Seismic Data Reconstruction Based on Double Sparsity Dictionary Learning With Structure Oriented Filtering

Lakshmi Kuruguntla, *Member, IEEE*, Vineela Chandra Dodda, *Member, IEEE*, Anup Kumar Mandpura, *Member, IEEE*, Sunil Chinnadurai, *Senior Member, IEEE*, and Karthikeyan Elumalai, *Member, IEEE*

***Abstract*—In seismic data processing, denoising and reconstruction are the two steps for identification of resources in the earth subsurface layers. The seismic data quality is affected by random noise and interference during acquisition. Further, the noisy data may be incomplete with missing traces. In this work, we propose a method for incomplete seismic data denoising and reconstruction based on double sparsity dictionary learning (DSDL) with structure oriented filtering (SOF). The main function of the DSDL step is denoising and SOF is used for residual noise attenuation and filling the missing data points. The proposed method is tested on 2-D synthetic and field datasets. The test results show that the DSDL-SOF method has better noise attenuation and reconstruction in terms of signal-to-noise ratio and mean squared error as compared to existing methods.**

***Index Terms*—Denoising, dictionary learning, double sparsity, seismic data reconstruction, structure oriented filtering (SOF).**

I. Introduction

IDENTIFICATION of resources present in the earth subsurface layers' is essential for the exploration of hydrocarbon and mineral deposits [1]. However, direct observation of these layers is difficult and uneconomical as the area under survey is usually vast. Seismic exploration is an efficient and economical method to determine the earth subsurface layers' information. During seismic exploration, data is collected using geophones arranged on the surface. However, the seismic data recorded by geophones is incomplete, irregular in spatial dimensions and contains anomalous traces due to imperfect instruments and the presence of noise [2], [3]. Removal of these traces from the data leads to loss of information. Furthermore, some recorded traces are lost due to obstacles in the field and ground surface limitations [4], [5], [6]. These incomplete and irregular recorded traces in seismic data severely effect the subsequent seismic data processing steps such as velocity analysis, multiples elimination, full waveform inversion, reverse time migration, amplitude variation with offset analysis, time-frequency analysis, and seismic interpretation [4], [5], [7], [8]. The missing data problems in seismic exploration are classified based on regular and irregular missing data. In regular missing data problem, the data is equidistant and missing at constant rate in a uniform grid. Whereas in irregular missing data problem, the data is randomly missing along the same uniform grid and therefore, challenging to reconstruct [2]. Therefore, reconstruction of missing data is important for accurate imaging of subsurface structures and removal of sampling artifacts.

In literature, four types of reconstruction (or) interpolation methods have been proposed for seismic data based on: Prediction Filtering (PF), Wave Equations (WE), Rank reduction (RR), and compressive sensing. The PF-based seismic data reconstruction exploits the linear predictability property of the seismic signal in different domains, namely the frequency-space (f-x), time-space (t-x), and frequency-wavenumber (f-k) domains. PF method in the f-x domain considers spatially sampled seismic data and performs the reconstruction in each frequency slice based on linear prediction (LP) theory [9]. In [10], the frequency-space domain PF method was extended to f-x-y domain in which data samples from each frequency slice are considered selectively for the designing the LP operator. Further in [11], a prediction error filter (PEF) was proposed for reconstruction of seismic data with multiple dip events having different slopes. The major limitation of PF-based methods is the need for equally spaced seismic data for reconstruction of missing traces [5].

The second category of seismic data reconstruction is based on WE techniques. The WE techniques used the Normal move out (NMO) and inverse dip move out (IDMO) jointly and performs the missing data reconstruction with the prior knowledge of the surface parameters. NMO and IDMO techniques utilized the seismic wave physical properties and depends on subsurface velocity model. The performance of seismic data reconstruction based on wave equations is dependent on the accurate velocity model which is difficult to obtain [3]. Also reconstruction

Manuscript received 31 March 2023; revised 31 May 2023, 18 July 2023, and 8 September 2023; accepted 27 September 2023. Date of publication 10 October 2023; date of current version 23 October 2023. This work was supported by the Department of Science and Technology, Science and Engineering Research Board (DST-SERB), India through Core Research under Grant CRG/2019/001234. *(Corresponding author: Karthikeyan Elumalai.)*

Lakshmi Kuruguntla, Sunil Chinnadurai, and Karthikeyan Elumalai are with the Department of Electronics and Communication Engineering, SRM University, Amaravathi 522502, India (e-mail: lakshmi_kuruguntla@srmap.edu.in; sunil.c@srmap.edu.in; imkarthi@gmail.com).

Vineela Chandra Dodda is with the Department of Electronics and Communication Engineering, Amrita School of Engineering Amaravati, Amrita Vishwa Vidyapeetham, Amaravathi 522503, India (e-mail: c_vineela@av.amrita.edu).

Anup Kumar Mandpura is with the Department of Electrical Engineering, Delhi Technological University, New Delhi 110042, India (e-mail: kanup@dtu.ac.in).

Digital Object Identifier 10.1109/JSTARS.2023.3323362

technique assumes a smooth spatial spectrum of the seismic data in WE-based techniques. Further, WE-based methods are computationally expensive and not preferred for high dimensional seismic data interpolation [12].

The third category of seismic data reconstruction is based on RR method with the prior assumption of rank. In RR method, complete data is assumed to have low rank and irregular data is assumed to have a high rank [2]. The Multichannel singular spectrum analysis (MSSA) method is a rank reduction method which transforms noisy and incomplete seismic data with individual frequency into Hankel matrix [13]. Then, a rank reduction operator known as truncated singular value decomposition (TSVD) approach is applied on the Hankel matrix to decompose the noisy-incomplete data into two different subspaces namely signal subspace and noise subspace corresponding to signal and noise [14]. The noise subspace consists of both noise and incomplete seismic data [4]. However, the MSSA method is computationally expensive for the complex data with large Hankel matrix. An efficient MSSA method, where TSVD approach is replaced with randomized QR decomposition was proposed in [15]. The MSSA method can decompose seismic data into a noise subspace and a signal-plus-noise subspace. Therefore in [16], a damped multichannel singular spectrum analysis (DMSSA) method was proposed for high dimensional seismic data reconstruction with a new type of SVD known as damped SVD was applied on Hankel matrix. DMSSA method can precisely separate the seismic data into noisy and signal subspaces. The DMSSA method performs well when the data has low signal-to-noise ratio (SNR) and high dimensionality. Therefore, the DMSSA method [16] is particularly attractive for solving the 5-D offset-vector-tile (OVT) seismic data reconstruction and regularization problems in complicated area. However, in RR-based methods the selection of rank is quite difficult. A good selection of rank values provide better reconstruction results for incomplete seismic data.

The fourth category of seismic data reconstruction method is based on compressive sensing which recovers the fully sampled data by sparsity promoting methods like sparse transforms and dictionary learning (DL). Compressive sensing methods assumes that the incomplete seismic data is less sparse in comparison to complete seismic data. The incomplete data is first transformed into sparse domain using fixed basis where useful seismic signals have large amplitude coefficients and incomplete traces along with noise have low amplitude coefficients. From these coefficients, the original seismic signal is recovered using a thresholding operator. A variety of sparse transforms with predefined basis such as Fourier [17], Wavelet [18], Curvelet [19], Radon [20], Shearlet [21], Seislet [22], and Dreamlet [23] have been studied for reconstruction of seismic data. Seislet and dreamlet are two types of fixed sparse bases commonly used in seismic data processing, specifically for denoising and interpolation tasks. Compared to f-k transform-based approaches, the seislet transform-based compressed sensing method offers significantly better results in terms of data recovery, as indicated by measures such as SNR, local similarities, and visual observations [24]. To incorporate sparsity constraints in the seislet transform domain, a fast projection onto convex sets (POCS) algorithm was proposed in [24]. However, accurate estimation of the local slope is crucial for interpolation of regularly missing traces using the seislet-based POCS algorithm. The iterative slope estimation strategy employed in the algorithm is not always be reliable and can lead to a degradation of the interpolation performance. To address this issue, a more dependable technique based on the velocity-slope transformation was introduced in [22]. This alternative approach reduces the number of iterations required for interpolation and significantly improves overall performance.

The dreamlet, is a physical wavelet introduced by Wu et al. [25] and has shown remarkable capabilities in representing seismic data. The dreamlet is localized on the light cone within the 4-D Fourier space, making it an efficient tool for estimating the true signal from noisy seismic data [26]. The high sparsity observed in the dreamlet domain further enhances its effectiveness in representing seismic signals. A damped dreamlet representation was introduced in [23] to further reduce the gap between the projected signal and the true signal. By incorporating a damping factor, the damped dreamlet representation achieves more accurate estimations of the true signal. However all the aforementioned sparse transforms use orthogonal dictionaries with fixed basis which cannot match with complex seismic data structures [27]. Hence, dictionary learning (DL) methods have been proposed in seismic data reconstruction with the dictionaries learnt from training data based on data driven tight frames (DDTF), K-means singular value decomposition (K-SVD), and sequential generalized K-means (SGK) [3], [28]. The SGK algorithm is computationally efficient and used for high dimensional seismic data reconstruction.

In addition to reconstruction, noise attenuation is also a major concern for subsequent steps in seismic exploration. Henceforth, simultaneous denoising and reconstruction were performed based on rank reduction methods in [13], [29], sparse transform methods in [26] and DL-based methods in [3], [27], [28], and [30]. However, all the aforementioned methods are unsuitable for complex structured events such as curved events in seismic data. In [31], structure-oriented filtering (SOF) was used for simultaneous removal of spike-like noise and interpolation of missing points in seismic data with complex structured events. However, double sparsity dictionary learning (DSDL) methods perform better in the presence of Gaussian distributed random noise and non-Gaussian noise in seismic data [32], [33]. In [30], a joint denoising and reconstruction method for seismic data was proposed using DSDL method in the presence of spiky noise. An extended sparse K-SVD method for dictionary update was used where the atoms are updated through weighted low rank approximation. However the method in [31] was computationally complex. In recent years, deep learning methods have gained significant attention in the field of seismic data reconstruction. By training the deep learning model on large volumes of seismic data, it is possible to reconstruct missing traces, suppress noise, and improve the overall resolution of seismic images [34], [35], [36]. Deep learning models require a large amount of labeled training data to learn and generalize effectively. However, acquiring labeled seismic data for training can be challenging, time-consuming and expensive. To address this problem, unsupervised learning or self-learning techniques are proposed in [37] and [38] for sparse time frequency analysis of seismic data.

