# A High-Resolution Airborne SAR Autofocusing Approach Based on SR-PGA and Subimage Resampling With Precise Hyperbolic Model 

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#### Abstract

High-resolution airborne synthetic aperture radar imaging requires long synthetic aperture time (LSAT). However, the LSAT invalidates the Fresnel approximation in the traditional autofocusing method, leaving a residual signal phase in the motion error. To solve the problem, a signal-reconstruction-based phase gradient autofocus (SR-PGA) and a subimage resampling (SIR) methods are proposed. They are developed on the hyperbolic model. First, a subaperture division strategy divides the full-aperture highorder error into multiple low-order subaperture errors (SPEs). Then, the SR-PGA is developed to estimate the SPE, in which the precise deramping is reconstructed to eliminate the residual signal phase in the SPE. Third, the SIR is proposed to eliminate residual Doppler-variant shift of the adjacent subimages, improving the accuracy of the SPE combination. Finally, simulation and actual data processing verify the effectiveness and validity of the algorithm.


Index Terms-Doppler subimage resampling, hyperbolic model, long synthetic aperture, SAR autofocusing, signal-reconstructionbased phase gradient autofocus (SR-PGA), synthetic aperture radar (SAR).

## I. Introduction

DUE to atmospheric disturbance or pilot error, the actual motion trajectory of an airborne synthetic aperture radar

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(SAR) may easily deviate from a straight line, and the acquired raw data cannot be directly focused due to the existence of motion errors [1], [2], [3], [4], [5], [6]. Such motion errors can be compensated by measuring the deviations, through the inertial navigation system (INS) and global positioning system (GPS) unit. However, these measurements are required to reach a precision of subwavelength scale to obtain high-resolution SAR images [5], [7]. Autofocus methods are usually employed to estimate motion error from the acquired radar data beyond the accuracy of the motion-measurement instrumentations [5], [8], [9], [10]. Thus, it is necessary to estimate and correct the motion errors using autofocus methods accurately from the acquired radar data [11], [12], [13].

Motion errors usually consist of an envelope error and a phase one, and both types of errors satisfy the linear mapping relationship, that is, the ratio of the phase error to the envelope error is $4 \pi / \lambda$. Some researchers calculate the phase error by estimating the envelope error [14], [15], [16], but others convert the phase error estimated by phase autofocusing methods into the envelope error [2], [6], [17], [18]. In the first-class methods, the cross-correlation method based on oversampled data of range profiles is employed to estimate the envelope error. However, the accuracy of the error estimation only reaches the envelope scale and cannot meet the imaging requirements. Therefore, the phase error estimation methods, whether parametric or nonparametric ones, are commonly used. In the former ones, a polynomial phase error model is established after the model parameterization. The map drift (MD) technique is the common and robust approach [2], [19]. It assumes the phase error is quadratic and estimates the quadratic coefficients of the phase error through the correlation processing of two adjacent subimages. The latter ones typically extract the phase or phase gradient of the motion error from the raw data directly, and among which, the phase gradient autofocus (PGA) method and its extended algorithms (e.g., [8], [9], [18], [20], [21], [22]) are representative and widely used.

The original PGA is suitable for the spotlight mode and the motion errors of scatterers in the processed scene are consistent. In this case, a full-aperture phase error can be directly estimated by PGA. However, for the stripmap mode, the scene illuminated by the radar changes with the movement of the aircraft, and
the motion error of scatterers varies with the different scenes. Therefore, PGA cannot be directly employed in stripmap long synthetic aperture processing [8]. Fortunately, the subaperture (SA) division can be employed to solve this problem. In the SA, it can be seen as a spotlight mode for an SA and PGA can be employed. However, another problem arises. With the increase of synthetic aperture interval, the number of SA becomes larger and the subaperture error (SPE) combination should be considered to obtain the full-aperture phase error.

Many scholars have done a lot of work on SA processing. Liu et al. [23] use the SA image coherent superposition to achieve high-resolution real-time imaging for spotlight SAR. Estimate the azimuth SA envelope and phase errors by the cross-correlation-based algorithm and map-drift method in the range-Doppler domain. Hai et al. [24] propose an optimal SA division method to guarantee the coherence of each SA data in the back-projection imaging. Yan et al. [25] achieve the along-track motion compensation based on the SA processing. Prats et al. [26] use SAs in the time domain to accommodate accurate topography and compensate for azimuth coregistration errors. Potsis et al. [27] propose an SA approach for residual motion compensation in azimuth processing. Bezvesilniy et al. [29] estimate and integrate the local quadratic phase error in the SA to obtain the full-aperture phase error.

For high-resolution airborne SAR imaging, the synthetic aperture interval is long and it is essential to estimate the full-aperture phase error accurately [6]. Therefore, the SA processing is used to estimate the full-aperture motion errors. As mentioned earlier, the accuracy of the SPE estimation and the SPE combination determines the accuracy of the full-aperture error estimation.

For the SPE estimation, Ding et al. [18] use the resampling weight PGA to estimate the SPEs in the SA back-projection imageries. Chen et al. [3] combine PGA and the weighted least square method to improve the robustness of PGA. Mao et al. [12] propose a knowledge-aided 2-D autofocus method using PGA, in which PGA is employed to estimate the azimuth-frequency phase error. Meanwhile, the SPE combination is employed to obtain the full-aperture motion errors. Thompson et al. [29] combine the SPEs after eliminating the mean difference between the SPEs' gradient obtained from the overlapped samples of the adjacent SAs. Zhu et al. [30] study a multi-subaperture PGA approach to obtain the full-aperture phase error, taking advantage of the Fresnel approximation.

However, in the aforementioned algorithms, the Fresnel approximation is employed in the SPE estimation and the SPE combination. In the SPE estimation, the Fresnel approximation used in PGA causes the residual signal phase. In addition, the Fresnel approximation introduces the residual Doppler-variant shift in the SPE combination, which decreases the estimation accuracy of the full-aperture phase error.

For the aforementioned problems in the SPE estimation and the SPE combination, an airborne SAR autofocusing approach based on the signal-reconstruction-based PGA (SR-PGA) and subimage resampling methods is proposed by introducing the precise hyperbolic model. The SR-PGA is proposed to precisely estimate the SPE, in which the precise deramping is reconstructed with the hyperbolic mode and the Doppler information


Fig. 1. Airborne SAR imaging geometry with motion errors.

