Extension for "Consistent Optical and Electrical Noise Figure"

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***Abstract*—This extends a published paper. Noise figure from optical down toward electrical domain is described and related issues are touched upon. The minimum noise figure of an electrical amplifier is $F_e = 1$. It is the SNR degradation factor in a linear transmission with two available receiver quadratures, i.e., one mode. E. Desurvire's traditional optical noise figure F_{pnf} in direct-detection receivers has the minimum value $F_{pnf} = 2$ for an amplifier with high gain. F_{pnf} is not the SNR degradation factor in a transmission with two available receiver quadratures. The photodetector acts like an extra squaring power meter inserted before a power meter. It does not fulfill the linearity condition. Other than transmissive devices it suppresses one signal degree-of-freedom. The latter is the reason for the minimum $F_{pnf} = 2$ instead of value 1. Optical amplifiers such as EDFAs cannot be held responsible for this, given that these are exact equivalents of electrical amplifiers. In the electrical domain there is intrinsic thermal source noise. But in the optical domain there is intrinsic detection shot noise which means that the number of quadratures matters. If F_{pnf} is a valid noise figure then power and gain need to be redefined and the system linearity requirement of noise figure definition must be overcome. F_{pnf} is in contradiction with physics and is incompatible with F_e. It is to be replaced by the correct optical noise figure $F_{o,IQ}$ for linear transmission with two available receiver quadratures. That is observable in coherent I&Q receivers, has the minimum $F_{o,IQ} = 1$ and is compatible with F_e. In the derivation of the consistent unified noise figure F_{IQ} for all frequencies, with the limit cases F_e and $F_{o,IQ}$, thermal noise energy is needed. Its usual simplified expression kT is now replaced by Nyquist's correct result. This holds also in a unified homodyne noise figure F_I, against which H. Haus' unified noise figure F_{fas} is discussed. When there is no I&Q power splitting and one degree-of-freedom is suppressed, relative strength of detection noise is halved, thereby doubling the SNR degradation introduced by an EDFA. Accordingly, the ideal F_I of a standard amplifier changes from 1 in electronics to $F_{o,I} = 2$ in optics. $F_{o,I}$ is the optical homodyne noise figure.**

***Index Terms*—Noise factor, noise figure, optical amplifiers, optical fiber communication.**

I. Introduction

EQUATION, figure and reference numbering of the original paper [17] is continued here. The correct optical I&Q noise figure $F_{o,IQ}$ as the 1:1 equivalent of the electrical noise figure F_e has been derived in [17]. Nothing needs to be changed there regarding optical noise figure (NF). The same is true for the optical homodyne NF $F_{o,I}$.

This paper adds to the description of the unified NF F_{IQ} for all frequencies. Thermal noise energy must be corrected from its usual simplified expression kT to Nyquist's accurate expression, in order to "avoid the UV catastrophe" (Section IV). For sake of completeness, the unified NF F_{fas} [6] of pioneer H. Haus is compared against the corrected unified homodyne NF F_I (Section V). But we start with distributed optical amplifiers (Section II) and a discussion of noise figure issues (Section III), to deliver a complete picture.

A caution about lab jargon: When NF is given as a factor and not in dB then in reality the noise factor (= SNR quotient) is meant. "noise figure" = (10 dB)$\cdot\log_{10}$("noise factor").

For introduction and summary, NF symbols are listed chronologically and with comments which are substantiated by [17] and this paper. Stated lower limits apply only for ideal amplifiers with high gain and 2 available quadratures! The expectation value of equivalent input-referred detectable noise photons per mode is $\tilde{\mu} = n_{sp}(1 - 1/G)$ [17].

- F_e = the NF [1], more precisely the electrical version of the NF, defined or understood as the SNR degradation factor $\geq$1 in a linear system with 2 available receiver (RX) quadratures (i.e., 1 mode)
- $F_{pnf} = 1/G + 2\tilde{\mu}$ = traditional optical NF $\geq$2 of detected photon number fluctuation, defined [3] for intensity modulation with direct detection (IM/DD), which is a nonlinear system that keeps only 1 quadrature (or degree-of-freedom)
- $F_{ase} = 1 + \tilde{\mu}$ = optical [5] and unified [6] NF $\geq$2, defined and intended for amplified spontaneous emission in a linear system with 2 available RX quadratures, but not equal to its optical SNR degradation factor
- $F_{fas} = 1/G + 2\tilde{\mu}$ = optical NF $\geq$2 with conflicting definitions [5], [6], in the latest version also a unified NF [6], slightly wrong regarding thermal noise at high frequencies, linear with seemingly 1 available RX quadrature in the optical domain, meant to become F_e in the electrical domain where there are 2 available RX quadratures
- $F_{o,IQ} = 1/G + \tilde{\mu}$ = the optical NF $\geq$1 [17] derived according to [1] as the SNR degradation factor in a linear system with 2 available RX quadratures
- F_{IQ} = the NF $\geq$1 [17] for all frequencies, with the limit cases of optical $F_{o,IQ}$ and electrical F_e
- $F_{o,I} = 1/G + 2\tilde{\mu}$ = optical homodyne NF $\geq$2 [17] derived as the SNR degradation factor in a linear system with 1