In this work, we propose to use a computationally efficient DSDL method as a reconstruction operator in weighted iterative projection on convex sets algorithm with analytic dictionary as fast discrete curvelet transform (FDCT) and adaptive dictionary as sequential generalized k-means (SGK). Sparse representation of the data in the dictionary domain does not always guarantee a successful separation between signal and noise. Spatial coherence of the training data has a significant impact on the dictionary learning process [39]. In data sets with poor spatial coherence, a learned dictionary will not be able to represent a complex structure. In order to alleviate this shortcoming, we further applied structure oriented filtering (SOF) on the reconstructed data obtained from DSDL. The mean filter is to applied along the structural direction of seismic events in an approximately flattened gather and helps to preserve the useful signals. Existing methods such as FDCT, SGK, and SOF, individually cannot accurately recover the missing traces. In DSDL reconstruction, the sparse dictionary is based on a sparsity model of the dictionary atoms over a fixed dictionary. However, the selection of parameters in DSDL is also a challenging task. Further, the output of DSDL is filtered using SOF for further improvement in SNR. Also, in the filtering step we fine tune the number of iterations to get the accurate slope estimation for the reconstructed output of DSDL. In SOF stage, we have applied the partial denoised and reconstructed data (rather than direct noisy decimated data) which helps to reconstruct the seismic data with high level of accuracy. When compared to existing methods, the proposed DSDL-SOF method applied to noisy and incomplete seismic data has a marked improvement in terms of SNR and mean squared error (MSE).

The rest of this article is organized as follows. In Section II, we illustrate the mathematical prerequisites for the seismic signal model with noise and missing traces, analytic (FDCT) and adaptive dictionary learning (Fast dictionary learning) methods. In Section III, we describe our proposed methodology. In Section IV, the reconstruction performance of the proposed method is compared to that of existing methods. Finally, Section V concludes the article.

II. Mathematical Prerequisites

This section describes the mathematical prerequisites that will be used in the subsequent sections.

A. Signal Model

In this section, we describe the signal model for incomplete seismic data with random noise. Let $X \in \mathbb{R}^{M\times N}$ be the complete seismic data with N traces having M data points each, mathematically expressed as

$$X = [T_1, T_2, \cdots T_N] \tag{1}$$

where $T_i \in \mathbb{R}^{M\times 1}$ is the clean ith trace. In the process of acquisition, the seismic signal recorded by geophones is afflicted by noise and also some traces are lost. Let $W \in \mathbb{R}^{M\times N}$ be the additive random noise with Gaussian distribution and $S \in \mathbb{R}^{M\times M}$ be the sampling matrix with L nonzero diagonal entries and $M - L$ diagonal elements with zeros. Then, the observed noisy and incomplete seismic dataset $O \in \mathbb{R}^{M\times N}$ can be expressed as

$$O = S(X + W) \tag{2}$$

where $M - L$ traces have been lost.

B. Fast Discrete Curvelet Transform

The FDCT is one of the efficient transform used for denoising and reconstruction of seismic data. FDCT is a multiscale and multidirectional localized transform used for interpolation with POCS [40]. FDCT is also used as sparsity promoting filter for noise attenuation in seismic data [33], [41] [42]. We assume that clean seismic data X has sparse representation d in FDCT domain C is represented as

$$X = C^T d. \tag{3}$$

Then, the (2) is reformulated as

$$O = S(C^T d + W) = Ad + \tilde{W} \tag{4}$$

where $A = SC^T$, denotes the measurement matrix and $\tilde{W} = SW$. The sparse signal that can be separated from noise is then solved by minimizing the cost function as follows:

$$\hat{d} = \arg\min_d \frac{1}{2}\|O - Ad\|_2^2 + \alpha\|d\|_1 \tag{5}$$

where α is the parameter used for regularization, $\|\|_1$ and $\|\|_2$ represents L_1 and L_2 norms, respectively. The solution of above (5), provides the denoised and reconstructed seismic data.

C. Fast Dictionary Learning

In this section, we describe the fast DL method for seismic data denoising and reconstruction. In DL, sparse coding and dictionary update are the two important steps which are iteratively performed until the convergence of data model is obtained. SGK algorithm is also known as fast DL wherein the atoms of the dictionary are updated through arithmetic average of training samples with accelerated sparse coding [27], [43]. The fast DL has been used to denoise and reconstruct seismic data at the same time [27].

In fast DL method, the observed data $O(M \times N)$ is divided into patches and the ith patch is denoted as O_i. Let $D \in \mathbb{R}^{M\times P}$ represents dictionary and $G \in \mathbb{R}^{P\times N}$ is the sparse coefficient matrix. Then, the sparse form of observed seismic data based on DL method is defined as the product D and $G = [g_1, g_2, \cdots g_N]$ and mathematically expressed as

$$O = D^n G \tag{6}$$

where n represents the iteration number. The first step of fast DL method is sparse coding which can be obtained by minimizing the cost function below.

$$\forall_j g_j^n = \arg\min_{g_j} \|O - D^n G\|_F^2$$
$$\text{subject to } \forall_j \|g_j\|_1 \leqslant q \tag{7}$$

where q denotes the sparsity level and g_j^n represents the sparse coefficient corresponding to jth column of sparse coefficient matrix, G and $\forall j$ represents all columns of sparse coefficient matrix G. To solve the optimization problem in (7), an orthogonal matching pursuit (OMP) algorithm can be used until the defined sparsity constraint level is met. However, the OMP method requires q iterations and hence, more computations. To address that problem, accelerated sparse coding method was used in SGK which computes the sparse coefficients in a single iteration [27]. After sparse coding, dictionary update is the second step in SGK where the sparse coefficients G are fixed from (7) and dictionary atoms are updated by solving the cost function given below.

$$D^{n+1} = \arg\min_{D} \|O - DG^n\|_F^2 \tag{8}$$

In SGK-DL, the cost function given in (8) is solved without performing singular value decompositions (SVD) operations. Instead, an arithmetic average of training data samples is applied for updating the dictionary atoms [27]. Let d_k be the kth atom in the dictionary, D and N^k represent the number of training samples corresponding to the kth atom, then update equation of kth atom via SGK-DL method can be defined as

$$d_k = \frac{1}{N^k} \sum_{i=1}^{N^k} O_i. \tag{9}$$

The above equation is used for updating each atom in the dictionary D. After the sparse coding and dictionary update through finite number of iterations, the denoised seismic data can be reconstructed as a linear combination of the updated atoms in the dictionary and updated sparse coefficients as follows:

$$\hat{O} = \hat{D}\hat{G} \tag{10}$$

where $\hat{D}$ and $\hat{G}$ are the updated dictionary and sparse coefficient matrices, respectively.

D. Structure-Oriented Filtering

In the presence of complex events, structure oriented filtering is performed in two steps: 1) local slope estimation; and 2) filtering. Local slope is an important characteristic that can help to separate the seismic signals from noise. Based on plane wave destruction algorithm (PWD) [44], [45], the local slope is calculated. Using this slope information, the seismic data are translated to the flattened domain. Then, structural filtering is performed using a mean or median operator applied in a specific local window that is driven by the local slope at the corresponding locations [46].

The structure oriented mean filter applied to 2-D seismic data is mathematically defined [31] as

$$\hat{w}_{i,j} = \arg\min_{w_u \in W_{i,j}} \sum_{l=1}^{L+1} (w_u - w_l)^2 \tag{11}$$

where $\hat{w}_{i,j}$ is the output for observed data point at (i,j)th position in which i and j are vertical and spatial axes, respectively. $L+1$ is the filter window length, u and l are indices in filter window. The above optimization equation is equivalent to determining the mean in the filtering window. The solution of optimization equation gives the output value for the particular position (i,j) of events, where $W_{i,j}$ represents searching window with respect to slope of observed data at specified positions defined as

$$W_{i,j} = \Big\{ w_{i,j-\left(\frac{L}{2}\right)\sigma_{i,j}}, \ldots, w_{i,j-\sigma_{i,j}}, w_{i,j}, w_{i,j+\sigma_{i,j}}, \ldots, w_{i,j+\left(\frac{L}{2}\right)\sigma_{i,j}} \Big\} \tag{12}$$

where $\sigma_{i,j}$ represents is the local slope of the structure pattern at position i, j.

III. Methodology

In this section, we demonstrate the methodology of our work for seismic data denoising and reconstruction based on DSDL-SOF. The block diagram of the proposed method is shown in Fig. 1 which is the cascade of DSDL and SOF.

In seismic data, most of the events have complex curved structure, hence in the first step of our proposed work, we imposed the double sparsity on noisy decimated data by computational efficient fast discrete curvelet transform and fast dictionary learning algorithm. FDCT is a one type of sparse transform with high sparseness can be able to separate the signal from noise [19]. The transform-domain thresholding-based filtering works best for random noise discussed in Section II-B. However, in our work we have considered the presence of random noise in the data along with missing traces which are randomly decimated. Hence, some residual noise remaining with partial interpolation.

Therefore, we further impose the sparsity using the adaptive dictionary learning method through sequential generalized k-means algorithm given in Section II-C for the removal of the residual noise. In adaptive dictionary learning method, the atoms in the dictionary are learned from the training data which is already sparse. Therefore, the data becomes double sparse and extracts the useful seismic signal from the noise. The DSDL is a promising method for Gaussian distributed random and non-Gaussian distributed erratic noise attenuation in seismic data [32], [33]. Let $\mathcal{G}$ represents the sparse coefficient matrix, B and F are the analytic and adaptive dictionaries, respectively. The sparse representation of the observed data O based on DSDL method is mathematically expressed as

$$O = \mathcal{D}\mathcal{G} \tag{13}$$

where $\mathcal{D}$ represents the double sparsity dictionary obtained by cascading of the analytic and adaptive dictionaries, i.e.,

$$\mathcal{D} = BF. \tag{14}$$

In double sparsity dictionary, the atoms in the analytic dictionary B are fixed and predefined, and the atoms in the adaptive dictionary are updated from the sparse data provided by the fixed dictionary in adaptive manner. The sparse coefficients $\mathcal{G}$ and dictionary $\mathcal{D}$ [32] are obtained by solving the cost function below:

$$\langle \hat{\mathcal{G}}, \hat{\mathcal{D}} \rangle = \arg\min_{\mathcal{G},\mathcal{D}} \frac{1}{2} \|O - \mathcal{D}\mathcal{G}\|_2^2 + \alpha \|\mathcal{G}\|_0. \tag{15}$$

