




Quantum Limits on the Capacity of Multispan Links With Phase-Sensitive Amplification

Karol Łukanowski , *Graduate Student Member, IEEE*, Konrad Banaszek , *Senior Member, IEEE*,
and Marcin Jarzyna 

Abstract—Long-distance fiber communication stands as a cornerstone of modern technology. One of the underlying principles, preventing signal levels from diminishing below the detectability threshold, is optical amplification. In particular, phase-sensitive amplifiers offer a promising solution as ideally they do not introduce any excess additive noise. Since such devices in principle operate at the quantum noise level, a natural question is whether one can further improve the capacity of amplified links using principles of quantum mechanics as it offers a much broader scope of signal modulations and detection schemes. We derive ultimate limits determined by the laws of quantum mechanics on the capacity of multispan links with phase sensitive amplification. We show that the quantum advantage over the standard approach based on optical quadrature detection is small and vanishes for long links.

Index Terms—Optical amplifiers, optical fiber communication, channel capacity, quantum communication, low-noise amplifiers.

I. INTRODUCTION

THE technological demand for the constantly raising amount of information exchanged between different entities puts extensive pressure on the increase of the communication rates of optical fiber links [1]. One of the main factors that limits the performance of a communication link is the reduction of the signal-to-noise ratio (SNR) due to the presence of losses in a fiber cable or other parts of an optical link. In order to overcome this issue, one may investigate various techniques such as changing the fiber structure or using different modulation formats [1], [2], [3], [4]. The primary way, however, is to incorporate signal amplification, by which one can restore signal power to a desired level. Standard phase-insensitive amplifiers allow to

bring back signal level at the cost of introducing additional noise. Importantly, this noise cannot be reduced below a certain value because of fundamental quantum mechanical effects [5]. This phenomenon causes a decrease of SNR with link length, since the noise introduced by each amplifier in the cable is amplified by subsequent ones. The overall effect is that one can indeed vastly improve communication rates with conventional phase-insensitive amplification, however, the rate still inevitably decreases with the link length [6], [7], [8].

A more sophisticated method of signal restoration is phase-sensitive amplification [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24]. A phase-sensitive amplifier (PSA) in general can amplify one of the signal quadratures while simultaneously reducing the other one [5]. At first sight, this is detrimental to the capacity, since one is able to efficiently transmit information encoded only in a single (amplified) quadrature of light whereas in the previous case both quadratures could carry the information. The advantage, however, is that PSAs are in principle noiseless devices, i.e., they do not introduce any additive noise. It is therefore expected that for large distances, when one would like to use a significant number of amplifiers, they can lead to an improved SNR as compared with the phase-insensitive scenario.

On the other hand, one may go beyond the standard picture of classical information theory and instead inspect a broader range of strategies allowed by the laws of quantum mechanics. In particular, novel detection schemes [25], such as the Dolinar receiver [26], [27] or various collective receivers [28] make it possible in certain scenarios to surpass classical capacity limits imposed by the Shannon-Hartley bound and attain the fundamental quantum capacity limit given by the Gordon-Holevo expression [29], [30], [31], [32]. Crucially, for such more general strategies one has to abandon SNR as a figure of merit and instead consider signal and noise separately. Another possibility for improvement is to use a non-standard modulation format that utilizes such quantum features of light as squeezing which in principle seems to be beneficial for phase sensitive channels [33].

In this article we analyze quantum capacity limits of multispan links in the presence of PSA. We identify two important regimes of amplification, depending on whether just the signal or the total power are restored to their initial values. We show that in the large distance limit in the former regime one obtains an exponential gain in the capacity with respect to the unamplified scenario. On the other hand, for total power restoration the capacity maintains its exponential decay known from the pure loss

Manuscript received 18 August 2022; revised 14 February 2023; accepted 1 March 2023. Date of publication 13 March 2023; date of current version 2 August 2023. This work was supported by the Foundation for Polish Science under the “Quantum Optical Technologies” project carried out within the International Research Agendas programme co-financed by the European Union under the European Regional Development Fund. (Corresponding author: Marcin Jarzyna.)

Karol Łukanowski is with the Centre for Quantum Optical Technologies, Centre of New Technologies, University of Warsaw, 02-097 Warszawa, Poland, and also with the Faculty of Physics, University of Warsaw, 02-093 Warszawa, Poland (e-mail: k.lukanowski@cent.uw.edu.pl).

Konrad Banaszek is with the Centre for Quantum Optical Technologies, Centre of New Technologies, University of Warsaw, 02-097 Warszawa, Poland, and also with the Faculty of Physics, University of Warsaw, 02-093 Warszawa, Poland (e-mail: k.banaszek@uw.edu.pl).

Marcin Jarzyna is with the Centre for Quantum Optical Technologies, Centre of New Technologies, University of Warsaw, 02-097 Warszawa, Poland (e-mail: m.jarzyna@cent.uw.edu.pl).

Color versions of one or more figures in this article are available at <https://doi.org/10.1109/JLT.2023.3256585>.

Digital Object Identifier 10.1109/JLT.2023.3256585

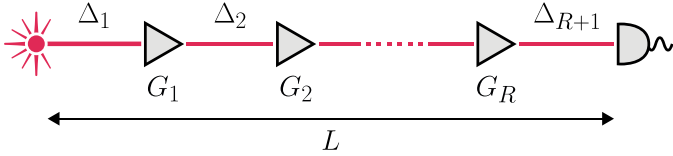


Fig. 1. Scheme of a lossy channel of length L with R amplifiers with gains G_r separated by distances $\Delta_r = l_r - l_{r-1}$ from each other.

channel instance but with an advantage in the form of an improved exponent when compared to both unamplified and phase-insensitive amplification cases. In the range of more typical distances, up to a few thousand kilometers, depending on the signal strength, one observes an exponential advantage in SNR and the capacities are approximately equal in both approaches. Importantly, the enhancement attainable by the most general quantum strategies over the standard Shannon information bound is present only for low distances and becomes negligible for large fiber lengths, meaning that quadrature detection is a nearly optimal detection strategy.

II. PSA FIBER LINK

A basic model of a PSA fiber link, presented schematically in Fig. 1, consists of a standard lossy channel characterized by an attenuation constant α , with total length L and with R PSAs inserted at regeneration nodes. The i -th amplifier is specified by a gain G_i and is located at a distance l_i from the input, $i = 1, 2, \dots, R$. An optical field is characterized by two orthogonal quadratures, denoted by x^Q and x^I . These quadratures can be further decomposed as

$$x^{Q/I} = x_S^{Q/I} + x_N^{Q/I}, \quad (1)$$

where $x_S^{Q/I}$ represents the contribution of the signal and $x_N^{Q/I}$ describes the noise. Since noise and signal are not correlated the quadratures variances can be similarly decomposed

$$\text{Var}[x^{Q/I}] = S^{Q/I} + N^{Q/I}, \quad (2)$$

where $S^{Q/I} = \text{Var}[x_S^{Q/I}]$ denotes the signal power in each quadrature and $N^{Q/I} = \text{Var}[x_N^{Q/I}]$ are the corresponding noise powers. Assuming that no external additive noise is introduced at any point of the link, the quadratures and their variances after the i -th amplifier are equal to

$$x_i^Q = x_{i-1}^Q \sqrt{G_i \tau_i} + \sqrt{1 - \tau_i} x_{N,i}^Q, \quad (3)$$

$$x_i^I = x_{i-1}^I \sqrt{\frac{\tau_i}{G_i}} + \sqrt{1 - \tau_i} x_{N,i}^I, \quad (4)$$