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available RX quadrature, equal to final version of optical F_{fas}, equal in value to F_{pnf}

- F_I = homodyne NF [17] for all frequencies, with the limit cases of optical homodyne $F_{o,I}$ and electrical homodyne, which is the same as F_e

II. Distributed Optical Amplification

It is worth investigating the NF of distributed optical amplifiers such as Raman. In these the whole fiber is the amplifier. Gain $G(\tau)$ until flight time τ and total gain of a distributed amplifier (ab) with group delay τ_g are

$$G(\tau) = e^{\int_0^\tau (a(\vartheta)-b(\vartheta))d\vartheta} \quad G_{\text{ab}} = G(\tau_g). \tag{65}$$

Raman gain causes a, the stimulated emission coefficient per time unit. Fiber loss without pumping strongly contributes to b, the absorption coefficient per time unit. Friis' cascading formula (45) can be rewritten as

$$F_{\text{ab}} - 1 = \int G^{-1}(\tau)\, d(F_{\text{i}}(\tau) - 1). \tag{66}$$

The differential $d(F_{\text{i}}(\tau) - 1) \to 0$ is the excess noise figure of an infinitesimally short amplifier (i) with group delay $d\tau \to 0$ and gain $G_{\text{i}}(\tau) = G(\tau)/G(\tau - d\tau) = 1 + (a(\tau) - b(\tau))d\tau$. Using (37), (42) and $n_{sp} = a/(a-b)$ we obtain $d(F_{o,IQ,\text{i}}(\tau) - 1)/d\tau = b(\tau)$ and $d(F_{o,I,\text{i}}(\tau) - 1)/d\tau = a(\tau) + b(\tau)$. This allows substituting the integration variable and results in

$$F_{o,IQ,\text{ab}} = 1 + \int_0^{\tau_g} b(\tau)\, e^{-\int_0^\tau (a(\vartheta)-b(\vartheta))d\vartheta} d\tau \geq 1, \tag{67}$$

$$F_{o,I,\text{ab}} = 1 + \int_0^{\tau_g} (a(\tau) + b(\tau))\, e^{-\int_0^\tau (a(\vartheta)-b(\vartheta))d\vartheta} d\tau = F_{pnf,\text{ab}} \geq 1. \tag{68}$$

We see that Raman amplifiers always have positive (in dB) NF.

The longitudinal coordinate L inside the amplifier is $L(t) = \int_0^t v_g(\tau)d\tau$, where v_g is the group velocity. The resulting function $L(t)$ can be inverted into $t(L)$. Using $dL/dt = v_g$ the integrations over time can be replaced by integrations over L. Often v_g is constant anyway.

If one tries to substitute the distributed amplifier (ab) by the unpumped fiber (b) with $G_{\text{b}} < 1$ and $F_{o,IQ,\text{b}} = 1/G_{\text{b}}$, followed by an equivalent concentrated amplifier (a) with $G_{\text{a}} = G_{\text{ab}}/G_{\text{b}}$ then it (a) must have an $F_{o,IQ,\text{a}} = G_{\text{b}} F_{o,IQ,\text{ab}}$, which is often unrealizable ($F_{o,IQ,\text{a}} < 1$, i.e., < 0 dB). The mistake here is that G_{a} occurs before the fiber end where $SNR_{in,\text{a}}$ is determined. But by NF definition G_{a} must occur behind the point where $SNR_{in,\text{a}}$ is determined. Similarly, the insertion of a transmitter booster amplifier (a) with $F_{o,IQ,\text{a}} > 1$ before a strongly attenuating fiber link (b) must not be interpreted as insertion of (a) with $F_{o,IQ,\text{a}} < 1$ behind the link (b). The RX input must be defined before the amplifier.

III. Discussion of Noise Figure Issues

Each mode (at frequency $f \neq 0$) has 2 quadratures. Why does F_e imply 2 available RX quadratures, even if a thermal or Schottky diode power detector is used? Well, such power detector could be equivalently replaced by a high-gain amplifier, followed by a 1:1 power splitter. At its two outputs, I and Q components of the signal could be synchronously detected and used, for NF measurement (after squaring, lowpass filtering and usual subtraction of power offsets) and in parallel for other purposes such as I&Q information transmission. In optics, a coherent I&Q RX works the same way, but a homodyne RX and an IM/DD RX don't.

$F_{ase} = 1 + \tilde{\mu}$ [5], [6] was defined for a linear RX with 2 quadratures. But F_{ase} differs from $F_{o,IQ} = 1/G + \tilde{\mu}$ which has been derived for the same scenario. This means, F_{ase} is not the SNR degradation factor in the intended scenario. Moreover, since F_{ase} contains the thermal noise term 1 instead of the shot noise term $1/G$ [17] it cannot be the SNR degradation factor in any optical RX. As a consequence, F_{ase} is not an optical NF.

