Hyperfine Intrinsic Magnitude and Phase Response Measurement of Optical Filters Based on Electro-Optical Harmonics Heterodyne and Wiener–Lee Transformation

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***Abstract*—An electro-optical heterodyne scheme is proposed and demonstrated for magnitude- and phase-frequency response measurement of optical filters based on harmonics heterodyne and Wiener–Lee transformation. The method consists of an acousto-optic frequency shifter and a phase modulator located in a frequency-shifted heterodyne interferometer. The minimum phase-frequency response is simultaneously extracted from the measured magnitude–frequency response with the help of Kramers–Kronig relations and the Wiener–Lee transformation. As compared with the single-sideband-based or double-sideband-based methods, our method eliminates small-signal assumption and features an immunity to undesired spurious sidebands, enabling harmonic sideband sweeping with even-folded measuring frequency range. Prior to the conventional frequency-shifted heterodyne methods, the method enables simultaneous extraction of intrinsic magnitude and phase frequency responses for most optical filters by introducing the Wiener–Lee transformation. A phase-shifted fiber Bragg grating is measured for extracting the frequency responses in the proof-of-concept experiment with the frequency resolution up to 50 kHz and frequency range of 80 GHz by using a frequency-swept modulation of 20 GHz.**

***Index Terms*—Electro-optical harmonics heterodyne, magnitude-frequency response, microwave photonics, optical filters, phase-frequency response, Wiener–Lee transformation.**

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I. Introduction

OPTICAL filters are basic but key components for signal processing in diverse applications such as in optical communications, optical sensing, spectroscopy, and optical imaging [1], [2]. High-resolution response function of optical filters in both magnitude and phase dimensions is highly desired in optical component fabrication and system characterization [3]. Many optical or electrical methods have been demonstrated for frequency response measurement of optical filters. Optical spectrum analysis method can be used to directly measure magnitude-frequency response in the optical domain with an amplified spontaneous (ASE) source or a super-continuum light source [4], [5]. Unfortunately, the optical spectrum analysis is only applicable for low-resolution magnitude measurement (about 1.25 GHz at 1550 nm), which is limited by the commercially available grating-based optical spectrum analyzer (OSA) [6], [7]. The phase-shift approach [8], [9] and the interferometry method [10], [11] enable both magnitude and phase response measurement by using an optical vector analyzer (OVA), which rely on the wavelength scan of a tunable laser source in the optical domain. However, the wavelength-swept optical source must be extremely stable and fast tunable, if high-resolution is involved [12]. The electro-optical frequency-swept methods are proposed based on electro-optical single-sideband (SSB) [13]–[16] or double-sideband (DSB) modulation [17]–[19]. The SSB-based method uses single optical sideband sweeping and provides an equivalent mapping of frequency response from optical to electrical domain, which achieves high-resolution measurement (up to 78 kHz) with the help of an electrical vector network analyzer (EVNA) [13]. However, it requires complex microwave modulation to suppress the undesired harmonic sidebands as much as possible resulted by the modulation nonlinearity [14]. Moreover, it is not efficient to measure optical bandpass response due to the degraded optical carrier and limited dynamic range [16]. The electro-optical DSB-based method is proposed by two-step DSB modulation at two known bias conditions, which doubles the measuring frequency range in the cost of two-step scan [17]. It should be noted that both SSB- and DSB-based methods are established under the assumption of small-signal operation, so they are sensitive to the bias voltage drifting and harmonic sideband suppression. In contrast, the electro-optical heterodyne methods are proposed for measuring magnitude-frequency response by frequency-shifted optical heterodyne between the frequency-swept first-order sidebands and frequency-shifted optical carrier [20]–[22]. These methods feature doubled measuring frequency range and immunity to modulation nonlinearity, benefiting from the equivalent observation of the frequency-swept sideband from optical domain to electrical domain. Nevertheless, the major difficulty for these methods lies in the incapability of phase response measurement due to the fact that the frequency-shifted heterodyne is only related to the magnitude of optical sidebands. Therefore, methods that enable hyperfine magnitude and phase response measurement for most optical filters, and at the same time avoid the complicated operation to suppress the undesired sidebands are of great importance.