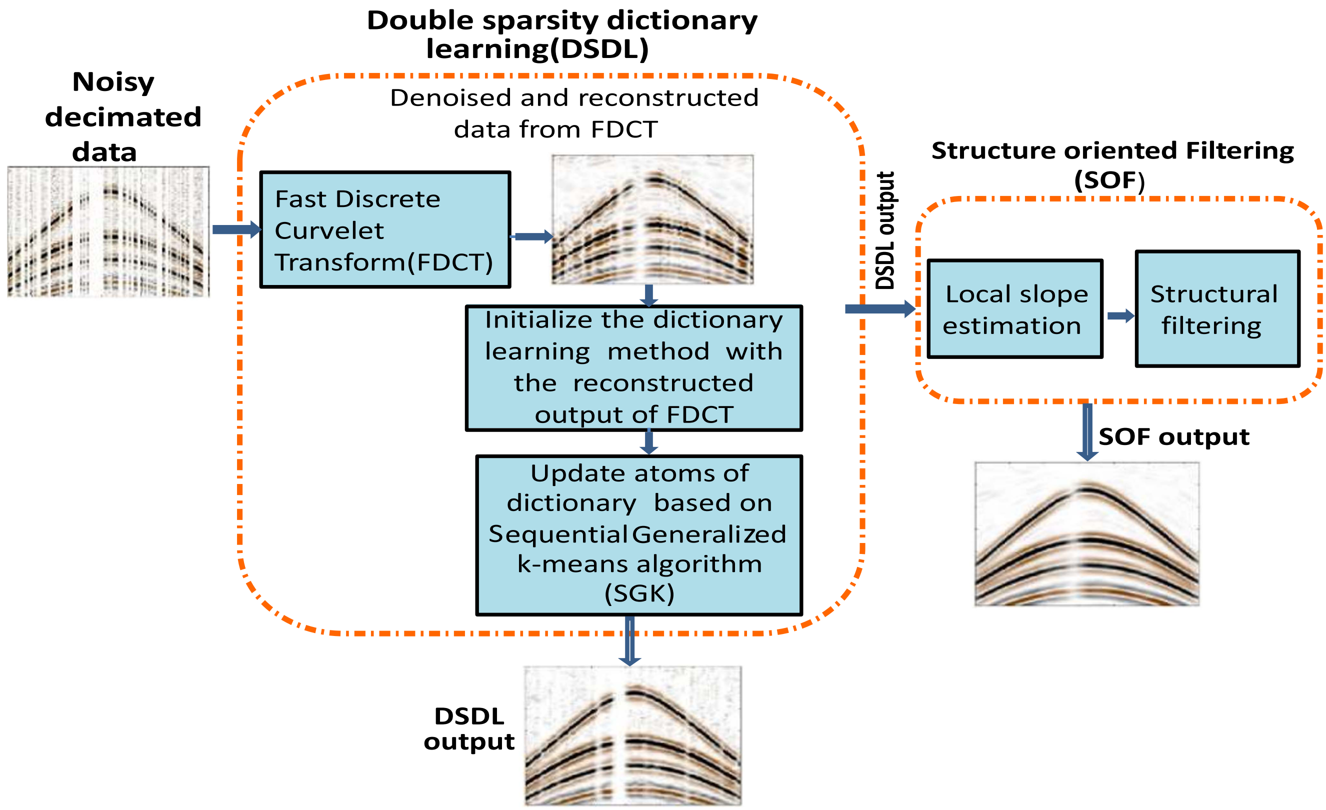

Fig. 1. Proposed DSDL with SOF method's block diagram.

The denoised seismic data R can be reconstructed using DSDL as follows:

$$R = \hat{\mathcal{D}}\hat{\mathcal{G}} \tag{16}$$

where $\hat{\mathcal{D}}$ and $\hat{\mathcal{G}}$ are updated dictionary and sparse coefficient matrices obtained using DSDL method. This DSDL method is used as a reconstruction operator in the weighted projection on convex sets algorithm for simultaneous denoising and missing traces reconstruction like in [3]. Even though, data interpolation need to be improved for further reconstruction of noiseless data with good signal preservation. Hence, finally we applied structure oriented filtering on the DSDL output with iterative slope estimation and structural filtering (Section II-D).

Let U be the SOF initialized by the DSDL output R given in (16), then the denoised and reconstructed data Y is mathematically expressed as

$$Y = U(R_{q+1}) \tag{17}$$

where q indicates the iteration number and R_{q+1} is the reconstructed data after $q+1$ iterations. The reconstructed data at $(q+1)$th iteration is obtained using the weighted iterative projection on convex sets algorithm [3] with DSDL as a reconstruction operator given as

$$R_{q+1} = \beta_q SO + (I - \beta_q S)\Gamma(O_q) \tag{18}$$

where Γ is the reconstruction operator and β_q is the relaxation parameter which decreases to 0 from 1 as the number of iterations increase. The DSDL-based reconstruction operator with sparsity constraint is represented mathematically as

$$\Gamma(O_q) = \arg\min_{\mathcal{D},\mathcal{G}} \|O - \mathcal{D}\mathcal{G}\|_2^2$$
$$\text{subject to } \forall_j {g_1}_j = e_v \tag{19}$$

where e_v represents the unit vector with only vth element as 1 and remaining elements zero which is considered as sparsity constraint for DSDL method. $\mathcal{D}$ and $\mathcal{G}$ are the double sparsity dictionary and sparse coefficient matrix defined in (13), where g_1 is the coefficient vector of matrix $\mathcal{G}$.

IV. Results and Discussion

In this section, we illustrate the performance of the proposed DSDL-SOF method on incomplete synthetic and field data. We generated hyperbolic and linear crossed events similar to [47] and [48], respectively. Also, two field data examples are also used from [49] and [50]. The efficacy of our work using DSDL-SOF on synthetic data is compared based on signal to noise ratio (SNR), MSE, and local similarity map with energy [3], [33] metrics. Whereas for field data, we analyzed based on visual quality and frequency spectra. The SNR of reconstructed data can be mathematically expressed as

$$\text{SNR} = 10\log_{10}\left(\frac{\sum_k X^2(k)}{\sum_k (Y(k) - X(k))^2}\right) \tag{20}$$

where k represents the trace number, $X(k)$ is the clean seismic data, and $Y(K)$ is the reconstructed data.

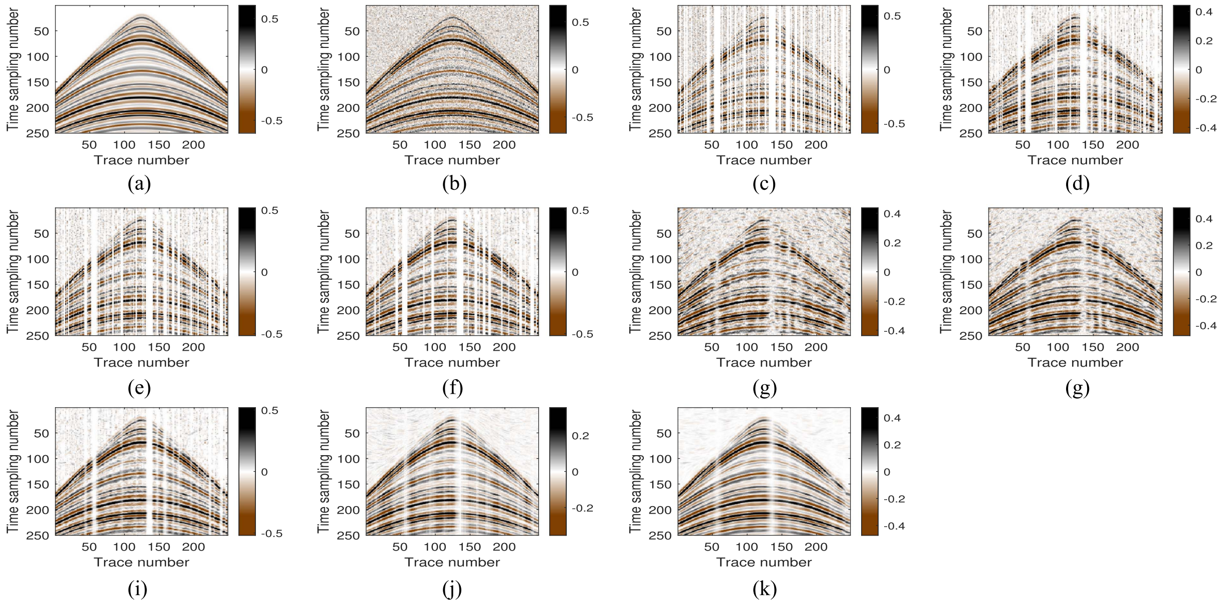

Fig. 2. Denoised and reconstruction results for existing and proposed methods on synthetic seismic data with complex hyperbolic events: (a) Clean data, (b) Noisy data, (c) noisy and incomplete seismic data with SNR 1.9863 dB, denoised and reconstructed results using:(d) DDTF, (e) K-SVD, (f) SGK, (g) MSSA, (h) DMSSA, (i) DSDL, (j) SOF, and (k) proposed DSDL-SOF.

The MSE of the reconstructed data $Y(k)$ with N traces is given by

$$\text{MSE} = \frac{1}{N}\sum_{k=1}^{N}\left(Y(k) - X(k)\right)^2. \tag{21}$$

The proposed method is studied on four different datasets as follows.

1) Synthetically generated data with complex and simple hyperbolic events;
2) synthetically generated data with linear crossed events;
3) field data-I;
4) field data-II.

A. *Study-I: Comparision of DSDL-SOF With Existing Methods on Synthetic Data With Complex Hyperbolic Events*

In this study, we analyzed the performance of proposed method with existing DL methods, structure oriented filtering method and rank-reduction methods (MSSA and DMSSA) [3] on synthetically generated seismic data with complex (more number of) hyperbolic events. The generated clean hyperbolic structured seismic events contain 250 traces with sampling duration of 4 ms as depicted in Fig. 2(a). The clean data is distorted with Gaussian distributed random noise presented in Fig. 2(b) and decimated by 50% which is shown in Fig. 2(c). Then, the noisy and incomplete synthetic data with hyperbolic events SNR is of 1.98 dB. We applied both the proposed DSDL-SOF and existing methods (DDTF, K-SVD, SGK, rank reduction methods based on MSSA and DMSSA) and individual DSDL, SOF methods on the noisy and incomplete data. Then, the obtained output SNR, MSE, energy, and computation time are tabulated in Table I.

