The Effect of the Density of Measurement Points Collected From a Multibeam Echosounder on the Selection of IDW Interpolation Points in the Process of Creating Seabed Models

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***Abstract*—The article presents the results of research on the effect of the density of measurement points (depth) from the multibeam echosounder (MBES) on the accuracy of the generated sea bottom models and the effect of this density of points on the optimal parameters of data interpolation using inverse distance weighting (IDW) interpolation. To carry out the above research, the existing standards related to hydrographic works were described, and then the difficulties in precise calculation of the accuracy of the created seabed models were presented. To solve the problem of assessing the accuracy of created seabed models, the author proposed a new method of preparing test data based on the author's virtual sea survey program. The presented approach enables the accurate calculation of errors that arise in the digital terrain model creation process based on measurement data, and thus, the study of the impact of selected transformation parameters on the accuracy of the generated models. Using this method, the author prepared a set of six sets of test data with different densities (referring them to existing measuring devices), and on their basis, calculated the effect of the density of measurement points on the accuracy of the generated models and the effect of the density of measurement points on the optimal parameters of the IDW transformation. When analyzing the results, the computation time was also considered, which may be crucial when selecting the parameters of the IDW transformation. A possible approach of hydrographic system operators when processing data files with different densities was also proposed. The presented research and results may be helpful in the practical processing of data from MBES during the sea survey.**

***Index Terms*—Digital bathymetric models, digital terrain model (DTM) bathymetry creation, inverse distance weighting (IDW) interpolation, multibeam echosounder (MBES) data density.**

I. INTRODUCTION

KNOWLEDGE of the shape of the seabed is one of the basic tasks performed in hydrography. Accurate depth models enable the creation of accurate nautical charts. They are necessary when planning underwater works, in analyzing changes in the shape of the seabed over time, in protecting the marine environment or in detailed visualization. These models are the basic information layer in hydrographic and GIS systems for further analysis and visualization. When creating models of the shape of the seabed, two aspects should be taken care of above all: the validity of the data and their high accuracy. The first of them forces institutions and entities performing marine measurements to perform them frequently, and the second to ensure the highest possible accuracy throughout the entire process of collecting and processing measurement data, to create high-precision models.

Currently, multibeam echosounders (MBES) are most often used to collect measurement (bathymetric) data. They allow to collect a huge amount of measurement data with 100% seabed coverage in a given area in a relatively short time, which in turn allows the calculation of accurate digital terrain models (DTM) based on them [1]. Survey data files contain millions and sometimes even tens of millions of individual measurements consisting of a measurement position (x, y) and a depth value (z) at that point. These points, although distributed approximately evenly, are not spatially regular, and their large number makes their direct analysis or visualization difficult. Therefore, an important stage in the processing of bathymetric data are the calculation of a regular grid, which will be a model of the bottom shape in a given area [2]. For these purposes, interpolation methods are used, the task of which is to calculate the regular structure, data reduction and their certain averaging (smoothing). Such a data model (grid) enables their further processing or analysis, and data reduction significantly speeds up the operation of GIS systems.

Depending on the MBES measuring device used and sea survey parameters, we obtain measurement data with different densities of points. Modern systems allow to perform up to 50 pings per second and 512 beams in every single swath (based on device EM2040C MKII) [3]. Using two heads allows to collect up to 184 million points in 1 h of the survey. Depending on the speed of the measuring unit and the depth, this gives a density of about 200–500 points per 1 m^2. Making additional profiles in the measured area can multiply this value even more. Slightly older measurement systems (still commonly used) or measurements in deeper areas cause the density of measurement points to decrease significantly. For example, the EM 3000 device [4] has 10 pings per second and 128 beams in every single swath, which allows

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to collect of about 5 million points in 1 h of survey and a density of about 5–20 points per 1 m^2. There are also situations where we use data from a single beam echosounder (SBES) or other measurements that are characterized by a much smaller amount of data, and thus also their density, to create DTM. As can be seen, the density of measurement points can be varied. When creating models based on data of different densities, we should not use the same interpolation parameters (in particular, the number of points or search *radius size*). It can be argued that for sets with different densities of measurement data, the optimal method of interpolation (taking into account the accuracy of the generated models and calculation time) will use a different number of measurement points for calculations. Greater density of measurement data should also ensure higher accuracy of the generated models, but it is difficult to estimate how much higher. The use of the newest echosounder devices, two heads, and additional profiles increasing the amount of measurement data have a significant impact on the accuracy of the generated models. The above aspects also have been thoroughly checked in these studies.