$$S_i^Q = G_i \tau_i S_{i-1}^Q, \quad N_i^Q = G_i \left(\tau_i N_{i-1}^Q + \frac{1 - \tau_i}{2} \right), \quad (5)$$

$$S_i^I = \frac{\tau_i}{G_i} S_{i-1}^I, \quad N_i^I = \frac{1}{G_i} \left(\tau_i N_{i-1}^I + \frac{1 - \tau_i}{2} \right), \quad (6)$$

where $\tau_i = e^{-\alpha(l_i - l_{i-1})}$ and $x_{N,i}^{Q/I}$ denote respectively transmission and noise variables contributed by the i -th span. Importantly, since amplifiers are phase sensitive, only one quadrature is amplified; the Q quadrature for $G_i \geq 1$ and the I quadrature

for $G_i \leq 1$. Note also that we assumed in (3)-(6) that the amplification process is aligned with the quadrature basis of the signal, i.e., it does not introduce correlations between x^Q and x^I . This requires some kind of phase stabilization between the source and the PSA.

In the absence of losses, $\tau_i = 1$, PSA just rescales quadratures and their variances correspondingly. This means that it can amplify a particular quadrature without introducing any additive noise. Crucially, however, when the channel is nonideal, $\tau_i < 1$, there appears a second term in the expression for $N^{Q/I}$ in (5) and (6) which increases the noise. This term originates from the amplification of the vacuum fluctuations that entered the signal during losses in the i -th span in (3) and (4). Note that these fluctuations are then amplified by each remaining PSA in the link which can make their contribution substantial at the output. Therefore, even though PSA is a noiseless process, due to the interaction between amplification and losses in the link, one cannot completely eliminate this phase sensitive noise.

In the quantum mechanical description, the optical field is characterized by quantum states of light and the quadrature random variables are promoted to operators [27]. A particularly important class of states, which we will consider here, are Gaussian quantum states which are fully characterized by the first and second quadrature operators moments. They include a wide variety of practically relevant states of light such as coherent states, which are quantum analogues of classical electromagnetic waves, and squeezed states. Crucially, both phase-sensitive and insensitive amplification as well as losses are examples of Gaussian quantum operations which preserve the Gaussian features of quantum states, meaning that if the input state is Gaussian the output of the link is Gaussian as well [34].

Assuming one considers only Gaussian states, the classical description in (1) and (2) can be also used to describe the quantum signal since one can take $x_S^{Q/I}$ as the first moment of the state and $x_N^{Q/I}$ as a value of a Gaussian random variable with distribution $x_N^{Q/I} \sim \mathcal{N}(0, V^{Q/I})$, where $V^{Q/I}$ denotes the fundamental quantum noise level corresponding to each quadrature. For coherent states $V^{Q/I} = 1/2$ is the typical shot noise limit present in quadrature detection, but in general the noise just has to satisfy $V^Q V^I \geq 1/4$ due to the Heisenberg uncertainty principle. One can therefore fully characterize the state of the received light pulses by looking at the quadrature values and their variances at the channel output which are equal to

$$x_{\text{out}}^Q = x_{\text{in}}^Q \sqrt{\tau_{\text{tot}} G_{\text{tot}}}, \quad x_{\text{out}}^I = x_{\text{in}}^I \sqrt{\frac{\tau_{\text{tot}}}{G_{\text{tot}}}}, \quad (7)$$

$$S_{\text{out}}^Q = S_{\text{in}}^Q \tau_{\text{tot}} G_{\text{tot}}, \quad S_{\text{out}}^I = S_{\text{in}}^I \frac{\tau_{\text{tot}}}{G_{\text{tot}}}, \quad (8)$$

$$N_{\text{out}}^Q = \tau_{\text{tot}} G_{\text{tot}} N_{\text{in}}^Q + \frac{\tau_{R+1}}{2} \sum_{i=1}^R G_i (1 - \tau_i) \prod_{j=i+1}^R G_j \tau_j + \frac{1 - \tau_{R+1}}{2}, \quad (9)$$

$$N_{\text{out}}^I = \frac{\tau_{\text{tot}}}{G_{\text{tot}}} N_{\text{in}}^I + \frac{\tau_{R+1}}{2} \sum_{i=1}^R \frac{1 - \tau_i}{G_i} \prod_{j=i+1}^R \frac{\tau_j}{G_j} + \frac{1 - \tau_{R+1}}{2}, \quad (10)$$

where $\tau_{\text{tot}} = \prod_{i=1}^{R+1} \tau_i$, $G_{\text{tot}} = \prod_{i=1}^R G_i$, $x_{\text{in}}^{Q/I}$, $S_{\text{in}}^{Q/I}$ and $N_{\text{in}}^{Q/I}$ are the initial values of quadratures and signal and noise powers. In the quantum mechanical language one can interpret the above phase sensitive noise as an effect of a series of beam splitters that mix the signal with increasingly squeezed vacuum states [11].

III. INFORMATION THEORY

In the standard information theory picture a general memoryless communication channel is characterized by a conditional probability distribution $p(y|x)$ which describes the statistical dependence of output symbols y on the input ones x . The sender uses symbols x with some prior probability distribution $p(x)$. In such a picture the communication rate is bounded by the mutual information

$$I(X, Y) = H(Y) - H(Y|X), \quad (11)$$

where $H(Y) = -\sum_y p(y) \log_2 p(y)$ and $H(Y|X) = -\sum_{x,y} p(x)p(y|x) \log_2 p(y|x)$ are the output and conditional Shannon entropies. One can optimize mutual information over the input probability distribution $p(x)$ in order to get the best performance and obtain the channel capacity

$$C = \max_{p(x)} I(X, Y), \quad (12)$$

which specifies the best achievable rate for a given information-theoretic channel.

In optical communication it is customary to impose some form of constraint on the input modulation, otherwise the capacity may become infinite. Typically, it is the average energy of the signal that cannot exceed some given value. This constraint can be expressed in terms of the average number of photons in the signal \bar{n} and reads

$$S_{\text{in}}^Q + S_{\text{in}}^I = 2\bar{n}. \quad (13)$$

Under the above constraint and assuming heterodyne detection performed at the output one can derive the well known Shannon-Hartley bound

$$C_{S2} = \frac{1}{2} \log_2 (1 + \text{SNR}^Q) + \frac{1}{2} \log_2 (1 + \text{SNR}^I), \quad (14)$$

where $\text{SNR}^{Q/I} = S_{\text{out}}^{Q/I} / N_{\text{out}}^{Q/I}$ are the SNRs corresponding to Q and I quadratures respectively. PSA amplifies the signal in just one direction of the optical phase space while it reduces it in the orthogonal one. Therefore, for large distances, when the total accumulated losses are considerable, the PSA would reduce SNR in one direction in (14) to low values. Thus, it is beneficial to spend all energy on modulating just a single quadrature and perform homodyne detection. The resulting capacity is equal to

$$C_{S1} = \frac{1}{2} \log_2 (1 + \text{SNR}^Q), \quad (15)$$

where we decided to amplify the Q quadrature and $S_{\text{in}}^Q = 2\bar{n}$. Importantly, the capacity in the above expressions (14) and (15) is given solely by the signal-to-noise ratio (SNR) in respective quadratures.

