Multistage Stochastic Programming for VPP Trading in Continuous Intraday Electricity Markets

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***Abstract*—The stochastic nature of renewable energy sources has increased the need for intraday trading in electricity markets. Intraday markets provide the possibility to the market participants to modify their market positions based on their updated forecasts. In this paper, we propose a multistage stochastic programming approach to model the trading of a Virtual Power Plant (VPP), comprising thermal, wind and hydro power plants, in the Continuous Intraday (CID) electricity market. The order clearing in the CID market is enabled by the two presented models, namely the Immediate Order Clearing (IOC) and the Partial Order Clearing (POC). We tackle the proposed problem with a modified version of Stochastic Dual Dynamic Programming (SDDP) algorithm. The functionality of our model is demonstrated by performing illustrative and large scale case studies and comparing the performance with a benchmark model.**

***Index Terms*—Continuous intraday electricity market, stochastic dual dynamic program, trading strategy, virtual power plant.**

Nomenclature

Indices and Sets

$\mathcal{D}$	Set containing the delivery products.
$\mathcal{P}^a$	Set containing the ask price levels p^a.
$\mathcal{P}^b$	Set containing the bid price levels p^b.
$\mathcal{T}_d$	Set of the trading stages t for delivery product d.

Parameters

$x_{t_{opn}d}$	Day-ahead position of the VPP for delivery product d.
H^{in}	Initial hydro reservoir volume.
λ^F	Future value of hydro reservoir content.
$\overline{Q}$	Maximum production for hydropower plant.
$\underline{Q}$	Minimum production for hydropower plant.
x_d^{DA}	Market commitment from the Day-Ahead market.
$\bar{S}_{t,d}^{ask}$	Maximum ask order volume that can be submitted by the VPP at trading stage t, for the delivery product d.
$\bar{S}_{t,d}^{bid}$	Maximum bid order volume that can be submitted by the VPP at trading stage t for the delivery product d.
T_d	Number of stages of the delivery product d.
ρ^+, ρ^-	Penalty for positive/negative deviations.
$P_{p^a,t,d}^{ask}$	Selling price at level p^a.
$P_{p^b,t,d}^{bid}$	Buying price at at level p^b.
$\overline{R}$	Ramp-up limit of thermal power plant.
$\underline{R}$	Ramp-down limit of thermal power plant.

Stochastic parameters

$I_{t,d}$	Inflow at stage t for delivery product d.
$A_{p^b,t,d}^{bid}$	Volume of ask orders available in SOB below price level p^b, at trading stage t, for delivery product d.
$B_{p^a,t,d}^{ask}$	Volume of bid orders available in SOB above price level p^a, at trading stage t, for delivery product d.
$W_{t,d}$	Wind generation at stage t for delivery product d.

Variables

$k_{p^a,t,d}^{ask}$	Ask volume at price level p^a at trading stage t, for delivery product d in POC model.
$f_{p^a,t,d}^{ask}$	Bid volume in the SOB at price level p^a at trading stage t, for delivery product d in POC model.
$\psi_{t,d}^{ask}$	Binary variable to account for the ask order posted by VPP at trading stage t for delivery product d.
$\psi_{t,d}^{bid}$	Binary variable to account for bid order posted by VPP at trading stage t for delivery product d.
$v_{p^a,t,d}^{ask}$	Cleared ask order volume at price level p^a, at trading stage t, for delivery product d.
$v_{p^b,t,d}^{bid}$	Cleared bid order volume at price level p^b, at trading stage t, for delivery product d.
$\delta_{t,d}^+$	Positive imbalance volume for delivery product d at time t.
$\delta_{t,d}^-$	Negative imbalance volume for delivery product d at stage t.
$\alpha_{p^a,t,d}^{ask}$	Order clearing variable at price level p^a at trading stage t for delivery product d in the IOC model.
$\beta_{p^b,t,d}^{ask}$	Order clearing variable at price level p^b at trading stage t for delivery product d in the POC model.
$q_{t,d}$	Production quantity for the hydro power plant for delivery product d.
$x_{t,d}$	Position of VPP at stage t for delivery product d.
$s_{p^a,t,d}^{ask}$	Posted ask order volume placed at price level p^a, at trading stage t, for delivery product d.
$s_{p^b,t,d}^{bid}$	Posted bid order volume placed at price level p^b at trading stage t, for delivery product d.
u_d	Positive increase of production of the thermal power plant for delivery product d.
d_d	Positive decrease of production of the thermal power plant for delivery product d.

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$h_{t,d}$ Reservoir volume at stage t, for delivery product d.
$o_{t,d}$ Spillage from reservoir at stage t, for delivery product d.
$g_{t,d}$ Thermal generation at t, for delivery product d.

Acronyms

VPP Virtual Power Plant
SIDC Single intraday coupling
CID Continuous intraday
MTU Market time unit
SOB Shared order book
DP Delivery product
GO Gate opening
GC Gate closure
MSSP Multi-Stage Stochastic Programming
MSSiP Multi-Stage Stochastic integer Programming
SDDP Stochastic Dual Dynamic Programming
SDDiP Stochastic Dual Dynamic integer Programming
ADDP Approximate Dual Dynamic Programming
IOC Immediate order clearing
POC Partial order clearing
DER Distributed energy resources
VRES Variable Renewable Energy Sources

I. Introduction

Due to the inherent stochasticity of Variable Renewable Energy Sources (VRES) the traded volumes in the short-term electricity markets are on the rise [1]. In several electricity markets around the world, the short-term trading time frames can be divided into Day-Ahead (DA), Intraday (ID) – within the day of delivery of electricity, and real-time (RT). ID markets provide the capability to the market participants to modify their day-ahead position (committed generation/consumption) through a market-based procedure. The deviations in the physical delivery from the final position at the gate closure of the ID market are subjected to imbalance penalties.

The intraday electricity markets in Europe take the form of either a continuous market clearing mechanism, or a discrete auctions scheme [1]. Continuous intraday (CID) trading is similar to the stock market and is based on the pay-as-bid principle. The traders can submit orders of the volumes that they are willing to buy (bid orders) or sell (ask orders) at a specific price, at any time during the trading horizon. For example, a bid order can be transacted whenever there is an ask order available with a price less than or equal to the price of the submitted bid order [2]. Discrete auction trading follows a merit-order approach with the auctions taking place only at prespecified times. As per December 2021, 23 European countries are coupled to trade in the intraday market through a single trading platform called *Single Intraday Coupling (SIDC)* [3]. The trade in this SIDC platform is organized using a CID mechanism, making this mechanism relevant to the market participants across Europe [3]. The volume traded through EPEX SPOT has increased from 47 TWh in 2014 to 111 TWh in 2020 [4].

The Market Time Unit (MTU) is the time granularity into which the market is operated. It is being planned to reduce the MTU from 60 min to 15 min [5] in the European electricity markets. As a result, the ID traders who manage a Virtual Power Plant (VPP) comprising a portfolio of assets for example, hydro, wind, thermal power plants, etc., are required to take faster decisions, while considering the technical limitations and uncertainties in the production of their assets. As a result, the complexity of the trading problem is becoming increasingly challenging for the human traders to handle, thus creating a paradigm shift towards a more automated trading environment [6].

In this paper, we model a sequential optimization problem to study the VPP participation in the CID market in a variable and uncertain environment. Throughout the paper, we refer to the term *VPP* as a trading entity with a portfolio of wind power, hydropower and thermal generation. The proposed framework would also be applicable to generation utilities with the aforementioned assets. The decision problem is modeled as a Multi-Stage Stochastic integer Programming (MSSiP) problem where the decision at any given stage depends on the present stage and the expectation of the future stages. We solve the MSSiP problem using a modified version of Stochastic Dual Dynamic Programming (SDDP) algorithm. SDDP algorithm is a multi-stage Benders decomposition algorithm, which utilizes sampling approaches to counter the *curse of dimensionality*, considering the stage coupling and uncertainties. It was proposed in the seminal work of [7].