TABLE I
COMPARISON OF PROPOSED METHOD USING DSDL-SOF ON COMPLEX HYPERBOLIC EVENTS (FIG. 2) WITH EXISTING METHODS IN TERMS OF SNR,MSE, ENERGY, AND COMPUTATION TIME

Methods	**SNR(dB)**	**MSE**	Energy	Computation time (sec)
Noisy and incomplete data	1.98	0.0241	-	-
DDTF	3.09	0.0187	0.4703	376.04
K-SVD	3.47	0.0171	0.4858	431.06
SGK	4.48	0.0136	0.5310	36.77
MSSA	4.65	0.0130	0.5618	10.32
DMSSA	5.96	0.0096	0.5922	5.76
DSDL	7.18	0.0073	0.6262	139.98
SOF	5.15	0.0116	0.6708	51.49
Proposed DSDL-SOF	8.15	0.0058	0.6764	175.17

According to Table I, DDTF, K-SVD, SGK, MSSA, DMSSA, SOF, and DSDL methods improved the SNR to 3.09 dB, 3.47 dB, 4.48 dB, 4.65 dB, 5.96 dB, 5.15 dB, and 7.18 dB, respectively. In addition, the proposed method enhances the SNR to 8.15 dB and also shows least MSE value as compared to the existing methods. The reconstructed plots of existing methods are shown in Fig. 2(d)–(h) and individual DSDL and SOF method plots are shown in Fig. 2(i)–(j) where noise can be seen in the reconstructed data and missing traces have been reconstructed partially. In Fig. 2(k), the plot of reconstructed seismic data corresponding to the proposed method based on DSDL-SOF method is presented and it is observed that it has performed better denoising and reconstruction of hyperbolic events.

It can be also observed from Fig. 2(i) that the DSDL method does not reconstruct the missing traces 55 to 59. In Fig. 2(j), SOF has a slightly better performance in reconstructing the missing

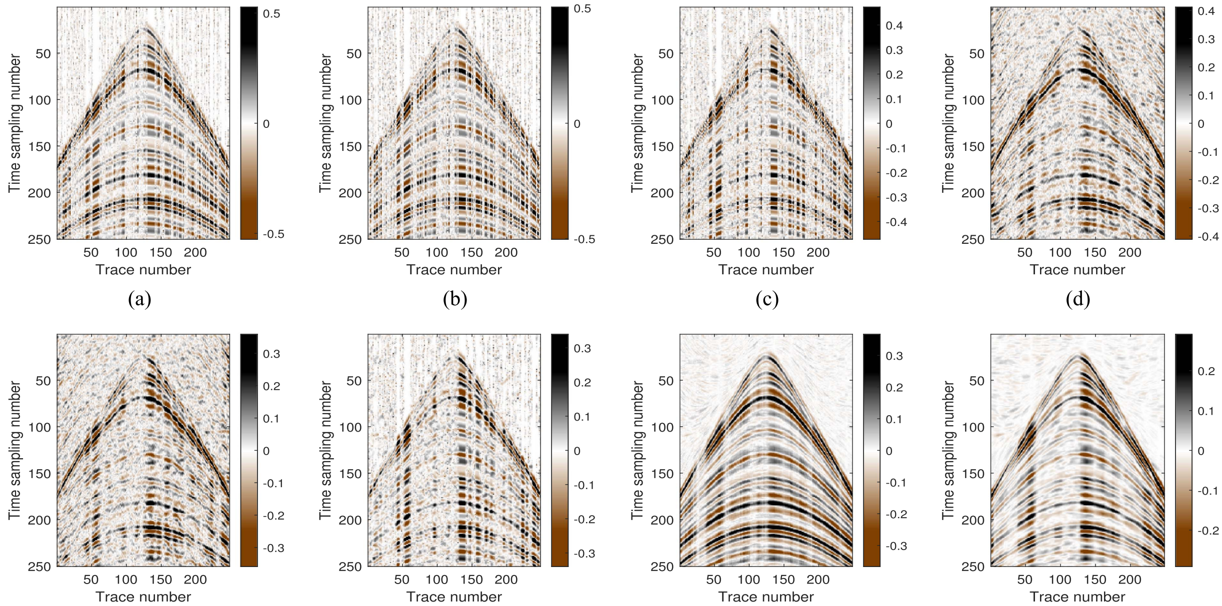

Fig. 3. Difference sections between clean and Reconstruction results for existing and proposed methods on synthetic seismic data with more hyperbolic events: Difference section of clean and reconstructed section of : (a) DDTF; (b) K-SVD; (c) SGK; (d) MSSA; (e) DMSSA (f) DSDL; (g) SOF; and (h) proposed DSDL-SOF.

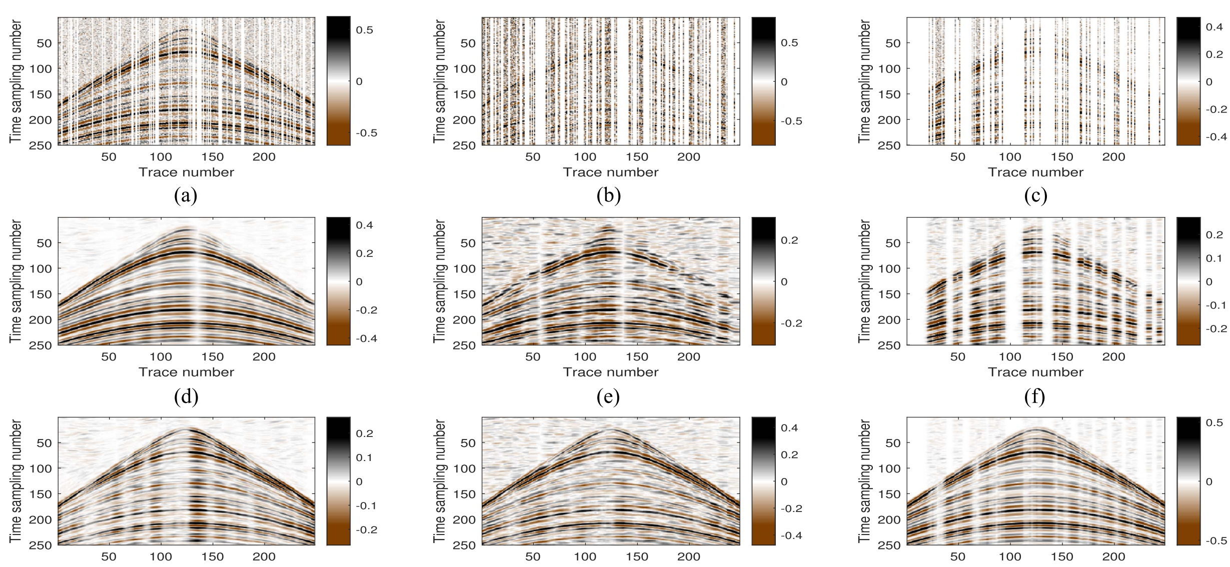

Fig. 4. Proposed method results for data with different noise levels and decimation: (a) Noisy seismic data decimated by 30% with SNR of 3.12 dB; (b) noisy seismic data decimated by 60% with SNR of –1.88 dB; (c) Noisy seismic data decimated by 80% with SNR of 0.68 dB; (d), (e), and (f) are reconstructed data of (a), (b) and (c), respectively, using the proposed method; (g), (h), and (i) are the difference sections of clean and reconstructed data of 30, 60, and 80% decimation levels.

traces 55 to 59. Similarly, the reconstruction performance for the traces 49, 50, 51, 52, 92, 93, 94, DSDL is slightly better than SOF. In Fig. 2(k), the proposed method combines the advantages of both the methods and has better reconstruction than either DSDL or SOF. The difference sections between clean and reconstructed data of all the existing and proposed methods are shown in Fig. 3(a)–(h), respectively. Further, we analyzed the performance of the proposed method on seismic data with different noise levels and sampling ratios. We generated noisy and decimated data with 30%, 60%, and 80% decimation having different noise levels (5.76 dB, –3.7 dB, and 5.82 dB). The resulting noisy and decimated data with SNR values 3.12 dB, –1.88 dB, and 0.68 dB, respectively, are shown in Fig. 4(a)–(c). The results obtained by applying the proposed method on the

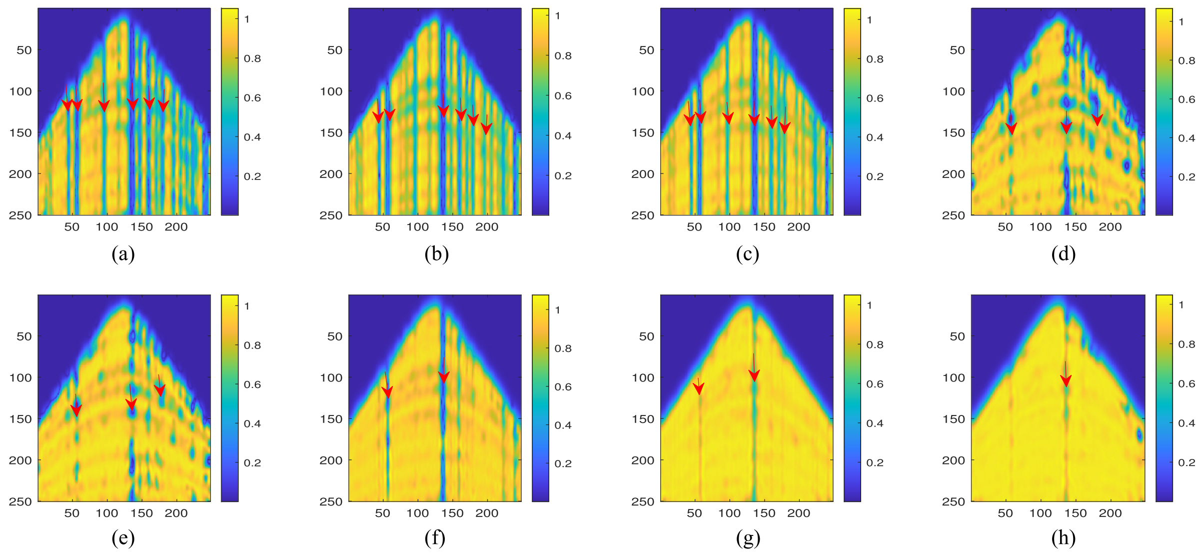

Fig. 5. Local similarity map between input and reconstructed data by (a) DDTF, (b) K-SVD, (c) SGK, (d) MSSA, (e) DMSSA, (f) DSDL, (g) SOF, and (h) proposed DSDL-SOF.

noisy decimated data are shown in Fig. 4(d)–(f), respectively. In order to illustrate the loss we consider noisy data with 50% decimation and plot the local similarity map between input and reconstructed data using the various methods which are shown in Fig. 5. The respective energy values are tabulated in Table I. From the similarity maps and Table I, we infer that the proposed method outperforms the existing methods in terms of energy. The loss in valid signal is indicated by arrows in Fig. 5 and it can be observed that signal loss in the proposed method given in Fig. 5(h) is small as compared to the existing methods. In addition, simulation results demonstrate that the DSDL method performs second best to the proposed method in terms of reconstruction and denoising. From Table I, the computation time for DSDL method is approximately 139 s and the proposed method takes approximately 175 s. While the proposed method takes slightly longer time, it achieves an improvement in SNR of nearly 1 dB as compared to DSDL method.

