Hamming Window to the Digital World

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Inspired by the awarding of the 30th jubilee IEEE medal "for exceptional contributions to information sciences and systems," this article is dedicated to the memory of Richard W. Hamming, mathematician and computer scientist. If Shannon's mathematical theory of communication, including coding, entropy, and channel capacity, can be assumed as cornerstone of the digital world, error-detecting and error-correcting codes, at the same time, pave the way to its redundancy and reliability. The role of the interaction between human and computer in the world of contemporary communication is considered, together with an emphasis on Hamming's fundamental contributions from information theory to computer technology. Hamming code, Hamming distance, Hamming metric, Hamming window, Hamming numbers, and sphere packing are now standard terms that reflect him as a leader in the development of coding theory. His ideas in coding theory continue to have practical use in computer design.

This month's article takes a look at the research and teaching of Richard W. Hamming with particular focus on Hamming's fundamental contributions from information theory to computer technology.

I. SURVEY OF HAMMING'S LIFE

Richard Wesley Hamming (Fig. 1) was born on February 11, 1915 in Chicago, IL. His father Richard J. Hamming, a native of Holland, was a credit manager. Richard Jr. spent his childhood with his mother, Mabel G. Redfield, and his father in Chicago, where he attended technical high school and college. His early years fostered a love of mathematics. Owing to immense talent, he turned out to be a more able mathematician than his teacher. In spite of his strong interest in mathematics, Hamming intended to study engineering. The main reason for it was to find out how the many different specific results may be fitted together in a common framework of mathematical ideas and techniques. However, the only scholarship offer that he received came from the University of Chicago, which had no engineering department. Therefore, he majored in mathematics (1937) and received his Bachelor of Science degree. Hamming continued his education at the University of Nebraska, where he received his Master's degree in mathematics in 1939, and then earned his Doctorate degree in mathematics from the University of Illinois at Urbana-Champaign in 1942. His seven-page doctoral dissertation, "Some Problems in the Boundary Value Theory of Linear Differential Equations," was supervised by Waldemar J. Trjitzinsky, who studied linear differential equations with two-point boundary conditions. In his thesis, Hamming extended the results of this work. In particular, he investigated the Green function and the characteristic solutions, for which he obtained asymptotic expressions.

He worked as an instructor of mathematics for a short time at the University of Illinois. In 1944, he became an Assistant Professor at the J. B. Speed Scientific School of Engineering at the University of Louisville. Next year, Hamming left Louisville for the Los Alamos Scientific Laboratory in New Mexico. As most of his colleagues, Hamming was pacifist by his conviction, but he joined a project which goal was to end the war. As part of Hans Bethe division, he programmed IBM machines that computed solutions to researchers' equations through the Manhattan project.

In 1946, Hamming began working in the Mathematics Department of the

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Fig. 1. **Richard W. Hamming (reprinted with permission of Nokia Corporation).**

Bell Telephone Laboratory in Murray Hill, NY. He did not leave Los Alamos for good, continuing to make two-week visits each summer as a consultant. Working for Bell Labs was an excellent choice because it gave him the opportunity to work with Claude E. Shannon and John W. Tukey. Hamming believed that most credit should go to the people at Bell Labs who have created an environment where it was natural to do research and creative work. A great deal of credit goes to Tukey for the education he gave through his patient teaching of the computing elements and for setting an example of excellence in his own work. Hamming was involved in nearly all of the most prominent achievements of the laboratories over the next 30 years. From 1958 to 1960, he served as President of the Association for Computing Machinery (ACM). Between 1960 and 1976, while still working at Bell Labs, he became interested in teaching and held a position of a visiting or Adjunct Professor at Stanford University, the City College of New York, the University of California at Irvine, and Princeton University. This period of his career was frustrating for him because aging researchers and scientists were using up space and resources that, in his opinion, could have been put to better use by younger ones like himself. Moreover, he believed that mathematicians were most productive early in their careers and that their productivity dropped off rapidly with their age. That was the main reason why he resolved, while still young, to retire early at age 61, in 1976. He gave up his research career and decided to start another one, as an author and a teacher. He went on to write a number of books on computing theory. Also, he formed the belief that the way mathematics was being taught was wrong and that the only way to invoke a new approach was to write textbooks. His teaching work included various evening classes at Bell Labs. To expand that experience, in 1976, he accepted a professorship of computer science at the Naval Postgraduate School (NPS) in Monterey, CA, where, in his opinion, the knowledge of the students was marvelous. He delivered his last lecture in 1997. Shortly before he retired that June as Distinguished Professor Emeritus, he said [1]: “When I left Bell Labs, I knew that this was the end of my scientific career. When I retire from here, in another sense, it’s really the end.” Hamming died seven months later, in January 1998.