Research on the influence of the density and spatial distribution of measurement points and the interpolation method used in the process of creating MBES based on SBES data were described in [5]. In this research, the authors used less than 18 000 points, and the interpolation methods were not optimized.

There are several fast interpolation methods used in DTM creation based on MBES data. Some commonly used are nearest neighbor interpolation, bilinear interpolation, or moving average interpolation (MA). Calculating a grid based on millions of measurement points is really fast with their use, but the created models are slightly less accurate [6]. On the other hand, we have a few methods, which are known for providing more precise results: higher order interpolation (which uses higher order polynomials or curves tend), radial basis function interpolation (which uses a combination of radial basis functions, such as Gaussian or thin-plate splines) or kriging interpolation (based on the principles of geostatistics), which is a powerful interpolation method for spatial data. The above methods can provide high precision and smoothness models, but this usually comes at the cost of exceptionally long computation time [7]. Another emerging disadvantage of these methods is overfitting. Precise interpolation methods, particularly those using higher order polynomials, are susceptible to overfitting, which occurs when the interpolation curve excessively captures noise or local fluctuations in the data, which was described in [8]. Between fast and the most accurate methods, we can also indicate ones in between, which means they interpolate local data quite accurately with an acceptable calculation time (much shorter than precise methods). The best example of such a method is the inverse distance weighting (IDW), the most often used one in interpolating various measurement data in hydrographic and GIS systems. A comparison of many bathymetric data interpolation methods can be found, e.g., in the work in [9], [10], [11], and [12]. However, these studies aim to compare different methods with each other but do not investigate the impact of the number and density of measurement points on the accuracy of the created models.

There are also studies examining the impact of the interpolation or grid resolution methods used on the accuracy of the DTM models of the seabed. For example, in the already mentioned work [12], the advantages and disadvantages of several familiar fast gridding methods were compared, including the distance-weighted method, multipoints average method and Gauss-weighted average method using MBES data from the sea test. In these studies, however, standard methods were used that were not optimized for large amounts and high density MBES data. In [13], a novel method for reducing bathymetric geodata was presented. This method processes data via a novel artificial neural network approach. However, these methods cannot cope with large data sets. In [14], an approach to integrate statistical controls, such as minimum error, variance into inverse distance interpolation was tested, including the sensitivity of the IDW interpolation to the number of input data. Unfortunately, in these studies, the number of test points was very small (several hundred), and the interpolated data did not contain random measurement errors that occur and which we should reduce in the interpolation process. Huang and Yang [15] configured an optimized grid computing platform for the geospatial analysis using IDW interpolation. Also, in this case, the IDW method was not optimized, and the datasets were not very dense. In some cases, models with multiresolution grid are created. This process requires multiple interpolations with variable grid resolution. This method is described, for example, in [16].

Similar researches focused on creating accurate DTM models using data collected by other devices, e.g., InSAR and LIDAR has been done [17], [18], but in this case, the source data properties (points density and distribution) and, above all, surface shape are so different. The standard approach in using the IDW method assumes that when calculating the value of the interpolated point, we consider all measurement points in the vicinity of the given size (radius size parameter). With this approach, the user sets this parameter, and it usually does not depend on the density of the input points.

Therefore, it can be said that despite the existence of many articles in this field, most of them describe research using small data sets and use the standard version of the IDW (with search radius) method.

Maleika [19] showed that we will get better results when we consider a fixed number of these points, specified by the user, instead of the criterion of the size of the neighborhood. Thanks to this, we obtain slightly more accurate models, with a slightly shorter calculation time. The natural question in this case is: how many of these points should we use in interpolation and whether their number depends on their density?

To sum up, the aim of the research is to examine two important aspects related to the creation of bathymetric models based on measurement data from MBES using IDW interpolation: the selection of the optimal number of measurement points used for interpolation of subsequent mesh nodes depending on the density measurement points and the effect of measurement data density on the accuracy of the generated DTM. In both cases, we strive to create the most accurate models in the shortest possible time.