TABLE I
System Parameters in Simulation

| Parameters | Value |
| :---: | :---: |
| Central frequency | 9.63 GHz |
| Signal bandwidth | 1.3 GHz |
| Center slant range | 8600 m |
| Platform velocity | $110 \mathrm{~m} / \mathrm{s}$ |
| Pulse repetition frequency | 2000 Hz |

of the main lobe center. The residual signal phase is eliminated by precise deramping. After the precise deramping, we can obtain subimages in the range-time and azimuth-frequency domains. The SIR method is proposed to resample the subimage in the Doppler domain. According to the method, the residual Doppler-variant shift of the adjacent subimages caused by the signal is directly eliminated, and the real shift caused by the linear motion error can be precisely estimated. Therefore, the method improves the combination accuracy of the SPEs. Then, the full-aperture phase error is adequately obtained with the SPE combination. Finally, the residual RCM and phase error in the raw data are compensated for, and a well-focused airborne SAR image is obtained.

This article is organized as follows. In Section II, the motion errors in an SA are analyzed. The algorithm is detailed in Section III. In Section IV, the algorithm is validated with acquired SAR data. The applicability of the algorithm is discussed in Section V. Finally, Section VI concludes this article.

## II. Analysis of Motion Errors in a Subaperture

## A. Signal Model in Subaperture

Fig. 1 shows the SAR geometric model with motion errors. The $X$-axis denotes a straight and ideal trajectory in data acquisition. The antenna phase center of airborne SAR moves along the nominal trajectory with a constant velocity $v$. The blue curve is the actual flight path. $P$ is a point target located within an observed ground scene. $R$ is the instantaneous slant range from SAR to $P$. $R_{b}$ denotes the nearest slant range between the trajectory and $P$, and $O$ is the central moment of azimuth time. $x_{c}$ is the azimuth distance between $O$ and $P$. After the initial

MOCO using the INS and GPS data, the slant range of $P$ as a function of full-aperture azimuth time $t_{a}$ can be written as

$$
\begin{equation*}
R\left(t_{a}, R_{b}\right)=R_{0}\left(t_{a}, R_{b}\right)+\Delta R\left(t_{a}, R_{b}\right) \tag{1}
\end{equation*}
$$

where $t_{a} \in\left(-n_{a} / 2, n_{a} / 2-1\right) / P R F, n_{a}$ is the full-aperture azimuth sample number, $n_{r}$ is the range sample number, $F_{s}$ is the range sampling rate, and the instantaneous slant range on the straight trajectory $R_{0}\left(t_{a}, R_{b}\right)=\sqrt{R_{b}^{2}+\left(v t_{a}-x_{c}\right)^{2}}$. $\Delta R\left(t_{a}, R_{b}\right)$ represents the slant range error caused by motion errors. The transmitted waveform is assumed to be

$$
\begin{equation*}
s_{t}\left(t_{r}\right)=\operatorname{rect}\left(\frac{t_{r}}{T_{p}}\right) \exp \left(j \pi\left(\gamma_{r} t_{r}^{2}+2 f_{c} t_{r}\right)\right) \tag{2}
\end{equation*}
$$

where $\operatorname{rect}(\cdot)$ represents the range modulation. The range fasttime $t_{r} \in\left(-n_{r} / 2, n_{r} / 2-1\right) / F_{s}, T_{p}$ is the pulsewidth, $\gamma_{r}$ is the chirp rate, and $f_{c}$ is the central frequency.

With $P$ as an example, the received range time-domain signal, after modulating to the baseband, is

$$
\begin{align*}
& s_{r}\left(t_{a}, t_{r}, R_{b}\right)=\operatorname{rect}\left(\left(t_{r}-\frac{2 R\left(t_{a}, R_{b}\right)}{c}\right) / T_{p}\right) W_{a}\left(t_{a}\right) \\
& \times \exp \left(j\left(\gamma_{r} \pi\left(t_{r}-\frac{2 R\left(t_{a}, R_{b}\right)}{c}\right)^{2}-\frac{4 \pi R\left(t_{a}, R_{b}\right)}{\lambda}\right)\right) \tag{3}
\end{align*}
$$

where $W_{a}(\cdot)$ is the azimuth amplitude modulation. $c$ is the speed of light and the radar wavelength. After using an SAR imaging algorithm (e.g., extended range migration algorithm (ERMA) [31]) to achieve the range pulse compression and the range cell migration correction (RCMC), one has the 2-D time-domain signal

$$
\begin{align*}
& s\left(t_{a}, t_{r}, R_{b}\right)=\operatorname{sinc}\left(B_{r}\left(t_{r}-\frac{2\left(R_{b}+\Delta r^{S M}+\Delta R\left(t_{a}, R_{b}\right)\right)}{c}\right)\right) \\
& \times W_{a}\left(t_{a}\right) \exp \left(-j \frac{4 \pi}{\lambda}\left(R_{0}\left(t_{a}, R_{b}\right)+\Delta R\left(t_{a}, R_{b}\right)\right)\right) \tag{4}
\end{align*}
$$

where $B_{r}$ is the signal bandwidth, $\Delta r^{S M}$ is an additional nonsystem envelope error introduced by the Stolt mapping in the ERMA [8]. For $\Delta r^{S M}$, the down-sampling operation in range dimension and the iterative strategy will decrease the effect of the additional envelope errors and frequency domain distortion. The down-sampling operation is achieved by summing up adjacent range cells into one cell or extracting a part of the range frequency band. Fortunately, the Stolt mapping does not change the phase error in the data. Therefore, only the exponential terms of the signal are expressed for the sake of simplicity in the following derivations.

To estimate the motion errors in azimuth, one divides the full-aperture into $N$ overlapped SAs. The SA azimuth sample number is $n_{a}^{S A}$, the adjacent SA spacing is $\Delta T=n_{a}^{o l} / P R F$, and $n_{a}^{o l}$ is the overlapping sample number of the adjacent SAs. Then, considering $P$, one analyzes the signal in each SA. The $n$th SA signal phase of $P$ in the azimuth time domain is defined
as

$$
\begin{align*}
& s_{n}\left(t_{\text {sub }}, R_{b}\right) \\
& =\exp \left(-j \frac{4 \pi}{\lambda} R_{\mathrm{sub}}\left(t_{\mathrm{sub}}, R_{b}\right)+j \varphi_{n}\left(t_{\mathrm{sub}}, R_{b}\right)\right) \\
& =\exp \left(-j \frac{4 \pi}{\lambda} \sqrt{R_{b}^{2}+\left(v\left(\Delta t_{n}+t_{\text {sub }}\right)\right)^{2}}+j \varphi_{n}\left(t_{\mathrm{sub}}, R_{b}\right)\right) \\
& \quad \times 1 \leq n \leq N \tag{5}
\end{align*}
$$