II. HAMMING’S CONTRIBUTIONS TO CODING THEORY

Hamming was, and still is, best known for his contribution in the area of error-detecting and error-correcting codes. At Bell Labs, Hamming spent much of his time with computers, working on particular problems with his colleagues. He comprehended that if a computer could detect when an error had occurred, it must be able to detect where it had occurred. Error detection was done by using a parity check on each block of symbols. This means that an extra digit was added so that the sum of digits in the block was even. Thus, an incorrect entry could be detected because the parity check would fail. Hamming introduced a concept of the number of positions in which two codewords differ and how many changes are required to transform one codeword into another. This is known as the Hamming distance. Consider, for example, a binary codeword, i.e., a sequence with n symbols of length. The Hamming distance $d(s_i, s_j)$ or d_{ij} between codewords s_i and s_j is defined as the number of positions in which these two codewords differ. It follows that the Hamming distance can be written in terms of the Hamming weights as $d_{ij} = w(s_i \oplus s_j)$, where symbol $\oplus$ denotes modulo 2 addition. Let $s_1 = 101101$ and $s_2 = 001100$, respectively. Because $101101 \oplus 001100 = 100001$, $d_{12} = w(100001) = 2$, which means that these two codewords differ in two positions.

Codes can either correct or merely detect errors depending on the amount of redundancy contained in the check symbols. Hamming’s techniques for finding and correcting a single error in a stretch of data and for finding two errors and correcting one of them became known as Hamming codes. With his paper “Error Detecting and Error Correcting Codes,” published in the April 1950 issue of the *Bell System Technical Journal*, a new era within information theory began. All the coding theory described in this paper had appeared over a year and one-half earlier in the interdepartmental memorandums, but the publication was postponed due to patent consideration. In order to patent the code, the Legal and Patent Division needed diagrams and descriptions of the switching circuitry necessary for implementation in addition to

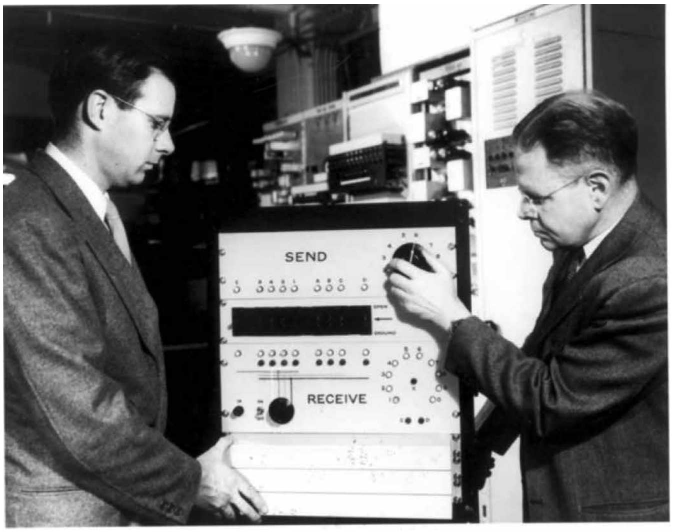

Fig. 2. **Hamming and Holbrook during the demonstration of error detecting and correcting system (reprinted with permission of Nokia Corporation).**

Hamming's mathematical work. Bernard D. Holbrook, from the Switching Research Department at Bell Labs, was asked to draw up the appropriate documentation. Hamming–Holbrook patent "Error-Detecting and Correcting System" (US2,552,629A) was issued in 1951 (Fig. 2). Hamming codes and Hamming distance became cornerstones in coding theory, used not only in many other fields of mathematics but also in practical computer design.