In this work, we propose an electro-optical heterodyne method that allows high-resolution and wideband frequency response measurement of optical filters based on harmonics optical heterodyne and Wiener-Lee transformation. The method consists of an acousto-optic frequency shifter and a phase modulator located in a frequency-shifted heterodyne interferometer (FHI). The probing harmonic sidebands are frequency-swept through the optical filter under test and then mapped from optical domain to electrical domain by heterodyning with the frequency-shifted optical carrier, respectively, which allows measuring the intrinsic magnitude-frequency response of the optical filter in the electrical domain. The minimum phase-frequency response is self-extracted from the measured magnitude-frequency response with the help of Kramers-Kronig relations and Wiener-Lee transformation. The method features immunity to bias voltage drifting and the undesired spurious sideband due to the bias-free phase modulation and frequency-shifted heterodyning, which largely improves the measurement accuracy. Moreover, it increases the measuring frequency range by even-fold through harmonic sideband heterodyne. As compared with the SSB-based methods, our method features the immunity to the undesired spurious sidebands. Unlike the DSB-based modulation methods, the proposed method eliminates small-signal assumption and enables harmonic sideband sweeping with even-folded frequency range. Prior to the conventional frequency-shifted heterodyne methods, the method enables simultaneous extraction of intrinsic magnitude and phase frequency responses of most optical filters. The proposed method is theoretically analyzed and experimentally studied in this paper. Frequency response of a phase-shifted fiber Bragg grating (FBG) is measured based on the proposed method with a frequency range up to 80 GHz and a frequency resolution as high as 50 kHz by using a frequency-swept modulation of 20 GHz. The measured results are compared with those utilizing the conventional SSB-based and OVA-based methods in order to check the consistency and accuracy of the proposed method.

II. Theoretical Description

In our scheme as shown in Fig. 1(a), the proposed harmonics optical heterodyne is based on the frequency-shifted heterodyne interferometer (FHI) consisting of an electric-optic phase modulator (PM) and an acousto-optic frequency shifter (FS) located in upper and lower arm of a Mach-Zehnder interferometer (MZI). The optical filter under test is inserted after the PM on the upper arm of the FHI. The phase modulated optical sidebands are swept by electro-optical modulation and then fed into the device under test (DUT), and then mapped from optical domain to electrical domain by heterodyning with the frequency-shifted optical carrier in the low arm of FHI. After photodetection, the magnitude response of the DUT can be obtained by analyzing the electrical spectra of the beat notes with an electrical spectrum analyzer (ESA).