A. Literature Review

There are several papers that study VPP bidding in DA and RT markets. In [8], a stochastic bilevel model to decide the optimal VPP participation in the DA market with the aim to maximize DA profit and minimize the anticipated Real-Time (RT) imbalance costs is presented. In [9], a day-ahead self-scheduling problem of a VPP trading in both energy and reserve electricity markets is proposed. The research work in [10] studies the effect of risk-averse behavior of the VPP on its profitability in the DA and spinning reserve market. A control strategy to maximize the RES usage for the inelastic demand managed by a VPP and minimize the DA and RT cost is proposed in [11]. Recent literature on intraday markets includes [12], where the strategy for the wind power plant to participate in the ID market is proposed based on the forecasts of up- and down-balancing prices. In [13], a bilevel optimization model is presented for a strategic producer to participate in the DA and ID markets. However, the aforementioned works neither consider the multiple stages of decision making in the CID market nor the specific details of the CID market including the evolution of the limit order book, changes in market liquidity, and the possibility to trade simultaneously for multiple delivery products. The problem of energy storage bidding in the CID market has been tackled using reinforcement learning in [14] and [15]. A detailed literature review on recent works on ID markets is available in [16]. None of the research works so far have addressed the participation of a portfolio of the power plants in the CID market as an MSSiP problem.

The Stochastic Dynamic Programming (SDP) approach has been utilized in some research works to solve Multi-Stage

Stochastic Programming (MSSP) problems [17]. However, the SDP approach suffers the curse of dimesionality making it difficult to be applied on large-scale problems. This is the main motivation behind employing the SDDP algorithm to model the VPP trading in a CID market as a sequential decision making problem. This algorithm has been widely used for long-term planning problems. A long-term operation model for hydro-dominated system is presented in [18] where the impact of short-term variability of the wind power on the system operation is considered. In [19], the long-term hydrothermal scheduling problem is tackled under water resources uncertainty to determine the optimal policy over a multi-annual planning horizon. In [20], with the aim to ensure security of supply through a risk averse approach, the conditional value at risk modeling is integrated in the long-term operation planning problem and solved using SDDP. The SDDP algorithm has also been commonly applied to medium-term planning problems. Such a medium-term hydro power scheduling problem is studied in [21] taking into account the risk aware operation, the provision of spinning reserves and short-term production flexibility with uncertainty in the water inflows [22].

Some research works leverage SDDP for solving models related to optimal short-term planning and operation problems under uncertainty. In [23], SDDP was used to solve a multi-stage stochastic transmission-constrained economic dispatch problem focusing on pumped hydro storage. The operation of a Distributed Energy Resources (DER) aggregator in RT markets has been modeled by using a modified version of SDDP in [24]. An MSSP model for minimizing the expected energy cost of operating a microgrid is proposed in [25]. SDDP requires the random data process, that acts as an input to the model, to be stagewise independent. Two approaches to cope with this in SDDP have been proposed in [26]; the first one models the data using autoregressive time series, while the other uses Markov chains to discretize the random data processes. In [27], an optimal bidding strategy for a VPP, comprising wind power parks, in the Spanish DA and six discrete auctions in ID market is solved with a variant of SDDP. However, none of the aforementioned works on SDDP has focused on modeling the trading of a VPP in the CID market.

B. Contributions & Organization

In this paper, we address the problem of participation of VPP in the European CID market, within a stochastic environment. We formulate the CID trade as an MSSiP problem. It is then solved by a modified SDDP approach. Specifically, the main contributions of our work are summarized in the following points:

- We model the problem of a VPP with a portfolio of hydropower, wind and thermal generation, participating in the CID market, as an MSSiP problem. The proposed model warrants the trading decisions concurrently for all the 24 hourly delivery products, considering the forward-looking approach enabled by dynamic programming;
- We propose two order clearing models, namely the Immediate Order Clearing (IOC) and the Partial Order Clearing (POC), to capture two possibilities by which the submitted orders can be cleared in the CID market. The volume available in the Shared Order Book (SOB), which is the compilation of buy (bid) and sell (ask) orders that are submitted by market participants, is modeled using stochastic processes;
- A modified SDDP algorithm is leveraged to solve the problem of VPP participation in the CID market considering stochastic wind generation, hydro inflow, and SOB volumes, using autoregressive (AR) processes. The proposed concept is displayed with an illustrative example with varied number of stages and transaction costs. We demonstrate the scalability and usefulness of our proposed models through case studies, comparing the results with the deterministic equivalents.

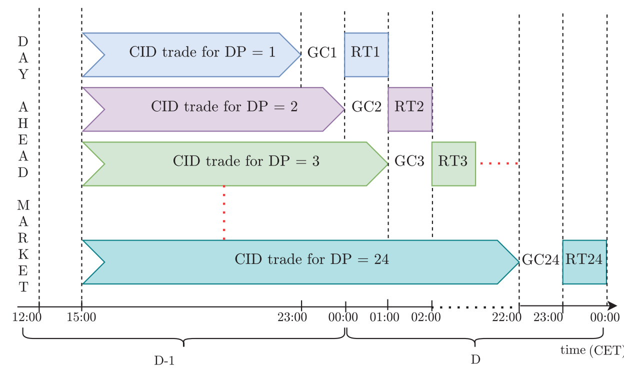

Fig. 1. Illustration of the timeline of the European CID market.

II. Background

In the European CID market, a Delivery Product (DP) refers to the time of physical delivery of electricity. For example, the first hourly DP of the day (D) would be the hour from 00:00 to 01:00. So, the CID trade for this product can take place on the D-1 (day before operation) starting at 15:00 CET. The CID market gate closure (GC) takes place a few minutes before the physical delivery of electricity. For example in Sweden, the GC for the first DP on day D takes place one hour before the delivery (23:00 on D-1). Similarly, there are 24 hourly DPs corresponding to the 24 hours of the day. Fig. 1 shows an illustration of the hourly DPs in the CID market. It is possible to trade simultaneously for multiple DPs. A VPP participates in the CID market with the objective to maximize its profit and update its position, with respect to the DA position, based on new information arriving on its stochastic parameters. Given the setup of the CID market, the VPP submits its orders directly to the Market Operator (MO) who clears the market and informs the VPP if its orders were accepted or not.

The problem of modeling and solving the optimal participation of a VPP in the CID market encompasses certain challenges due to the design of the CID market and the technical constraints of the VPP. In a CID market, trades can take place simultaneously for multiple DPs even though for the same VPP, the problem is coupled across the various DPs as shown in Fig. 1. At the same time, the position of the VPP evolves with time due to

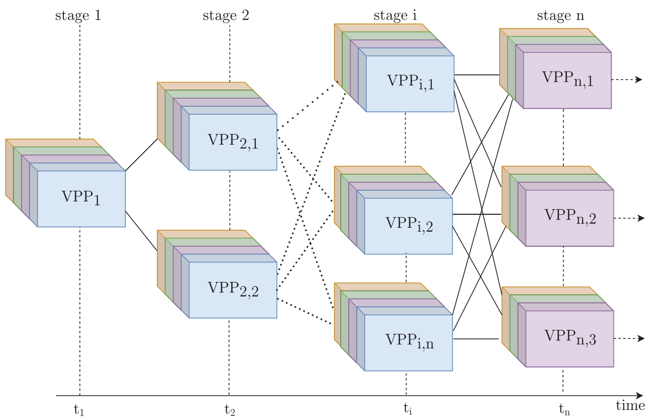

Fig. 2. Representation of a multistage model for VPP participation in the CID market.

the trading that it is involved in based on new forecasts of the hydro inflow and wind. As a result, the model needs to capture these time-interlinked dynamics. Fig. 2 illustrates the multistage nature of the VPP participation in the CID market. Consider the VPP at a given stage and scenario, then each block (blue, violet, green, orange color) in Fig. 2 represents the state (including VPP position, portfolio generation, CID trades, and imbalances) of the VPP corresponding to a given DP. The reduction in the number of blocks at stage 'n' denotes that the gate-closure time of the first DP has passed. This demonstrates a decrement in the state space of the VPP, as the time proceeds in the CID market.