TABLE I
MINIMUM BATHYMETRY STANDARDS FOR THE SAFETY OF NAVIGATION HYDROGRAPHIC SURVEYS (M = METRES, ALL UNCERTAINTIES AT 95% CL)

Criteria	Order 2	Order 1b	Order 1a	Special Order	Exclusive Order
Area description (Generally)	Areas where a general description of the sea floor is considered adequate.	Areas where underkeel clearance is not considered to be an issue for the type of surface shipping expected to transit the area.	Areas where underkeel clearance is considered not to be critical but features of concern to surface shipping may exist.	Areas where underkeel clearance is critical	Areas where there is strict minimum underkeel clearance and manoeuvrability criteria
Depth THU [m] + [% of Depth]	20 m + 10% of depth	5 m + 5% of depth	5 m + 5% of depth	2 m	1 m
Depth TVU (a) [m] and (b)	a = 1.0 m b = 0.023	a = 0.5 m b = 0.013	a = 0.5 m b = 0.013	a = 0.25 m b = 0.0075	a = 0.15 m b = 0.0075
Feature Detection [m] or [% of Depth]	Not Specified	Not Specified	Cubic features > 2 m, in depths down to 40 m; 10% of depth beyond 40 m	Cubic features > 1 m	Cubic features > 0.5 m
Feature Search [%]	Recommended but Not Required	Recommended but Not Required	100%	100%	200%
Bathymetric Coverage [%]	5%	5%	≤ 100%	100%	200%

II. SOURCE OF ERRORS AND IHO STANDARDS

A. International Hydrographic Organization (IHO) Standards

All hydrographic works performed by authorized entities should be characterized by high accuracy of measurements and generated models. Countries associated in the IHO adhere to jointly agreed standards described in the document *Standards for Hydrographic surveys (S-44)* [20]. It contains a detailed description of the minimum requirements for all hydrographic works (mainly in terms of accuracy), including those related to the course of sea survey and DTM creation. By following certain standards, we can be sure that the generated models and products have high accuracy and allow users to use them safely (e.g., nautical charts, ECDIS systems, GIS systems, etc).

Accuracy requirements directly related to bathymetric measurements introduce the concept of the maximum allowable total vertical uncertainty (TVU) calculated at a 95% confidence level (CL = 95%). It is defined by the formula:

$$\mathrm{TVU}_{\mathrm{MAX}}\ (d) = \sqrt{a^2 + (b \cdot d)^2} \qquad (1)$$

where a represents that portion of the uncertainty that does not vary with the depth, b is a coefficient that represents that portion of the uncertainty that varies with the depth, and d is the depth. Other requirements related to bathymetric measurements (including the values of parameter a and b) are presented in Table I.

The test surface used in the presented tests as well as all generated models belong to the *special order class*, therefore in the given formula $a = 0.25$ and $b = 0.0075$, and taking into account that the depths of the test data are in the range of 8–13 m, it is easy to calculate that for the tests described in this article $\mathrm{TVU}_{\mathrm{MAX}} = {\sim}0.26$ m.

CL is a statistical measure of the percentage of test results that can be expected to be within a specified range. For example, a CL of 95% means that the result of an action will probably meet expectations 95% of the time.

All bathymetric works (sea survey, models creation, data analysis and processing, nautical charts, etc.), including the impact of used interpolation method on additional inaccuracies of the model, should be done under the IHO standards.

B. Accuracy of Depth Measurement and Methods of Assessing the Accuracy of Generated Models

Among the various devices used to measure depths in the sea, MBES is generally considered the most accurate. Multibeam systems provide high-resolution bathymetric data by emitting multiple sound beams simultaneously and capturing the returning signals. This allows for a wider coverage area and more detailed seabed mapping. The accuracy of MBES can vary depending on factors, such as the specific system used, calibration, survey conditions, seafloor composition, the presence of obstacles or features that can affect sound wave reflections and data processing techniques. However, modern MBES systems can achieve depth accuracies within a few centimetres or even better under ideal conditions [1], [4], [21]. Also, the author's earlier research showed that the accuracy of the depth measurement also depends on the beam angle and the depth, and for the EM3000 probe is approx. 3 cm at 2—8 m depth, 4 cm at 8—16 m depth, 5 and 6 cm at 16—20 m depth, and 7 and 8 cm at a depth of 22 m [22].