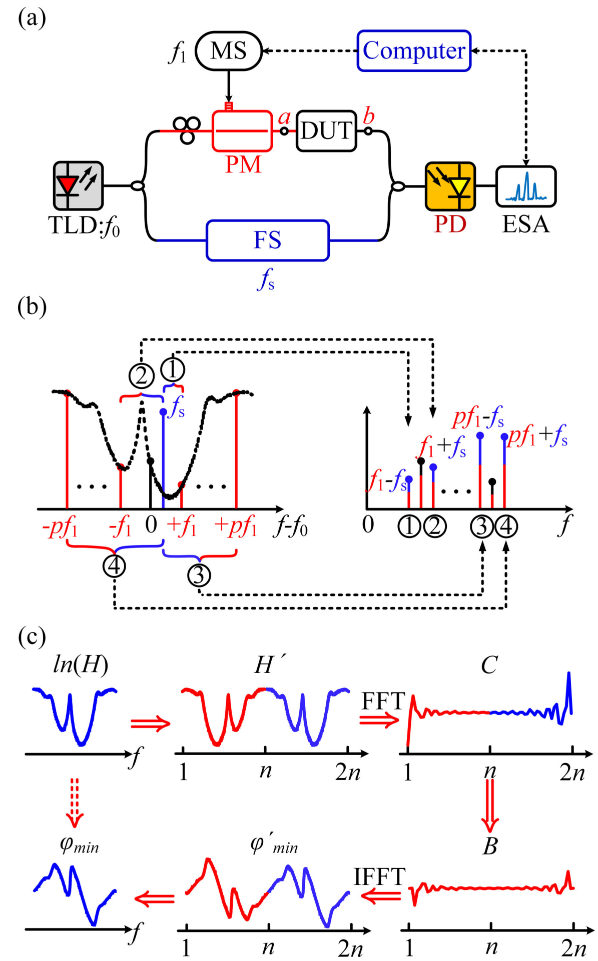

Fig. 1. Schematic diagram of the proposed method. (a) Systematic setup. (b) Electro-optical harmonics heterodyne mapping. (c) Minimum-phase response reconstruction. TLD: tunable laser diode, PM: phase modulator, MS: microwave source, FS: frequency shifter, DUT: device under test, PD: photodetector, ESA: electrical spectrum analyzer.

The phase modulated signals generated by the PM can be written by

$$E_1(t) = Ae^{j(2\pi f_0 t + \beta_1 \sin 2\pi f_1 t)} \tag{1}$$

where A and f_0 are the amplitude and frequency of the optical carrier in the upper arm of FHI. f_1 is the frequency of the sinusoidal microwave signal and β_1 is the modulation index of the PM corresponding to the microwave frequency of f_1. With the help of the Jacobi-Anger theorem, Eq. (1) can be reorganized as

$$E_1(t) = A\sum_{p=-\infty}^{+\infty} J_p(\beta_1)\, e^{j2\pi(f_0+pf_1)t} \tag{2}$$

where $J_p(\cdot)$ is the pth-order Bessel function of the first kind. The phase-modulated optical signals are used as a frequency-swept optical source to probe the DUT, and the affected optical field after the DUT is given by

$$E'_1(t) = A\sum_{p=-\infty}^{+\infty} J_p(\beta_1)\, H(f_0+pf_1) \cdot e^{j2\pi(f_0+pf_1)t+j\varphi(f_0+pf_1)} \tag{3}$$

where $H(f)$ and $\varphi(f)$ are the magnitude- and phase-frequency response functions of the DUT, respectively. In the lower arm of FHI, the optical carrier is frequency-shifted by f_s in the acousto-optic FS, given by

$$E_2(t) = Be^{j2\pi(f_0+f_s)t} \tag{4}$$

with the amplitude B of the frequency-shifted optical carrier.

The phase-modulated optical signal are combined with the frequency-shifted optical carrier, which can be expressed by the sum of the two optical signals from the two arms of FHI as

$$\begin{aligned} E(t) &= E'_1(t) + E_2(t)\cdot e^{j\phi} \\ &= A\sum_{p=-\infty}^{+\infty} J_p(\beta_1)H(f_0+pf_1)e^{j2\pi(f_0+pf_1)t+j\varphi(f_0+pf_1)} \\ &\quad + Be^{j2\pi(f_0+f_s)t+j\phi} \end{aligned} \tag{5}$$

where ϕ denotes the phase difference between the two arms of FHI. The combined optical signal is detected by the PD to generate an instantaneous photocurrent as shown in Fig. 1(b), given by

$$\begin{aligned} i/R &= E(t)\,E(t)^* \\ &= B^2 + 2A^2\sum_{p=-\infty}^{+\infty}\sum_{q=-\infty}^{+\infty} J_p(\beta_1)\,J_q(\beta_1) \\ &\quad \times H(f_0+pf_1)\,H(f_0+qf_1) \\ &\quad \cdot\cos[2\pi(p-q)f_1 t + \varphi(f_0+pf_1) - \varphi(f_0+qf_1)] \\ &\quad + 2AB\sum_{p=-\infty}^{+\infty} J_p(\beta_1)\,H(f_0+pf_1) \\ &\quad \cdot\cos[2\pi(pf_1-f_s)t + \varphi(f_0+pf_1) - \phi] \end{aligned} \tag{6}$$

where R is the responsivity of PD. From Eq. (6), the phase modulated optical sidebands at $f_0 + pf_1$ in the optical domain are mapped into to the spectrum components at $pf_1 - f_s$ in electrical domain as

$$i(pf_1-f_s) = 2ABJ_p(\beta_1)\,H(f_0+pf_1)\,R(pf_1-f_s) \tag{7}$$

In order to eliminate the impact from other devices like the PM and the PD, the reference measurement can be obtained through connecting port ***a*** and port ***b*** directly without DUT, as given by