where $R_{\text {sub }}\left(t_{\text {sub }}, R_{b}\right)$ is the slant range within the $n$th SA, the divided SA number is $N=\left(\left(n_{a}-n_{a}^{S A}\right) / n_{a}^{o l}\right)+1$, and $t_{\text {sub }} \in$ $\left(-n_{a}^{S A} / 2, n_{a}^{S A} / 2\right) / P R F$ is the SA azimuth time. $t_{n}$ is the central time of the $n$th SA, and $t_{c}=x_{c} / v . \Delta t_{n}$ is the azimuth-time spacing between $t_{c}$ and $t_{n}$. Note that the hyperbolic mode is shown as the first term of the second line in (5). One expands $R_{\text {sub }}$ into a Taylor series as

$$
\begin{equation*}
R_{\mathrm{sub}}\left(t_{\mathrm{sub}}, R_{b}\right)=\sum_{i=0}^{M} \eta_{i}\left(\Delta t_{n}, R_{b}\right) \cdot t_{\mathrm{sub}}^{i}, M \geq 2 \tag{6}
\end{equation*}
$$

where the zeroth, first, and second components are

$$
\begin{align*}
& \eta_{0}\left(\Delta t_{n}, R_{b}\right)=\sqrt{R_{b}^{2}+\left(v \Delta t_{n}\right)^{2}}  \tag{7a}\\
& \eta_{1}\left(\Delta t_{n}, R_{b}\right)=\frac{v^{2} \Delta t_{n}}{\sqrt{R_{b}^{2}+\left(v \Delta t_{n}\right)^{2}}}, \text { and }  \tag{7b}\\
& \eta_{2}\left(\Delta t_{n}, R_{b}\right)=\frac{v^{2} R_{b}^{2}}{2 \sqrt[3]{R_{b}^{2}+\left(v \Delta t_{n}\right)^{2}}} \tag{7c}
\end{align*}
$$

respectively. In addition, the $n$th SPE in the azimuth time domain is

$$
\begin{equation*}
\varphi_{n}\left(t_{\text {sub }}, R_{b}\right)=-4 \pi \Delta R_{n}\left(t_{\text {sub }}, R_{b}\right) / \lambda \tag{8a}
\end{equation*}
$$

and its expansion as a Taylor series is

$$
\begin{equation*}
\varphi_{n}\left(t_{\mathrm{sub}}, R_{b}\right)=a_{n}+b_{n} t_{\mathrm{sub}}+\theta_{n}\left(t_{\mathrm{sub}}, R_{b}\right) \tag{8b}
\end{equation*}
$$

where $a_{\mathrm{n}}$ is a constant and $b_{n}$ is a coefficient of the first-order term. $\theta_{n}\left(t_{\text {sub }}\right)$ represents all higher order components.

Then, the $n$th SA signal phase (5) in the azimuth time domain can be rewritten as

$$
\begin{align*}
& s_{n}\left(t_{\text {sub }}, R_{b}\right)=\exp \\
& \quad \times\left(\begin{array}{l}
-j\left(\frac{4 \pi}{\lambda} \eta_{0}\left(\Delta t_{n}, R_{b}\right)-a_{n}\right) \\
-j 2 \pi\left(\frac{2}{\lambda} \eta_{1}\left(\Delta t_{n}, R_{b}\right)-\frac{b_{n}}{2 \pi}\right) t_{\text {sub }} \\
-j\left(\frac{4 \pi}{\lambda} \sum_{i=2}^{M} \eta_{i}\left(\Delta t_{n}, R_{b}\right) \cdot t_{\text {sub }}^{i}-\theta_{n}\left(t_{\text {sub }}, R_{b}\right)\right)
\end{array}\right) \tag{9}
\end{align*}
$$

After applying a Fourier transform (FT) to (9) in the azimuthtime domain, the transformed signal in the range-Doppler domain is expressed as

$$
\begin{align*}
& S_{n}\left(f_{a}, R_{b}\right)=\exp \left(-j\left(\frac{4 \pi}{\lambda} \eta_{0}\left(\Delta t_{n}, R_{b}\right)-a_{n}\right)\right) \\
& \cdot \operatorname{sinc}\left(T_{\text {sub }}\left(f_{a}-\left(\frac{2}{\lambda} \eta_{1}\left(\Delta t_{n}, R_{b}\right)-\frac{b_{n}}{2 \pi}\right)\right)\right)  \tag{10}\\
& \otimes \Omega\left(f_{a}, R_{b}\right) \otimes E\left(f_{a}, R_{b}\right), 1 \leq n \leq N
\end{align*}
$$

where $\otimes$ denotes the convolution operation, and the azimuth frequency $f_{a} \in\left(-n_{a}^{S A} / 2, n_{a}^{S A} / 2-1\right) / n_{a}^{S A} \cdot P R F$. The first term has a constant phase, which does not affect the SA imaging focus. The second one is the azimuth-compressed expression


Fig. 2. Residual as a function of $\Delta t_{n}$.
of the SA data in the range-Doppler domain. $\Omega\left(f_{a}, R_{b}\right)$ is the FT result of $\exp \left(-j \frac{4 \pi}{\lambda} \sum_{i=2}^{M} \eta_{i}\left(\Delta t_{n}, R_{b}\right) \cdot t_{\text {sub }}^{i}\right) \cdot E\left(f_{a}, R_{b}\right)$ is $\exp \left(j \theta_{n}\left(t_{\text {sub }}, R_{b}\right)\right)$ after the FT. $\Omega\left(f_{a}, R_{b}\right)$ and $E\left(f_{a}, R_{b}\right)$ defocuses the azimuth-compressed SA image. To obtain the full-aperture phase error precisely, one should estimate $b_{n}$ and $\theta_{n}\left(t_{\text {sub }}, R_{b}\right)$ in each SA.