III. Problem Formulation

A general multistage stochastic framework comprises the sequential decision making with the given uncertainty, where new information is assumed to be revealed at each stage. The evolution in time of an uncertain parameter can be generally modeled as a stochastic process. As shown in (1), the objective of an MSSP problem is to maximize the current profit while considering the expected future profit according to the dynamic programming concept [28].

$$\max_{(i_1,j_1)\in F}\Big\{f_1(i_1,j_1)+\mathbb{E}_{\zeta'_{[2,T]}|\zeta_{[1,1]}} \times \Big[\max_{(i_2,j_2)\in F_2(i_1,\zeta_2)}\Big\{f_2(i_2,j_2,\zeta_2)\ldots+\mathbb{E}_{\zeta'_{[T,T]}|\zeta_{[1,T-1]}} \times \Big[\max_{(i_T,j_T)\in F_T(i_{T-1},\zeta_T)}\{f_T(i_T,j_T,\zeta_T)\}\Big]\Big\}\Big]\Big\} \quad (1)$$

where i_t is the state variable that is used for interlinking the decision to the previous stage and j_t is the stage variable that is only defined for a particular stage. The decision (i_t) taken at any stage t, depends on the realization of the uncertainty set (ζ_t) and the decision taken at the previous stage $t-1$.

We study the problem of VPP participation in the CID market as an MSSiP problem – where each stage of the MSSiP problem (except the last stage) corresponds to a CID trading decision of the VPP for a DP. The last stage of each DP represents the physical delivery (the RT stage) of that DP. The VPP takes a decision at any trading stage by maximizing its profits, while considering the expected future profit at all the upcoming stages. For example, a trading decision of the VPP at stage t would be taken by solving an optimization problem considering profits in stages $t,\ t+1,\ t+2,\ldots T$. The outcome of the problem would denote an optimal set of actions over the trading horizon, however, at stage t, the immediate action to perform would be for the given stage. As the time progresses, it is possible to run the entire model at stage $t+1$ while taking into account all the remaining stages and so on. Therefore, the decision that the VPP takes is based on a forward-looking approach which would take into account the upcoming opportunities in the CID market to buy or sell at a profitable price later on.

A. Multi-Stage Stochastic Programming Model

The objective of the VPP trading in the CID market is to maximize its profits by updating its position in the market from the DA position, while minimizing its imbalances considering the availability of updated forecasts. We assume the VPP to be a price taker, that submits orders with certain volumes at pre-specified price levels. All the variables and stochastic parameters discussed next are defined over scenarios but the indices for scenarios are dropped for brevity.

$$\underset{\theta^D}{\text{Maximize}} \sum_{d\in\mathcal{D}}\Bigg\{\sum_{t\in\mathcal{T}}\Bigg[\sum_{p^a\in\mathcal{P}^a}\left(P^{ask}_{p^a,t,d}\cdot v^{ask}_{p^a,t,d}-C^c\cdot v^{ask}_{p^a,t,d}\right) \quad (2a)$$

$$-\sum_{p^b\in\mathcal{P}^b}\left(P^{bid}_{p^b,t,d}\cdot v^{bid}_{p^b,t,d}+C^c\cdot v^{bid}_{p^b,t,d}\right) \quad (2b)$$

$$-\delta^-_{t,d}\cdot\rho^-_d-\delta^+_{t,d}\cdot\rho^+_d-o_{t,d}\cdot\rho^o\Bigg] \quad (2c)$$

$$-C^r\cdot u^{rt}_d+C^r\cdot d^{rt}_d\Bigg\} \quad (2d)$$

$$+\lambda^f h_{T_d,|\mathcal{D}|} \quad (2e)$$

$\theta^D=\{s^{ask}_{p^a,t,d},s^{bid}_{p^b,t,d},v^{ask}_{p^a,t,d},v^{bid}_{p^b,t,d},\delta^-_{t,d},\delta^+_{t,d},o_{t,d},u^{rt}_d,d^{rt}_d\}$. The objective term (2a) maximizes the profit of the VPP by selling in the CID market while minimizing the transaction cost (imposed by the market platform). Similarly, (2b) refers to the buying trade by the VPP in the CID market. Additionally, the trading decision of the VPP at each stage is also driven by the motivation to minimize its deviations given by $\delta^+_{t,d}$ and $\delta^-_{t,d}$, which is accounted by (2c). The latter part in (2c) is to minimize the water spillage. The cost of the thermal generation at the RT stage is considered by (2d). A linear cost term is assumed for the thermal generation [29]. In the objective function, we only consider the change in thermal generation instead of the overall cost as in the CID market, the DA position acts as a reference since the decision for changing the thermal position further is of primary interest. The future value of the final hydro reservoir content is imposed by (2e). The objective function given in (2) is subjected to the constraints described next.

VPP position constraints: Let the VPP position for a DP, d, at the time of gate opening (t_{opn}) of the CID market be denoted by $x_{t_{opn},d}$. As this is the position of the VPP before trading in

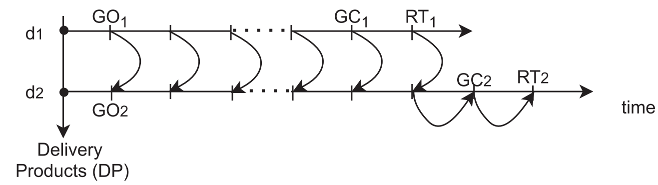

Fig. 3. Illustration of the interlink between the hydropower constraints for two consecutive DPs of CID market (GO_1: gate-opening time for d_1, GC_1: gate-closure time for d_1).

the CID market, it corresponds to its DA position (x_d^{DA}) for the same DP:

$$x_{t_{opn,d}} = x_d^{DA}, \forall\, d \in \mathcal{D} \tag{3}$$

The position of the VPP at a given time, t, corresponding to a DP, d, is given by $x_{t,d}$. It is updated based on the volume of the accepted bid order ($v_{p^b,t,d}^{bid}$) or ask order ($v_{p^a,t,d}^{ask}$) in the CID market at any stage t for each DP, d.

$$x_{t,d} = x_{t-1,d} - \sum_{p^b \in \mathcal{P}^b} v_{p^b,t,d}^{bid} + \sum_{p^a \in \mathcal{P}^a} v_{p^a,t,d}^{ask}, \forall\, t \in \{2, \ldots, T_d\} \tag{4}$$

B. Technical Constraints of the VPP's Portfolio

Hydropower constraints: The hydro inflow is considered to be a stochastic input with a different inflow uncertainty for each DP, which is represented by $I_{t,d}$. The hydropower reservoir content at a given stage, t, for a DP, d, depends on the content for DP, $d-1$, at the same stage, t, as well as the inflow, outflow, and hydropower generation corresponding to the same stage and DP. However, this relation holds only until the last stage of the previous DP. For the remaining stages of the DP, d, the hydropower reservoir content corresponding to that DP at stage, t, is impacted by the reservoir content in the previous stage, $t-1$, for the same DP, d. Along with that, the reservoir content is also influenced by the inflow, outflow, and hydropower generation corresponding to the same DP. Fig. 3 demonstrates the CID timeline with two DPs showing the interlinking between the hydropower reservoir content with stage and DPs. The first stage of the multistage model for a DP, d corresponds to the gate-opening (GO) of the CID market for that DP and the last stage to the RT of the DP, d. We assume that the last opportunity to trade in the CID market is given by the penultimate stage, $T_d - 1$, which corresponds to the gate-closure (GC) of the CID market for the same DP, d.