Furthermore, we compare the performance of our proposed method with existing methods on the synthetic data of 128 traces with simple hyperbolic events for different noise levels such as –2 dB, 0.28 dB, and 2.42 dB. The SNR and MSE of the reconstructed data with existing state-of-the-art methods and the proposed method are tabulated in Table II. From Table II, it is observed that the proposed method has an improved performance over a wide range of noise levels and decimation of seismic data.

In Fig. 6, we presented the comparative results of existing and proposed methods on synthetic seismic data with hyperbolic events of 128 traces. In Figs. 6(a)–(c) depicts noise free clean data, noisy data, noisy and incomplete data with 50% decimation having SNR of 2.42 dB, respectively. The proposed method and existing methods are applied for three different noise levels of data and the performance in terms of SNR and MSE is given in Table III. The plots of reconstructed outputs of existing and proposed methods for noisy level of 2.42 dB are shown in Fig. 6(d)–(k).

From Table III, it can be inferred that the proposed method has superior performance while compared to the existing methods

TABLE II
SNR AND MSE COMPARISON OF PROPOSED METHOD USING DSDL-SOF WITH EXISTING METHODS FOR THREE DIFFERENT NOISE LEVELS AND DECIMATION PERCENTAGES ON HYPERBOLIC EVENTS GIVEN IN FIG. 2(A)

Noisy and incomplete data with SNR(dB)	Method	Denoised and reconstructed data SNR(dB)	MSE
3.12	DDTF	4.96	0.0186
	K-SVD	5.93	0.0097
	SGK	6.54	0.0084
	MSSA	5.58	0.0105
	DMSSA	7.30	0.0071
	DSDL	7.68	0.0065
	SOF	8.61	0.0052
	Proposed DSDL-SOF	8.93	0.0050
-1.88	DDTF	-0.24	0.0403
	K-SVD	-0.28	0.0407
	SGK	1.16	0.0291
	MSSA	1.39	0.0276
	DMSSA	2.09	0.0235
	DSDL	2.10	0.0234
	SOF	2.91	0.0194
	Proposed DSDL-SOF	3.20	0.0182
0.68	DDTF	0.76	0.0320
	K-SVD	0.94	0.0306
	SGK	1.55	0.0266
	MSSA	1.25	0.0285
	DMSSA	1.34	0.0279
	DSDL	1.85	0.0024
	SOF	1.47	0.0271
	Proposed DSDL-SOF	2.11	0.0234

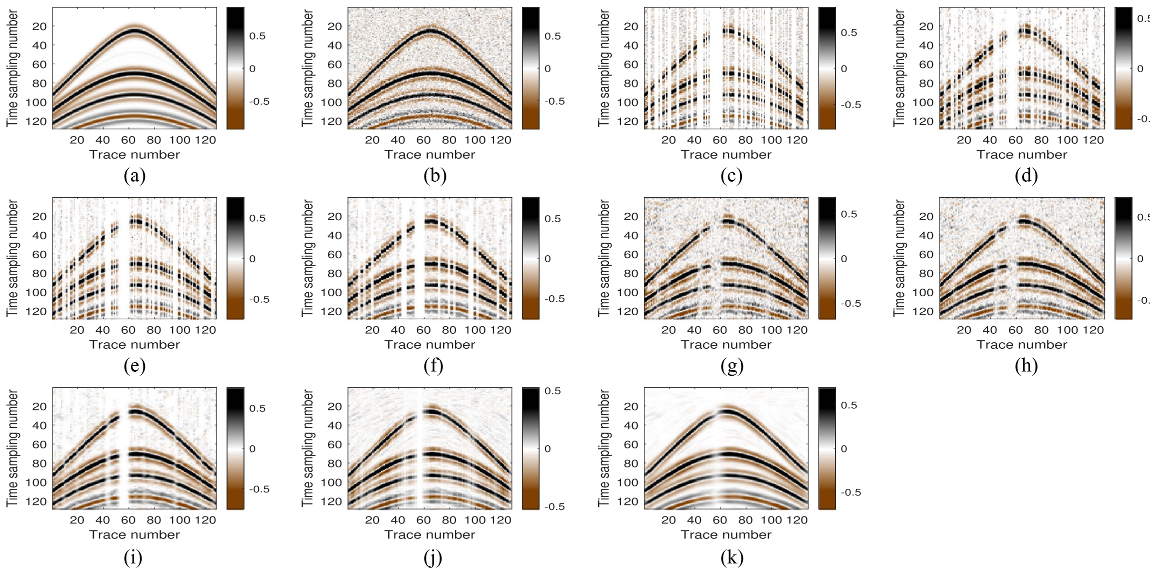

Fig. 6. Performance analysis of existing and proposed methods for data with less number of hyperbolic events: (a) Clean data with hyperbolic events, (b) noisy seismic data, (c) noisy and incomplete seismic data with SNR of 2.42 dB, denoised and reconstructed results using (d) DDTF, (e) K-SVD, (f) SGK, (g) MSSA, (h) DMSSA, (i) DSDL, (j) SOF, and (k) proposed DSDL-SOF.

TABLE III
SNR AND MSE COMPARISON OF PROPOSED METHOD USING DSDL-SOF WITH EXISTING METHODS FOR THREE DIFFERENT NOISE LEVELS ON SIMPLE HYPERBOLIC EVENTS DATA GIVEN IN FIG. 6(A)

Noisy and incomplete data SNR(dB)	Method	Denoised and reconstructed data SNR(dB)	MSE
-2	DDTF	-0.05	0.0735
	K-SVD	0.10	0.0708
	SGK	1.72	0.0488
	MSSA	1.46	0.0518
	DMSSA	1.77	0.0482
	DSDL	3.60	0.0317
	SOF	3.43	0.1173
	Proposed DSDL-SOF	5.66	0.0197
0.28	DDTF	1.23	0.0547
	K-SVD	2.24	0.0433
	SGK	3.53	0.0322
	MSSA	3.55	0.0320
	DMSSA	4.14	0.0280
	DSDL	6.03	0.0181
	SOF	4.45	0.0260
	Proposed DSDL-SOF	7.69	0.0124
2.42	DDTF	2.55	0.0403
	K-SVD	3.52	0.0323
	SGK	4.97	0.0231
	MSSA	5.39	0.0210
	DMSSA	6.65	0.0157
	DSDL	8.24	0.0109
	SOF	5.06	0.0415
	Proposed DSDL-SOF	9.05	0.0090

over wide range of noise levels in the seismic data. Further, we also compared with local similarity map between clean and reconstructed data and respective energy values are tabulated in Table IV. From Table IV, it is clear that our proposed method has high energy value which indicates the high similarity between the clean input data and reconstructed data. The respective local similarity maps between clean and reconstructed data of existing and proposed methods are shown in Fig. 7(a)–(h).

TABLE IV
ENERGY COMPARISON OF PROPOSED DSDL-SOF METHOD WITH EXISTING METHODS ON SIMPLE HYPERBOLIC EVENTS DATA GIVEN IN FIG. 6(A)

Methods	**Energy**
DDTF	0.4468
K-SVD	0.5046
SGK	0.5784
MSSA	0.5597
DMSSA	0.5912
DSDL	0.6819
SOF	0.6846
Proposed DSDL-SOF	0.7400

TABLE V
SNR AND MSE COMPARISON OF PROPOSED METHOD USING DSDL-SOF ON LINEAR CROSSED EVENTS WITH EXISTING METHODS

Methods	**SNR(dB)**	**MSE**
Noisy and incomplete data	0.44	0.0110
DDTF	1.73	0.0082
K-SVD	2.14	0.0075
SGK	3.55	0.0054
MSSA	2.47	0.0069
DMSSA	3.65	0.0053
DSDL	5.12	0.0038
SOF	5.68	0.0033
Proposed DSDL-SOF	5.85	0.0032

B. Study-II: Comparision of DSDL-SOF With Existing Methods on Synthetic Data With Linear Crossed Events

In this study, we compared the proposed method results to that of existing methods on synthetic data having linear crossed

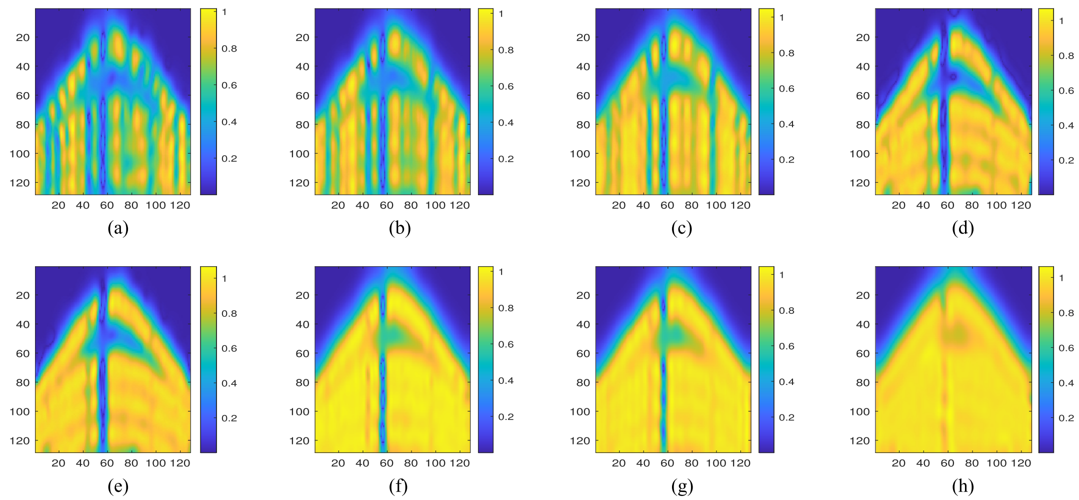

Fig. 7. Local similarity map between input and reconstructed data by (a) DDTF, (b) K-SVD, (c) SGK, (d) MSSA, (e) DMSSA, (f) DSDL, (g) SOF, and (h), proposed DSDL-SOF.