## B. Analysis of Residual Signal Phase Caused by Fresnel Approximation

The azimuth time-domain SA signal phase history in the first term of (5) is hyperbolic and relates to the azimuth time spacing $\Delta t_{n}$. Note that $\Delta t_{n}$ increases as beamwidth increases. Unlike the hyperbolic model, the Fresnel approximation considers that the signal history is quadratic. According to the Fresnel approximation, one removes the quadratic signal phase by a deramping function in PGA before estimating the SPE in the azimuth time domain. The function is

$$
\begin{equation*}
s_{d e}\left(t_{\mathrm{sub}}, R_{b}\right)=\exp \left(j \pi \gamma t_{\mathrm{sub}}^{2}\right) \tag{11}
\end{equation*}
$$

with $\gamma=2 v^{2} /\left(\lambda R_{b}\right)$. A residual signal phase of the $n$th SA in the azimuth time domain can be expressed as
$\phi_{\text {resi }}\left(t_{\text {sub }}, R_{b}\right)=\exp \left(-j \frac{4 \pi}{\lambda} \sum_{i=2}^{M} \eta_{i}\left(\Delta t_{n}, R_{b}\right) \cdot t_{\text {sub }}^{i}+j \pi \gamma t_{\text {sub }}^{2}\right)$
To understand the residual signal phase $\phi_{\text {resi }}\left(t_{\text {sub }}, R_{b}\right)$, we simulated with the main parameters given in Table I. The results are shown in Fig. 2. The larger $\left|\Delta t_{n}\right|$ is, the larger the residual phase. Thus, $\phi_{\text {resi }}\left(t_{\text {sub }}, R_{b}\right)$ caused by the Fresnel approximation can significantly affect the accuracy of the SPE estimation.

## C. Analysis of Residual Doppler-Variant Doppler Shift Caused by Fresnel Approximation

After obtaining the multiple SPEs, one should estimate the linear error in each SA to achieve the SPE combination, and one can obtain the linear error by estimating $b_{n}$ in the $n$th SA. According to (10), it can be known that a coefficient $b_{n}$ of the first-order error shifts the Doppler SA image. Thus, $b_{n}$ can be


Fig. 3. Doppler shifts of the same scatterer between the adjacent SA images.


Fig. 4. Flowchart of the proposed algorithm.
estimated by evaluating the Doppler shift between the adjacent SA images [9], [30].

With (7b) and the second term in (10), the focused Doppler frequency position of $P$ in the $n$th SA image in the range-Doppler domain is

$$
\begin{equation*}
f_{n}=\frac{2}{\lambda} \frac{v^{2} \Delta t_{n}}{\sqrt{R_{b}^{2}+\left(v \Delta t_{n}\right)^{2}}}-\frac{b_{n}}{2 \pi} \tag{13}
\end{equation*}
$$

Similarly, the focused position of $P$ in the $m$ th SA image in the range-Doppler domain can be expressed as

$$
\begin{equation*}
f_{m}=\frac{2}{\lambda} \frac{v^{2} \Delta t_{m}}{\sqrt{R_{b}^{2}+\left(v \Delta t_{m}\right)^{2}}}-\frac{b_{m}}{2 \pi} \tag{14}
\end{equation*}
$$



Fig. 5. Diagram of a line signal consisting of signals of eight-point targets.
where $\Delta t_{m}=\Delta t_{n}+\Delta t$. The azimuth time interval $\Delta t$ is $(m-n) T_{\text {sub }}$, and $T_{\text {sub }}$ is the azimuth duration of each SA. Then, the Doppler shift between the $n$th and $m$ th SA images in the range-Doppler domain is

$$
\begin{equation*}
\Delta f_{m n}=\left|f_{m}-f_{n}\right|=\left|\Delta f_{a d}\right|+\frac{1}{2 \pi}\left|b_{n}-b_{m}\right| \tag{15}
\end{equation*}
$$

In (15), $\frac{1}{2 \pi}\left|b_{n}-b_{m}\right|$ is the relative Doppler shift between the $n$th and $m$ th SA images caused by the linear phase error, and it can be used to estimate the linear phase error in the azimuth time domain. And there is an inherent Doppler-variant Doppler shift of the adjacent SA images $\Delta f_{a d}$ or

$$
\begin{equation*}
\Delta f_{a d}=\left|\frac{2}{\lambda} \frac{v^{2} \Delta t_{m}}{\sqrt{R_{b}^{2}+\left(v \Delta t_{m}\right)^{2}}}-\frac{2}{\lambda} \frac{v^{2} \Delta t_{n}}{\sqrt{R_{b}^{2}+\left(v \Delta t_{n}\right)^{2}}}\right| . \tag{16}
\end{equation*}
$$

However, under the Fresnel approximation [30], the inherent Doppler shift $\Delta f_{a d}$ is replaced by $\Delta f_{a d}^{\text {Fres }}$ or

$$
\begin{equation*}
\Delta f_{a d}^{\mathrm{Fres}}=\left|\frac{2 v^{2}}{\lambda R_{b}} \Delta t\right| \tag{17}
\end{equation*}
$$

and it is Doppler-invariant. Note that there is a residual Dopplervariant Doppler shift between $\Delta f_{a d}$ and $\Delta f_{a d}^{\mathrm{Fres}}$, and the shift is

$$
\begin{equation*}
\Delta f_{\mathrm{resi}}=\left|\Delta f_{a d}-\Delta f_{a d}^{\mathrm{Fres}}\right| \tag{18}
\end{equation*}
$$

It should be noted that the difference is due to the Fresnel approximation and the difference will cause a biased estimation of $b_{n}$.

Fig. 3 demonstrates $\Delta f_{a d}, \Delta f_{a d}^{\mathrm{Fres}}$, and $\Delta f_{\text {resi }}$ derived from the same scatterer of the adjacent SA images as functions of $\Delta t_{n}$. In the figure, the solid red curve shows the time-frequency line of a real echo signal phase with the hyperbolic form, and the solid blue line represents the time-frequency line with the Fresnel approximation. The two cyan boxes indicate the $m$ th and $n$th SAs, respectively. After the deramping operation, the focusing positions in the Doppler domain of the same scatterer in the $m$ th and $n$th SAs are the green and black curves, respectively. $\Delta f_{a d}$ in Fig. 3 is the inherent Doppler shift of the focusing positions. $\Delta f_{a d}^{\mathrm{Fres}}$ is the calculated Doppler shift with the Fresnel approximation. According to Fig. 3, there is a residual Dopplervariant Doppler shift $\Delta f_{\text {resi }}$ between $\Delta f_{a d}$ and $\Delta f_{a d}^{\text {Fres }}$, and the


Fig. 6. SA image alignment results based on the (a) Fresnel approximation and the (b) SIR method.
residual shift is indicated by a purple line with bidirectional arrows. Thus, the residual Doppler-variant Doppler shift caused by the Fresnel approximation cannot be ignored during the SPE combination.