$$i'(pf_1-f_s) = 2ABJ_p(\beta_1)\,R(pf_1-f_s) \tag{8}$$

According to Eqs. (7) and (8), the magnitude-frequency response of the DUT at frequencies $f_0 \pm pf_1$ can be expressed as

$$H(f_0 \pm pf_1) = \frac{i(pf_1 \mp f_s)}{i'(pf_1 \mp f_s)} \tag{9}$$

The frequency response of the DUT consists of magnitude and phase parts in the type of complex exponential and usually expressed as

$$\tilde{H}(f) = H(f)\, e^{j\varphi_{\min}(f)+j2m\pi} \tag{10}$$

where $\varphi_{\min}(f) = \varphi(f) - 2m\pi$ is the minimum-phase response in the range of $(-\pi\ \pi]$.

In most cases, an optical filter can be considered as a causal, linear and time-invariant system said to be with a minimum phase filter, including a uniform or apodized fiber Bragg grating (FBG), a PS-FBG, a Fabry-Perot filter, a WGM resonator and a microring resonator [23]. In these cases, the minimum-phase response of optical filters can be related to the magnitude response by the well-known Kramers-Krönig relation as [24], [25]

$$\varphi_{\min}(f) = -\frac{f}{\pi}P\int_{-\infty}^{+\infty}\frac{\ln H(f)}{f'^2 - f^2}df' \tag{11}$$

where P means principal part. Direct calculation of the integral in Eq. (11) is complicated and time-consuming. Alternatively, the Wiener-Lee transform is usually employed to numerically calculate the integral, which is based on the assumption of an even function ln $H(f)$ and an odd function $\varphi_{\min}(f)$ written in trigonometric series as [23], [26]–[29]

$$\ln H(f) = \sum_{n=0}^{+\infty} a_n \cos 2\pi n f \tag{12a}$$

$$\varphi_{\min}(f) = \sum_{n=1}^{+\infty} b_n \sin 2\pi n f \tag{12b}$$

where a_n and b_n are the coefficients of the cosine and sine series with the relationship of $a_n = -b_n$ $(n \geq 1)$, respectively. The function $\varphi_{\min}(f)$ can be solved based on Eq. (12b), and the intrinsic phase-frequency response of the DUT can be extracted.

In the first step, we apply the natural logarithm on the measured magnitude response H as $\ln(H) = [h_1, h_2, h_3, \ldots, h_{n-2}, h_{n-1}, h_n]$. In order to apply the Wiener-Lee transformation, we build a transitional magnitude response $H' = [h_n, h_{n-1}, h_{n-2}, \ldots, h_3, h_2, h_1, h_1, h_2, h_3, \ldots, h_{n-2}, h_{n-1}, h_n]$ with even symmetry by using ln(H), as shown in Fig. 1(c). In the second step, we apply the Fast Fourier transformation (FFT) of H', and obtain the Fourier expansion coefficients $C = [c_1, c_2, c_3, \ldots, c_{2n-1}, c_{2n}]$ and the cosine series coefficients in Eq. (12a) for even function H' is $[a_0, a_1, a_2, \ldots, a_{n-2}, a_{n-1}] = 2[c_1, c_2, c_3, \ldots, c_{n-1}, c_n]$. In the third step, let $a_n = -b_n$ $(n \geq 1)$ and $b_0 = 0$, the Fourier expansion coefficients for a transitional phase response $\varphi'_{\min}$ can be built as $B = [0, -c_2, -c_3, \ldots, -c_{n-1}, -c_n, 0, c_n, c_{n-1}, \ldots, c_3, c_2]$. In the fourth step, we apply inverse FFT of B, the transitional phase response $\varphi'_{\min}$ can be obtained as $\varphi'_{\min} = [d_1, d_2, d_3, \ldots, d_{2n-1}, d_{2n}]$, which is odd symmetry. Finally, the phase response $\varphi_{\min}$ is recovered as $\varphi_{\min} = [d_{n+1}, d_{n+2}, d_{n+3}, \ldots, d_{2n-1}, d_{2n}]$, that is, the phase-frequency response of DUT under the minimum group delay.