This opened up a new field of study that included addressing important problems in telecommunication and computer science. Hamming also investigated a variety of codes for the binary channel and found some basic properties of the binary symmetric channel. He described the coding procedure by using geometric language. This means that each signal was taken as a point or a vector in an N-dimensional space, with coordinates equal to values of 0 or 1 for N binary symbols. Then, the distance between two points was defined as the number of coordinates in which they differed. In such language, the noisy signal was decoded as the nearest of the signal points, and the corresponding message was chosen.

Next, he introduced the Hamming bound, known as sphere packing or volume bound. This represents a limitation on the parameters of an arbitrary block code, a member of the large family of error-correcting codes that encode data in blocks. The limitations took the form of bounds that relate different parameters of the block code to each other, such as its rate and ability to detect and correct errors. This was an important limitation on the efficiency with which any error-correcting code can utilize the space in which its codewords are embedded. A code that attained the Hamming bound was said to be a perfect code. Some of Hamming's solutions were used by Bell Labs in AT&T's computer systems and telephone switching systems.

Hamming received frequent recognition for his work and contribution to coding theory. In the paper "A Comparison of Signalling Alphabets," published in the *Bell System Technical Journal*, Edgar N. Gilbert, a mathematician, coding theorist, and long-time researcher at Bell Labs, acknowledged Hamming for his many helpful suggestions during the investigations summarized in the preparation of the mentioned paper. In the well-known paper "Coding for Noisy Channels," Peter Elias, a pioneer in the field of information theory at MIT, credited Hamming with investigating a variety of codes for the binary channel and forming some basic properties of the binary symmetric channel. The conventional Hamming distance for the binary symmetric channel was referenced in the coding theory fundamental work [2]. Communication and computer technologies may come and go, but the concepts of Hamming codes and Hamming distance remain, together with the basic principles that provide the necessary background for the current digital era.

III. HAMMING'S INFLUENCE ON THE COMPUTER REVOLUTION

The emergence of computers required Hamming to dedicate almost all of his time to them. As a consequence, his research became centered on improving computer functionality. In 1956, Hamming worked on IBM 650, an early computer, and with Ruth A. Weiss developed the L2 programming language. This was one of the earliest computer languages and was in use for many years. His work led to the development of programming languages, which later evolved into high-level computer languages. One such language was widely used within Bell Labs and by external users, who knew it as Bell 2. In 1957, Bell 2 was superseded by FORTRAN when IBM 650 at Bell Labs was replaced by IBM 704. Also, Hamming continued to deal with numerical analysis, integration of differential equations, and the Hamming spectral window, which is used a lot in computation for smoothing data before applying Fourier analysis. In *A Discipline of Programming*, Edsger W. Dijkstra attributed the problem of efficiently finding regular numbers to Hamming. The problem became known as the Hamming problem, where regular numbers are often referred to as the Hamming numbers. The Hamming problem requires generating, in ascending order, all numbers divisible only by a given set of primes, usually 2, 3, and 5. It is a standard problem with standard solutions, but the solution types can vary according to the language of a programmer.