The constraint to account for interlinking between the DPs for the hydropower reservoir content, $h_{t,d}$, until the last stage of previous DP, $d-1$, is given by:

$$h_{t,d} = \begin{cases} H^{in} - q_{t,d} - o_{t,d} + I_{t,d}, \forall\, t \in \{1, .., T_{d-1}\}, d = 1 \\ h_{t,d-1} - q_{t,d} - o_{t,d} + I_{t,d}, \forall\, t \in \{1, .., T_{d-1}\}, d > 1 \end{cases}$$

Interlinking the hydropower reservoir content for the remaining stages in the trading horizon of DP, d, yields:

$$h_{t,d} = h_{t-1,d} - q_{t,d} - o_{t,d} + I_{t,d}$$
$$\forall t \in \{T_{d-1}+1, .., T_d\}, d \in \mathcal{D}$$

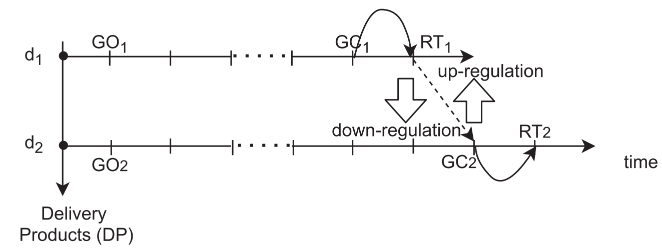

Fig. 4. Illustration of the interlinking of thermal generation for two consecutive delivery products.

The variable domains for the hydro reservoir content and hydropower production are:

$$\begin{aligned} &\underline{Q} \le q_{t,d} \le \overline{Q}, \\ &\underline{H} \le h_{t,d} \le \overline{H}, \quad \forall\, t \in \mathcal{T}_d, d \in \mathcal{D} \end{aligned} \tag{5}$$

Thermal production constraints: We define thermal generation ($g_{t,d}$) as a function of stage t and DP, d to provide a flexible portfolio for participation of the VPP in the CID market trade. It is constrained by generation limits as per:

$$\underline{G} \le g_{t,d} \le \overline{G}\, \forall\, t \in \mathcal{T}_d, d \in \mathcal{D} \tag{6}$$

The thermal power generation (g_d^{rt}) at the RT stage (T_d) for a DP, d is equal to that at the gate closure time (T_{d-1}) for the same DP, d. It follows from the assumption that the thermal power generation is not subjected to variability and can deliver the same energy in the RT, that it has committed at the GC of the CID market.

In order to interlink the thermal generation to be delivered for DP, d (g_d^{rt}) with that delivered for the previous DP, $d-1$, we account for the ramp-up (u_d^{rt}) and ramp-down (d_d^{rt}) of the thermal power plant corresponding to DP, d as given in (7). For the first DP, the thermal generation of the last hour of previous day (G^{init}) is taken as the reference for calculating the changes in thermal generation. As G^{init} acts as an input in determining the actual thermal generation level, the cost for ramp-up or ramp-down is included in the objective function (2). For the remaining DPs, their respective RT stages occur after the thermal power plant has been dispatched for their corresponding prior DP. Therefore, the real generation of the thermal unit for the prior DP is considered while deciding for the ramp-up or ramp-down of any DP other than for $d = 1$.

$$g_d^{rt} = \begin{cases} G^i + u_d^{rt} - d_d^{rt}, & \text{if } d = 1 \\ g_{d-1}^{rt} + u_d^{rt} - d_d^{rt}, & \text{otherwise} \end{cases} \tag{7}$$

Fig. 4 demonstrates this interlinking of the thermal generation between DPs, d_1 and d_2. It also shows how the activated ramp-up and ramp-down contribute in determining the thermal generation for a DP, d given the thermal generation of the previous DP, $d-1$.

The schedule of the thermal plant is obtained based on the CID trade. The ramp-up and ramp-down constraints for the thermal generation unit are given by $0 \le u_d^{rt} \le \overline{R}$ and $0 \le d_d^{rt} \le \underline{R}$ for all DPs.

Fig. 5. Representation of the bid orders in the SOB and ask orders of the VPP for the IOC model (hatched area: $B^{ask}_{1,t,d}$, cross-hatched area: $B^{ask}_{2,t,d}$).

VPP imbalances: The relation between the VPP position ($x_{t,d}$), generation from wind power ($W_{t,d}$), hydropower ($h_{t,d}$) and thermal ($g_{t,d}$) can result into imbalances at each time and each DP, d. Therefore, we introduce non-negative variables, one for the positive imbalances, $\delta^{+}_{t,d}$ and the other for the negative imbalances, $\delta^{-}_{t,d}$, for each stage and DP:

$$W_{t,d} + q_{t,d} + g_{t,d} - x_{t,d} - \delta^{+}_{t,d} + \delta^{-}_{t,d} = 0,$$
$$\delta^{+}_{t,d}, \delta^{-}_{t,d} \geq 0,\ \forall\, d \in \mathcal{D},\ t \in \mathcal{T}_d \tag{8}$$

C. Order Clearing Models

The volume submitted in the market can be either completely or partially transacted. The volume that is not cleared at the stage it is submitted, can either be cancelled or added in the SOB. This depends on the type of the order [30]. Hence, we present two distinct order clearing models to account for the two possible types of orders by the VPP. For enabling the VPP to adopt the forward-looking approach, we further discuss how the VPP estimates the order volume available in the SOB at any stage. Note that the price level index in this Subsection III-C refers to p^a unless mentioned otherwise.

1) Immediate Order Clearing (IOC) Model: In this model, it is assumed that the order volume submitted by the VPP to the CID market can only be cleared at that time instant t, while any remaining order volume is cancelled. This is similar to the *market order* that can be placed by participants in the CID market. Firstly, we discuss about the ask orders submitted by the VPP at predefined price levels. An example with two price levels is shown in Fig. 5. The bid order volume available in the SOB that can be matched with the ask order submitted by the VPP at a price level p is given by the summation of all the bid order volume available at a price above price level p at time t, and is denoted by $\sum_{n=1}^{p} B^{ask}_{n,t,d}$. This volume is considered as a stochastic input and obtained from the market data which is elaborated in Section IV.

Based on the order clearing principle of the CID market, ask order volume can be cleared with the bid order volume that is available at a price greater than or equal to the price at which the ask order is submitted. This is realized at the price level, $p = 1$, by considering the minimum of the submitted ask order volume $s^{ask}_{p,t,d}$ and the bid volume ($B^{ask}_{p,t,d}$) available in the SOB at a price greater than that at price level p.

$$\alpha^{ask}_{1,t,d} = \min\left\{B^{ask}_{1,t,d}, s^{ask}_{1,t,d}\right\}, \forall\, t \in \{1,\ldots,T_d-1\}, d \in \mathcal{D} \tag{9}$$

For the next price levels, the entire bid order volume available at the price level above p, which is not cleared with the submitted VPP orders at $p-1$ can be matched with the VPP ask order at price level p, $s^{ask}_{p,t,d}$.

$$\alpha^{ask}_{p,t,d} = \min\left\{\sum_{n=1}^{p} B^{ask}_{n,t,d} - \sum_{n=1}^{p-1} s^{ask}_{n,t,d}, s^{ask}_{p,t,d}\right\},$$
$$\forall\, p \in \mathcal{P} \setminus \{1\}, t \in \{1,\ldots,T_d-1\}, d \in \mathcal{D} \tag{10}$$

If $\alpha^{ask}_{p,t,d}$ turns out to be negative, then it indicates that no volume is cleared, which is imposed by:

$$v^{ask}_{p,t,d} = \max\left\{\alpha^{ask}_{p,t,d}, 0\right\},$$
$$\forall\, p \in \mathcal{P},\ t \in \{1,\ldots,T_d-1\}, d \in \mathcal{D} \tag{11}$$

To ensure that the VPP does not submit any order beyond the GC of CID market for a DP, we impose $s^{ask}_{p,t,d} = 0$ for the last stage of each DP.