Our method features the following advantages. Firstly, both of the magnitude- and phase-frequency response measurements are free of the bias drifting, the undesired spurious sidebands and the power imbalance and phase difference of FHI, thanks to the bias-free phase modulator and heterodyne interferometer. Secondly, from Eq. (9), the microwave signal at f_1 can be used to measure the magnitude-frequency response of DUT by both the upper and lower harmonic sidebands, i.e., $H(f_0 - pf_1)$ and $H(f_0 + pf_1)$, so the proposed method enables the even-fold ($2 \times p$) measuring frequency range. Last but not least, ours allows simultaneously extracting the magnitude- and phase-frequency response with high-resolution and wide frequency range by the harmonics heterodyne and Wiener-Lee transformation.

III. Experimental Demonstration

A proof-of-concept experiment is demonstrated based on the setup shown in Fig. 1(a). An optical carrier with the wavelength of 1550.125 nm and output power of 11.54 dBm comes from a narrow-linewidth tunable laser diode (TLD). In the upper arm of FHI, the optical carrier is firstly phase modulated by a microwave signal at the frequency of f_1 from a microwave source (MS, R&S SMA 100A) using a z-cut $LiNbO_3$ phase modulator (PM, COVEGA 10027), and then modified by a phase-shifted FBG, which is employed as the DUT. For a compromised measuring resolution and sweeping time, the MS is swept up to 20 GHz with a frequency step of 50 MHz for a 80-GHz measuring frequency range. The optical carrier in the lower arm of FHI is frequency-shifted by 70 MHz utilizing an acousto-optic frequency shifter (FS, CETC F-FSG70). The optical signals from upper and lower arms are combined through an optical coupler at the end of the FHI, and detected by a 45-GHz PD (New Focus 1014). The generated electrical signals are then received by an electrical spectrum analyzer (ESA, R&S FSU50). The MS and ESA are connected to a computer and controlled by a MATLAB program via NI-VISA protocols.

Fig. 2 shows a typical electrical spectrum of the heterodyning signals with the DUT in the case of $f_1 = 10$ GHz and $f_s = 70$ MHz, where the resolution bandwidth (RBW) of the ESA is set to be 50 kHz. The electrical spectrum holds extremely narrow spectrum lines due to the inherent coherence of the optical heterodyning signals originating from the same optical carrier. The desired heterodyning components for measuring magnitude-frequency response $H(f_0 \pm f_1)$ and $H(f_0 \pm 2f_1)$ are at $f_1 \pm f_s$ and $2f_1 \pm f_s$, respectively, as shown in the inset of Fig. 2. For the calibration, the electrical spectra of the heterodyning signals without and with DUT are both obtained for reference. As shown in Fig. 3(a) and 3(b), the electrical power at the frequency of $f_1 - f_s = 9.93$ GHz is measured to be -11.69 dBm

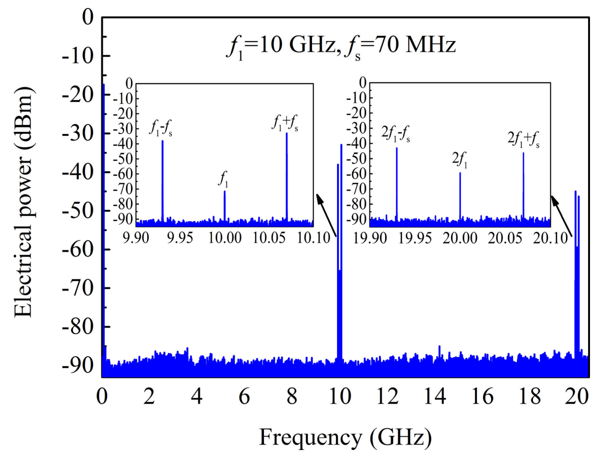

Fig. 2. Measured electrical spectrum of the heterodyning signals after PD in the case of $f_1 = 10$ GHz and $f_s = 70$ MHz, where the inset shows a zoom-in of the spectrum lines at f_1, $f_1 \pm f_s$ and $2f_1 \pm f_s$.