## III. Autofocusing Algorithm

In Section II, we analyzed the effects of the Fresnel approximation on the SPE estimation and the SPE combination for airborne SAR autofocusing. The effects directly decrease the estimation accuracy of the full-aperture phase error and decrease the resolution of the SAR image. In this section, the SR-PGA method is proposed to eliminate the effect of the Fresnel approximation on the SPE estimation and improve the estimation accuracy. The method employs the hyperbolic model and reconstructs a precise deramping function to compensate for the residual signal phase in the SA. Then, to solve the residual Doppler-variant shift caused by the Fresnel approximation during the SPE combination, the SIR method based on the hyperbolic model is proposed. According to the developed subimage resample in the method, the residual Doppler-variant shift of the adjacent subimages is directly eliminated. Thus, the method can precisely estimate the linear phase error in each SA, further combining multiple SPEs into the high-order full-aperture motion error. Compared with the traditional autofocusing algorithm, the proposed autofocusing method is more suitable for long synthetic aperture autofocusing.

Finally, the method flowchart is shown in Fig. 4, and the detailed processing for the proposed autofocusing algorithm is introduced next.

## A. SR-PGA Method

To eliminate the residual signal phase caused by the Fresnel approximation and improve the precision of the SPE estimation, the SR-PGA method is proposed. According to (9), if the phase of the higher order terms of the SA signal is compensated for and the SA signal is transformed into the range-Doppler domain, the peak response's position $\hat{f}_{n}$ of point target $P$ in the $n$th SA image in the range-Doppler domain is

$$
\begin{equation*}
\hat{f}_{n}=\frac{2}{\lambda} \eta_{1}\left(\Delta \hat{t}_{n}, R_{b}\right)-\frac{b_{n}}{2 \pi} \tag{19}
\end{equation*}
$$


(b)

Fig. 7. Schematic diagram of the SPE combination. (a) Obtained SPE gradients. (b) Estimated Doppler shifts. (c) Combined full-aperture phase error gradient. (d) Full-aperture phase error.

TABLE II
Subaperture Image Quality Evaluation Results

| Items | IRW (m) |  | PSLR (dB) |  | ISLR (dB) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Range | Azimuth | Range | Azimuth | Range | Azimuth |
| PGA | 0.1221 | 2.0358 | -12.32 | -10.82 | -10.21 | -8.95 |
| SR- <br> PGA | 0.1219 | 2.0351 | -12.80 | -13.03 | -10.36 | -9.39 |

The azimuth time spacing $\Delta \hat{t}_{n}$ can be calculated as

$$
\begin{equation*}
\Delta \hat{t}_{n}=-\frac{R_{b}}{v} \frac{\left(\left(\hat{f}_{n}+\frac{b_{n}}{2 \pi}\right) / f_{a m}\right)}{\sqrt{1-\left(\left(\hat{f}_{n}+\frac{b_{n}}{2 \pi}\right) / f_{a m}\right)^{2}}} \tag{20}
\end{equation*}
$$

where $f_{a m}=2 v / \lambda$. Then, the signal phase of $P$ in the $n$th SA reconstructed by the estimated $\Delta \hat{t}_{n}$ in the azimuth time domain is

$$
\begin{equation*}
s^{\prime}\left(t_{\mathrm{sub}}, R_{b}\right)=\exp \left(-j \frac{4 \pi}{\lambda} \sqrt{R_{b}^{2}+\left(v\left(\Delta \hat{t}_{n}+t_{\mathrm{sub}}\right)\right)^{2}}\right) \tag{21}
\end{equation*}
$$

According to (6), the constant and first-order terms in (21) are eliminated and the higher order terms of the SA signal phase in the azimuth time domain are reconstructed as

$$
\begin{equation*}
s_{d e}^{\prime}\left(t_{\mathrm{sub}}, R_{b}\right)=\exp \binom{-j \frac{4 \pi}{\lambda} \sqrt{R_{b}^{2}+\left(v\left(\Delta \hat{t}_{n}+t_{\mathrm{sub}}\right)\right)^{2}}}{+j \frac{4 \pi}{\lambda} \sum_{i=0}^{1} \eta_{i}\left(\Delta \hat{t}_{n}, R_{b}\right) \cdot t_{\mathrm{sub}}^{i}} . \tag{22}
\end{equation*}
$$

Here, we call (22) the precise deramping function to distinguish it from the coarse deramping function in (11). Compared

TABLE III
Image Quality Evaluation Results

| Items | IRW (m) |  | PSLR (dB) |  | ISLR (dB) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Range | Azimuth | Range | Azimuth | Range | Azimuth |
| Initial <br> MOCO | 0.1652 | 0.6688 | -5.08 | -0.22 | -1.62 | -7.64 |
| PGA- <br> MD | 0.1347 | 0.2328 | -3.40 | -0.32 | -2.99 | -1.28 |
| SRPG <br> A-SIR | 0.1235 | 0.1110 | -10.09 | -13.32 | -9.15 | -9.40 |

with (11), (22) reconstruct the signal phase with a precise hyperbolic model and eliminate the residual signal phase history of the SA in the 2-D time domain.

Multiplying (22) by (9) to match the SA signal phase in the azimuth time domain, one obtains the SA signal after the precise deramping

$$
\begin{align*}
& s^{\prime}{ }_{n}\left(t_{\text {sub }}, R_{b}\right)=\exp \left(-j\left(\frac{4 \pi}{\lambda} \eta_{0}\left(\Delta \hat{t}_{n}, R_{b}\right)-a_{n}\right)\right) \\
& \times \exp \left(-j 2 \pi\left(\frac{2}{\lambda} \eta_{1}\left(\Delta \hat{t}_{n}, R_{b}\right)-\frac{b_{n}}{2 \pi}\right) t_{\text {sub }}+j \theta_{n}\left(t_{\text {sub }}, R_{b}\right)\right) . \tag{23}
\end{align*}
$$

Thus, the azimuth-variant higher order terms of the SA signal phase are matched precisely. Also, (23) only has the SPE that a conventional PGA algorithm can estimate. With the PGA approach [21], the proposed SR-PGA method consists of the following six steps.

Coarse Deramping: The first step is to multiply the coarse deramping function (11) by (9). The product can match the SA signal phase uniformly. An FT in the azimuth dimension is applied to the SA signal after the deramping, and the coarsefocused SA image is obtained in the range-Doppler domain.