Similar to ask orders, the bid orders submitted by the VPP at t can also be cleared by the IOC model and the part of the bid volume that cannot be cleared at t is eliminated from the SOB. The bid volume posted at $P^{bid}_{p^b,t,d}$ available at stage t is given by $s^{bid}_{p^b,t,d}$. This is cleared with the ask volume available in the CID market at a price equal to or lower than $P^{bid}_{p^b,t,d}$, to give the bid order volume ($v^{bid}_{p^b,t,d}$) accepted at stage t.

In order to make sure that bids and asks are not posted by the VPP at the same stage, we impose the following constraints:

$$0 \leq s^{ask}_{n,t,d} \leq \psi^{ask}_{t,d}\,\bar{S}^{ask}_{t,d},$$
$$0 \leq s^{bid}_{n,t,d} \leq \psi^{bid}_{t,d}\,\bar{S}^{bid}_{t,d},$$
$$\psi^{ask}_{t,d} + \psi^{bid}_{t,d} \leq 1 \quad \forall\, t \in \{1,\ldots,T_d-1\}, d \in \mathcal{D} \tag{12}$$

where $\psi^{ask}_{t,d}$ and $\psi^{bid}_{t,d}$ are binary variables corresponding to $\bar{S}^{ask}_{t,d}$ and $\bar{S}^{bid}_{t,d}$ respectively. The volumes cleared ($v^{ask}_{p^a,t,d}, v^{bid}_{p^b,t,d}$) are non-negative variables.

2) Partial Order Clearing (POC) Model: In this case, we assume that the order volume posted by the VPP in the market might not be cleared at the time t when it is submitted. It is similar to the *limit order* that can be submitted by market participants in the CID market. This way, any submitted order in the CID market at t is available for transaction at t and if it is not cleared then it is available in the SOB at $t+1$. For the sake of simplicity, it is assumed to be eliminated from the SOB beyond $t+1$. Considering the above assumption, we can allow for a partial order clearing in two consecutive stages and therefore denote the order volume available at price level p^a (hereafter denoted as $p \in \mathcal{P}$), $k^{ask}_{p,t,d}$ by:

$$k^{ask}_{p,t,d} = s^{ask}_{p,t,d} + k^{ask}_{p,t-1,d} - v^{ask}_{p,t-1,d},$$
$$\forall\, p \in \mathcal{P}, t \in \{1,\ldots,T_d-1\}, d \in \mathcal{D} \tag{13}$$

Fig. 6. Representation of bid orders in SOB and ask orders of VPP for POC model (hatched area: $B^{ask}_{1,t,d}$, cross-hatched area: $B^{ask}_{2,t,d}$).

As shown in Fig. 6, the bid volume available to be cleared with the ask order at price level p_1 is given by $B^{ask}_{1,t,d}$, which is the bid order volume available at a price above p_1. The ask order volume submitted by the VPP at p_1 would be cleared with the available bid order volume which is represented by:

$$\beta^{ask}_{1,t,d} = \min\left\{B^{ask}_{1,t,d}, k^{ask}_{1,t,d}\right\},\ t \in \{1,\ldots,T_d-1\}, d \in \mathcal{D} \tag{14}$$

Similarly, the bid volume above p_2 and remaining bid volume above price level p_1 and can be transacted with the ask order volume of the VPP at price level p_2 and so on. The generalized expression for the bid volume available for clearing with the VPP ask order at price level p is given by:

$$f^{ask}_{p,t,d} = \sum_{n=1}^{p} B^{ask}_{n,t,d} - \sum_{n=1}^{p-1}\left(s^{ask}_{n,t,d} + k^{ask}_{n,t-1,d} - v^{ask}_{n,t-1,d}\right),$$
$$\forall p \in \{2,\ldots,n\}, t \in \{1,\ldots,T_d-1\}, d \in \mathcal{D}$$

For each price level $\{2,\ldots,n\}$, the volume that can be potentially transacted is given by:

$$\beta^{ask}_{p,t,d} = \min\{f^{ask}_{p,t,d}, k^{ask}_{p,t,d}\},$$
$$\forall p \in \{2,\ldots,P\}, t \in \{1,\ldots,T_d-1\}, d \in \mathcal{D} \tag{15}$$

Again, if the value of $\beta^{ask}_{p,t,d}$ is negative according to (15) then it indicates that no volume is cleared, which is ensured by imposing the following constraint:

$$v^{ask}_{p,t,d} = \max\left\{\beta^{ask}_{p,t,d}, 0\right\},\ \forall p \in \mathcal{P}, t \in \{1,\ldots,T_d-1\}, d \in \mathcal{D} \tag{16}$$

The total bid order volume ($k^{bid}_{p^b,t,d}$) at t can be divided into two components, the submitted bid volume at t and the remaining bid volume from $t-1$. Similar to the ask POC model, the bid volume cleared at any price level considers the bid volume, $k^{bid}_{p^b,t,d}$ and the bid volume available at a lower price than that price level. Additionally, the set of equations in (12) are also imposed in the POC clearing model to ensure non-simultaneous submission of ask and bid orders to the CID market.

As the POC model involves additional variables and equations to allow for partial clearing of orders, it increases the computational complexity. However, the theoretical comparison of IOC and POC models would require the discussion about whether it is profitable for the VPP to submit a market order or a limit order. This topic is well studied in financial market literature [31]. It is an interesting direction for future work in the context of CID electricity markets.

D. Mixed-Integer Reformulation

Equations (9)–(11), (14)–(16) comprise the min and max functions, making them non-linear. We reformulate those equation using the Big M method. This is illustrated for (9) below:

$$\alpha^{ask}_{1,t,d} \leq s^{ask}_{1,t,d}, \tag{17a}$$
$$\alpha^{ask}_{1,t,d} \leq B^{ask}_{1,t,d} \tag{17b}$$
$$M(1-\phi) - s^{ask}_{1,t,d} + \alpha^{ask}_{1,t,d} \geq 0 \tag{17c}$$
$$M\phi - B^{ask}_{1,t,d} + \alpha^{ask}_{1,t,d} \geq 0 \tag{17d}$$

where ϕ is a binary variable and M is a large positive number. The other non-linear expressions can also be reformulated in the similar way as in (17).

The updated set of variables for problem 2 is: $\theta^U = \{\theta^D \cup g_{t,d}, h_{t,d}, q_{t,d}\}$.

E. Solution Methodology

We leverage the Approximate Dual Dynamic Programming (ADDP) algorithm [26], a modified version of SDDP, which is a two-step procedure. Central to this algorithm is the construction of the lattice, i.e., a graph with layers where each layer represents a stage. It comprises nodes that denote a state of the stochastic process, while arcs denote the probability of transition from one node to the other. In a lattice, unlike scenario trees, each node can have multiple predecessors. The first step of the algorithm is to create a discrete lattice out of the continuous Markov process in such a way so that its optimal policy is as close as possible to the optimal policy of the true process. The second step is to solve the discrete Markov Decision Process (MDP) using SDDP, a simulation-based iterative algorithm that conducts a forward pass to determine the approximate optimal policy by sampling a sequence of state transition from the scenario lattice. We denote the sample average reward collected during the forward pass as the *simulated reward*, which is in the form of a distribution with a standard deviation that reduces with iterations. In the backward pass, new hyperplanes are added to the set of supporting hyperplanes to determine the approximate post-decision value function. This value function acts as an upper bound to the true value function and is utilized in the next iteration of the forward pass to determine the approximate optimal policy. *Expected reward* is the approximate reward obtained using the value function approximation that enters the first-stage problem. It includes the first-stage objective function and the cost-to-go approximated by hyperplanes. Since the proposed model in Section III comprises of integer variables, we utilize Stochastic Dual Dynamic integer Programming (SDDiP) algorithm, a further extension of ADDP, which is proposed in [28]. We adopt the strengthened Benders' cut for our model to be used in the SDDiP algorithm and follow the procedure as per the first step of [32]. However, we do not proceed with the second step which involves the binarization of the continuous, time-interlinked variables to be able to apply Lagrangian cut as it adds significant computational burden, which is not favourable to our application.

