Equivalence of Virtual Synchronous Machines and Frequency-Droops for Converter-Based MicroGrids

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***Abstract*—Over the last decade, frequency-droop-based control schemes have become the preferred solution in microgrids dominated by power electronic converters. More recently, the concept of virtual synchronous machines (VSMs) has emerged as an effective method for adding virtual inertia to the power system through the control of power electronic converters. These two approaches have been developed in two separate contexts, but present strong similarities. In fact, they are equivalent under certain conditions, as demonstrated in this letter. Analysis of this equivalence provides additional physics-based insight into the tuning and operation of both types of controllers.**

***Index Terms*—Droop control, power electronic converters, virtual synchronous machine.**

I. INTRODUCTION

A "**Vir**tual **S**ynchronous **Ma**chine" (VSM), labeled as "VISMA," was first presented in English by Beck and Hesse in 2007 [1]. Later, the VSM concept for emulating the behavior of synchronous machines (SMs) by controlling voltage source converters (VSCs) has been pursued by several other authors [2]–[4]. In the future perspective of decentralized generation with increasing penetration of converter-interfaced energy sources, VSMs are being recognized as an effective method for addressing potential stability issues by adding virtual inertia to the power system.

In the adjacent research area of converter-dominated microgrids, droop-based schemes have become the preferred solution for control of VSCs [5]. These control schemes can ensure stand-alone operation and load sharing among parallel connected VSC units both in steady state and during transients, similarly to what is achieved with traditional SMs.

The VSM and the droop-based schemes have emerged almost independently and in two separate contexts, but present strong similarities. In fact, the two approaches are equivalent under certain conditions, as proven in this letter.

II. VIRTUAL SYNCHRONOUS MACHINES

Several VSM implementations have been presented in literature, with significant differences both in the applied control structure and in the SM model [1]–[4]. However, all VSM controllers include more or less explicitly a numerical model of the SM to be emulated. This model provides the references to the control algorithms for operating the actual VSC unit. Indeed, the various models of the SM reproduce the behavior of the real machine with different levels of accuracy, but a common feature is the emulation of the mechanical inertia effect. Arguably, the simplest model for SM emulation in a VSM is the traditional swing equation, which can provide the voltage phase angle to be used as reference in the VSC control system. Hence, the inertial dynamics of a VSM can be approximately represented by the SM per unit power balance in the Laplace domain according to (1)[6].

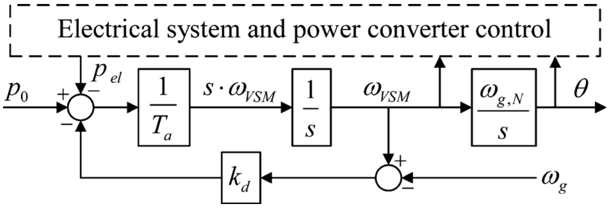

Fig. 1. Inertia emulation by virtual synchronous machines.

$$T_a \cdot s \cdot \omega_{VSM} \approx p_0 - p_{el} - k_d \left(\omega_{VSM} - \omega_g\right). \qquad (1)$$

In (1), $T_{\mathrm{a}}(= 2H)$ is the mechanical time constant representing the rotor inertia, p_0 the active power reference, p_{el} the electric power output from the VSM, and k_{d} is the damping coefficient. The rotating speed of the VSM is given by ω_{VSM}, while ω_{g} is the actual angular grid frequency when the VSM is connected to a strong grid. If the VSM is operated in stand-alone mode or in a MicroGrid, ω_{g} will be the angular grid frequency reference, possibly provided by a secondary controller. The angular position θ of the VSM rotor, corresponding to the phase angle of the VSC voltage reference, is given by the integral of the angular frequency ω_{VSM}. The VSM voltage amplitude is assumed to be given by a separate reactive power control loop, which can be considered as decoupled from the VSM-based emulation of the mechanical inertia. A block diagram illustrating the swing equation used for implementing the VSM, together with its interface to the rest of the converter control system and to the electric power system, is shown in Fig. 1.

III. DROOP CONTROL FOR MICROGRIDS

The control system design in converter dominated microgrids is commonly based on droop regulators for the active and the reactive power according to (2)[5], [7].

$$\omega^* = \omega_g - m_p \left(p_m - p_0\right), v^* = v_g - m_q \left(q_m - q_0\right). \qquad (2)$$

In these equations, the $*$ is indicating reference values for angular frequency ω and the voltage v, while m_{p} and m_{q} are

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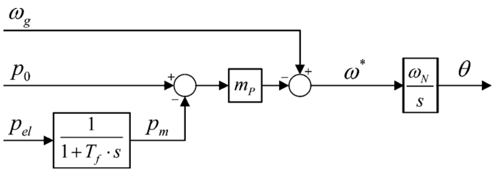

Fig. 2. Frequency-droop controller for microgrids.