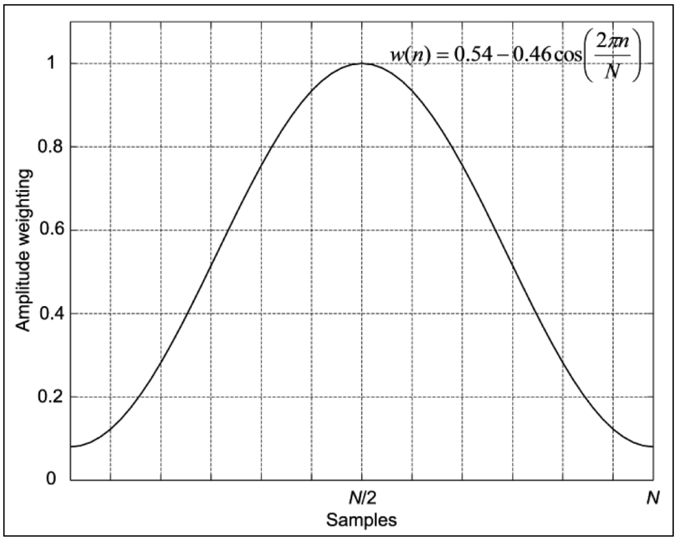

Fig. 3. **Hamming window function.**

Hamming had enormous foresight in predicting, half a century ago, that computers could soon change everyday life, which then only few people could have imagined. His observations were cited during the "Man and the Computer" symposium at the annual meeting of the American Association for the Advancement of Science held in Denver, CO, in December 1961. Two years later, in *The American Mathematical Monthly*, he published the paper with the title "Intellectual Implications of the Computer Revolution," in which the main thesis was that computers were producing a revolution in the world of ideas. Some of the new effects on engineering, science, language, music, and process of thinking were discussed, while the consequences on education were mentioned as well.

IV. DIGITAL FILTER THEORY AND HAMMING WINDOW

Besides error-correcting codes, Hamming became very interested in digital filter theory. It was very important to him to join this research field because, in signal processing, a digital filter was a system that performed mathematical operations on a sampled discrete-time signal to enhance certain characteristics of that signal. His focus was on filter design, particularly on the conversion of analog data to digital form for various computer and communication applications. Hamming's contribution to digital filters arose out of his concern for teaching analog computing specialists the new digital way of thinking. He was even encouraged to write on the subject, learning the field from such experts as Tukey and James F. Kaiser.

Writing about digital filters led Hamming to a new design method and also to a certain "window" being named after him, thanks to Tukey who recognized his contribution in [3] and [4]. Fig. 3 presents the Hamming window function. In signal processing, a window function is a mathematical function that is zero valued outside some chosen interval. It represents a statistical tool that lets users look at a small region of a signal, often a stream, with the least amount of leakage from any other part of the signal. The applications of window functions include spectral analysis, finite impulse response (FIR) filter design, and beamforming.

Characterized by good frequency resolution, reduced spectral leakage, and acceptable noise performance, the Hamming window is used in communication and computation for smoothing data before applying Fourier analysis. It has been applied extensively in telephone signal processing, in which 8-b codecs were the standard for many decades. For high-quality audio signal processing, more efficient windows may be required, particularly when those windows act as low-pass filters (e.g., the Blackman–Harris window).

V. PUBLISHING AND TEACHING WORK

In addition to more than 70 papers and three patents, Hamming also published nine significant books, some of them in several editions, and continuously kept in print today. Armed with his knowledge of basic statistics and the simplest elements of information theory,

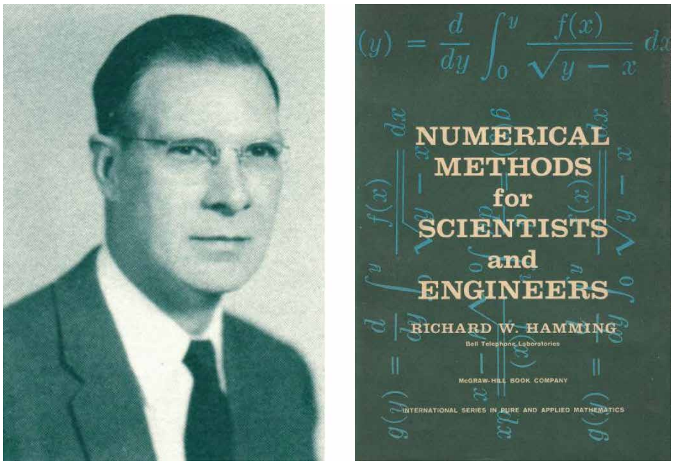

Fig. 4. **(Left) Hamming from the early period of his writing activities. (Right) The cover page of Hamming's first book.**

combined with his 15 years of experience in machine computation, his writing activates started in 1962 (Fig. 4).