IV. Stochastic Process Modeling

We employ a stochastic process modelling and simulation methodology to model the evolution of the uncertain factors in time. The uncertainties of interest for the VPP under the proposed framework are the wind power, hydro inflow forecast errors, bid and ask volumes available in the SOB within the different price levels. The VPP is assumed to receive forecasts for both wind power production and hydro inflow, which improve in stages closer to the time of physical delivery. The autoregressive (AR) processes are introduced to capture the forecast errors at each stage (ξ_t) associating them with their lag values (ξ_{t-i}), serially independent and a identically distributed (i.i.d.) noise, ϵ_t [33].

$$\xi_t = \zeta + \sum_{i=1}^{l} R_i \xi_{t-i} + \epsilon_t \tag{18}$$

R_i the autocorrelation coefficients (calculated from the historical data), and l the number of lags for the AR model. Without reducing the generality of our methodology, we make certain simplifying assumptions for the practical implementation of the case study. We assume a first-order AR process, i.e., AR(1), and we disregard the correlation coefficients between the wind and inflow processes. Also, it is assumed that the AR noise follows a normal distribution (μ, σ). Following this method, the actual wind generation and inflow can be expressed as:

$$\begin{aligned} W_{t,d} &= \hat{W}_{t,d} + \xi^W_{t,d} \\ I_{t,d} &= \hat{I}_{t,d} + \xi^I_{t,d} \end{aligned} \tag{19}$$

$\hat{W}_{t,d}$ and $\hat{I}_{t,d}$ are the wind and hydro inflow forecasts at the GO of the CID market for each DP, and $\xi^W_{t,d}$, $\xi^I_{t,d}$ their forecast errors respectively.

Additionally, we also consider the bid ($B^{ask}_{p^a,t,d}$) and ask ($A^{bid}_{p^b,t,d}$) volumes available in the SOB from the other traders as stochastic inputs. For these volumes, the historical ID trade data is analysed from Nord Pool for the period 2019-2020 [34]. A number of price levels are selected and the temporal evolution of the total volume within those ranges from the historical CID trades is studied. For each price range, defined by two consecutive price levels, we calculate the total accepted ask and bid volumes for a specific price area combination and interpolate this data to create an evenly-spaced time series. Subsequently, this time series is deployed to fit the AR models to create scenarios for the available order volume at a specific time. We use AR(1) models for creating scenarios for each of the ask and bid volume time series corresponding to each DP, assuming that there are no correlations between them and the time series of other DPs.

TABLE I
Input Data for the Autoregressive Processes (Values in MWh)

	ξ^W_t	ξ^I_t	A^{bid}_1	A^{bid}_2	B^{ask}_1	B^{ask}_2
μ	0.5	0.8	0.7	0.4	0.5	0.6
σ	8	5	4	4	4	4
In	0	0	0.5	0.5	0.5	0.5
lag	0.7	0.5	0.6	0.3	0.7	0.4
ζ	0	0	20	25	30	30

V. Results and Discussion

The simulations for the trade of VPP in the CID market have been carried out in Python 3.7 and the SDDiP model has been solved with the help of QUASAR [26] using the FICO Xpress solver. All the simulations were performed with an Intel(R) Core(TM) i7-10850H CPU @2.70 GHz machine.

A. *Illustrative Example*

In this example, we consider the gate-opening of the CID market to be at 15:00 CET of the D-1 (one day before the day of physical delivery) while the gate-closure is at 23:00 CET for DP, $d = 1$. The gate-opening is same for $d = 2$ while the gate-closing happens at 00:00 CET. For the sake of simplicity, we assume one trade per hour. Therefore, the total number of trades for $d = 1$ are 7 and for $d = 2$ are 9 while the 8^{th} stage for $d = 1$ and 10^{th} stage for $d = 2$ are the RT stages for the two DPs respectively. We obtain a lattice for 10 stages with 901 nodes with 1 root node and 100 nodes per stage and 62555 arcs [35]. The average computation time for the case studies of the illustrative examples were 40 seconds.

The inputs considered for the autoregressive (AR) processes are shown in Table I for both the DPs. The minimum and maximum reservoir content is 0 and 100 MWh respectively. The up- and down-regulation capacity of the thermal unit is considered to be 10 MW/h. The minimum and maximum generation is taken to be 0 and 20 MWh respectively. While we provide a general formulation for multiple price levels in the SOB, the case study is limited to two price levels each for ask and bid volumes. The prices at which the VPP is able to buy are €20/MWh and €25/MWh, while it sells at €45/MWh and €40/MWh respectively (price levels $p = 1$ and $p = 2$). The future hydro reservoir value (λ^f) is €100/MWh. The Big M parameter was set to 1000, while the thermal cost was €40/MW. The penalty terms ρ^+ and ρ^- were kept at 100. The transaction cost for the CID trades was considered to be €10/MWh. The DA positions for $d = 1$ and $d = 2$ were 60 MWh and 70 MWh respectively.

1) VPP Trade for Two Delivery Products: Fig. 7, demonstrates the results for the CID trades to be carried out by the VPP for $d = 1$ and $d = 2$ where for the fanchart the obtained values are divided in 19 quantiles (0.05,...,0.95). The wind and inflow scenarios corresponding to both DPs are shown in Fig. 7. On deploying the SDDiP algorithm in our model, the expected reward at the first stage and the mean of the distribution of simulated rewards is checked for convergence. The obtained value functions at the end of the simulation are used as the decision policy which provides a solution for a given stage,

