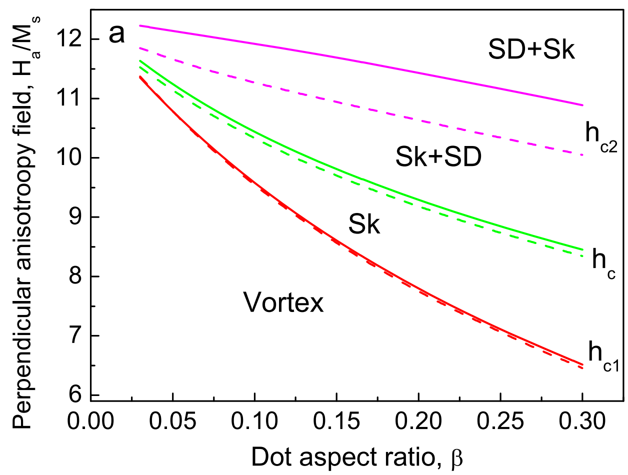


Fig. 2. Equilibrium skyrmion radius in units of the dot radius R. The dot parameters: $R = 100$ nm, $\beta = 0.1$ (red dot line), $R = 250$ nm, $\beta = 0.1$ (solid line), and $R = 100$ nm, $\beta = 0.3$ (blue dashed line). The critical fields are shown by arrows. Inset: profile of the magnetization component m_z for $h_a = 9.0$ (1), $h_a = 10.0$ (2), $h_a = 10.5$ (3), $R = 100$ nm, $\beta = 0.3$.

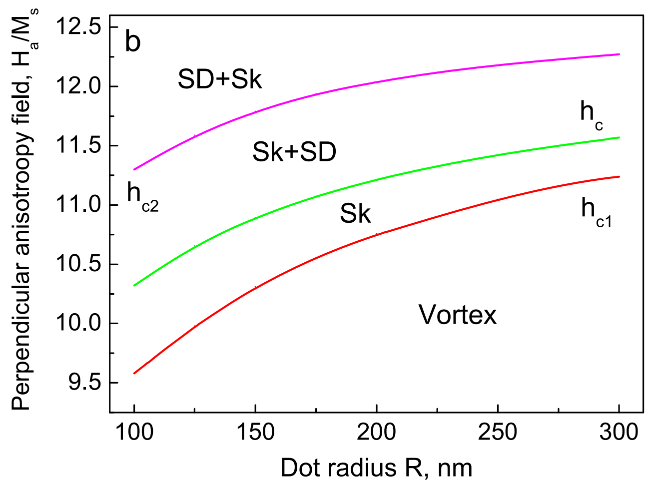
mapping of xOy plane to the surface of unit sphere $\mathbf{m}^2 = 1$ defined by the function $\mathbf{m}(x,y)$. In the case of magnetic skyrmion trapped to a circular dot we get using (5), the topological charge $S = (m_z(0) - m_z(R))/2 = 1/(1 + c^2)$. As shown below, $c^2 << 1$ for a stable skyrmion; therefore, the skyrmion charge is $S \approx 1$. The charge determines gyrotropic dynamics (gyrovector) of the magnetic skyrmions.

III. CALCULATION OF SKYRMION STABILITY

Any magnetization configuration can be characterized as stable state (the ground state or global energy minimum), metastable state (local energy minimum), or unstable state in some parameter space. There are several stable and metastable magnetization states in a circular magnetic nanodot as a function of the dot parameters L_e, h_a, R and $\beta = L/R$.



(a)



(b)

Fig. 3. Skyrmion stability area $h_{c1} < h_a < h_{c2}$ versus dot aspect ratio $\beta = L/R$ for different dot radii, $R = 250$ nm (solid lines), $R = 100$ nm (dashed lines) (a), and versus dot radius for the dot thickness $L = 10$ nm (b). The anisotropy field h_c marks the border of metastability of the perpendicular SD state. The stable state is listed first.

A. Comparison of Skyrmion Energy With Energies of Other Magnetization Configurations

The Sk state energy is lower than the V energy for the anisotropy field $h_a > h_{c1}$ and lower than the perpendicular SD state energy if $h_a < h_{c2}$ (see Fig. 1). That is, the skyrmion state is stable in the anisotropy field region $h_{c1} < h_a < h_{c2}$.

B. Equilibrium Skyrmion Radius

The equilibrium Sk radius $c(h_a, \beta, R)$ is determined from the equation $\partial E/\partial c = 0$ and shown in Fig. 2 for typical dot parameters.