The physical channel over which the information is transmitted at the fundamental level is characterized by the laws of quantum mechanics. In particular, the information about symbols x is encoded in quantum states ρ_x of physical information carriers, which in case of optical communication are photons. These states then undergo an evolution described by a quantum channel Λ and are detected by the receiver using a measurement described by a positive operator valued measure (POVM) Π_y . The classical information theoretic channel can be then reconstructed using the Born rule $p(y|x) = \text{Tr}[\Lambda(\rho_x)\Pi_y]$ which allows to evaluate both mutual information and channel capacity through (11) and (12) respectively.

Crucially, in the quantum mechanical description one explicitly includes measurement and quantum states of the signal, meaning there are more degrees of freedom which can be used to boost information transfer rate of a physical channel Λ . In particular, optimization of the mutual information over measurements results in the Holevo bound [30]

$$C_{\text{GH}} = S \left[\sum_x p(x) \Lambda(\rho_x) \right] - \sum_x p(x) S[\Lambda(\rho_x)], \quad (16)$$

where $S(\rho) = -\text{Tr}(\rho \log_2 \rho)$ is the von Neumann entropy of a state ρ . The Holevo bound is in principle saturable but in general it requires exotic collective measurements performed on a very large number of channel outputs.

In order to find the upper bound on the capacity attainable for the general quantum measurements and states, one needs to evaluate (16) and optimize it over all prior ensembles of states at the input. In the case of the pure loss channel the result is given by the Gordon-Holevo capacity and equal to [29], [35], [36]

$$C_{\text{GH}}(\bar{n}) = g(\tau_{\text{tot}}\bar{n}), \quad (17)$$

where the function $g(x) = (x+1) \log_2(x+1) - x \log_2 x$. This formula can be further generalized to the case of general phase-insensitive Gaussian bosonic channels [37]. However, since the quantum channel implied by PSA is phase-sensitive, standard expressions for the capacity do not apply in this case. Instead, one rather needs to perform quite cumbersome calculations [33] resulting in

$$C_{\text{GH}}(\bar{n}) = g(\bar{M}_{\text{out}}) - g(M_{\text{out}}), \quad (18)$$

where \bar{M}_{out} and M_{out} are quantities implicitly depending on the properties of the link which we discuss in the Appendix A. Note that in the case of the pure loss channel, when the total transmission $\tau_{\text{tot}} = e^{-\alpha L}$ is small, both classical and quantum bounds predict capacity scaling as $C \sim e^{-\alpha L}$, which prevents communication on large distances.

A crucial difference between (18) and (14) and (15) is that the quantum mechanical bound does not depend solely on the SNR. One needs to separately consider signal and noise in both quadratures in order to find the Gordon-Holevo capacity. For large noise, however, one may simplify the Gordon-Holevo

bound. To see this, note that the function $g(x)$ for large x can be approximated as

$$g(x) \approx \log_2 e(1+x) + O\left(\frac{1}{x}\right). \quad (19)$$

For large phase sensitive noise, i.e. $N^Q \gg 1$, one can approximate (50) and (51) in Appendix A by

$$\bar{M}_{\text{out}} \approx y\sqrt{1 + \text{SNR}^Q} - \frac{1}{2}, \quad M_{\text{out}} \approx y - \frac{1}{2}, \quad (20)$$

where $y \approx \sqrt{N^Q N^I}$. Plugging these values into (18) gives

$$C_{\text{GH}}(\bar{n}) \approx \frac{1}{2} \log_2 (1 + \text{SNR}^Q), \quad (21)$$

which is exactly (15). Therefore, in the regime of large noise quantum effects are not relevant and the Gordon-Holevo capacity can be attained by the homodyne measurement irrespectively of the signal strength [8].

IV. CAPACITY OF THE PSA LINK

The capacity of the PSA channel is given by either (15) if one insists on quadrature detection or (18) if general quantum measurements are allowed. The signal and noise variances entering these formulas are given by (7)-(10). In order to find the ultimate bound on the capacity in both scenarios one needs to perform an optimization over the ensemble of input states and locations and gains of amplifiers. Assuming Gaussian input states, the first task can be done by simply optimizing the input signal and noise variances. In particular, the coherent state ensemble, which is most readily available, can be considered by just taking $N_{\text{in}}^{Q/I} = 1/2$. On the other hand, optimization over the amplifiers has to be somehow constrained in order to avoid infinite gains of PSAs which would require large power at each amplifier. We will consider two types of constraints that have a simple physical interpretation: the amplitude restoration and total power restoration regimes.

In the first scenario, the amplitude restoration regime, each PSA restores the amplitude of the Q quadrature to its initial value. In terms of gains and losses in each span this requirement translates into equality constraint $G_i = 1/\tau_i$. The signal and noise Eqs. (5),(6) therefore read

$$S_i^Q = S_{\text{in}}^Q, \quad N_i^Q = N_{i-1}^Q + \frac{1 - \tau_i}{2\tau_i}, \quad (22)$$

$$S_i^I = \tau_i^2 S_{i-1}^I, \quad N_i^I = \tau_i^2 N_{i-1}^I + \frac{\tau_i(1 - \tau_i)}{2}. \quad (23)$$

Note that even though the signal in the Q quadrature remains constant, the noise increases with each subsequent span of the link. This is because not only the signal is amplified at each PSA but also the noise. A consequence of this fact is the growth of the total power traveling through the link with each passed PSA.

The constraints in the amplitude restoration regime can be easily formulated mathematically, but as argued above, in general they may lead to large total power in the fiber. This may be problematic since if the combined intensity of signal and noise becomes too strong, various non-linear effects begin to play a role in the propagation, which decreases the communication

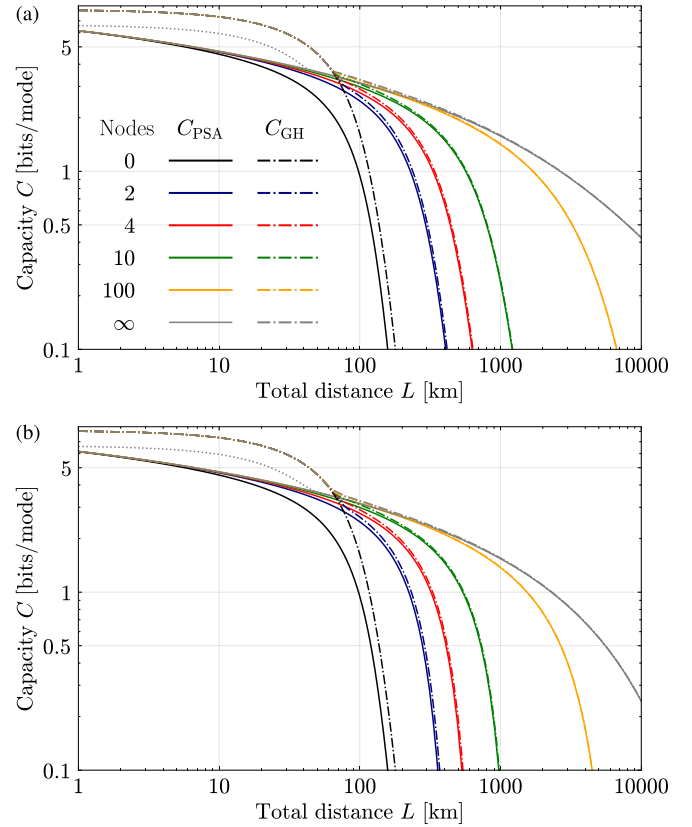


Fig. 2. Capacity attainable with homodyne detection C_{PSA} and the Gordon-Holevo bound C_{GH} in the (a) amplitude and (b) total power restoration regimes as a function of the link length for $\alpha = 0.05\text{km}^{-1}$ and $\bar{n} = 100$. The gray dotted curves represent capacity in the distributed amplification scenario for a hybrid double homodyne scheme with optimized transmissivity of the first beam splitter and signal power distribution between the quadratures.

capacity [38]. The workaround to avoid this issue is provided by the total power restoration regime in which it is the total power that cannot exceed its initial value at any point in the link. The relevant constraints read

$$S_i^Q + S_i^I + N_i^Q + N_i^I \leq 2\bar{n} + 1, \quad (24)$$

for every amplifier i , which in terms of PSA gains is given by a set of complicated nonlinear inequalities. In general, the constraints in the total power restoration regime are stricter than in the amplitude restoration scenario, leading to a weaker output signal.