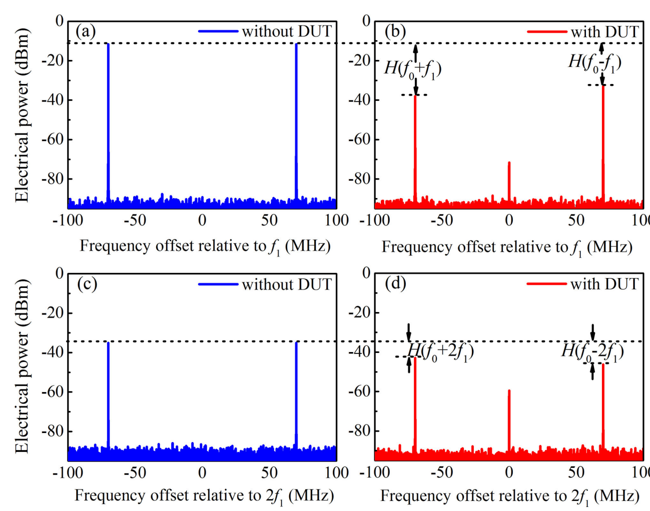

Fig. 3. Measured electrical spectra in the case of $f_1 = 10$ GHz and $f_s = 70$ MHz, (a) without and (b) with DUT for first-order harmonic heterodyning, (c) without and (d) with DUT for second-order harmonic heterodyning.

and 38.06 dBm, respectively, from the first-order harmonic heterodyne. Based on Eq. (9), the magnitude-frequency response $H(f_0 + f_1)$ of DUT is calculated to be 26.37 dB at the frequency $f_0 + f_1$. Similarly, $H(f_0 - f_1)$ is solved to be 19.82 dB at $f_0 - f_1$. At the same time, the magnitude-frequency response $H(f_0 + 2f_1)$ and $H(f_0 - 2f_1)$ can be also obtained with the second-order harmonic signals at $2f_1 - f_s$ and $2f_1 + f_s$, which are measured to be -7.89 dB and -11.19 dB, respectively, as shown in Fig. 3(c) and 3(d). Therefore, the microwave signal at f_1 can be used to measure the magnitude response at $f_0 \pm f_1$ and $f_0 \pm 2f_1$, so our method enables a four-fold ($4\times$) measuring frequency range compared with the SSB-based method. The measurement can be easily operated at other frequencies by changing the microwave frequency f_1.

Fig. 4(a) shows the magnitude-frequency response of DUT. The measuring frequency range of 80 GHz is realized in single frequency-swept range of 20 GHz. It should be note that the sweeping frequency range of MS is about quarter of the measuring frequency range with the second-order harmonic hetero-

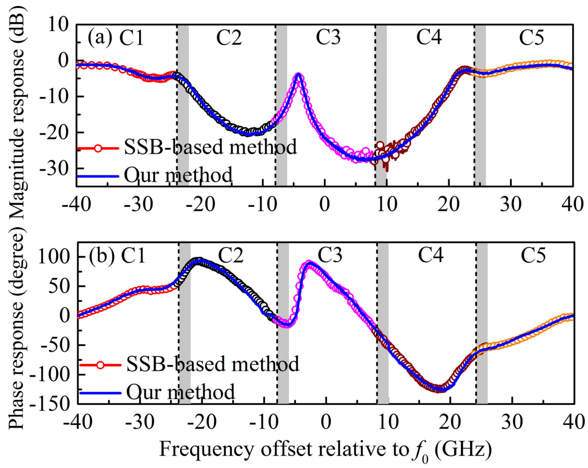

Fig. 4. Measured (a) magnitude-frequency response and (b) phase-frequency response with SSB-based method and our method.