Returning toward $F_{o,IQ}$, we look at the NF in an ideal optical heterodyne RX. If it is optically broadband then the SNR is half that of an ideal true homodyne RX. For NF calculation, SNR quotients are taken. This means $F_{o,het} = F_{o,I}$ for a heterodyne (*het*) RX. The heterodyne SNR can be improved with an optical image rejection filter. Amplifier noise is then only half as high, like in an I&Q RX. SNR and NF become the same as for I&Q, i.e., $F_{o,het+ir} = F_{o,IQ}$ for a heterodyne RX with image rejection (*ir*).

Next consider an optical receiver with a 1:1 power splitter at its input. At each power splitter output an ideal phase-sensitive amplifier is connected. One of them amplifies in-phase signals. The other amplifies quadrature signals. This can be achieved by suitable pumping. The phase-sensitive amplifiers yield sensitivities like homodyne receivers. But referred to the receiver input (before power splitting) they have sensitivities like an I&Q RX. So, also in this setup $F_{o,IQ}$ is the relevant NF.

Let us switch now to the primary topic of [17] and this work. The above makes clear that $F_{o,IQ}$ is observed in any ideal linear optical RX with two available quadratures, just like F_e is observed in the corresponding electrical case. So, $F_{o,IQ}$ is not a (new) definition. Rather it results from applying, at optical frequencies, the existing original NF definition, which yields the SNR degradation factor observed in an ideal linear RX with two available quadratures. Whether both RX quadratures are used does not matter; it suffices that they are available.

With respect to tradition, the need for deriving $F_{o,IQ}$ [17] is illustrated in Fig. 5. The insertion of a photodiode into the signal path [3] as a kind of extra power meter is opposed to NF definition (linear channel, 2 available RX quadratures; minimum amplifier NF normally equals 1) [18]. As will be seen, the resulting traditional optical NF of E. Desurvire [3], called F_{pnf} in [5], is in contradiction with ~150 years of science.

High-frequency engineers would reject the idea of inserting an extra squaring power meter into the linear signal path. And so should optical engineers. But the photodiode needed in F_{pnf} definition acts as a squarer and power meter and eliminates

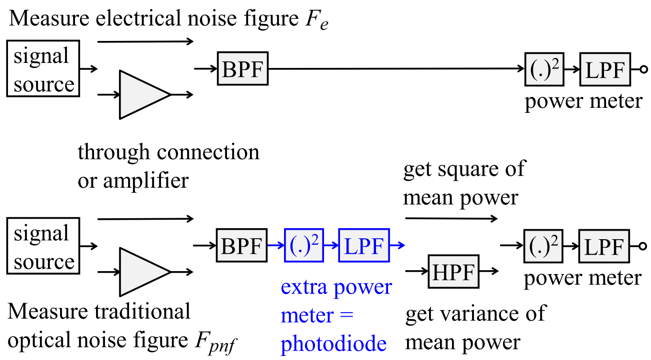

Fig. 5. Measurement of electrical and of traditional optical noise figure. LPF/BPF/HPF = lowpass/bandpass/highpass filter.

linearity. It also eliminates one of the signal's two quadratures or degrees-of-freedom, like a homodyne RX.

Subsequently the needed "power" = "P" $\sim I^2 \sim P^2$, i.e., electrical power of a photocurrent I flowing through a load resistor, is proportional to the square of the optical power P. It holds $P \sim |\mathbf{E}|^2$ where $\mathbf{E}$ is the optical field. This means "P" $\sim |\mathbf{E}|^4$ and work = sqrt("P") · time! Power must be redefined! By definition,

$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{P_{signal,in}P_{noise,out}}{P_{noise,in}P_{signal,out}} = \frac{G_{noise}}{G_{signal}}. \tag{69}$$

The expression $F = G_{noise}/G_{signal}$ is correct because NF definition assumes a linear system.

Science is systematic and exact and does not tolerate contradictions. Definitions such as power, gain, NF must not depend on measurement method or f.

If we set $F = F_{pnf}$ and derive (69) using "P" $\sim I^2$ then we find that G_{noise} is nonlinear and signal-dependent and any amplifier with gain $G \equiv G_{signal}$ has a supposed gain $G_{signal} = G^2$! As a consequence, G is replaced by G^2, fiber loss 0.2 dB/km becomes 0.4 dB/km. No more (optical) dB are allowed, only "electrical" dB!

When measuring F_{pnf} the photodiode may be replaced by a thermal power detector (bolometer). Now "P" $\sim (\Delta T)^2$ is proportional to the square of the detected temperature difference ΔT. Thermal detectors can be built at all f. With appropriate preamplifiers they can detect even smallest powers. We decrease f until we arrive in the electrical domain. Here we find "P" $\sim |U|^4$ where U is the input voltage, and again $G \equiv G_{signal} = G^2$! All powers can be converted into thermal power and can be compared this way. Therefore F_{pnf} with its non-standard power definition implies more false statements, for instance: Mechanical power is no longer force · velocity v, or rolling resistance · v^2. Instead it becomes "P" $\sim v^4$. Also, work = $\sqrt{''\mathrm{P}''}$· time.