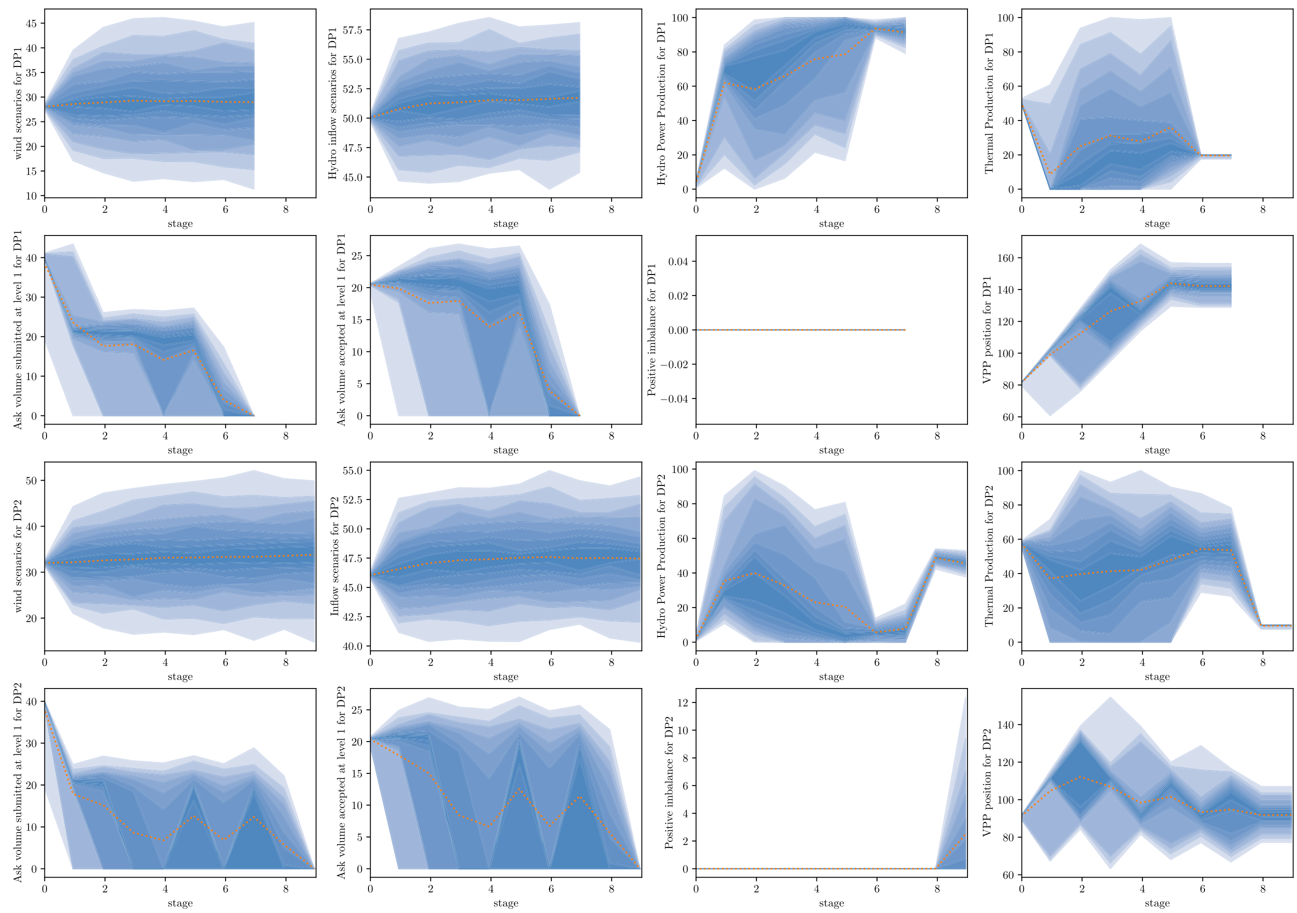

Fig. 7. Results of the VPP trade in the CID market for first and second DP with the IOC model (all y-axis values are in MWh, dotted curve: mean).

state, and relevant decision history. The decision variables as shown in Fig. 7 are obtained by simulating 2000 scenarios from the AR processes with the decision policy. The outcome for the hydropower and thermal production at each stage of the CID market is demonstrated for both the DPs. As a result of the submitted order volume (s_{ask}) and the available volume in the CID market for realizing the trade, the cleared volume (v_{ask}) is illustrated in the figure. For example, at stage 1, the submitted ask order was 40 MWh while only 20 MWh were cleared according to the IOC model described in Section III-C1 as the average value of bid volume in SOB was 20 MWh. The evolution of the VPP position over stages as a result of the realized trades is shown in Fig. 7. The positive imbalances of the VPP for both the DPs are shown in the plots and they were found to be 0 for all the stages except the last stage for $d = 2$ where a small amount of positive imbalance was found at the RT stage due to uncertain parameters.

For the $d = 1$, in the beginning of the CID market, the VPP predominantly tries to sell in the CID market with the expectation of increased inflow. This leads to an increase in the VPP position for the $d = 1$. The thermal generation is envisaged to be reduced to minimize the eventual cost of generation whereas the portfolio of the VPP is balanced by increasing the hydropower generation. On average, the VPP sells further as it receives a higher price in the CID market than its expected cost of thermal generation. Furthermore, the presumption of an increase in hydropower inflow and availability of reservoir volume supports the VPP's decision to sell. However, in some of the scenarios, where the VPP comes across reduction in wind and inflow, it buys from the market, reducing its position. Beyond stage 6, as the RT stage approaches, the VPP is not willing to trade further and tries to maintain its position so as to minimize its expected imbalances at the RT stage.

It can be observed from Fig. 7 that the number of stages for $d = 2$ are more than that for $d = 1$ as the gate-opening time for the DPs is the same while the gate-closure time is later for $d = 2$ as compared to $d = 1$. In this DP too, the VPP attempts to sell with the anticipated increase in inflow. As we also consider the future water value of the hydro reservoir, this restricts the usage of the hydro reservoir content further reducing the volume sold by the VPP after stage 3. Moreover, the VPP predominantly buys at stage 4 and beyond as it is less expensive to buy back energy rather than produce it using the thermal unit. However, its position is almost unchanged from stage 7. At the RT stage, the hydropower generation compensates for the reduction in thermal generation that allows to avoid the additional generation cost.

We evaluate the rewards obtained by the VPP as per the IOC model when the number of stages (trades) are increased for both the DPs as shown in Table II. Thereafter, we analyze the impact of different transaction costs on the rewards of the VPP with the

TABLE II
COMPARISON OF THE EXPECTED (EXP.) AND SIMULATED (SIM.) REWARDS FROM THE IOC MODEL FOR DIFFERENT NUMBER OF STAGES

	Trans. cost = 10 €/MWh			Trans. cost = 20€/MWh		
Stages	Exp. reward (€)	Sim. reward (€)	Std. dev. (€)	Exp. reward (€)	Sim. reward (€)	Std. dev. (€)
9, 10	13472.4	13003.7	49.3	11164.3	11069.2	81.3
18, 20	16503.3	15870.1	23.0	12090.1	11992.9	80.3
27, 30	18010.6	17043.9	54.8	12821.6	12830.3	21.2
36, 40	19740.8	17745.8	115.4	12859.7	12802.8	21.4
45, 50	20549.6	18929.7	122.3	12858.8	12832.1	19.0
54, 60	21430.6	20483.1	150.6	12855.7	12850.8	18.5
63, 70	25299.5	21041.5	135.3	12863.4	12825.0	16.3
72, 80	24011.9	22741.0	134.4	12858.2	12801.2	20.9
81, 90	25459.6	24080.1	50.7	12863.7	12881.5	17.6
90, 100	27454.1	24945.1	62.3	12860.5	12796.9	20.7

TABLE III
SENSITIVITY ANALYSIS OF VPP PERFORMANCE FOR DIFFERENT FORECASTING UNCERTAINTY LEVELS OF WIND POWER PRODUCTION AND HYDRO INFLOW WITH VARYING NUMBER OF STAGES

Stages	σ^W	σ^I	Exp. reward (€)	Sim. reward (€)	Std. dev. (€)
9,10	2	5	13509.08	13046.48	35.95
	5	2	13420.83	12898.54	87.32
	8	8	12954.98	12407.15	209.32
18, 20	2	5	16191.87	15610.2	45.12
	5	2	16229.13	15542.9	119.95
	8	8	15371.89	14598.45	297.01
27,30	2	5	17402.61	16902.37	41.72
	5	2	17531.78	16804.88	44.78
	8	8	16692.28	16128.4	171.55
36, 40	2	5	18776.8	17996.49	79.78
	5	2	18666.64	18017.55	41.88
	8	8	17849.25	17290.99	227.9
45, 50	2	5	19908.56	19343.8	40.26
	5	2	19953.01	19279.78	53.08
	8	8	19587.9	18243.4	223.74

TABLE IV
COMPARISON OF VPP PERFORMANCE FOR DIFFERENT CID MARKET LIQUIDITY WITH VARYING NUMBER OF STAGES

Stages	A_1^{bid}	A_2^{bid}	B_1^{ask}	B_2^{ask}	Exp. rew. (€)	Sim. rew. (€)	Std. dev. (€)
18, 20	10	10	30	30	14743.2	14484	85.4
	50	50	30	30	17487.1	15730.2	228.1
	70	70	30	30	17669.8	16191.8	149.7
	30	30	10	10	14438.7	14048.5	84.2
	30	30	50	50	17372.6	15981.7	177.6
	30	30	70	70	18116.7	16348.2	153.8
27, 30	10	10	30	30	15590.0	15221.8	81.7
	50	50	30	30	19581.8	17540.2	321.0
	70	70	30	30	20187.2	17998.2	138.9
	30	30	10	10	15330.5	14956.3	73.8
	30	30	50	50	19415.8	17546.6	178.2
	30	30	70	70	20409.3	18022.0	177.4
36, 40	10	10	30	30	16499.7	16017.1	171.6
	50	50	30	30	21658.2	19134.8	213.5
	70	70	30	30	23185.3	19727.7	188.1
	30	30	10	10	16313.5	15597.4	191.4
	30	30	50	50	21540.1	19735.6	180.7
	30	30	70	70	23789.2	19620.7	174.7
45, 50	10	10	30	30	17366.6	16777.9	174.8
	50	50	30	30	23481.5	21135.0	205.1
	70	70	30	30	26277.8	19315.2	994.6
	30	30	10	10	17049.5	16479.3	75.4
	30	30	50	50	24085.0	19315.9	760.7
	30	30	70	70	25650.1	21767.8	203.0

increase in the number of stages. It can be seen that the increase in the number of trades does not provide any further benefit beyond six trades per hour (54, 60 stages) for 10 €/MWh transaction cost. However, for a higher transaction cost (20 €/MWh) no significant added benefit is observed beyond three trades per hour (27, 30 stages).