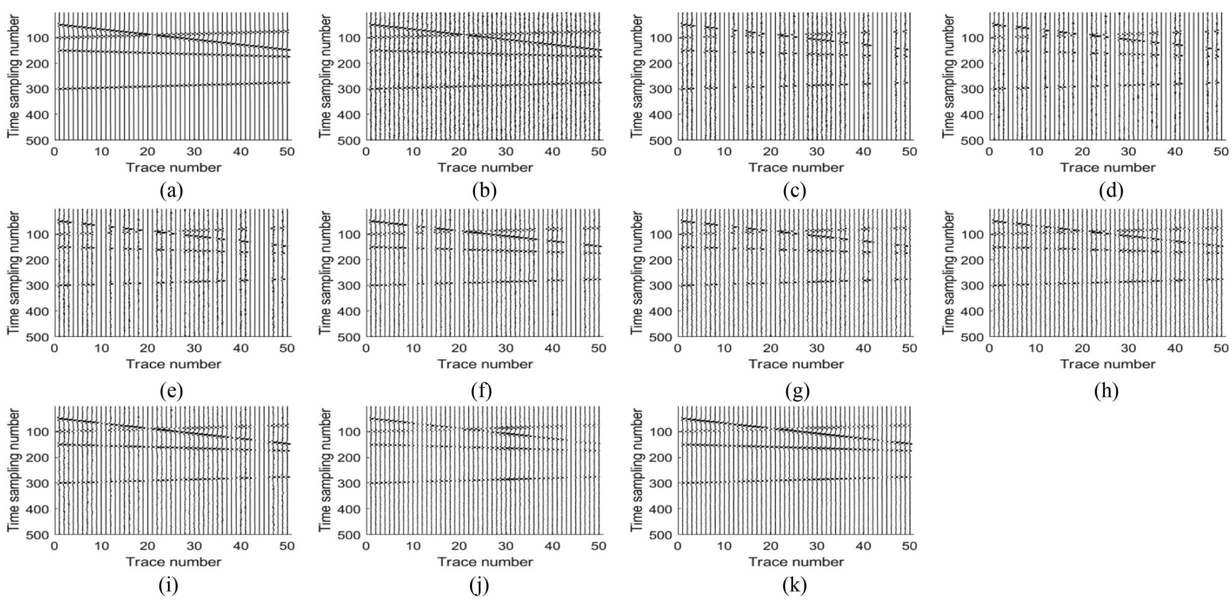

Fig. 8. Comparative results for linear-crossed events : (a) Clean data with linear-crossed events, (b) noisy seismic data with SNR 0.89 dB, (c) noisy and incomplete seismic data with SNR 0.44 dB, denoised and reconstructed data using (d) DDTF, (e) K-SVD (f) SGK, (g) MSSA, (h) DMSSA, (i) DSDL, (j) SOF, and (k) proposed DSDL-SOF.

events shown in Fig. 8. In Fig. 8(a), we presented the synthetically generated clean seismic data with linear crossed events. The clean data consists of 50 traces with sampling duration of 4 ms. The noisy data is generated by adding the Gaussian distributed Random noise to the clean data which is shown in Fig. 8(b). The noisy data is then decimated by 50% to obtain noisy incomplete data with SNR of 0.44 dB and is presented in Fig. 8(c). We applied the DDTF, K-SVD, SGK, MSSA, DMSSA, DSDL, SOF methods and the proposed method on the noisy and incomplete seismic data and the reconstructed results are shown in Figs. 8(d)–(k), respectively. The corresponding SNR and MSE values are given in Table V.

Table V clearly shows that the proposed method outperforms existing methods. The respective plot of proposed method is as shown in Fig. 8(k), where we clearly notified that the reconstructed data is more similar to the original noise free data in Fig. 8(a).

C. Study-III: Comparision of DSDL-SOF With Existing Methods on Field Data-I

In this study, we examine the performance of the proposed and existed methods on field data-I [49] is analyzed. The field data-I is presented in Fig. 9(a). The field data is decimated by 50% and the resultant incomplete field data is presented in Fig. 9(b). The existing and proposed methods are applied on decimated field data for noise attenuation and reconstruction. The denoised and reconstructed data of the DDTF, K-SVD, SGK, MSSA, DMSSA, DSDL, SOF and the proposed methods are presented in Fig. 9(c)–(j), respectively. From Fig. 9(j), we observed that

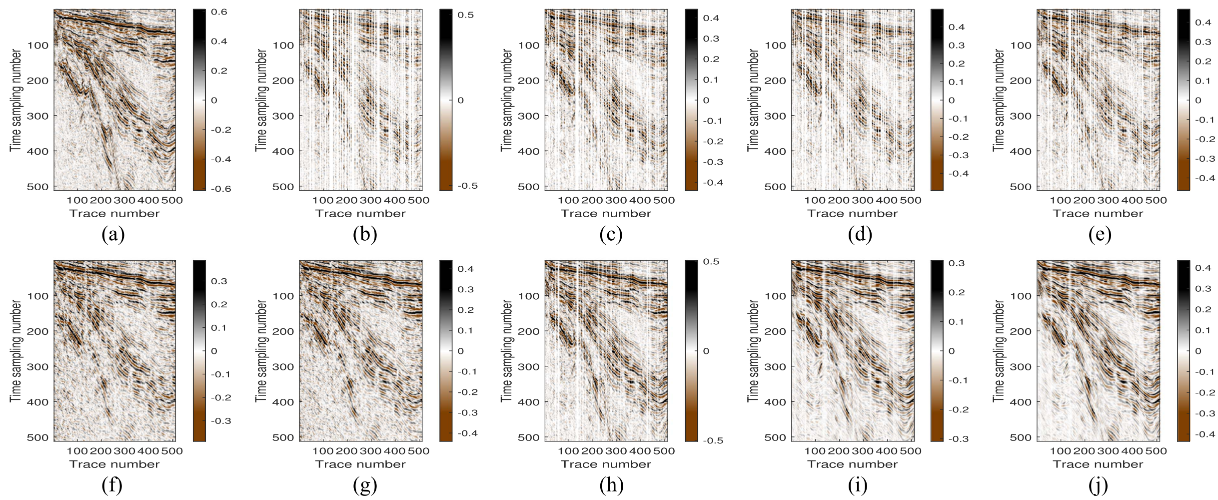

Fig. 9. Denoised and reconstructed results for field data-I (a) field data-I, (b) incomplete field data with 50% decimation, denoised and reconstructed field data using (c) DDTF, (d) K-SVD, (e) SGK, (f) MSSA, (g) DMSSA, (h) DSDL, (i) SOF, and (j) proposed DSDL-SOF.

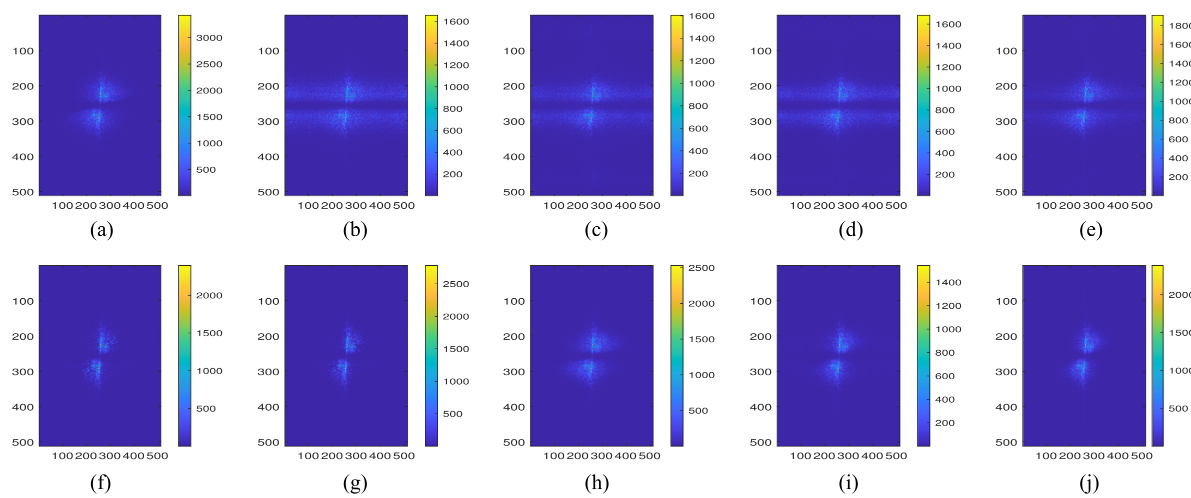

Fig. 10. Frequency spectra for input field data-I and reconstructed data through existing and proposed methods: (a) field data-I, (b) incomplete field data with 50% decimation, and reconstructed field data using (c) DDTF, (d) K-SVD, (e) SGK, (f) MSSA, (g) DMSSA, (h) DSDL, (i) SOF, and (j) proposed DSDL-SOF.

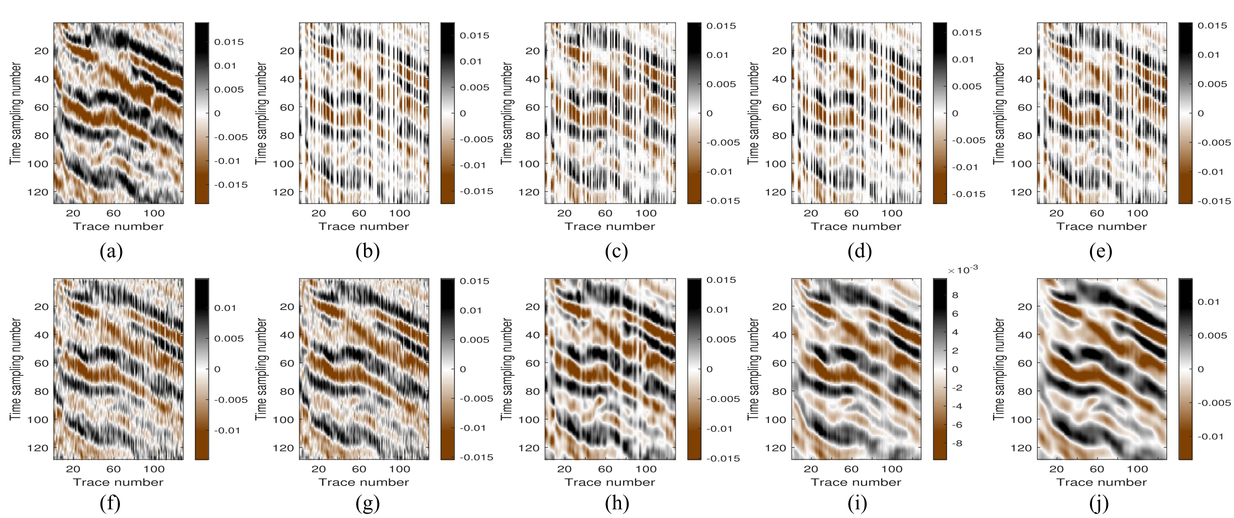

Fig. 11. Denoised and reconstructed results for (a) field data-II, (b) incomplete field data with 50% decimation, denoised and reconstructed field data using (c) DDTF, (d) K-SVD, (e) SGK, (f) MSSA, (g) DMSSA, (h) DSDL, (i) SOF, and (j) proposed DSDL-SOF.