All this is direct consequence of calling F_{pnf} a NF. So, either F_{pnf} is a NF. Or, as we know, the NF has been defined for linear systems, and basic physics $P \sim |U|^2$ and G are correct.

All the same, F_{pnf} [3] has great historic merits in the development of optical communication.

$F_{pnf} = 2$ is found for an ideal optical amplifier with large gain. This seems inappropriate because the sensitivity of an ideal optical receiver with 2 available quadratures is not degraded by an ideal optical preamplifier.

While H. Haus exposed F_{pnf} to violate NF definition [5], his proposed solutions [5], [6] F_{fas} (rare case, optical homodyne NF for 1 quadrature, other than F_e for 2 quadratures), F_{ase} (no NF because it is not the SNR degradation factor in any optical receiver) also have a minimum value of 2 for an ideal amplifier, unlike F_e. Seemingly, the optical NF definitions prior to the electrically/physically correct derivation of $F_{o,IQ}$ [17] (linear RX with 2 available quadratures, like known from F_e) have all been guided by the idea that the NF of a standard ideal optical amplifier should become equal to 2 (instead of 1). Usually the value 2 is explained by claiming that optical amplifiers be special.

But standard optical amplifiers such as EDFAs are not special. They amplify 2 quadratures and add Gaussian amplitude (field) noise in the 2 quadratures [17], like standard electrical amplifiers. Also not special is optical detection noise (photon/particle aspect manifests) [19] as opposed to electrical source noise (thermal origin). Special in combination with optical detection noise are true homodyne detection and direct detection because these keep only 1 degree-of-freedom or quadrature and suppress the other. For an ideal optical amplifier one heuristically finds [18] the minimum

$$F_{opt,\min} = \frac{\text{number of available quadratures in amplifier}}{\text{number of available quadratures in receiver}} \tag{70}$$

where F_{opt} is any optical NF. Even though optical direct detection is nonlinear, also F_{pnf} obeys (70) with 1 available quadrature in receiver. F_{ase} is not covered by (70), given that F_{ase} is not the NF in any optical RX. Clearly it makes normal sense to have the same number of available quadratures in amplifier and RX, and to choose this number equal to 2 (i.e., 1 mode) like in the electrical case F_e. This is confirmed by the correct NF $F_{o,IQ} \geq 1$ of a standard optical amplifier [17].

Instead of claiming $F_{o,\min} = 2$ to be normal (F_{pnf}, $F_{fas} = F_{o,I}$, also F_{ase}, all with standard optical amplifier) one could with the same right claim $F_{opt,\min} = 1/2$ to be normal (degenerate parametric optical preamplifier with $F_{o,IQ} = 1/2$ blocks one quadrature and increases I&Q RX sensitivity to that of a true homodyne RX). Both is mathematically correct, but none should be considered as the normal case.

An EDFA improves practical IM/DD RX sensitivity by typically >10 dB even though F_{pnf} suggests that it is degraded by >3 dB. In contrast, change of practical coherent I&Q RX sensitivity by an EDFA is indeed on the order of $F_{o,IQ}$, i.e., close to 0 dB.

F_{pnf} needs to be replaced by the correct $F_{o,IQ}$, i.e., the optical NF; for conversion see (38).

It has been argued that $F_{o,IQ}$, $F_{o,I}$ [17] be not correct because they were derived in semiclassical description. There, a photocurrent I has a one-sided noise power spectral density (PSD) $2eI$ due to a Poisson distribution of photoelectrons. Indeed one can alternately assume zero-point fluctuations with energy

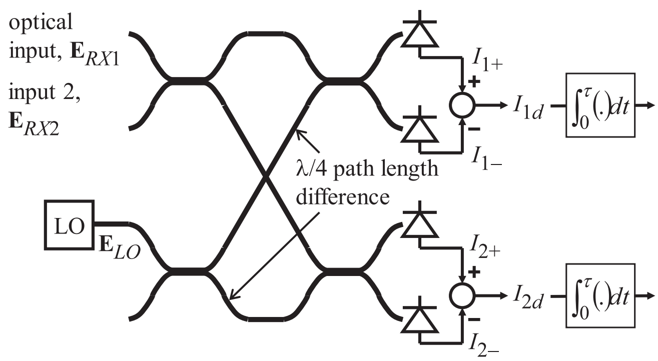

Fig. 6. Coherent I&Q receiver with polarization matching.