The reduced Sk equilibrium radius depends mainly on the dot aspect ratio β and only weakly depends on R due to the small contribution of the exchange energy at $R >> L_e$. The Sk radius $R_c(h_a)$ decreases with h_a increasing from h_{c1} to h_{c2}. It has also physical sense beyond this range, up to borders of the skyrmion metastability. A calculation of the skyrmion metastability region (where Sk is a local minimum of the magnetic energy) is beyond the scope of this paper.

C. Stability of Skyrmion as a Function of the Dot Sizes

The critical lines of Sk stability $h_{c1,2}(\beta, R)$ are determined as solutions of the equations $E_V = E_{Sk}$ and $E_{SD} = E_{Sk}$ (see Fig. 3).

The critical field $h_{c1}(\beta, R)$ depends mainly on the dot aspect ratio $\beta = L/R$ and can be essentially lower than 4π ($h_{c1} \approx 2\pi$, $Q \approx 0.5$) for thick dots [see Fig. 3(a)], whereas the critical field $h_{c2}(R, \beta)$ depends also on the dot radius, increases slowly with R increasing, and is close to 4π. For typical $M_s = 10^3$ G, the interval of the perpendicular anisotropy fields (h_{c1}, h_{c2}) in absolute units varies as function of L and R from 1 to 5 kOe.

Sk state can be also stabilized for $h_a > 4\pi(Q > 1)$, for instance, in epitaxial FePt dots with $Q = 4.7$ [Moutafis 2007]. However, we want to find stabilization of Sk state in circular dots made of relatively low anisotropy materials, like hcp Co or tetragonally distorted Ni, where $Q \approx 0.4 - 0.5$ is expected [Moutafis 2006]. Moutafis [2006] numerically calculated the transition from V to Sk state for a fixed value of $h_a^0 = 5.03$ ($Q = 0.4$) as a function of β in the range 0.33–0.46 for $R = 75$ nm. They found the V ground state for $\beta < 0.33$ and some intermediate V–Sk state at $0.33 < \beta < 0.46$. This is in good agreement with our calculations of V–Sk equilibrium line $h_{c1}(\beta, R)$: $h_a^0 < h_{c1}(0.33, 75) = 6.13$ and $h_a^0 > h_{c1}(0.46, 75) = 4.98$.

IV. CONCLUSION

The skyrmion is the ground state (stable state) of a circular ferromagnetic nanodot without the DM exchange interaction and bias magnetic field for moderate perpendicular anisotropy fields h_a ($Q < 1$) within the range $h_{c1} < h_a < h_{c2} < 4\pi$. The critical anisotropy fields of the skyrmion stability $h_{c1,2}(\beta, R)$ are functions of the dot thickness L and radius R. The obtained skyrmion stability phase diagrams can serve as a benchmark for calculations of magnetic skyrmion dynamics in restricted geometry as well as a guide to experimentalists in preparation of samples having the magnetic skyrmion ground state.

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REFERENCES

Abanov Ar, Pokrovsky V L (1998), "Skyrmion in real magnetic film," *Phys. Rev. B*, vol. 58, pp. R8889–R8892, doi: 10.1103/PhysRevB.58.R8889.

Beg M, Chernyshenko D, Bisotti M-A, Wang W, Albert M, Stamps R L, Fangohr H, "Finite size effects, stability, hysteretic behavior and reversal mechanism of skyrmionic textures," (2013). [Online]. Available: http://arxiv.org/abs/1312.7665].

Du H, Ning W, Tian M, Zhang Y (2013), "Magnetic vortex with skyrmionic core in a thin nanodisk of chiral magnets," *Europhys. Lett.*, vol. 101, 37001, doi: 10.1209/0295-5075/101/37001.

Ezawa M (2010), "Giant skyrmions stabilized by dipole-dipole interaction in magnetic films," *Phys. Rev. Lett.*, vol. 105, 197202, doi: 10.1103/PhysRevLett.105.197202.

Fert A, Cros V, Sampaio J (2013), "Skyrmions on the track," *Nat. Nanotechnol.*, vol. 8, pp. 152–156, doi: 10.1038/nnano.2013.29.

Finazzi M, Savoini M, Khorsand A R, Tsukamoto A, Itoh A, Duò L, Kirilyuk A, Rasing Th., Ezawa M (2013), "Laser-induced magnetic nanostructures with tunable topological properties," *Phys. Rev. Lett.*, vol. 110, 177205, doi: 10.1103/PhysRevLett.110.177205.