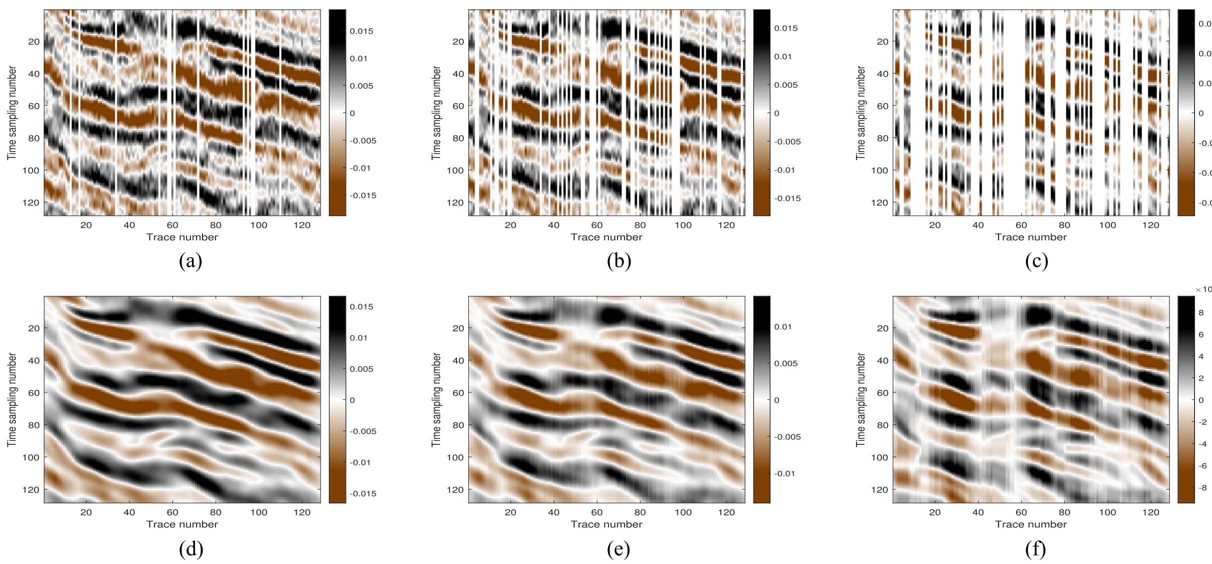

Fig. 12. Proposed method results for data with different levels of decimation: (a) Field seismic data decimated by 10%; (b) field seismic data decimated by 30%; (c) field seismic data decimated by 60%; (d), (e), and (f) are reconstructed data of (a), (b), and (c), respectively, using the proposed method.

the events in the output of proposed method are clearly visible with better noise attenuation and reconstruction. Furthermore, we also provided the frequency spectra of input field data and reconstructed data of all the existing and proposed methods in Fig. 10.

D. Study-IV: Comparision of DSDL-SOF With Existing Methods on Field Data-II

In this study, we compared the proposed method on field data-II in [50] with existed DL methods (DDTF, K-SVD, SGK, and DSDL), RR methods using MSSA and DMSSA, and filtering method based on SOF. The field data-II is depicted in Fig. 11(a). The field data decimated by 50% is presented in Fig. 11(b). The outputs of DDTF, K-SVD, SGK, MSSA, DSDL, SOF and proposed DSDL-SOF methods on incomplete field data are shown in Fig. 11(c)–(j), respectively.

The performance of the proposed method on the field data-II with three different decimation levels 10%, 30%, and 60%, respectively, shown in Fig. 12(a)–(c). The results of reconstruction from low decimation to high decimation are presented in Fig. 12(d)–(f), respectively. In Fig. 12(f), which corresponds to reconstructed data for highly decimated data, the features present in the actual are also visible.

E. Discussion

In practical scenario, the recorded seismic data is noisy and consists of missing traces. The reconstruction of irregularly decimated and noisy seismic data is a challenging problem. To address this issue, a novel method is proposed by combining the benefits of DSDL and SOF. We used the DSDL method as a reconstruction operator in weighted iterative projection on convex sets algorithm. However, the sparse representation of data in the dictionary domain does not always guarantee a successful separation between signal and noise. Spatial coherence of the training data also has a significant impact on the dictionary learning process. In datasets with poor spatial coherence, a learned dictionary will not be able to represent a complex structure. In order to alleviate this shortcoming, we further applied SOF on the reconstructed data obtained from DSDL. The mean filter is applied along the structural direction of seismic events in an approximately flattened gather and it helps to preserve the useful signals. Therefore, by combining the DSDL method and SOF method the overall performance of denoising and reconstruction has been improved. When compared to existing methods, the proposed DSDL-SOF method applied to noisy and incomplete seismic data has a marked improvement in terms of SNR and MSE. In addition, we observed aliasing in regular decimated (consecutive traces missing) seismic data with high percentage, nearly 79%. Due to aliasing, the high frequency components cannot be recovered and the edges in the reconstructed data are not sharp. Novel methods for accurate reconstruction of missing data with aliasing will be considered in the future work.

V. Conclusion

In this article, we proposed a novel method for seismic data denoising and reconstruction based on DSDL with SOF in the presence of incomplete and noisy seismic data. In our proposed method, we perform denoising and reconstruction through two cascaded steps: 1) DSDL; and 2) SOF. The first step performs denoising and reconstruction leveraging the benefits of analytic dictonary (FDCT) and adaptive dictionary (SGK), respectively. In the second step, the output of DSDL method is filtered through SOF with accurate slope estimation. Therefore, the residual noise is attenuated and simultaneous reconstruction of missing traces is performed with improved SNR. The efficacy of the proposed double sparsity DL method with SOF is studied using synthetically generated data and field data. The studies with synthetic and field data prove that the proposed method has better denoising and reconstruction performance when compared to existing methods and has better SNR, MSE, and local similarity.

Acknowledgment

The authors would like to give special thanks to Prof. Yangkang Chen and his research group at the University of Texas at Austin, for their great help in providing research data, professional suggestions.

References

[1] D. W. Steeples and R. D. Miller, "Seismic reflection methods applied to engineering, environmental, and ground-water problems," in *Proc. SEG Geotech. Environ. Geophys.*, pp. 409–461, 1988.

[2] X. Chai, H. Gu, F. Li, H. Duan, X. Hu, and K. Lin, "Deep learning for irregularly and regularly missing data reconstruction," *Sci. Rep.*, vol. 10, no. 1, pp. 1–18, 2020.

[3] H. Wang, W. Chen, Q. Zhang, X. Liu, S. Zu, and Y. Chen, "Fast dictionary learning for high-dimensional seismic reconstruction," *IEEE Trans. Geosci. Remote Sens.*, vol. 59, no. 8, pp. 7098–7108, Aug. 2021.

[4] W. Huang, D. Feng, and Y. Chen, "De-aliased and de-noise cadzow filtering for seismic data reconstruction," *Geophysical Prospecting*, vol. 68, no. 2, pp. 553–571, 2020.

[5] L. Fu, M. Zhang, Z. Liu, and H. Li, "Reconstruction of seismic data with missing traces using normalized Gaussian weighted filter," *J. Geophys. Eng.*, vol. 15, no. 5, pp. 2009–2020, 2018.

[6] S. Gan, S. Wang, Y. Chen, X. Chen, W. Huang, and H. Chen, "Compressive sensing for seismic data reconstruction via fast projection onto convex sets based on seislet transform," *J. Appl. Geophys.*, vol. 130, pp. 194–208, 2016.

[7] J. Wu, M. Bai, D. Zhang, H. Wang, G. Huang, and Y. Chen, "Fast and robust low-rank approximation for five-dimensional seismic data reconstruction," *IEEE Access*, vol. 8, pp. 175501–175512, 2020.

[8] J. Wu and M. Bai, "Adaptive rank-reduction method for seismic data reconstruction," *J. Geophys. Eng.*, vol. 15, no. 4, 2018, Art. no. 1688.

[9] S. Spitz, "Seismic trace interpolation in the FX domain," *Geophysics*, vol. 56, no. 6, pp. 785–794, 1991.

[10] Y. Wang, "Seismic trace interpolation in the FXY domain," *Geophysics*, vol. 67, no. 4, pp. 1232–1239, 2002.

[11] J. Claerbout and S. Fomel, "Geophysical image estimation by example: Stanford exploration project," Stanford Exploration Project, 2014.

[12] W. Huang and J. Liu, "Robust seismic image interpolation with mathematical morphological constraint," *IEEE Trans. Image Process.*, vol. 29, pp. 819–829, 2020.

[13] V. Oropeza and M. Sacchi, "Simultaneous seismic data denoising and reconstruction via multichannel singular spectrum analysis," *Geophysics*, vol. 76, no. 3, pp. V25–V32, 2011.

[14] M. A. N. Siahsar, S. Gholtashi, E. O. Torshizi, W. Chen, and Y. Chen, "Simultaneous denoising and interpolation of 3-D seismic data via damped data-driven optimal singular value shrinkage," *IEEE Geosci. Remote Sens. Lett.*, vol. 14, no. 7, pp. 1086–1090, Jul. 2017.

[15] J. Cheng and M. D. Sacchi, "Fast dual-domain reduced-rank algorithm for 3D deblending via randomized QR decomposition," *Geophysics*, vol. 81, no. 1, pp. V89–V101, 2016.

[16] W. Huang, R. Wang, Y. Chen, H. Li, and S. Gan, "Damped multichannel singular spectrum analysis for 3D random noise attenuation," *Geophysics*, vol. 81, no. 4, pp. V261–V270, 2016.

[17] H. Zhang, X.-H. Chen, and X.-M. Wu, "Seismic data reconstruction based on CS and Fourier theory," *Appl. Geophys.*, vol. 10, pp. 170–180, 2013.

[18] N. Guo, M. Chen, and Y. Cui, "Prestack seismic data reconstruction method based undecimated discrete wavelet transform," in *Proc. 77th EAGE Conf. Exhib.*, vol. 2015, no. 1. EAGE Publications BV, 2015, pp. 1–5.

[19] H. Yang, Y. Long, J. Lin, F. Zhang, and Z. Chen, "A seismic interpolation and denoising method with curvelet transform matching filter," *Acta Geophysica*, vol. 65, no. 5, pp. 1029–1042, 2017.