We performed numerical optimization over PSA gains and their locations in both approaches mentioned above. We chose the average number of photons $\bar{n} = 100$ which corresponds to $\text{SNR}^Q = 26\text{dB}$ at the input. It is seen in Fig. 2 that the Gordon-Holevo capacity C_{GH} can be improved by using PSA for distances $L \gtrsim 70\text{km}$ for the typical value of the fiber attenuation coefficient $\alpha = 0.05\text{km}^{-1}$. The advantage grows with the number of amplifiers and saturates at the curve representing the distributed amplification case, $R \rightarrow \infty$, which is discussed in detail in the next section. On the other hand, it is seen that the quantum advantage from using general POVMs is small when PSAs are applied and becomes negligible when the length of

the link becomes considerable. Therefore, in this case one can attain the quantum bound using just the homodyne detection, as discussed in the previous section. In the regime of small distances $L \lesssim 70\text{km}$ it is highly beneficial to utilize general quantum measurements, as the Gordon-Holevo bound is much larger than the Shannon-Hartley bound. However, in this limit, the capacity cannot be increased by signal amplification. The threshold value of distance for which this happens is derived in Appendix C. One can improve the capacity also by considering a more general classical strategy such as double homodyne detection with an adjustable beam splitter redirecting appropriate fractions of signals to each of the detectors, optimized over the distribution of signal power between the quadratures and beam splitter transmissivity. Still, as can be seen in Fig. 2, such receiver leaves a gap with respect to the Gordon-Holevo bound.

V. CONTINUOUS AMPLIFICATION

In general, finding the exact expressions for quadratures variances for optimal distribution of amplifiers is very complicated. However, one can derive simple formulas for the case of continuous amplification, when $R \rightarrow \infty$. In such a case, the discrete transmission of each span and PSA gains can be approximated as $\tau_i = 1 - \alpha\Delta l$ and $G_i = 1 + \gamma(l)\Delta l$, respectively. Note that although this approximation is performed assuming equal spans with length $\Delta l = L/(R + 1)$, it remains valid in the general case. As discussed in [39] in the regime $\Delta l \rightarrow 0$, the propagation is approximated by

$$\frac{dS^Q}{dl} = (\gamma(l) - \alpha) S^Q, \quad (25)$$

$$\frac{dS^I}{dl} = -(\gamma(l) + \alpha) S^I, \quad (26)$$

$$\frac{dN^Q}{dl} = (\gamma(l) - \alpha) N^Q + \frac{\alpha}{2}, \quad (27)$$

$$\frac{dN^I}{dl} = -(\gamma(l) + \alpha) N^I + \frac{\alpha}{2}. \quad (28)$$

In case of amplitude restoration, one has $\gamma(l) = \alpha$, which, assuming coherent state modulation in the Q quadrature, gives

$$S_{\text{out}}^Q = 2\bar{n}, \quad S_{\text{out}}^I = 0, \quad (29)$$

$$N_{\text{out}}^Q = \frac{1 + \alpha L}{2}, \quad N_{\text{out}}^I = \frac{1 + e^{-2\alpha L}}{4}, \quad (30)$$

which agrees with previously obtained results [9], [11]. Plugging these results into (15) gives in the large distance regime $\alpha L \gg 1$

$$C \approx \frac{2\bar{n}}{\alpha L \ln 2}. \quad (31)$$

This is an exponential gain with respect to the $C \sim e^{-\alpha L} \bar{n}$ value obtained for the pure loss channel without PSA.

For total power restoration the situation is more complicated since the gain function $\gamma(l)$ is given implicitly by the constraint in (24). Nevertheless, one can still solve (25)-(28) analytically. For coherent state modulation in the Q quadrature the optimal

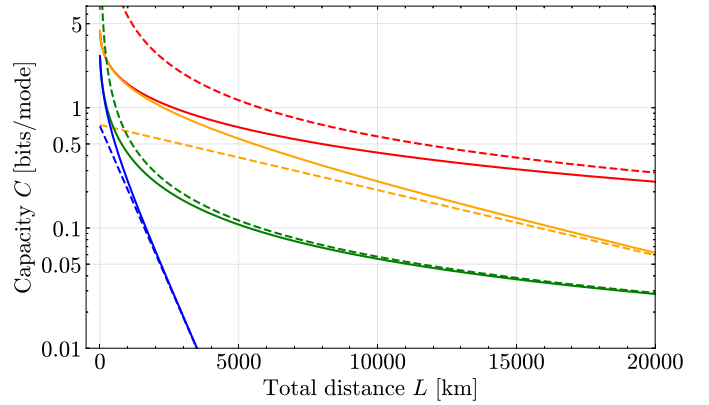


Fig. 3. Optimized capacity as a function of the link length for $\alpha = 0.05\text{km}^{-1}$. Power restoration regime: orange curve - $\bar{n} = 100$, blue curve - $\bar{n} = 10$; amplitude restoration regime: red curve - $\bar{n} = 100$, green curve - $\bar{n} = 10$. Dashed lines of respective colors represent corresponding asymptotic expressions, (31) for amplitude restoration and (34) for power restoration.

gain profile reads

$$\gamma(l) = \frac{2\alpha\bar{n}}{\sqrt{4\bar{n}^2 + 2\bar{n}(1 - e^{-2\alpha l})}}, \quad (32)$$

and the optimal quadrature variances are given in Appendix B. The capacity attainable with homodyne detection can then be approximated to

$$C = \frac{1}{2} \log_2 \left(1 + \frac{4\bar{n}e^{-\frac{\alpha L}{4\bar{n}+1}}}{4\bar{n} \left(1 - e^{-\frac{\alpha L}{4\bar{n}+1}} \right) + 1} \right). \quad (33)$$

In the large distance regime the above formula can be further approximated as

$$C \sim \frac{2\bar{n}}{(4\bar{n} + 1) \ln 2} e^{-\frac{\alpha L}{4\bar{n}+1}}. \quad (34)$$

Crucially, this represents an improvement in the exponent by factors of $1/(4\bar{n} + 1)$ and $1/4$ with respect to the situation without PSA and when the amplifiers are phase-insensitive, respectively [8], [39]. Note that the latter scenario includes for example distributed Raman amplification. On the other hand, the exponential decay of the capacity is still present, even in the optimal case.