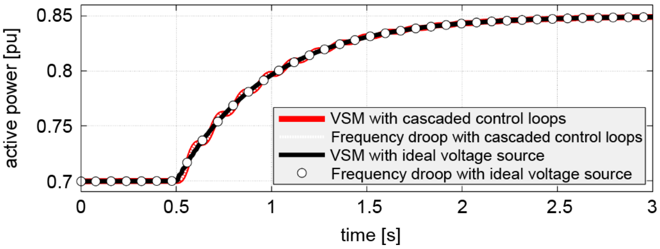

Fig. 3. Comparison of results from simulation of various concepts for VSM and droop controllers ($\mathrm{Ta} = 1.8s$ and $\mathrm{kd} = 5.7103$).

the droop gains for the active and reactive power control respectively. A block diagram illustrating the implementation of the frequency-droop equation is shown in Fig. 2. As seen from the figure, the active power, p_{el}, measured at the grid interface of the power electronic converter is low pass filtered before it is used as the measurement feedback signals p_{m} in (2)[5]. The same filtering is also used for the reactive power feedback, although this loop is not further discussed here. These filters will be proved necessary to stabilize the control loops even though their introduction is normally justified only as a solution to reject disturbances and oscillations present in every power measurement. Similarly as for the VSM model, the instantaneous voltage phase angle reference resulting from the droop controllers is given by the integral of the frequency reference as shown in Fig. 2.

IV. Equivalence Between Virtual Synchronous Machines and Microgrid Frequency-droop Control

Assuming a constant set-point ω_{g} for the grid angular frequency and a constant reference for the active power, p_0, it can be proved that the droop regulator presented in Section III is equivalent to the VSM model based on the swing equation described in Section II. This can be demonstrated by combining (2) with the low-pass filter indicated in Fig. 2, and isolating the expression for p_{el}, resulting in:

$$p_{el} = (1 + T_f \cdot s)\left(\frac{1}{m_p}(\omega_g - \omega^*) + p_0\right). \qquad (3)$$

Expanding the products of (3) results in:

$$p_{el} = \frac{T_f\left(\cancel{s\cdot\omega_g} - s\cdot\omega^*\right)}{m_p} + \frac{\left(\omega_g - \omega^*\right)}{m_p} + \cancel{T_f\cdot s\cdot p_0} + p_0 \qquad (4)$$

As indicated, the expression in (4) can be simplified by eliminating the derivatives of constant terms, leading to:

$$\underbrace{T_f\frac{1}{m_p}s\cdot\omega^*}_{\text{Inertia term}} = p_0 - p_{el} - \underbrace{\frac{1}{m_p}(\omega^* - \omega_g)}_{\text{Damping term}}. \qquad (5)$$

Equation (5) has exactly the same form as (1). The formal equivalence between the VSM model from (1) and the droop regulator from (2) can then be explicitly expressed by:

$$T_a = T_f \cdot \frac{1}{m_p}, \qquad k_d = \frac{1}{m_p}. \qquad (6)$$

The relations in (6) provide a further insight in the functional meaning of the terms in (1) and (2). Indeed, the damping gain k_{d} in the VSM is inversely linked to the droop gain m_{p}. Moreover, the time constant T_{f} of the first order low pass filter on the active power measurement serve an analogous function of the virtual inertia. Thus, a power-frequency droop without low-pass filtering in the power measurement will correspond to a VSM with zero inertia, which would be inherently unstable. By using (6), a droop regulator can, however, be easily tuned in order to emulate the inertia and damping effects of a specific synchronous machine. Moreover, simplified stability analysis based on the swing equation of traditional SMs or VSMs can be directly applied to conventional droop controllers.

The equivalence of the control schemes in Fig. 1 and in Fig. 2 when the conditions specified in (6) are fulfilled has been verified by numerical simulations. The response to a small step in power reference has been simulated for the two schemes directly controlling an ideal voltage source. Results are shown in Fig. 3 and appear as perfectly overlapping. The same transient has also been repeated when replacing the ideal voltage source with a VSC average model controlled by conventional cascaded voltage and current loops in the synchronous reference frame, similar to the structure analyzed in [7]. The responses of the two schemes are again identical as shown in Fig. 3. As can be seen from the figure, the more complex control structure does not change the overall behavior of the VSM, but only introduces small additional oscillations superimposed to the inertial response.

V. Conclusions

This letter has demonstrated the equivalence between VSM-based control and frequency-droop controllers. The demonstrated equivalence links into a single theoretical frame two well established concepts that have been developed so far in separate contexts. This provides a new perspective for the VSM and a deeper physical insight into the interpretation of the gain and filter parameters for droop controllers in converter-based microgrids.

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