All of his books, mainly related to the areas of applied mathematical analysis, computer science, and information theory, are permeated with original philosophy on scientific computing which can be summarized in one sentence: "The purpose of computing is insight, not numbers." Hamming's books are written for those who expect to use large-scale computers as a tool for research and development. Hamming believed that such people did not need a book of formulas or a collection of isolated results, but rather an original presentation of the main ideas in the computing field so that they could understand how the number computed by a machine is related to a particular problem.

Although from the published papers and books, one can have an impression that Hamming was not a team player, he appreciated his colleagues and did not miss the opportunity to acknowledge their support and help. In his books, he regularly cited the management and his colleagues at Bell Labs, who provided him an environment that encouraged independent thinking. In particular, he referred to Professor Roger S. Pinkham of Stevens Institute of Technology, a consultant at Bell Labs, as a constant source of stimulation and inspiration. He often had discussions with Pinkham, mostly about mathematical problems, which resulted in a joint paper "A Class of Integration Formulas," published in the *Journal of the ACM*, that dealt with the generalization of Gregory's formula and examined special cases that were highly suitable for computer library use.

After leaving the intellectual environment of Bell Labs, but being still too young to retire, Hamming ended his scientific work and accepted a professorship of computer science at NPS (Fig. 5). He was very satisfied with the selection of students and liked the fact that he was teaching people who in 25–30 years would be very important in the military organizations of many nations. He was trying to teach a "style" of thinking in science and engineering.

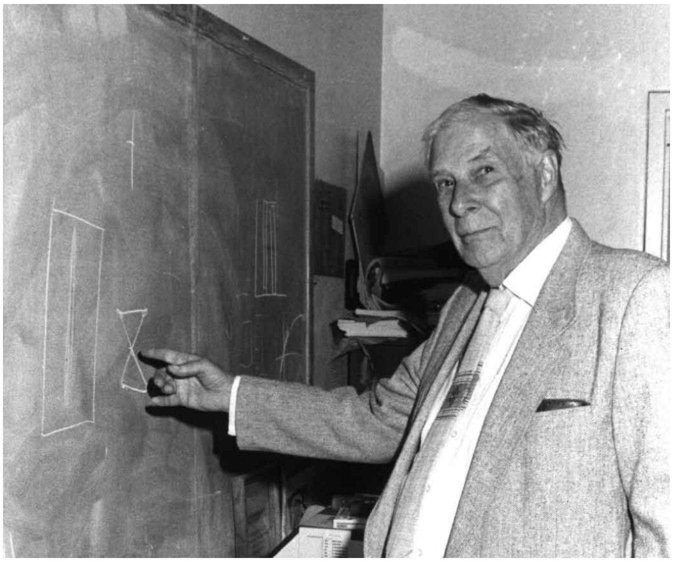

Fig. 5. **Hamming at the Naval Postgraduate School.**

In the beginning of teaching circuit theory, Hamming took his students through a text full of formulas line by line, with very little writing on the blackboard [5]. Hamming encouraged his students to look at problems from various and new angles. He always had in mind that the difficulty of knowing the future does not absolve the teacher from trying to help the student to be ready for it when it arrives. His opinion that "the teachers should prepare the student for the student's future, not for the teacher's past" has been cited many times in the open literature.

The subtitle of his last book *Learning to Learn* was the main solution that Hamming offered to help students cope with the rapid changes in their field of interest. That was the reason why the course centered on how to look at and think about knowledge. By telling stories of his own experiences, Hamming tried to impart general lessons, both personal and technical, to help students achieve a successful technical career. He stated that the course was sometimes called "Hamming on Hamming" because "it draws heavily on my own past experiences, observations, and wide reading." His idea was to continue teaching in the years to come. Soon, he became distinguished not only for his research but also for his full-time teaching work.