In practical realizations, however, the average number of photons in the signal is large, $\bar{n} \gg 1$. This means that if $L \lesssim 4\bar{n}/\alpha$, the exponential term in (33) in the denominator is negligible and for strong signals one can approximate it by $e^{-\alpha L/(4\bar{n}+1)} \approx 1 - \frac{\alpha L}{4\bar{n}}$. Therefore, the capacity is rather equal to $C \sim \frac{1}{2} \log_2 \frac{4\bar{n}}{\alpha L}$ which is also the value obtained under the amplitude restoration constraint in this regime. One can intuitively understand this fact by inspecting the optimal gain profile in (32). Upon Taylor expansion for large \bar{n} values one obtains $\gamma(l) \approx \alpha + O(1/\bar{n})$ which is very close to the gain in the amplitude restoration approach $\gamma_{\text{amp}} = \alpha$. Therefore, the capacities in these two instances should also be equal. This is seen in Fig. 3 in which the attainable capacities in the amplitude and total power restoration regimes are equal for small and

moderate distances and start to diverge for large link length, when they saturate respective asymptotic expressions.

VI. CONCLUSION

In conclusion, we derived the ultimate quantum limits for the capacity of communication links with PSA. The advantage offered by general quantum mechanical measurement strategies against conventional protocols is small and becomes negligible in the long link regime. We showed that, depending on the constraints imposed on the amplification, one can consider two physically relevant regimes - amplitude and total power restoration. The asymptotic capacity values obtained in these two approaches differ significantly. Importantly, compared to pure loss and phase-insensitive amplification scenarios, the exponential decay rate of the capacity in the power restoration regime is improved by the factors of $1/(4\bar{n})$ and $1/4$ respectively, representing a huge advantage in the capacity. An even stronger enhancement can be found in the amplitude restoration regime in which the capacity scaling changes from exponential to inversely proportional in the link length. On the other hand, for strong signals, typically encountered in optical fiber links, both approaches turn out to be equivalent, offering exponential enhancement of the SNR. Crucially, our results suggest that quantum enhanced measurement strategies are beneficial only for short-haul links in which noise effects are not predominant.

APPENDIX A

GORDON-HOLEVO BOUND FOR THE PSA CHANNEL

In this section we outline the method to calculate the Gordon-Holevo capacity bound for phase-sensitive Gaussian channels described and proven in [33]. A general phase-sensitive single mode quantum Gaussian channel is specified by its action on the vector of the first moments \mathbf{d} and the covariance matrix Σ of quadrature operators [34]. This can be described as

$$\mathbf{d}_{\text{out}} = X\mathbf{d}_{\text{in}}, \quad \Sigma_{\text{out}} = X\Sigma_{\text{in}}X^T + Y, \quad (35)$$

where X and Y are real and symmetric matrices, satisfying certain conditions [34]. The von Neumann entropy of a general Gaussian state with a covariance matrix Σ is given by

$$S(\rho) = g(M), \quad M = \sqrt{\det \Sigma} - 1/2. \quad (36)$$

Assuming that the signal is modulated by displacing a certain Gaussian state with a given covariance matrix Σ_{in} in phase space according to some Gaussian input distribution, the averaged input state in (16) is also Gaussian and is specified by the covariance matrix $\bar{\Sigma}_{\text{in}}$. Therefore, the Gordon-Holevo capacity in such case is given by

$$C_{\text{GH}} = g(\bar{M}_{\text{out}}) - g(M_{\text{out}}), \quad (37)$$

with

$$\bar{M}_{\text{out}} = \sqrt{\det \bar{\Sigma}_{\text{out}}} - 1/2, \quad \bar{\Sigma}_{\text{out}} = X\bar{\Sigma}_{\text{in}}X^T + Y, \quad (38)$$

$$M_{\text{out}} = \sqrt{\det \Sigma_{\text{out}}} - 1/2, \quad \Sigma_{\text{out}} = X\Sigma_{\text{in}}X^T + Y. \quad (39)$$

This expression depends on the average input energy which reads

$$\bar{n} = \frac{1}{2} (\text{Tr } \bar{\Sigma}_{\text{in}} - 1). \quad (40)$$

One can show that for a given Gaussian channel one can come up with a so-called fiducial channel with the same capacity as the original one and diagonal matrices X , Y [40]. The problem can then be described by just three parameters τ , y , ω given by equations

$$\tau = \det X = \det \begin{pmatrix} x_1 & 0 \\ 0 & x_2 \end{pmatrix}, \quad y = \sqrt{\det Y} \quad (41)$$

and

$$Y = y \begin{pmatrix} \frac{x_1}{x_2}\omega^{-1} & 0 \\ 0 & \frac{x_2}{x_1}\omega \end{pmatrix}. \quad (42)$$

For the PSA channel link in consideration the parameters of the fiducial channel are given by $\tau = \tau_{\text{tot}}$, $y = \sqrt{(N_{\text{out}}^Q - x_1^2 N_{\text{in}}^Q)(N_{\text{out}}^I - x_2^2 N_{\text{in}}^I)}$, $\omega = G_{\text{tot}} \sqrt{(N_{\text{out}}^I - x_2^2 N_{\text{in}}^I)/(N_{\text{out}}^Q - x_1^2 N_{\text{in}}^Q)}$. Note that y and ω do not depend on the initial values of the quadratures noise but rather solely on the amount of noise added in the course of signal propagation. These parameters calculated for a given channel allow one to specify a threshold energy

$$\bar{n}_{\text{thr}} = \frac{1}{2\omega} \left(1 + \frac{y}{|\tau|} |1 - \omega^2| \right) - \frac{1}{2}. \quad (43)$$

If one wants to further optimize the Gordon-Holevo capacity formula in (18) over ensembles of input states, the calculation proceeds based on whether the input energy is *above* this threshold, $\bar{n} \geq \bar{n}_{\text{thr}}$, or *below* it, $\bar{n} < \bar{n}_{\text{thr}}$.

A. Above Threshold

We define

$$\bar{w}_{\text{in}} = \sqrt{\frac{|\tau|(2\bar{n} + 1) + y(\omega^{-1} - \omega)}{|\tau|(2\bar{n} + 1) - y(\omega^{-1} - \omega)}}. \quad (44)$$

The optimal input covariance matrices read then

$$\Sigma_{\text{in}} = \frac{1}{2} \begin{pmatrix} \frac{1}{\omega} & 0 \\ 0 & \omega \end{pmatrix}, \quad \bar{\Sigma}_{\text{in}} = \frac{2\bar{n} + 1}{\frac{1}{\bar{w}_{\text{in}}} + \bar{w}_{\text{in}}} \begin{pmatrix} \frac{1}{\bar{w}_{\text{in}}} & 0 \\ 0 & \bar{w}_{\text{in}} \end{pmatrix}, \quad (45)$$

which corresponds to displacing in phase space a squeezed vacuum state according to a symmetric Gaussian distribution. The corresponding Gordon-Holevo capacity is given by

$$C_{\text{GH}}(\bar{n}) = g \left[|\tau| \left(\bar{n} + \frac{1}{2} \right) + \frac{1}{2} (y(\omega^{-1} + \omega) - 1) \right] - g \left[\frac{|\tau|}{2} + y - \frac{1}{2} \right]. \quad (46)$$