2) Sensitivity Analysis of Forecasting Uncertainty: The rewards obtained by the VPP for different forecast uncertainties of wind power production and hydro inflow with varying number of CID market stages is shown in Table III. We assume the IOC model for this analysis. It can be observed from the Table III that as the forecast errors are increased, the rewards obtained by the VPP are reduced for the same number of stages. Moreover, the standard deviation of the simulated reward is also found to be increasing with higher uncertainty level. This trend, of decrease in rewards with the increase in uncertainties of wind and hydro inflow, continues with the increase in the number of stages in the CID market.

3) Sensitivity to Market Liquidity: In Table IV, the variation in the VPP performance with different CID market liquidity, on the ask and bid side of the SOB, is provided. The results are obtained with the IOC model. It can be observed that both the ask and bid side liquidity can have a significant influence on the reward earned by the VPP for a given number of stages in the CID market. As the number of stages are increased in the analysis, the observation about increase in reward with higher liquidity still holds true. Whether the rewards earned by the VPP are more sensitive to the changes in liquidity in the ask or the bid side of the SOB would depend on whether the VPP is more likely to post bid or sell orders.

4) Partial Order Clearing Results: According to the CID market principle, if the submitted order volume is not completely cleared then it can be added in the SOB. This feature of the CID market proposed in the POC model described in Section III-C2. Fig. 8 demonstrates the results of the submitted order (s^{ask}), available bid order in the order book (f^{ask}), net ask order from the VPP (k^{ask}), and the resulting cleared volume (v^{ask}). It can be seen that the ask volume cleared in the CID market is the minimum of f^{ask} and k^{ask}. Consider the stage 2 where the average submitted ask volume is 44 MWh, in which however, only 30 MWh is cleared as that is the maximum buy volume available in the SOB. The remainder volume (14 MWh) that could not be cleared at stage 2 is then carried forward to the next stage where it is added together with the incoming ask order (8 MWh) at stage 3. The total ask volume at stage 3 is 22 MWh, which can be cleared with the buy volume at stage 3. Therefore, there would be no volume carried forward to stage 4 in this case.

B. Comparison With the Deterministic Solution

With the aim to demonstrate the scalability of our model, the CID trading model of the VPP is implemented in an more detailed case study including all the 24 hourly DPs. As shown in Table V, the *cases 1a* and *1b* have 10 stages for the first DP, adding one stage for each consecutive DP, making it 10, 11,..., 33 stages. The *cases 2a* and *2b* had 10, 12, 14,..., 56 stages.

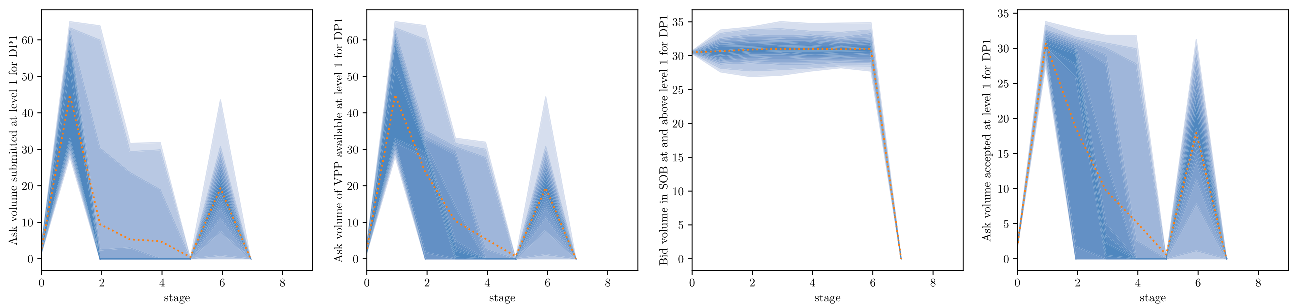

Fig. 8. Demonstration of the ask orders submitted, cleared and bid volume in the SOB for POC model (all y-axis values are in MWh) (dotted curve: mean).

TABLE V
REWARDS FOR THE PROPOSED MODEL WITH DIFFERENT NUMBER OF STAGES AND OF THE DETERMINISTIC MODELS

Case	Expected reward (€)	Simulated reward (€)	Standard dev. (€)	Time (s)	LPs solved
1a	653049.38	638088.79	404.35	58.17	77693
1b	648032.25	636243.83	1511.76	18.49	6432
2a	1010823.40	1006344.33	358.50	220.97	114906
2b	1010664.84	1004435.22	1442.67	24.26	7388
3a	1374730.46	1362476.49	197.32	274.55	160778
3b	1223114.88	1217431.58	948.35	24.62	8597
4a	1733591.19	1730021.58	60.80	373.03	210348
4b	1732932.72	1720080.62	1468.74	34.39	10217

Similarly, *cases 3a* and *3b* considered 10, 13, 16,.., 79 stages while *cases 4a* and *4b* had 10, 14, 18,..., 102 stages. All the 1b, 2b, 3b, and 4b cases represent the deterministic equivalent of the 1a, 2a, 3a, and 4a models respectively. In the deterministic model, the AR processes are fixed to their respective mean values. It can be seen from the Table V that the our proposed model provides higher rewards and lesser standard error in the simulated reward as compared to the corresponding deterministic equivalent. However, it comes at the cost of added computation time and the number of linear programs (LPs) solved. No final imbalances were found in any of the aforementioned cases.

VI. CONCLUSION

In this paper, we have modeled the problem of a VPP, with wind, hydro and thermal power generation portfolio, participation in the continuous intraday market as a multistage stochastic integer programming problem (MSSiP). The proposed MSSiP problem has been tackled using a modified version of Stochastic Dual Dynamic Programming (SDDP) algorithm where the stochastic inputs are modeled as autoregressive processes. Our model enables simultaneous order posting for multiple delivery products at any given time in the CID market trading horizon. Furthermore, two order clearing models, namely the Immediate Order Clearing (IOC) and the Partial Order Clearing (POC) have been presented to model the trading process in the CID market. It was found that the increase in the number of trading stages does not add any further reward beyond a particular number of stages and also depends on the transaction costs. The effectiveness of our approach is demonstrated by comparing the performance of the VPP through our proposed model for all the 24 hourly delivery products with the corresponding deterministic equivalent. The proposed model was found to provide higher rewards at the cost of higher computation time with respect to the deterministic model.

VII. FUTURE SCOPE OF WORK

The proposed model can be extended to take into account the behavior of the VPP in the day-ahead electricity market and thereby study the correlation between the DA and CID behavior of the VPP. Further, an interesting future work could be to study the full cycle performance of the VPP in the forward, day-ahead, and balancing market along with the CID market. Battery energy storage can also be integrated in the VPP portfolio to model and study its behavior in the CID market. The comparison of proposed multistage stochastic programming problem for the VPP trading in the CID market, solved with the SDDiP algorithm, to a stochastic model predictive control approach is another potential future work. Even though the major focus of our work is to model the VPP trading in the European CID market as a MSSiP, the proposed concept can be applied to other markets where it is possible to take sequential decisions. It would require modifications in the market clearing mechanism.

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