Electrical $kT>>hf$	Unified/generalized	Optical $kT<<hf$
kT	$\tilde{\mu} = n_{sp}(1-1/G)$ $k'T = hf/\left(e^{hf/(kT)}-1\right)$	$\tilde{\mu} = n_{sp}(1-1/G)$
2 available RX quadratures (I&Q); the noise figure:		
$F_e \equiv F_{e,IQ}$ $= 1+T_{ex}/T$	$F_{IQ} = \dfrac{k'(T+T_{ex})+(\tilde{\mu}+1/G)hf}{k'T+hf}$	$F_{o,IQ} = \tilde{\mu}+1/G$
1 available RX quadrature; optical homodyne (and IM/DD without sp-sp):		
$F_{e,I} = 1+T_{ex}/T$	$F_I = \dfrac{k'(T+T_{ex})+(2\tilde{\mu}+1/G)hf/2}{k'T+hf/2}$	$F_{o,I} = 2\tilde{\mu}+1/G$

Fig. 7. Noises, I&Q, and homodyne noise figures in linear receivers.

hf/2 per mode. They give rise to the same noise PSD 2*eI* in a photocurrent *I*. Fig. 6 is similar to Fig. 1, but the Y splitters are replaced by 2×2 couplers and each of the 4 inputs gets zero-point fluctuations. Interferences of zero-point fluctuations from the LO coupler inputs with the LO signal eventually cancel upon photocurrent subtraction. Interferences of all the zero-point fluctuations with the received signal are negligible since the LO is strong. Interferences of zero-point fluctuations in $\mathbf{E}_{RX1,2}$ from the signal coupler inputs with the LO signal occur with opposite signs in $I_{1\pm}$. They add upon photocurrent subtraction, just like for any other received signal. Using zero-point fluctuations, exactly our known $F_{o,IQ}$ and $F_{o,I}$ are obtained in the end. Likewise, F_{pnf} can be calculated with either semiclassical shot noise or zero-point fluctuations.

Why does intensity modulation with direct detection (IM/DD) and infinite signal power behave like true homodyne? LO shot noise in a homodyne RX and shot noise in an IM/DD RX with strong input signal can be taken into account by a co-polarized in-phase zero-point fluctuation *u*. If one squares the output signal of the homodyne RX it becomes equivalent to an IM/DD RX: Using our convenient definition $P = |\mathbf{E}|^2$ and the approximation $|\mathbf{E}| = \sqrt{P_S} + u$ for large signal powers P_S we get $P \approx P_S + 2\sqrt{P_S}u$. This results in

$$SNR_{o,I,in} = \frac{P_{S,in}}{\langle u_{in}^2 \rangle} \quad SNR_{o,I,out} = \frac{P_{S,out}}{\langle u_{out}^2 \rangle} \tag{71}$$

$$SNR_{pnf,in} = \frac{P_{S,in}^2}{4\langle u_{in}^2 \rangle P_{S,in}} \quad SNR_{pnf,out} = \frac{P_{S,out}^2}{4\langle u_{out}^2 \rangle P_{S,out}} \tag{72}$$

$$F_{pnf} = \frac{SNR_{pnf,in}}{SNR_{pnf,out}} = \frac{P_{S,in}\langle u_{out}^2 \rangle}{P_{S,out}\langle u_{in}^2 \rangle} = F_{o,I}. \tag{73}$$

Essentially the same, namely $|\mathbf{E}| \sim \sqrt{P}$ and insertion of a square-root device at the output of an IM/DD receiver, has been explained in [22]. Due to the nonlinear (squaring) operation, the traditional F_{pnf} is not even a special NF. Yet F_{pnf} is closely related to the special NF $F_{o,I}$ and is equal to it in the usual approximation of infinite signal power.

Fig. 7 tabularizes noise formulas in linear receivers, which are further discussed in Sections IV, V. Electrical homodyne and electrical I&Q are identical because there is source noise. Optical homodyne is different or special because two things come together: (i) Only 1 available RX quadrature instead of the usual 2. (ii) Shot noise occurs upon (any) optical detection and does not come from the source.

The nonlinear direct detection of intensity modulation (IM/DD) without spontaneous-spontaneous beat noise behaves like optical homodyne.

The unified/generalized NF F_{IQ}, F_I are valid for all frequencies. For $kT << hf$ they become $F_{o,IQ}$, $F_{o,I}$, valid in the optical domain. For $kT >> hf$ they become $F_{e,I} = F_e \equiv F_{e,IQ}$, valid in the electrical domain. The NF F_{IQ}, valid for 2 available RX quadratures or 1 mode as is usual in the electrical domain, has the limit cases $F_{o,IQ}$, F_e.