B. Below Threshold

To calculate the classical capacity below threshold, one has to solve the following transcendental equation for ω_{in} :

$$g'(\bar{M}_{\text{out}}) \left(\frac{1}{\bar{w}_{\text{out}}} - \bar{w}_{\text{out}} \right) = g'(M_{\text{out}}) \left(\frac{1}{\omega_{\text{out}}} - \frac{\omega_{\text{out}}}{\omega_{\text{in}}^2} \right) \quad (47)$$

where $g'(x)$ is the derivative of $g(x)$ and we define

$$\omega_{\text{out}} = \sqrt{\frac{\tau \frac{\omega_{\text{in}}}{2} + y\omega}{\tau \frac{1}{2\omega_{\text{in}}} + \frac{y}{\omega}}}, \quad (48)$$

$$\bar{\omega}_{\text{out}} = \sqrt{\frac{\tau \frac{\omega_{\text{in}}}{2} + y\omega}{\tau (2\bar{n} + 1 - \frac{\omega_{\text{in}}}{2}) + \frac{y}{\omega}}}, \quad (49)$$

$$M_{\text{out}} = -\frac{1}{2} + \sqrt{\left(\tau \frac{1}{2\omega_{\text{in}}} + \frac{y}{\omega}\right) \left(\tau \frac{\omega_{\text{in}}}{2} + y\omega\right)}, \quad (50)$$

$$\bar{M}_{\text{out}} = -\frac{1}{2} + \sqrt{\left(\tau (2\bar{n} + 1 - \frac{\omega_{\text{in}}}{2}) + \frac{y}{\omega}\right) \left(\tau \frac{\omega_{\text{in}}}{2} + y\omega\right)}. \quad (51)$$

After finding ω_{in} one then has

$$\bar{\omega}_{\text{in}} = \frac{1}{\sqrt{2(2\bar{n} + 1)\omega_{\text{in}}^{-1} - 1}}. \quad (52)$$

The optimal average input covariance matrix reads

$$\Sigma_{\text{in}} = \frac{1}{2} \begin{pmatrix} \frac{1}{\omega_{\text{in}}} & 0 \\ 0 & \omega_{\text{in}} \end{pmatrix}, \quad \bar{\Sigma}_{\text{in}} = \frac{2\bar{n} + 1}{\frac{1}{\bar{\omega}_{\text{in}}} + \bar{\omega}_{\text{in}}} \begin{pmatrix} \frac{1}{\bar{\omega}_{\text{in}}} & 0 \\ 0 & \bar{\omega}_{\text{in}} \end{pmatrix}, \quad (53)$$

which corresponds to displacing a squeezed vacuum state across a single quadrature. The Gordon-Holevo capacity below the threshold is finally given by

$$C_{\text{GH}}(\bar{n}) = g(\bar{M}_{\text{out}}) - g(M_{\text{out}}). \quad (54)$$

Note that one can easily find what is the optimal capacity below the threshold assuming coherent state modulation by taking $\omega_{\text{in}} = 1$ in (50) and (51) and plugging the result in (54).

APPENDIX B

EXACT SOLUTION FOR DISTRIBUTED AMPLIFICATION

In order to find the exact quadrature variances with the constraint of total power (24) one can sum (25)-(28) resulting in

$$S^I + N^I = \frac{1}{\gamma(l) + \alpha} (\alpha + (\gamma(l) - \alpha)(S^Q + N^Q)), \quad (55)$$

where we have used the fact that $dS^Q/dl + dS^I/dl + dN^Q/dl + dN^I/dl = 0$ due to constant total power. Plugging this into (24) one obtains

$$\gamma(l) = -\frac{2\alpha\bar{n}}{2\bar{n} + 1 - 2(S^Q + N^Q)}. \quad (56)$$

Let us now sum (25) and (27) and introduce $Z^Q = S^Q + N^Q$:

$$\frac{dZ^Q}{dl} = -\alpha Z^Q \left(1 + \frac{2\bar{n}}{2\bar{n} + 1 - 2Z^Q}\right) + \frac{\alpha}{2}. \quad (57)$$

The solution to this equation is given by

$$Z^Q(l) = \frac{1}{2} \left[1 + 2\bar{n} + \sqrt{4\bar{n}^2 + 2\bar{n} - Ae^{-2\alpha l}}\right], \quad (58)$$

where A is a constant specified by initial conditions. For coherent state modulation in the Q quadrature one has $A = 2\bar{n}$. One can

now plug this result in (56) and find the optimal gain profile

$$\gamma(l) = \frac{2\alpha\bar{n}}{\sqrt{4\bar{n}^2 + 2\bar{n} - Ae^{-2\alpha l}}}, \quad (59)$$

which for $A = 2\bar{n}$ gives (32). Using the derived value of $\gamma(l)$ one can now solve (25)-(28) resulting in

$$S^Q = 2\bar{n}e^{-\alpha l} \left[\frac{1 - \sqrt{\frac{1+2\bar{n}}{2\bar{n}}}}{1 + \sqrt{\frac{1+2\bar{n}}{2\bar{n}}}} \times \frac{1 + \sqrt{\frac{1+2\bar{n}}{2\bar{n}+1-e^{-2\alpha l}}}}{1 - \sqrt{\frac{1+2\bar{n}}{2\bar{n}+1-e^{-2\alpha l}}}} \right]^{\sqrt{\frac{\bar{n}}{4\bar{n}+2}}}, \quad (60)$$

$$N^Q = Z^Q - S^Q, \quad (61)$$

$$S^I = 0, \quad (62)$$

$$N^I = \frac{1}{2} \left[1 + 2\bar{n} - \sqrt{4\bar{n}^2 + 2\bar{n} - 2\bar{n}e^{-2\alpha l}}\right], \quad (63)$$

where we have taken $A = 2\bar{n}$. By assuming large distances and then large power one can approximate the expressions for S^Q and N^Q by

$$S^Q \approx 2\bar{n}e^{-\frac{\alpha L}{4\bar{n}+1}}, \quad (64)$$

$$N^Q \approx 2\bar{n}(1 - e^{-\frac{\alpha L}{4\bar{n}+1}}) + \frac{1}{2}, \quad (65)$$

which results in capacity given in (33). On the other hand, by expanding (60)-(63) for large input power $\bar{n} \gg 1$ these equations can be approximated by

$$S^Q \approx 2\bar{n} - \frac{\alpha L}{2} + \frac{1 - e^{-2\alpha l}}{4}, \quad (66)$$

$$N^Q \approx \frac{1 + \alpha L}{2}, \quad (67)$$

$$S^I = 0, \quad (68)$$

$$N^I \approx \frac{1 + e^{-2\alpha l}}{4}. \quad (69)$$

Note that this makes sense only when $\bar{n} \ll \alpha L/4$.

APPENDIX C

QUANTUM THRESHOLD DISTANCE

It is seen in Fig. 2 that for short communication links, when the length of the link is smaller than a threshold value $L < L_{\text{th}}$ the Gordon-Holevo capacity exceeds the performance of the homodyne setup. In this section we derive an approximate expression for the threshold length L_{th} below which this effect occurs. We will derive this distance for the distributed amplification scenario in the amplitude restoration regime. Crucially, the Gordon-Holevo bound for L_{th} for this case is equal to the bound on the pure loss channel. Therefore, based on (29) one needs to solve the following transcendental equation

$$g(\bar{n}e^{-\alpha L_{\text{th}}}) = \frac{1}{2} \log_2 \left(1 + \frac{4\bar{n}}{1 + \alpha L_{\text{th}}}\right). \quad (70)$$

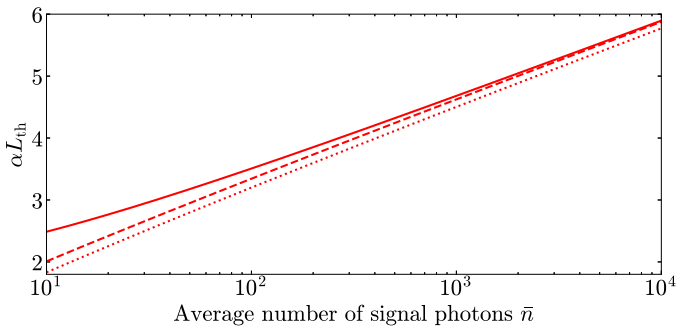


Fig. 4. Product of the fiber attenuation coefficient α and the threshold distance L_{th} above which homodyne detection suffices to attain the quantum Gordon-Holevo bound, plotted as a function of the average number of photons in the input signal \bar{n} . The solid line is the exact solution of the transcendental equation (70), and the dashed and dotted curves are the analytic approximations given by (72) and (73) respectively.