Hamming's many years of research experience led him to a number of observations. As a result, he gained introspection while studying theories of creativity. Dick Hamming, as he was known by his colleagues, gave a very interesting and stimulating talk entitled "You and Your Research" to an audience of some 200 Bellcore staff members and visitors at the Morris Research and Engineering Center. The talk focused on his observations and research on the question: "Why do so few scientists make significant contributions and why are so many forgotten in the long run?" His lectures, including that talk, continue to inspire young researchers, as indicated by the enormous number of views of his videos online.

VI. AWARDS AND PROFESSIONAL ACCOMPLISHMENTS

Hamming was an intellectual who fully enjoyed the pursuit of research and the challenges of teaching. He developed a number of simple theoretical and practical approaches that have found their way into many other universal coding, communication, and computer schemes. Hamming earned numerous awards and honors for his inspiring leadership as a researcher and a teacher, as well as his exceptional contributions to coding theory and computer science. He was a Fellow of the IEEE and the Association for Computing Machinery (ACM), and a member of the National Academy of Engineering.

First, he received the A. M. Turing Award for "work on numerical methods, automatic coding systems, error-detecting and error-correcting codes," in 1968. In 1979, he received the IEEE Emanuel R. Piore Award for "introduction of error-correcting codes, pioneering work in operating systems and programming languages, and the advancement of numerical computation." Next year, he received the IEEE Computer Society Pioneer Award as chapter recipient for error-correcting codes. In 1981, he was awarded by the University of Pennsylvania for complete research and scientific work. In 1988, the IEEE established the Richard W. Hamming Medal, to be given each year in recognition "for exceptional contributions to information sciences and systems" (Fig. 6). Hamming was the first recipient of the IEEE medal named after him. For work on error-correcting codes, Eduard Rhein Foundation gave him an award in 1996. His colleagues from NPS assigned to him a valuable recognition in the form of a new supercomputer using Hamming's name, in 2009.

VII. CONCLUSION

There are two main periods in the career of Richard W. Hamming: scientific research and teaching. Starting as a programmer of one of the earliest digital computers, he became best known for his scientific work on error-detecting and error-correcting codes. This work motivated and helped him to introduce a new subject within information theory, as well as standard terms, such as Hamming codes, Hamming distance, and Hamming matrix, which are used today not only in coding theory but also in many areas of mathematics. Hamming established a connection between coding theory and computer design, which was of practical interest. The idea was "a consideration of large scale computing machines in which a large number of operations must be performed without a single error in the end result." His work on codes is linked to packing, whereas the error-correcting codes he discovered led directly to the development of a programming language that has since evolved into computer languages. Hamming's results in the field of coding theory are still included in textbooks on data communications. He was a brilliant mathematician and computer scientist who had the ability to find the most direct and simplest way to solve a complex problem.

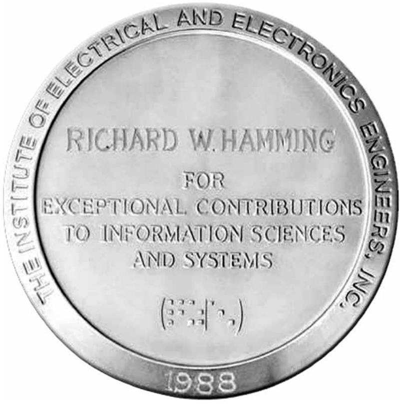

Fig. 6. **Back of the IEEE Richard W. Hamming Medal.**

The second period of his work was devoted to teaching. In this field, his philosophy was extremely practical. He often said: "Whatever subject you are teaching is really a class of learning to learn." Born as a scientist in his mind and as a teacher in his soul, Hamming paved the way for the future for his students. Firmly believing that Hamming's lectures are as relevant today as they were when first offered, one of his students, Donald P. Brutzman, from the NPS Department of Information Sciences, leads an effort to relaunch Hamming's course to distance learning. ■

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