[20] J. Shao and Y. Wang, "Seismic data antialiasing interpolation using sparse radon transform and dynamic mask function," *Geophysics*, vol. 87, no. 5, pp. V437–V449, 2022.

[21] L. Jicheng, C. Yongxin, and Z. Jianjiang, "Interpolating seismic data via the POCS method based on shearlet transform," *J. Geophys. Eng.*, vol. 15, no. 3, pp. 852–876, 2018.

[22] W. Liu, S. Cao, S. Gan, Y. Chen, S. Zu, and Z. Jin, "One-step slope estimation for dealiased seismic data reconstruction via iterative seislet thresholding," *IEEE Geosci. Remote Sens. Lett.*, vol. 13, no. 10, pp. 1462–1466, Oct. 2016.

[23] W. Huang, R.-S. Wu, and R. Wang, "Damped dreamlet representation for exploration seismic data interpolation and denoising," *IEEE Trans. Geosci. Remote Sens.*, vol. 56, no. 6, pp. 3159–3172, Jun. 2018.

[24] S. Gan*, S. Wang, Y. Chen, and X. Chen, "Seismic data reconstruction via fast projection onto convex sets in the seislet transform domain," in *Proc. SEG Tech. Prog. expanded Abstr. Soc. Exploration Geophysicists*, 2015, pp. 3814–3819.

[25] R.-S. Wu, Y. Geng, and B. Wu, "Physical wavelet defined on an observation plane and the dreamlet," in *Proc. SEG Int. Expo. Annu. Meeting*, 2011, pp. SEG–2011.

[26] B. Wang, R.-S. Wu, X. Chen, and J. Li, "Simultaneous seismic data interpolation and denoising with a new adaptive method based on dreamlet transform," *Geophysical J. Int.*, vol. 201, no. 2, pp. 1182–1194, 2015.

[27] J. Wu, Q. Chen, Z. Gui, and M. Bai, "Fast dictionary learning for 3D simultaneous seismic data reconstruction and denoising," *J. Appl. Geophys.*, vol. 194, 2021, Art. no. 104446.

[28] S. Yu, J. Ma, X. Zhang, and M. D. Sacchi, "Interpolation and denoising of high-dimensional seismic data by learning a tight frame," *Geophysics*, vol. 80, no. 5, pp. V119–V132, 2015.

[29] Y. Chen et al., "Simultaneous denoising and reconstruction of 5-D seismic data via damped rank-reduction method," *Geophysical J. Int.*, vol. 206, no. 3, pp. 1695–1717, 2016.

[30] L. Zhu, E. Liu, and J. H. McClellan, "Joint seismic data denoising and interpolation with double-sparsity dictionary learning," *J. Geophys. Eng.*, vol. 14, no. 4, pp. 802–810, 2017.

[31] Y. Zhou and S. Li, "Simultaneous deblending and interpolation using structure-oriented filters," *J. Appl. Geophys.*, vol. 150, pp. 230–243, 2018.

[32] Y. Chen, J. Ma, and S. Fomel, "Double-sparsity dictionary for seismic noise attenuation," *Geophysics*, vol. 81, no. 2, pp. V103–V116, 2016.

[33] L. Kuruguntla, V. C. Dodda, A. K. Mandpura, and K. Elumalai, "Erratic noise attenuation using double sparsity dictionary learning method," *IEEE Trans. Geosci. Remote Sens.*, vol. 60, 2022, Art. no. 5913613.

[34] N. Liu, L. Wu, J. Wang, H. Wu, J. Gao, and D. Wang, "Seismic data reconstruction via wavelet-based residual deep learning," *IEEE Trans. Geosci. Remote Sens.*, vol. 60, 2022, Art. no. 4508213.

[35] N. Liu, J. Wang, J. Gao, S. Chang, and Y. Lou, "Similarity-informed self-learning and its application on seismic image denoising," *IEEE Trans. Geosci. Remote Sens.*, vol. 60, 2022, Art. no. 5921113.

[36] N. Liu et al., "NS2NS: Self-learning for seismic image denoising," *IEEE Trans. Geosci. Remote Sens.*, vol. 60, 2022, Art. no. 5922311.

[37] N. Liu, Y. Lei, R. Liu, Y. Yang, T. Wei, and J. Gao, "Sparse time-frequency analysis of seismic data: Sparse representation to unrolled optimization," *IEEE Trans. Geosci. Remote Sens.*, vol. 61, 2023, Art. no. 5915010.

[38] Y. Yang, Y. Lei, N. Liu, Z. Wang, J. Gao, and J. Ding, "SparsetFNet: A physically informed autoencoder for sparse time–frequency analysis of seismic data," *IEEE Trans. Geosci. Remote Sens.*, vol. 60, 2022, Art. no. 4512812.

[39] H. Lv, "Sparse dictionary learning for noise attenuation in the exactly flattened dimension," *J. Seismic Exploration*, vol. 28, no. 5, pp. 449–474, 2019.

[40] H. Zhang, H. Zhang, J. Zhang, Y. Hao, and B. Wang, "An anti-aliasing pocs interpolation method for regularly undersampled seismic data using curvelet transform," *J. Appl. Geophys.*, vol. 172, 2020, Art. no. 103894.

[41] Q. Zhao, Q. Du, X. Gong, and Y. Chen, "Signal-preserving erratic noise attenuation via iterative robust sparsity-promoting filter," *IEEE Trans. Geosci. Remote Sens*, vol. 56, no. 6, pp. 3547–3560, Jun. 2018.

[42] H. Wang et al., "High-precision seismic data reconstruction with multi-domain sparsity constraints based on curvelet and high-resolution radon transforms," *J. Appl. Geophys.*, vol. 162, pp. 128–137, 2019.

[43] Y. Chen, "Fast dictionary learning for noise attenuation of multidimensional seismic data," *Geophysical J. Int.*, vol. 209, no. 1, pp. 21–31, 2017.

[44] S. Fomel, "Local seismic attributes," *Geophysics*, vol. 72, no. 3, pp. A29–A33, 2007.

[45] S. Fomel, "Applications of plane-wave destruction filters," *Geophysics*, vol. 67, no. 6, pp. 1946–1960, 2002.

[46] H. Wang et al., "A matlab code package for 2D/3D local slope estimation and structural filtering," *Geophys.*, vol. 87, no. 3, pp. 1–102, 2022.

[47] Y. Zhou, J. Yang, H. Wang, G. Huang, and Y. Chen, "Statistics-guided dictionary learning for automatic coherent noise suppression," *IEEE Trans. Geosci. Remote Sens.*, vol. 60, 2020, Art. no. 5900117.

[48] G. Huang, M. Bai, Q. Zhao, W. Chen, and Y. Chen, "Erratic noise suppression using iterative structure-oriented space-varying median filtering with sparsity constraint," *Geophysical Prospecting*, vol. 69, no. 1, pp. 101–121, 2020.

[49] S. Beckouche and J. Ma, "Simultaneous dictionary learning and denoising for seismic data," *Geophysics*, vol. 79, no. 3, pp. A27–A31, 2014.

[50] O. M. Saad and Y. Chen, "Deep denoising autoencoder for seismic random noise attenuation," *Geophysics*, vol. 85, no. 4, pp. V367–V376, 2020.

Lakshmi Kuruguntla (Member, IEEE) received the B.Tech. degree in electronics and communication engineering (ECE) from Jawaharlal Nehru Technological University (JNTU), Hyderabad, India, in 2008, the M.Tech. degree in digital electronics and communication systems (DECS) from JNTU, Kakinada, India, in 2012, and the Ph.D. degree in seismic data denoising and reconstruction based on dictionary learning from SRM University, Amaravathi, India, in 2023.

She has ten years of teaching experience in reputed institutions. She is currently an Assistant Professor with Koneru Lakshmaiah Education Foundation (KLEF) deemed to be University, Guntur, India. Her research interests include seismic signal processing, compressive sensing, dictionary learning, and machine learning.

Vineela Chandra Dodda (Member, IEEE) received the B.Tech. degree in electronics and communication engineering (ECE) from Amrita Vishwa Vidyapeetham, Coimbatore, India, in 2015, the M.Tech. degree in embedded systems from KL Deemed to be University, Guntur, India, in 2019, and the Ph.D. degree in seismic data processing, inversion and interpretation from SRM University, Amaravathi, India.

From 2015 to 2017, she was an Associate Software Engineer with Robert Bosch Engineering and Business Solutions Ltd, Coimbatore, India. She is currently a Faculty Associate with the Department of ECE, Amrita Vishwa Vidyapeetham, Amaravati, India. Her research interests include neural networks, machine learning, and image processing.

Anup Kumar Mandpura (Member, IEEE) received the B.Tech degree in electronics engineering from Banaras Hindu University, Varanasi, India, in 2006, the M.Tech degree in digital signal processing from the Indian Institute of Technology, Guwahati, India, in 2009, and the Ph.D. degree in wireless communication, from the Indian Institute of Technology, Delhi, India, in 2018.

He is currently an Assistant Professor with the Department of Electrical Engineering, Delhi Technological University, Delhi, India. His research interests include performance analysis of wireless communication systems, energy harvesting communication systems, microwave/millimeter-wave systems, and signal processing.

Sunil Chinnadurai (Senior Member, IEEE) received the B.E degree in electronics and communication engineering (ECE) from Anna University, Chennai, India, in 2009, the M.S. degree in electronics and communication engineering from Mid Sweden University, Sweden, in 2012, and the Ph.D. degree in electronics and communication engineering from Jeonbuk National University, South Korea, in 2017.

For six months, he was a Postdoctoral Research Fellow with Jeonbuk National University. He was with the Signal Intelligence Research Center, Hanyang University, Seoul, South Korea, for a year, working as a Postdoctoral Research Scientist. From 2019, he is an Assistant Professor with the Department of ECE, SRM University AP, Andhra Pradesh, India. His research interests include 5G and B5G communications, intelligent reflecting surfaces, massive MIMO, NOMA, mm wave communications, machine learning, and hyperspectral image processing.

Dr. Chinnadurai was the recipient of the Best Paper Award at the 24th MSPT International Symposium in 2016. He is also a Fellow of the IETE.

Karthikeyan Elumalai (Member, IEEE) received the B.E. degreee in electronics and communication engineering and M.E. degree in communication systems from Anna University, Chennai, India, in 2007 and 2009, respectively, and the Ph.D. degree in signal processing from the Indian Institute of Technology Delhi (IIT-D), Delhi, India, in 2018.

He is currently a Assistant Professor with the Department of ECE, SRM University-AP, Amaravathi, India. Also, he received a research grant from Department of Science and Technology (DST), India. His research interests include seismic signal processing, machine learning, and seismic modeling and inversion.