Assuming large input power \bar{n} , one can approximate both sides of the above equation as

$$1 + \log \bar{n} - \alpha L_{\text{th}} = \frac{1}{2} \log 4\bar{n} - \frac{1}{2} \log (1 + \alpha L_{\text{th}}), \quad (71)$$

which has a solution given in terms of Lambert W_{-1} function

$$L_{\text{th}} = -\frac{1}{2\alpha} \left[2 + W_{-1} \left(-\frac{8}{\bar{n}e^4} \right) \right]. \quad (72)$$

One can further approximate L_{th} by noting that $W_{-1}(x) \approx \log(-x) - \log(-\log(-x))$ for small x , which gives

$$L_{\text{th}} \approx \frac{1}{\alpha} \left(1 + \frac{1}{2} \log \left[\frac{\bar{n}}{8} \left(\log \frac{\bar{n}}{8} + 4 \right) \right] \right). \quad (73)$$

The relation between the product αL_{th} and \bar{n} is shown in Fig. 4. As expected, for a given value of the fiber attenuation coefficient α , increasing the input signal power extends the region in which no amplification is beneficial.

ACKNOWLEDGMENT

The authors would like to thank Peter Andrekson, René-Jean Essiambre, Magnus Karlsson and Jochen Schröder for insightful discussions.

REFERENCES

- [1] R.-J. Essiambre and R. W. Tkach, "Capacity trends and limits of optical communication networks," *Proc. IEEE*, vol. 100, no. 5, pp. 1035–1055, May 2012.
- [2] K. Kikuchi, "Fundamentals of coherent optical fiber communications," *J. Lightw. Technol.*, vol. 34, no. 1, pp. 157–179, Jan. 2016.
- [3] J. M. Kahn and K.-P. Ho, "Spectral efficiency limits and modulation/detection techniques for DWDM systems," *IEEE J. Sel. Topics Quantum Electron.*, vol. 10, no. 2, pp. 259–272, Mar./Apr. 2004.
- [4] P. J. Winzer, "High-spectral-efficiency optical modulation formats," *J. Lightw. Technol.*, vol. 30, no. 24, pp. 3824–3835, Dec. 2012.
- [5] C. M. Caves, "Quantum limits on noise in linear amplifiers," *Phys. Rev. D*, vol. 26, pp. 1817–1839, Oct. 1982, doi: [10.1103/PhysRevD.26.1817](https://doi.org/10.1103/PhysRevD.26.1817).
- [6] A. Yariv, "Signal-to-noise considerations in fiber links with periodic or distributed optical amplification," *Opt. Lett.*, vol. 15, no. 19, pp. 1064–1066, Oct. 1990. [Online]. Available: <http://opg.optica.org/ol/abstract.cfm?URI=ol-15-19-1064>
- [7] C. Antonelli, A. Mecozzi, M. Shtaif, and P. J. Winzer, "Quantum limits on the energy consumption of optical transmission systems," *J. Lightw. Technol.*, vol. 32, no. 10, pp. 1853–1860, May 2014.
- [8] M. Jarzyna, R. García-Patrón, and K. Banaszek, "Ultimate capacity limit of a multi-span link with phase-insensitive amplification," in *Proc. IEEE 45th Eur. Conf. Opt. Commun.*, 2019, pp. 1–4, doi: [10.1049/cp.2019.0742](https://doi.org/10.1049/cp.2019.0742).
- [9] R. Loudon, "Theory of noise accumulation in linear optical-amplifier chains," *IEEE J. Quantum Electron.*, vol. 21, no. 7, pp. 766–773, Jul. 1985.
- [10] I. Bar-Joseph, A. A. Friesem, R. G. Waarts, and H. H. Yaffe, "Parametric interaction of a modulated wave in a single-mode fiber," *Opt. Lett.*, vol. 11, no. 8, pp. 534–536, Aug. 1986. [Online]. Available: <https://opg.optica.org/ol/abstract.cfm?URI=ol-11-8-534>
- [11] A. Mecozzi and P. Tombesi, "Parametric amplification and signal-to-noise ratio in optical transmission lines," *Opt. Commun.*, vol. 75, no. 3, pp. 256–262, 1990. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/0030401890905282>
- [12] J. A. Levenson, I. Abram, T. Rivera, and P. Grangier, "Reduction of quantum noise in optical parametric amplification," *J. Opt. Soc. Amer. B*, vol. 10, no. 11, pp. 2233–2238, Nov. 1993. [Online]. Available: <https://opg.optica.org/josab/abstract.cfm?URI=josab-10-11-2233>
- [13] C. J. McKinstrie and S. Radic, "Phase-sensitive amplification in a fiber," *Opt. Exp.*, vol. 12, no. 20, pp. 4973–4979, Oct. 2004. [Online]. Available: <https://opg.optica.org/oe/abstract.cfm?URI=oe-12-20-4973>
- [14] M. Vasilyev, "Distributed phase-sensitive amplification," *Opt. Exp.*, vol. 13, no. 19, pp. 7563–7571, Sep. 2005. [Online]. Available: <https://opg.optica.org/oe/abstract.cfm?URI=oe-13-19-7563>
- [15] T. Torounidis, P. A. Andrekson, and B.-E. Olsson, "Fiber-optical parametric amplifier with 70-dB gain," *IEEE Photon. Technol. Lett.*, vol. 18, no. 10, pp. 1194–1196, May 2006.
- [16] K. Croussore and G. Li, "Phase and amplitude regeneration of differential phase-shift keyed signals using phase-sensitive amplification," *IEEE J. Sel. Topics Quantum Electron.*, vol. 14, no. 3, pp. 648–658, May/Jun. 2008.
- [17] J. Kakande et al., "QPSK phase and amplitude regeneration at 56 Gbaud in a novel idler-free non-degenerate phase sensitive amplifier," in *Proc. Opt. Fiber Commun. Conf./National Fiber Optic Engineers Conf.*, 2011, Art. no. OMT4. [Online]. Available: <http://opg.optica.org/abstract.cfm?URI=OFC-2011-OMT4>
- [18] Z. Tong et al., "Towards ultrasensitive optical links enabled by low-noise phase-sensitive amplifiers," *Nature Photon.*, vol. 5, pp. 430–436, 2011.
- [19] B. Corcoran, S. L. Olsson, C. Lundström, M. Karlsson, and P. Andrekson, "Phase-sensitive optical pre-amplifier implemented in an 80 km dqpsk link," in *Proc. Opt. Fiber Commun. Conf.*, 2012, Art. no. PDP5A.4. [Online]. Available: <https://opg.optica.org/abstract.cfm?URI=OFC-2012-PDP5A.4>
- [20] T. Umeki, M. Asobe, H. Takara, Y. Miyamoto, and H. Takenouchi, "Multi-span transmission using phase and amplitude regeneration in ppls-based psa," *Opt. Exp.*, vol. 21, no. 15, pp. 18170–18177, Jul. 2013. [Online]. Available: <http://opg.optica.org/oe/abstract.cfm?URI=oe-21-15-18170>
- [21] S. L. I. Olsson, C. Lundström, M. Karlsson, and P. A. Andrekson, "Long-haul (3465 km) transmission of a 10 GBd QPSK signal with low noise phase-sensitive in-line amplification," in *Proc. IEEE Eur. Conf. Opt. Commun.*, 2014, pp. 1–3.
- [22] S. L. I. Olsson, B. Corcoran, C. Lundström, T. A. Eriksson, M. Karlsson, and P. A. Andrekson, "Phase-sensitive amplified transmission links for improved sensitivity and nonlinearity tolerance," *J. Lightw. Technol.*, vol. 33, no. 3, pp. 710–721, Feb. 2015.
- [23] S. Olsson, H. Eliasson, E. Astra, M. Karlsson, and P. A. Andrekson, "Long-haul optical transmission link using low-noise phase-sensitive amplifiers," *Nature Commun.*, vol. 9, 2018, Art. no. 2513.
- [24] R. Kakarla, J. Schröder, and P. A. Andrekson, "One photon-per-bit receiver using near-noiseless phase-sensitive amplification," *Light Sci. Appl.*, vol. 9, 2020, Art. no. 153.
- [25] I. A. Burenkov, M. V. Jabir, and S. V. Polyakov, "Practical quantum-enhanced receivers for classical communication," *AVS Quantum Sci.*, vol. 3, no. 2, 2021, Art. no. 025301, doi: [10.1116/5.0036959](https://doi.org/10.1116/5.0036959).
- [26] S. J. Dolinar, "An optimum receiver for the binary coherent state quantum channel," *Quart. Prog. Rep.*, vol. 11, pp. 115–120, 1973.
- [27] K. Banaszek, L. Kunz, M. Jachura, and M. Jarzyna, "Quantum limits in optical communications," *J. Lightw. Technol.*, vol. 38, no. 10, pp. 2741–2754, May 2020.
- [28] S. Guha, "Structured optical receivers to attain superadditive capacity and the Holevo limit," *Phys. Rev. Lett.*, vol. 106, no. 24, 2011, Art. no. 240502.
- [29] J. P. Gordon, "Quantum effects in communications systems," *Proc. IRE*, vol. 50, no. 9, pp. 1898–1908, Sep. 1962.