Precise Deramping: By adequately selecting strong scatterers in the coarse-focused SA image, one obtains the frequency shift $\hat{f}_{n}$ of each scatterer and substitutes all $\hat{f}_{n} s$ into the precise deramping function (22). Then, one multiplies (22) by (9) to match the SA signal phase accurately. After that, an azimuth FT is applied to the processed SA signal, and the SA image after the precise deramping is obtained in the range-Doppler domain.

Circular Shifting: Selecting strong scatterers in the SA image and circularly shifting them to the position with zero Doppler frequency in the Doppler domain can remove the frequency shift caused by the linear term in (9). The circular shifting operation aligns the scatterers, subsequently improving the signal-to-noise ratio for the SPE estimation. The shifting can align regions with a low contrast ratio as well.

Windowing: Windowing can achieve the desired effect of preserving the width of the dominant blur of selected scatterers and discarding scatterers' data that do not contribute to the SPE estimation. With the assumption that the phase error is azimuthinvariant, the strong scatterers in the same SA image have an identical phase error $\theta_{n}\left(t_{\text {sub }}, R_{b}\right)$. Then, the scatterers are of an exact blurring function. We sum all the spreading functions to obtain a one-dimensional function in the Doppler domain whose width adequately captures the point spread function character and then use an energy threshold to determine the window size. The size decreases for subsequent iterations because the focus level of the SA image increases after each iteration.


Fig. 8. Estimated motion errors used for the initial MOCO.


Fig. 9. Range profiles (a) before and (b) after the initial MOCO.

Phase Gradient Estimation: The shifted and windowed SA image is denoted as $E\left(f_{a}, R_{b}\right)$, and the inverse FT of $E\left(f_{a}, R_{b}\right)$ is $\exp \left(j \theta_{n}\left(t_{\text {sub }}, R_{b}\right)\right)$. The phase gradient estimation of the $n$th SA image is given by

$$
\begin{equation*}
\hat{\dot{\theta}}_{n}\left(t_{\mathrm{sub}}, R_{b}\right)=\sum_{i=1}^{k} \dot{\theta}_{n, i}\left(t_{\mathrm{sub}}, R_{b}\right) \theta_{n, i}^{*}\left(t_{\mathrm{sub}}, R_{b}\right) \tag{24}
\end{equation*}
$$

where $k$ is the number of strong point scatterers in the $n$th SA image. $\dot{\theta}_{n}\left(t_{\text {sub }}, R_{b}\right)$ denotes the next azimuth time of $\theta_{n}\left(t_{\text {sub }}, R_{b}\right)$ and $\theta_{n}^{*}\left(t_{\text {sub }}, R_{b}\right)$ is the conjugation of $\theta_{n}\left(t_{\text {sub }}, R_{b}\right)$. The phase gradient $\hat{\dot{\theta}}_{n}\left(t_{\text {sub }}, R_{b}\right)$ is integrated to obtain the SPE.

Iterative Phase Correction: With an estimated SPE, the phase error is compensated for in the SA data in the azimuth-time domain. The estimation and compensation process is iterative. As the focus of the SA image improves, the spreading functions of individual scatterers become more compact. Once the removal of the Doppler shifts through, the circular shifting reaches the desired accuracy, the iteration stops.

## B. Doppler Shift Estimation Based on Doppler Subimage Resampling Method

As discussed in Section II-C, the Fresnel approximation causes a residual Doppler-variant shift in the estimation of $b_{n}$. The SIR method is proposed to eliminate the Doppler-variant shift. After the SPE estimated by the SR-PGA method is compensated for the SA data, the data are processed using the coarse


Fig. 10. (a) SA image after the MOCO with SPE obtained by the PGA algorithm. (b) SA image after the MOCO with SPE obtained by the SR-PGA algorithm. (c) Contour map of target $A$ in (a). (d) Contour map of target $A$ in (b). Impulse response curves of target A obtained by the PGA and SR-PGA algorithms. (e) Range profiles. (f) Azimuth profiles.
deramping function (11). A focused subimage can be obtained in the range-Doppler domain.

Considering $P$, one takes the $n$th subimage in the rangeDoppler domain as a reference subimage and considers there is no linear term of the SPE in it. According to the second term of (10) and (7b), the relation between the peak response's position $f_{n}$ of $P$ in the $n$th subimage and $\Delta t_{n}$ is

$$
\begin{equation*}
f_{n}=\frac{2 v^{2} \Delta t_{n}}{\lambda \sqrt{R_{b}^{2}+\left(v \Delta t_{n}\right)^{2}}} \tag{25}
\end{equation*}
$$

Since each $f_{n}$ corresponds to an azimuth time interval $\Delta t_{n}$, the azimuth time interval and the azimuth Doppler frequency in the subimage can be calculated from each other. Then, the focus position $f_{m}$ of $P$ in the $m$ th subimage is

$$
\begin{equation*}
f_{m}=\frac{2 v^{2}\left(\Delta t_{n}+(m-n) \Delta T\right)}{\lambda \sqrt{R_{b}^{2}+\left(v\left(\Delta t_{n}+(m-n) \Delta T\right)\right)^{2}}} \tag{26}
\end{equation*}
$$



Fig. 11. Phase error gradient in the stripmap data. (a) SPE gradients extracted by the PGA and SR-PGA algorithms. (b) SPE gradients combined by the PGAMD and SRPGA-SIR algorithms.

Thus, the resampling relationship of $f_{n}$ and $f_{m}$ is established as

$$
\begin{equation*}
f_{m}=\frac{2 v^{2}\left(-\frac{R_{b}}{v} \frac{\left(f_{n} / f_{a m}\right)}{\sqrt{1-\left(f_{n} / f_{a m}\right)^{2}}}+(m-n) \Delta T\right)}{\lambda \sqrt{R_{b}^{2}+\left(v\left(-\frac{R_{b}}{v} \frac{\left(f_{n} / f_{a m}\right)}{\sqrt{1-\left(f_{n} / f_{a m}\right)^{2}}}+(m-n) \Delta T\right)\right)^{2}}} . \tag{27}
\end{equation*}
$$

The focus positions in the $n$th and the $m$ th subimages of $P$ can be aligned by the subimage resampling relationship (27) in the range-Doppler domain, and the Doppler-variant shift is corrected. Here, we assume that the expression of the $n$th subimage of $P$ in the range-Doppler domain is $S_{n}^{\mathrm{img}}\left(f_{a}-f_{n}\right)$ and the expression of the $m$ th subimage of $P$ without the liner term $b_{m}$ of the SPE is $S_{m}^{\mathrm{img}}\left(f_{a}-f_{m}\right)$. According to (27), the $n$th subimage of $P$ can be resampled into the $m$ th subimage to achieve the alignment of the adjacent SA images or

$$
\begin{equation*}
S_{n}^{\mathrm{img}}\left(f_{a}-f_{n}\right) \xrightarrow{(27)} S_{m}^{\mathrm{img}}\left(f_{a}-f_{m}\right) \tag{28}
\end{equation*}
$$