IV. Correct Thermal Noise at All Frequencies

In the inspiring derivation of a unified NF F_{fas} [6] the mean value of detectable thermal photons per mode was given (in other, equivalent nomenclature) as $\langle n_\vartheta \rangle = kT/(hf)$; see (13) of [6]. Mean thermal noise energy or thermal PSD is hence $\langle n_\vartheta \rangle hf = kT$. In [17] I have adopted this and have derived the unified I&Q NF F_{IQ}, using the optical I&Q NF $F_{o,IQ}$. But in unified noise figures the term *kT* needs to be corrected at high frequencies. Total thermal power is finite. It is expedient to write

$$k'T = \frac{hf}{e^{hf/(kT)}-1} \quad \underline{\text{thermal power spectral density}}. \tag{74}$$

The right hand side is (7) in [20], by Nyquist. Only for $hf << kT$ it approaches *kT*. The left hand side $k'T$ is defined such that where *k* was written in the derivation of F_{IQ} in [17] this is now replaced by k'. k' is a frequency- and temperature-dependent function which approaches the Boltzmann constant *k* in the case $hf << kT$, and 0 in the case $hf >> kT$. This k' is needed in the compilation of Fig. 7, instead of *k* which was assumed in [17] and [6].

Now consider electrical NF measurement. Since a real power detector is thermally noisy the SNR degradation factor underestimates the true F_e. In particular, a sufficiently low-noise amplifier in front of a highly noisy detector will even improve the SNR. To get rid of thermal detector noise one puts the source at two different temperatures and measures noises with the power detector. Linear extrapolation of the measured noises to T =

0 K yields the own thermal noise of the power detector. It is subtracted in all NF calculations. This way the calculated SNRs and NF become higher. In practice, $T = 0$ K cannot be reached, and maybe the particular power detector wouldn't even work at $T = 0$ K. But this does not matter. The NF is simply the quotient of SNRs that one would achieve if the power detector had no thermal noise.

The same principle must be applied for the unified F_{IQ}. In practice, thermal noise occurs also at the unused input 2 (Fig. 6). It must not enter into the NF equations. To this purpose we assume an ideal cooled absorber with temperature 0 K at input 2. Thermal noise from the LO inputs and in electronics behind the photodiodes is likewise eliminated because we have assumed $P_{LO} \to \infty$. In a practical, nonideal receiver a few control measurements allow isolating and removing these thermal noises.

There is yet another idealization: In the photodiode responsivity $R = \frac{\eta e}{hf}$ the efficiency η has been set as $\eta = 1$. This maximizes the SNR. The same $\eta = 1$ is also used in the traditional F_{pnf}.

The corrected equations (k' from (74) instead of k) are:

$$
\begin{aligned}
SNR_{IQ,out} &= \frac{\overline{I_{1d}}^2}{\sigma_e^2 + \sigma_{I_{1d}}^2 + \sigma_{I_{1s}}^2} \\
&= \frac{R^2 G P_s P_{LO}}{R^2 P_{LO} G F_e k' T B_o/2 + R^2 P_{LO} \tilde{\mu} G h f B_o/2 + e R P_{LO} B_e} \\
&= \frac{G P_s}{G F_e k' T B_o/2 + \tilde{\mu} G h f B_o/2 + h f B_e} \\
&= \frac{P_s \tau}{F_e k' T/2 + F_{o,IQ} h f/2} = \frac{P_s \tau}{k' (T + T_{ex})/2 + (\tilde{\mu} + 1/G) h f/2}
\end{aligned} \tag{47}
$$

$$
SNR_{IQ,in} = \frac{P_s \tau}{k' T/2 + h f/2}, \tag{48}
$$

$$
\begin{aligned}
F_{IQ} &= \frac{SNR_{IQ,in}}{SNR_{IQ,out}} = \frac{F_e k' T + F_{o,IQ} h f}{k' T + h f} \\
&= \frac{k' (T + T_{ex}) + (\tilde{\mu} + 1/G) h f}{k' T + h f} \qquad \left(A = \frac{k' T}{k' T + h f}\right) \\
&= A + (1 - A)/G + (A T_{ex}/T + (1 - A)\, \tilde{\mu}).
\end{aligned} \tag{49}
$$

F_{IQ} is the noise figure valid for all frequencies. The change k' instead of k affects also Table I, Figs. 2, 3 and other places in [17]. The same change is needed in [18]. We recognize thermal source noise $k'T$, thermal noise $k'T_{ex}$ added in amplifier, spontaneous emission field noise $\tilde{\mu} h f$ added in amplifier and shot noise hf/G in detector, all input-referred and per mode. For an amplifier it is not important to know the individual contributions of F_e, $F_{o,IQ}$; only the resulting F_{IQ} counts.

Strength of shot noise is known. All other parameters needed to determine optical NF can be measured even with simple mean power measurements, without electrically broadband optical receivers. Fig. 8 sketches measurements with signal and amplifier, without signal, without optical amplifier. When there is no signal a quantum-based amplifier must be loaded with other

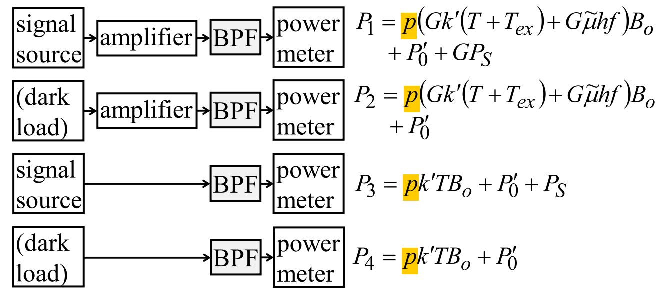

Fig. 8. Mean power measurements for determining optical NF.