Guslienko K Yu, Novosad V (2004), "Vortex state stability in soft magnetic cylindrical nanodots," *J. Appl. Phys.*, vol. 96, pp. 4451–4455, doi: 10.1063/1.1793327.

Hubert A, Schäfer R (1998), *Magnetic Domains*. Berlin, Germany: Springer, pp. 287–296.

Johnson P, Gangopadhyay A K, Kalyanaraman R, Nussinov Z (2012), "Demagnetization-borne microscale skyrmions," *Phys. Rev. B*, vol. 86, 064427, doi: 10.1103/PhysRevB.86.064427.

Kirakosyan A S, Pokrovsky V L (2006), "From bubble to skyrmion: Dynamic transformation mediated by a strong magnetic tip," *J. Magn. Magn. Mater.*, vol. 305, pp. 413–422, doi: 10.1016/j.jmmm.2006.01.113.

Kiselev N S, Bogdanov A N, Schäfer R, Rössler U K (2011), "Chiral skyrmions in thin magnetic films: New objects for magnetic storage technologies?" *J. Phys. D: Appl. Phys.*, vol. 44, 392001, doi: 10.1088/0022-3727/44/39/392001.

Li J, Tan A, Moon K W, Doran A, Marcus M A, Young A T, Arenholz E, Ma S, Yang R F, Hwang C, Qiu Z Q (2014), "Tailoring the topology of an artificial magnetic skyrmion," *Nat. Commun.*, vol. 5, 4704, doi: 10.1038/ncomms5704.

Metlov K L, Guslienko K Yu (2002), "Stability of magnetic vortex in soft magnetic nano-sized circular cylinder," *J. Magn. Magn. Mater.*, vols. 242–245, pp. 1015–1017, doi: 10.1016/S0304-8853(01)01360-9.

Moutafis C, Komineas S, Vaz C A F, Bland J A C, Shima T, Seki, Takanashi K (2007), "Magnetic bubbles in FePt nanodots with perpendicular anisotropy," *Phys. Rev. B*, vol. 76, 104426, doi: 10.1103/PhysRevB.76.104426.

Moutafis C, Komineas S, Vaz C A F, Bland J A C (2006), "Vortices in ferromagnetic elements with perpendicular anisotropy," *Phys. Rev. B*, vol. 74, 214406, doi: 10.1103/PhysRevB.74.214406.

Rössler U K, Bogdanov A N, Pfleiderer C (2006), "Spontaneous skyrmion ground states in magnetic metals," *Nature*, vol. 442, pp. 797–801, doi: 10.1038/nature05056.

Rohart S, Thiaville A (2013), "Skyrmion confinement in ultrathin film nanostructures in the presence of the Dzyaloshinskii-Moriya interaction," *Phys. Rev. B*, vol. 88, 184422, doi: 10.1103/PhysRevB.88.184422.

Sampaio J, Cros V, Rohart S, Thiaville A, Fert A (2013), "Nucleation, stability and current-induced motion of isolated magnetic skyrmions in nanostructures," *Nat. Nanotechnol.*, vol. 8, pp. 839–844, doi: 10.1038/nnano.2013.210.

Scholz W, Guslienko K Yu, Novosad V, Suess D, Schrefl T, Chantrell R W, Fidler J (2003), "Transition from single-domain to vortex state in soft magnetic cylindrical nanodots," *J. Magn. Magn. Mater.*, vol. 266, pp. 155–163, doi: 10.1016/S0304-8853(03)00466-9.

Skyrme T H R (1962), "A unified field theory of mesons and baryons," *Nucl. Phys.*, vol. 31, pp. 556–569, doi: 10.1016/0029-5582(62)90775-7.

Sun L, Cao R X, Miao B F, Feng Z, You B, Wu D, Zhang W, Hu A, Ding H F (2013), "Creating an artificial two-dimensional skyrmion crystal by nanopatterning," *Phys. Rev. Lett.*, vol. 110, 167201, doi: 10.1103/PhysRevLett.110.167201.

Thiele A A (1970), "Theory of the static stability of cylindrical domains in uniaxial platelets," *J. Appl. Phys.*, vol. 41, pp. 1139–1145, doi: 10.1063/1.1658846.

Tu Y-O (1971), "Determination of magnetization of micromagnetic wall in bubble domains by direct minimization," *J. Appl. Phys.*, vol. 42, pp. 5704–5709, doi: 10.1063/1.1660002.

Usov N A, Peschany S E (1993), "Magnetization curling in a fine cylindrical particle," *J. Magn. Magn. Mater.*, vol. 118, pp. L290–L294, doi: 10.1016/0304-8853(93)90428-5.