When the liner phase error exists in the $m$ th subimage, the expression of the $m$ th subimage of $P$ is $S_{m}^{\mathrm{img}}\left(f_{a}-f_{m}+\frac{b_{m}}{2 \pi}\right)$. The Doppler shift $\frac{b_{m}}{2 \pi}$ can be estimated via cross-correlating $S_{m}^{\mathrm{img}}\left(f_{a}-f_{m}+\frac{b_{m}}{2 \pi}\right)$ and $S_{m}^{\mathrm{img}}\left(f_{a}-f_{m}\right)$ obtained by the resampling of $S_{n}^{\mathrm{img}}\left(f_{a}-f_{n}\right)$.

To show the performance of the proposed SIR method, we carried out another simulation with the parameters in Table I. For simplicity, a line signal consisting of eight-point targets was simulated, and there was no phase error in the signal. The time-frequency diagram of the targets' signals is shown in Fig. 5. Two SAs shown in red and blue frames were selected. After coarse deramping, the refocused red and blue subimages were obtained in the range-Doppler domain, and they are called $S_{n}^{\mathrm{img}}\left(f_{a}-f_{n}\right)$ and $S_{m}^{\mathrm{img}}\left(f_{a}-f_{m}\right)$, respectively. After eliminating the inherent shift between the images, the aligned results of the SA images based on the Fresnel approximation and the proposed SIR method are as shown in Fig. 6. In Fig. 6(a), $S_{n}^{\text {img }}\left(f_{a}-f_{n}\right)$ and $S_{m}^{\text {img }}\left(f_{a}-f_{m}\right)$ are aligned by eliminating the Doppler-variant shift $\Delta f_{a d}^{\mathrm{Fres}}$ based on the Fresnel approximation. A noticeable shift exists in the magnified box. The estimated Doppler shift is biased. In comparison, the red image
$S_{n}^{\mathrm{img}}\left(f_{a}-f_{n}\right)$ in Fig. 6(b) is resampled into the blue image $S_{m}^{\text {img }}\left(f_{a}-f_{m}\right)$ using the proposed method. There is no Doppler shift, and the bias is eliminated.

After cross-correlating all subimages, the LS technique can be used to obtain the successive Doppler shifts. (The detailed LS technique can be found in [30]). The results are denoted as

$$
\mathbf{b}=\left[\begin{array}{llll}
b_{1} & b_{2} & \cdots & b_{n} \cdots  \tag{29}\\
\cdots & b_{N}
\end{array}\right]_{1 \times N}
$$

The combined SPE gradients with $b_{n}$ subtracted by the proposed method is

$$
\begin{equation*}
\hat{\dot{\theta}}_{n}^{\text {pro }}\left(t_{\text {sub }}, R_{b}\right)=\hat{\dot{\theta}}_{n}\left(t_{\text {sub }}, R_{b}\right)-b_{n}, 1 \leq n \leq N \tag{30}
\end{equation*}
$$

Then, we obtain the combined full-aperture phase error gradient $\hat{\dot{\theta}}^{\text {pro }}\left(t_{a}, R_{b}\right)$, and integrate it to get the full-aperture phase error $\theta^{\text {pro }}\left(t_{a}, R_{b}\right)$ in the azimuth time domain. The schematic diagram of the SPE combination is shown in Fig. 7. Fig. 7(a) shows the obtained SPE gradients. The Doppler shifts estimated by cross-correlating all subimages are presented in Fig. 7(b). Adding the SPE gradients and Doppler shifts yields the combined full-aperture phase error gradient shown in Fig. 7(c), and we can get the full-aperture phase error in Fig. 7(d).

## IV. Results

To validate the proposed algorithm, we analyze the highresolution airborne data acquired in a stripmap mode. The SAR system parameters are given in Table I. The coherent processing interval of the raw data was about 32 s , and an aircraft flew at $110 \mathrm{~m} / \mathrm{s}$. The motion error estimated by the INS/GPS data is shown in Fig. 8, ranging from about -1000 to 3000 rad for the entire integration time.

Then, the initial MOCO was applied to the raw data. After the RCMC and range compression for the compensated data, 3072 range cells in the range dimension of the data were analyzed. The profiles of the data before and after the initial MOCO are shown in Fig. 9. The phase errors [see Fig. 8] are large enough to cause the profiles to exceed one cell range, as shown in Fig. 9(a). The initial MOCO adequately contains the range error within about one range cell [see Fig. 9(b)]. Thus, after the initial MOCO, one can ignore the influence of range error on the SPE estimation. However, it should be emphasized that the phase errors are still sufficient to defocus the image, and the full-aperture phase error should be estimated further to obtain the refocused image.

In estimating the full-aperture phase error, an overlapping SA strategy was employed. The size of each SA was 1024 samples with the overlapped samples of 512. An SA image in the range-Doppler domain was selected to compare the accuracy of the PGA and SR-PGA algorithms in error estimations. Subimages after compensation of the SPE obtained by the PGA and SR-PGA methods are shown in Fig. 10. Both images are well-focused overall. Fig. 10(a) has an entropy value of 12.1743, whereas the entropy value of Fig. 10(b) is 12.1455. Meanwhile, the same strong scatterer with the red rectangles in Fig. 10(a) and (b) was selected to show its contour maps. The results are illustrated in Fig. 10(c) and (d), respectively. The contour lines are more symmetric after the SR-PGA method than


Fig. 12. $X$-band SAR images after the (a) initial MoCo, (b) PGA-MD, and (c) SRPGA-SIR algorithms. The image is about 3520 m in the azimuth and 300 m in the range direction.


Fig. 13. Close-up views of area 1 in Fig. 12 after the (a) initial MoCo, (b) PGA-MD, and (c) SRPGA-SIR algorithms.
the PGA method. Moreover, to evaluate the focusing results of the scatterer obtained by the SR-PGA and PGA methods, the quantitative results, including the impulse response width (IRW), peak side lobe ratio (PSLR), and integrated side lobe ratio (ISLR) of the scatter, are listed in Table II. The resolution is evaluated by the $3-\mathrm{dB}$ bandwidth of the major lobe. According to the impulse response curves of strong scatter in Fig. 10(f), the azimuth resolutions of both methods are 2.0351 and 2.0358 m , respectively. Because the synthetic aperture time for the SA is short, the azimuth resolution of the scatterer is low. Note that, although the azimuth resolutions obtained by both methods are similar, there is a high-order residual phase error in the PGA and it causes the sidelobes to be not symmetric. Thus, the subimage quality after the SR-PGA method is better than that after the PGA method.