dyne, and the bandwidth of PD and ESA are about half of the measuring frequency range. The measuring frequency range can be further improved by higher-order harmonics heterodyne. For comparison, the magnitude-frequency response of DUT is also measured based on the SSB-based method using an electrical vector network analyzer (EVNA, HP 8703A). The sweeping frequency range of 18 GHz is limited by the bandwidth of the 90-deg electrical hybrid coupler (ABACUS MICROWAVE 9-010180) in the electro-optical SSB modulation. The total 80-GHz measuring frequency range is divided into five 16-GHz channels (C1–C5), as shown in Fig. 4. The 2-GHz overlapping between neighboring channels is used for stitching and reconstructing the magnitude- and phase-frequency response of DUT. As the open circles shown in Fig. 4(a), a complete magnitude-frequency response of the DUT is presented by stitching the magnitude responses in the adjacent channels with a frequency offset of 18 GHz and a frequency overlapping of 2 GHz. It can be seen from Fig. 4(a), the measured magnitude-frequency responses obtained with our method and the SSB method agree well with each other. With the magnitude-frequency response of DUT, the minimum-phase frequency response can be calculated by using the Kramers-Krönig relations and Wiener-Lee transformation, as shown in Fig. 4(b), which has the similar tendency to that of the SSB-based method by using multiple channel stitching. It should be noted that the phase-frequency response measured with the SSB-based method often includes the linear phase response of DUT caused by the constant time-delay of the pigtail fiber of DUT, which requires extra measurement for subtracting the intrinsic phase-frequency response. The pigtail fiber is about 0.62 meter in length and its time-delay is measured to be 3130.2 ps. In order to further verify our method, the measured frequency response of DUT with our method is also compared, in terms of magnitude and group delay, with the method by using an OVA (LUNA, OVA 5000), as shown in Fig. 5. The relative group delay is directly obtained by differentiate the minimum phase response of DUT, which is similar but shifted by about 3137.5 ps with respect to the measured group delay by using the OVA method.

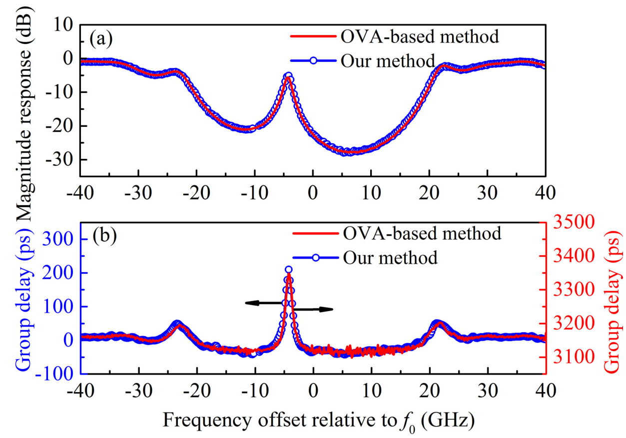

Fig. 5. Measured (a) magnitude-frequency response and (b) group delay with OVA-based method and our method.

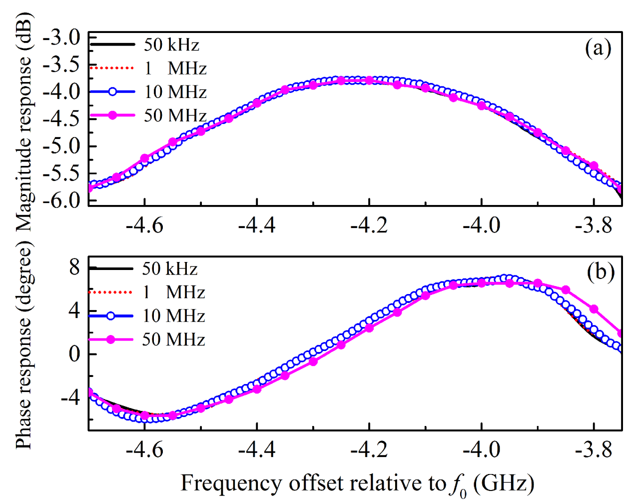

Fig. 6. Hyperfine frequency response measurement with different frequency-swept step. (a) Magnitude-frequency response. (b) Phase-frequency response.

It is also worthy noticing that the frequency resolution of our measurement is mainly limited by the tuning resolution of the MS, the RBW of ESA, and the linewidth of the laser source, which can be much less than 50 MHz. For hyperfine resolution, we demonstrate a detail measurement for the transmission peak with the frequency step of 50 MHz, 10 MHz, 1 MHz and even 50 kHz, respectively. As is shown in Figs. 6(a) and 6(b), the magnitude- and phase-frequency responses results are both very close to each other, indicating the high-resolution and repeatable measurement. Moreover, the measuring frequency range of the proposed method is four-fold the frequency-swept range of the MS, and can be further extended by multiple sweepings with different optical carriers and/or higher-order harmonic heterodyne.