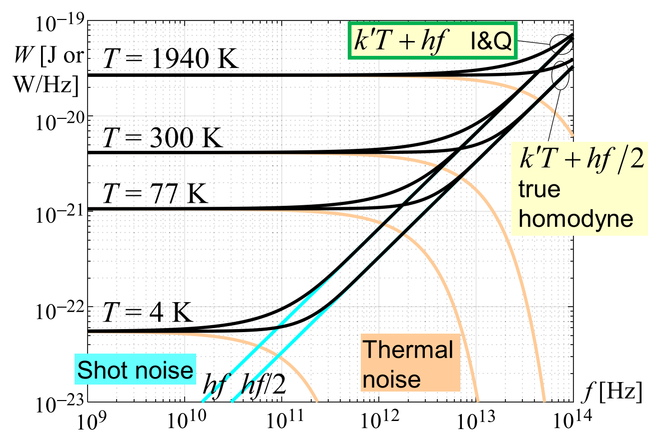

Fig. 9. Intrinsic noise energy W per mode (thermal: orange/brown; shot: cyan; total: black) vs. frequency f for different temperatures.

(signal) power, at other frequencies/time/polarization. Otherwise G would increase because of missing gain saturation. Like drawn, without polarizer, there are $p = 2$ noisy polarization modes. With inserted polarizer $p = 1$ holds. P'_0 is a power readout offset caused by noise generated inside the power meter. Source and added noises $k'T + k'T_{ex} + \tilde{\mu} h f$ can be measured as static powers, unlike scaled shot noise hf/G. Gain $G = (P_1 - P_2)/(P_3 - P_4)$ and added noise $k'T_{ex} + \tilde{\mu} h f = (P_2 - P_4)/(pGB_o) - (1 - 1/G)k'T$ are found. Then all NF can be calculated.

The crossover condition $hf = k'T$ of equally strong thermal and quantum noises yields $hf = kT \ln 2$. This requires $f =$ 194 THz / 28 THz / 4.3 THz / 1.1 THz / 58 GHz at $T = 13400$ K / 1940 K / 300 K / 77 K / 4 K, respectively. In [21], a 66 GHz electronic circuit operates at 4 K. Quantum noise plays a role here. Cryo and space electronics in the mm wave range and possible future THz applications need F_{IQ}. The same would hold for an extremely hot attenuator or amplifier at the CO_2 laser frequency 28 THz.

Fig. 9 shows noise power spectral densities vs. frequency. (In [17] and [6], thermal noise was wrongly assumed to be frequency-independent and not to decrease at high frequencies. Under that wrong assumption the orange/brown thermal noise would be horizontal lines and the black lines would have rounder bends.) Thermal/shot noise crossover conditions for I&Q and for homodyne occur where the orange/brown lines and the cyan straight lines (partly covered by black) intersect. Only half of these noise energies per arriving mode manifests per received quadrature. Note that thermal noise power can be measured also as the average detected power whereas shot noise only shows up as fluctuations during detection of an optical signal.

The total noises of Fig. 9 are found as (82a), (82b) in [22], where thermal and shot (called quantum) noises are already discussed. As we have seen, the difference between F_{pnf} and $F_{o,IQ}$ is not due to the square law action of the photodetector. It is due to outputting one (essentially, if signal is strong) vs. two available RX quadratures.

V. Unified Homodyne Noise Figure

This is investigated in order to complete the picture in the context of F_{fas} in [6]. According to (35) of [17], the signal power P_S in a true homodyne receiver appears multiplied by $4R^2GP_{LO}$. The same holds for the in-phase part $F_e k'TB_o/2$ of the total received thermal noise power $F_e k'TB_o$. We add the product to the denominator of (35) and obtain

$$
\begin{aligned}
&SNR_{I,out}\\
&= \frac{4R^2GP_sP_{LO}}{4R^2P_{LO}GF_ek'TB_o/2+4R^2P_{LO}\tilde{\mu}GhfB_o/2+2eRP_{LO}B_e}\\
&= \frac{2GP_s}{2GF_ek'TB_o/2+2\tilde{\mu}GhfB_o/2+hfB_e}\\
&= \frac{2P_s\tau}{F_ek'T+F_{o,I}hf/2} = \frac{2P_s\tau}{k'(T+T_{ex})+(2\tilde{\mu}+1/G)hf/2}
\end{aligned}
\tag{75}
$$