For the sake of description, the traditional autofocusing methods based on the Fresnel approximation are called the PGA-MD method in this article, and the proposed method is named the SRPGA-SIR method. The full-aperture phase errors are obtained by the PGA-MD method and the SRPGA-SIR method, respectively. Fig. 11(a) shows the SPE gradients obtained by the PGA and SR-PGA methods in all SAs, and the gradients differ. The SPE gradients after the PGA-MD and the proposed methods are shown in Fig. 11(b). As the number of azimuth samples
increases, the deviation in evaluating the Doppler shift increases and the variation leads to different phase error estimations in a full aperture.

After using the full-aperture phase errors to compensate for the raw data and completing the RCMC, the focused image after the PGA-MD algorithm is shown in Fig. 12(b) and the image after the SRPGA-SIR algorithm in Fig. 12(c). Meanwhile, the focused image after the initial MOCO is shown in Fig. 12(a). Each image is about 3520 m in the azimuth and 300 m in the range. Both are well-focused generally. The close-up views of area 1 and area 2 [see Fig. 12] are shown in Figs. 13 and 14, respectively. Fig. 13(a) is the close-up of area 1 in Fig. 12(a), Fig. 13(b) area 1 in Fig. 12(b), and Fig. 13(c) area 1 in Fig. 12(c). Fig. 14(a) is the close-up of area 2 in Fig. 12(a), Fig. 14(b) area 2 in Fig. 12(b), and Fig. 14(c) area 2 in Fig. 12(c). Buildings and open spaces with scattered trees are better delineated in Fig. 13(c) than in Fig. 13(a) and (b), and the street lamps are better well-focused in Fig. 14(c) than in Fig. 14(a) and (b). Fig. 13(c) is quantitatively better focused than Fig. 13(a) and (b). The image obtained by the initial MOCO is defocusing, which is caused by the lack of precision of the measurement data. Therefore, the full-aperture phase error obtained by the proposed SRPGA-SIR method is more precise than that obtained by the initial MOCO and PGA-MD methods.


Fig. 14. Close-up views of area 2 in Fig. 12 after the (a) initial MoCo, (b) PGA-MD, and (c) SRPGA-SIR algorithms.


Fig. 15. Impulse response curve comparison of the street lamp 3 in Fig. 14. (a) Range profile. (b) Azimuth profile.

To illustrate the azimuth resolution after the initial MOCO, the PGA-MD and SRPGA-SIR algorithms, we identified a street lamp within area 3 [see Fig. 14], and the street lamp can be seen as a strong scatter. The quantitative results, including the IRW, PSLR, and ISLR of the strong scatter, are listed in Table III. The resolution is evaluated by the $3-\mathrm{dB}$ bandwidth of the major lobe. According to the impulse response curves of strong scatter in Fig. 15, the 2-D spatial resolution is $0.1235 \mathrm{~m} \times 0.1110 \mathrm{~m}$ (range $\times$ azimuth). Furthermore, one can argue that the SRPGA-SIR algorithm could output an image with a high azimuth resolution.

## V. DISCUSSION

## A. Analysis of Influence of Subaperture Number

Generally, the more the number of the SAs is, the more serious the accumulation of phase errors is. There are two reasons for the accumulation of phase errors. The first reason is that the accuracy of the SPE estimation may not be high, and even the estimated SPE is wrong. The second one is the shift estimation of the adjacent SPEs, which depends on the accuracy of the estimated SPE. In the proposed algorithm, there are two aspects of improvement are proposed to minimize the influence of the number of SAs and avoid the accumulation of phase errors as soon as possible. The SR-PGA method is proposed to improve the accuracy of the estimated SPE, and the SIR method is used
to improve the accuracy of the estimated Doppler shift of the adjacent SA images.

## B. Applicability of Proposed Algorithm

To ensure well-focused results in the azimuth dimension, the phase error must be within $\pm \pi / 4$. As plotted in Fig. 2, the residual azimuth-variant signal phase $\phi_{\text {resi }}\left(t_{\text {sub }}, R_{b}\right)$ in an SA increases as $\left|\Delta t_{n}\right|$ increases. Thus, to obtain a well-focused image, the range resolution $\rho_{r}$ should meet

$$
\begin{align*}
& \max \left(\phi_{\text {resi }}\left(t_{\text {sub }}, R_{b}\right)\right) * \frac{R_{b} \lambda}{2 \rho_{r} v T_{\text {sub }}} \leq \frac{\pi}{4}  \tag{31a}\\
& \text { or } \max \left(\phi_{\text {resi }}\left(t_{\text {sub }}, R_{b}\right)\right) * \frac{2 R_{b} \lambda}{\pi v T_{\text {sub }}} \leq \rho_{r} \tag{31b}
\end{align*}
$$

where the SA azimuth duration $T_{\text {sub }}=n_{a}^{S A} / P R F$. When $\phi_{\text {resi }}\left(t_{\text {sub }}, R_{b}\right)$ and $\rho_{r}$ do not meet the aforementioned relationship, the proposed algorithm should be applied.

Future work includes extending this proposed algorithm to combine with the 2-D image autofocusing methods for obtaining the ultrahigh-resolution SAR image, in which the proposed algorithm can estimate and compensate for the spatial-invariant motion errors and the 2-D autofocusing methods can compensate for the 2-D spatial-variant motion errors, which exist in range
and azimuth dimensions. After that, a well-focused SAR image can be obtained.

## VI. CONCLUSION

In this article, high-resolution airborne SAR autofocusing with a precise hyperbolic model is studied. In terms of the SPE estimation, the SR-PGA method is proposed, in which the precise deramping based on the hyperbolic model is reconstructed to eliminate the residual signal phase and improve the estimation accuracy of the SPE. In addition, an SIR method based on the hyperbolic model is proposed to eliminate the residual Doppler-variant shift between the adjacent subimages and further improve the accuracy of the SPE combination. The proposed algorithm is effective as shown in simulations and analyses of acquired raw airborne SAR datasets. It is a viable option to form well-focused high-resolution images.

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