For the accuracy, the uncertainty of the measured magnitude-frequency response can be written as [30], [31]

$$\frac{\delta H\left(f_0 \pm p f_1\right)}{H\left(f_0 \pm p f_1\right)} = \frac{\delta i\left(p f_1 \mp f_s\right)}{i\left(p f_1 \mp f_s\right)} - \frac{\delta i'\left(p f_1 \mp f_s\right)}{i'\left(p f_1 \mp f_s\right)} \quad (13)$$

which is derived from the total derivative of Eq. (9). $\delta H(f_0 \pm pf_1)$ are the measurement errors of the magnitude-frequency

TABLE I
PERFORMANCE COMPARISON BETWEEN OUR METHOD AND OTHER WORK

	Method	MFR/GHz	FRMF	Magnitude- or phase-frequency response	Resolution	Small-signal assumption	Modulation nonlinearity
[5]	OSA	≥100	1	Mag only	1.25 GHz	Free	Independent
[9]	OWS	≥100	1	Both	125 MHz	Free	Independent
[13]	SSB	29	1	Both	78 kHz	Restricted	Dependent
[18]	DSB	80	2	Both	667 kHz	Restricted	Dependent
[20]	EOH	40	2	Mag only	10 MHz	Free	Independent
Ours	EOHH	≥80	≥4	Both	50 kHz	Free	Independent

MFR: measuring frequency range; FRMF: frequency range multiplication factor (defined by the ratio between the measuring frequency range and sweeping frequency range); OWS: optical wavelength-swept; EOH: electro-optical heterodyne; EOHH: electro-optical harmonics heterodyne.

response at the frequencies of $f_0 \pm pf_1$. $\delta i(pf_1 \mp f_s)$ and $\delta i'(pf_1 \mp f_s)$ are the measurement errors of the photocurrent at the frequencies of $pf_1 \mp f_s$ with and without DUT, respectively. In our experiment, the uncertainty of the measured electrical power is smaller than 0.1 dB according to the specification of ESA. Therefore, the measurement uncertainty is less than 0.2 dB, indicating that a total relative error of less than 2.33% might be delivered to the measured magnitude-frequency response in the worst case. With the worst magnitude-frequency response measurement error of 2.33%, the phase-frequency response measurement is estimated with an RMS phase error of 0.6 degree and peak phase error of 1.7 degree in the worst case.

We also make a comparison between our method and other works, as illustrated in Table I. Compared with the optical methods [4], [9], the measurement resolution level is improved from GHz to kHz and increased by 6 orders of magnitude. The proposed method enables hyperfine response measurement for most optical filters. Unlike the electro-optical SSB- or DSB-based methods [13], [18], our method eliminates the operation of small-signal assumption, which is immunity to the modulation nonlinearity. Prior to the conventional electro-optical heterodyne method, our method allows simultaneous extraction of magnitude and phase response for most optical filters. Moreover, the measuring frequency range is four-fold (4×) the sweeping frequency range.

IV. CONCLUSION

In summary, an electro-optical heterodyne method for measuring the intrinsic magnitude- and phase-frequency response of optical filters is proposed based on harmonics optical heterodyne and Wiener-Lee transformation. In the experimental demonstration, the intrinsic frequency response of a phase-shift FBG is measured with the measuring frequency range of 80 GHz and the frequency resolution of 50 kHz by using the frequency-swept modulation of 20 GHz. It should be noted that the measurement resolution can be up to Hz level by decreasing the frequency-swept step, benefiting from the coherence nature of the proposed frequency-shifted harmonics heterodyne. Moreover, the measuring frequency range can be extended by using ultra-wide phase modulation and heterodyne detection and/or stitched multiple segmental measurements with different optical carrier. Besides, the measuring frequency range can be further extended by more than four-fold (4×) through the high-order harmonic optical heterodyne. Therefore, we realize intrinsic magnitude- and phase-frequency response measurement for optical filters with wide frequency range and high-resolution based on harmonic optical heterodyne and Wiener-Lee transformation.

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