- [30] A. S. Holevo, "Bounds for the quantity of information transmitted by a quantum communication channel," *Problems Inf. Transmiss.*, vol. 9, no. 3, pp. 177–183, 1973.
- [31] B. Schumacher and M. D. Westmoreland, "Sending classical information via noisy quantum channels," *Phys. Rev. A*, vol. 56, pp. 131–138, Jul. 1997, doi: [10.1103/PhysRevA.56.131](https://doi.org/10.1103/PhysRevA.56.131).
- [32] A. S. Holevo, "The capacity of the quantum channel with general signal states," *IEEE Trans. Inf. Theory*, vol. 44, no. 1, pp. 269–273, Jan. 1998.
- [33] J. Schafer, E. Karpov, O. V. Pilyavets, and N. J. Cerf, "Classical capacity of phase-sensitive gaussian quantum channels," 2016, *arXiv:1609.04119*.
- [34] C. Weedbrook et al., "Gaussian quantum information," *Rev. Mod. Phys.*, vol. 84, pp. 621–669, May 2012, doi: [10.1103/RevModPhys.84.621](https://doi.org/10.1103/RevModPhys.84.621).
- [35] A. S. Holevo and R. F. Werner, "Evaluating capacities of bosonic Gaussian channels," *Phys. Rev. A*, vol. 63, Feb. 2001, Art. no. 032312.
- [36] V. Giovannetti, S. Guha, S. Lloyd, L. Maccone, J. H. Shapiro, and H. P. Yuen, "Classical capacity of the lossy bosonic channel: The exact solution," *Phys. Rev. Lett.*, vol. 92, Jan. 2004, Art. no. 027902.
- [37] V. Giovannetti, R. García-Patrón, N. J. Cerf, and A. S. Holevo, "Ultimate classical communication rates of quantum optical channels," *Nature Photon.*, vol. 8, pp. 796–800, 2014.
- [38] R.-J. Essiambre, G. Kramer, P. J. Winzer, G. J. Foschini, and B. Goebel, "Capacity limits of optical fiber networks," *J. Lightw. Technol.*, vol. 28, no. 4, pp. 662–701, Feb. 2010.
- [39] K. Łukanowski, M. Jarzyna, and K. Banaszek, "Quantum limits on the capacity of multispans links with phase-sensitive amplification," 2022, *arXiv:2206.14624*.
- [40] J. Schäfer, E. Karpov, R. García-Patrón, O. V. Pilyavets, and N. J. Cerf, "Equivalence relations for the classical capacity of single-mode gaussian quantum channels," *Phys. Rev. Lett.*, vol. 111, Jul. 2013, Art. no. 030503, doi: [10.1103/PhysRevLett.111.030503](https://doi.org/10.1103/PhysRevLett.111.030503).

Karol Łukanowski (Graduate Student Member, IEEE) received the B.Sc. and M.Sc. degrees in physics from the University of Warsaw, Warsaw, Poland, in 2020 and 2022, respectively, where he is currently working toward the Ph.D. degree in physics. Since 2019, he has been with the Centre of New Technologies, University of Warsaw. His research interests include the impact of quantum phenomena on the limits of communication and quantum key distribution in a device-independent setting. He was the recipient of the Junior KCIK Award by the National Quantum Information Centre operating at the University of Gdańsk for the best B.Sc. thesis on quantum information theory defended in Poland in 2020.

Konrad Banaszek (Senior Member, IEEE) received the M.Sc. and Ph.D. degrees in physics from the University of Warsaw, Warsaw, Poland, in 1997 and 2000, respectively. He held postdoctoral positions with the University of Rochester, NY, USA, and the University of Oxford, Oxford, U.K., followed by a Junior Research Fellowship with St. John's College, Oxford, U.K., and faculty appointments with the Nicolaus Copernicus University, Toruń, Poland, from 2005 to 2009, and has been with the University of Warsaw, since 2009. He is currently the Director of the Centre for Quantum Optical Technologies established in 2018 by the University of Warsaw in partnership with the University of Oxford under the International Research Agendas Programme operated by the Foundation for Polish Science. His research interests include quantum physics and optical sciences with a focus on novel approaches to communication, sensing, and imaging that enable operation beyond the standard quantum limits. Dr. Banaszek is a corresponding Member of the Polish Academy of Sciences and Fellow of Optica (formerly OSA). He was an Associate Editor for *Optics Express*. In 2001, he was the recipient of the European Physical Society Fresnel Prize for his contributions to the understanding of non-classical light and its applications in quantum information processing.

Marcin Jarzyna received the M.Sc. and Ph.D. degrees in physics from the University of Warsaw, Warsaw, Poland, in 2011 and 2016, respectively. Since 2016, he has been with the Centre of New Technologies, University of Warsaw. Research leading to his Ph.D. was focused mainly on quantum metrology and the impact of entanglement on the asymptotic precision limits under decoherence. His research interests include low power limits of communication, impact of signal amplification, superresolution effects in optical imaging, and realistic implementations of quantum key distribution.