In the absence of amplifier noise and gain this becomes

$$SNR_{I,in} = \frac{2P_s\tau}{k'T+hf/2}. \tag{76}$$

That can be rewritten as signal energy $P_s\tau$ divided by $k'T/2+hf/4$, i.e., the intrinsic homodyne noise energy in the received quadrature (as explained in the context of Fig. 9). Term $hf/4$ is the zero-point fluctuation energy per quadrature. We get the homodyne / in-phase / single-quadrature NF

$$
\begin{aligned}
F_I &= \frac{SNR_{I,in}}{SNR_{I,out}} = \frac{F_ek'T+F_{o,I}hf/2}{k'T+hf/2}\\
&= \frac{k'(T+T_{ex})+(2\tilde{\mu}+1/G)hf/2}{k'T+hf/2} \quad \left(A_I = \frac{k'T}{k'T+hf/2}\right)\\
&= A_I+(1-A_I)/G+(A_IT_{ex}/T+(1-A_I)2\tilde{\mu}).
\end{aligned}
\tag{77}
$$

Note that in the electrical domain, single-quadrature or homodyne analysis simply means that the other quadrature is suppressed. This can be done by downconversion to baseband in a multiplier/mixer, or by degenerate parametric amplification. As mentioned, the electrical homodyne NF $F_{e,I}$ equals the electrical I&Q NF, $F_{e,I} = F_e \equiv F_{e,IQ}$, because of (thermal) source noise. In (75), (77) one could write $F_{e,I}$ instead of F_e. The ideal F_I of a standard amplifier with high gain and 2 amplified quadratures varies between 1 (electrical, with thermal source noise) and 2 (optical, with detection shot noise).

F_I will probably not be needed in the electrical domain (mm waves at low temperatures) because single-quadrature electric receivers and amplifiers have (in a chosen quadrature) no intrinsic noise advantage over standard electrical receivers and amplifiers. F_I could be applied at the CO_2 laser frequency for an extremely hot device in front of a homodyne receiver.

For $hf >> kT$, H. Haus' unified NF F_{fas}, (18) in [6], becomes $F_{o,I}$. This means F_{fas} is a homodyne NF at optical frequencies. F_{fas} is very similar to F_I. Differences are:

- F_{fas} contains kT in $\langle n_\vartheta \rangle$ instead of the correct $k'T$ (74).
- No clear number of quadratures is defined for F_{fas}. Since F_{fas} should generalize the familiar F_e one is left to assume that in F_{fas} there is 1 quadrature at optical *f*, 1 …2 quadratures at intermediate/thermal *f* and the usual 2 quadratures at electrical *f*. Such transition is of course not possible.
- In F_{fas} it is defined $T_{ex} = 0$ (which is allowed), and all added thermal noise is assumed to be contained in a sufficiently large spontaneous emission factor n_{sp}. For a pure attenuator one must guess and set $n_{sp} = -k'T/(hf)$. Otherwise the needed $F_{fas} = 1/G$ is not reached. For comparison, $F_I = 1/G$ is easily obtained from the known $F_{e,I} = F_e = F_{o,I} = 1/G$ and $n_{sp} = 0$.

References

[1] H. A. Haus, Subcommittee 7.9 on Noise, Chairman, "IRE standards on methods of measuring noise in linear twoports, 1959," *Proc. IRE*, vol. 48, pp. 60–74, 1960.

[3] E. Desurvire, *Erbium Doped Fiber Amplifiers: Principles and Applications*. Hoboken, NJ, USA: Wiley, 1994.

[5] H. A. Haus, "The noise figure of optical amplifiers," *IEEE Photon. Technol. Lett.*, vol. 10, no. 11, pp. 1602–1604, Nov. 1998, doi: 10.1109/68.726763.

[6] H. A. Haus, "Noise figure definition valid from RF to optical frequencies," *IEEE J. Sel. Topics Quantum Electron.*, vol. 6, no. 2, pp. 240–247, Mar./Apr. 2000.

[17] R. Noe, "Consistent optical and electrical noise figure," *J. Lightw. Technol.*, vol. 41, no. 1, pp. 137–148, Jan. 2023, doi: 10.1109/JLT.2022.3212936.

[18] R. Noe, "Noise figure and homodyne noise figure," in *Proc. 24th ITG-Symp. Photonic Netw.*, 2023, pp. 85–91.

[19] R. Noe, "Do propagating lightwaves contain photons?," in *Proc. 24th ITG-Symp. Photonic Netw.*, 2023, pp. 113–121.

[20] H. Nyquist, "Thermal agitation of electric charge in conductors," *Phys. Rev.*, vol. 32, Jul. 1928, Art. no. 110, doi: 10.1103/PhysRev.32.110.

[21] Y. Zhang, X. Jin, W. Liang, P. Sakalas, and M. Schröter, "66 GHz 11.5 mW low-power SiGe frequency quadrupler operating at 300 K and 4 K," in *Proc. 14th German Microw. Conf.*, 2022, pp. 100–103.

[22] B. M. Oliver, "Thermal and quantum noise," *Proc. IEEE*, vol. 53, no. 5, pp. 436–454, May 1965.