In practice, the measurement data collected with MBES is distributed relatively evenly, but not regularly. The distance between points in one swath increases with increasing beam angle. In addition, there are many points collected during one sea survey session (5 to 50 million points per 1 km^2). Therefore, as mentioned earlier, models based on a regular grid with a specific resolution are generated on their basis. Thanks to this, we obtain a regular structure and a significant data reduction. However, in practice, we do not have the possibility to calculate the accuracy of the model generated in this way, understood as the difference in depth between the calculated grid model and the actual surface depth, because its actual shape (and depth) is unknown. We only have measurement data describing this surface in discrete points and in an approximate way (with some random error). It is therefore impossible to precisely determine the TVU for the calculated models and thus to check, which one is more accurate.

Several methods are used to assess the accuracy of interpolation methods and models generated with their help. The first consists of generating synthetic surfaces (based on mathematical formulas) and then generating a set of randomly distributed measurement points with depth values (z). Based on these points, we can calculate subsequent DTMs using various interpolations, and then compare them to the synthetically generated surface. The advantage of this approach is primarily a simple solution and implementation. The disadvantage is the fact that the distribution of measurement points is random (which does not correspond to reality), the depth values are set perfectly (we do not take into account the measurement error), and the shape of the surface itself is regular, mathematically determined, which also is far from reality.

The second, much more frequently used approach to verify the DTM accuracy is using a large set of measurement points from the actual sea survey. This set is then divided into two subsets: a larger one, used to create DTM models (using various interpolation methods), and a second, smaller one, to verify the accuracy of the generated DTMs (determination of depth errors). Such an approach to accuracy verification can be found, for example, in the work in [23] and [24]. The advantage of this approach is the use of actual data from measurements. However, for this method to give reliable results, it must be assumed that the collected measurement data are very accurate and describe the surface precisely, without measuring errors. Only in this case, the differences in the depth values between the data verification set and the generated model will result from data processing (used interpolation method). In fact, it is not. The actual measurement data are burdened with a certain random error and, as it will be shown later in this article, it is larger than the errors resulting from data interpolation and gridding. Therefore, we should not use this method of verifying the accuracy of models, because the results may be unreliable (disturbed by noisy measurement data).

To solve the above problem, the author proposed a novel approach to verify the accuracy of the generated DTMs, using the proprietary *virtual sea survey* simulator and *reference surfaces*. This is described in more detail in the Section IV-A.

III. BASIS OF THE IDW METHOD

IDW interpolation is a spatial interpolation method commonly used in GIS and hydrography systems. It estimates unknown values at specific locations based on the values of neighboring known data points. The IDW interpolation assigns weights to the known data points based on their proximity to the target location, with closer points receiving higher weights.

The IDW formulas are given as follows [24]:

$$\grave{R}_p = \sum_{i=1}^{N} w_i \; R_i \tag{2}$$

$$w_i = \frac{d_i^{-\alpha}}{\sum_{i=1}^{N} d_i^{-\alpha}} \tag{3}$$

where $\grave{R}_p$ represents the unknown depth data (cm), R_i represents the depth value measured by MBES (cm), N is the number of points used for interpolation, w_i represents the weighting of each depth, d_i is the distance from each depth to the grid node, α is the power (generally assumed to be two). Several studies, e.g., [19] and [25] have experimented with variations in power, examining its effects on the spatial distribution of information from precipitation observations.

The IDW method can also be used as a smoothing interpolator. Although the points closest to the interpolated one have the most significant impact on the calculated value for a given grid node, using a larger number of points (even several dozen) causes the calculated value to undergo a certain averaging. Since the measurement points are burdened with a certain random error (noise), using more points to calculate a new value partly eliminates this noise. Therefore, it seems reasonable to use more points, but only if they are located in the immediate vicinity of the mesh node. In the research presented in this article, the author undertook i.e., an attempt to estimate what number of points involved in interpolation is optimal (provides the most accurate models) and whether it depends on the density of measurement points.

IV. RESEARCH

A. Test Data Preparation Method

As described in Section II, a significant challenge when searching and evaluating the most effective and accurate interpolation method and its parameters for creating DTMs lies in the difficulty of calculating the model's error that occurs throughout this process.

The author has developed a solution that eliminates the issues described in Section II-B and which he successfully uses in research related to assessing the accuracy of interpolation methods in creating seabed DTMs. It consists in creating, on the basis of a large set of real measurement data (derived from MBES), two types of surfaces: a *reference surface* with a standard (tested) resolution, which will be used to calculate the errors of the

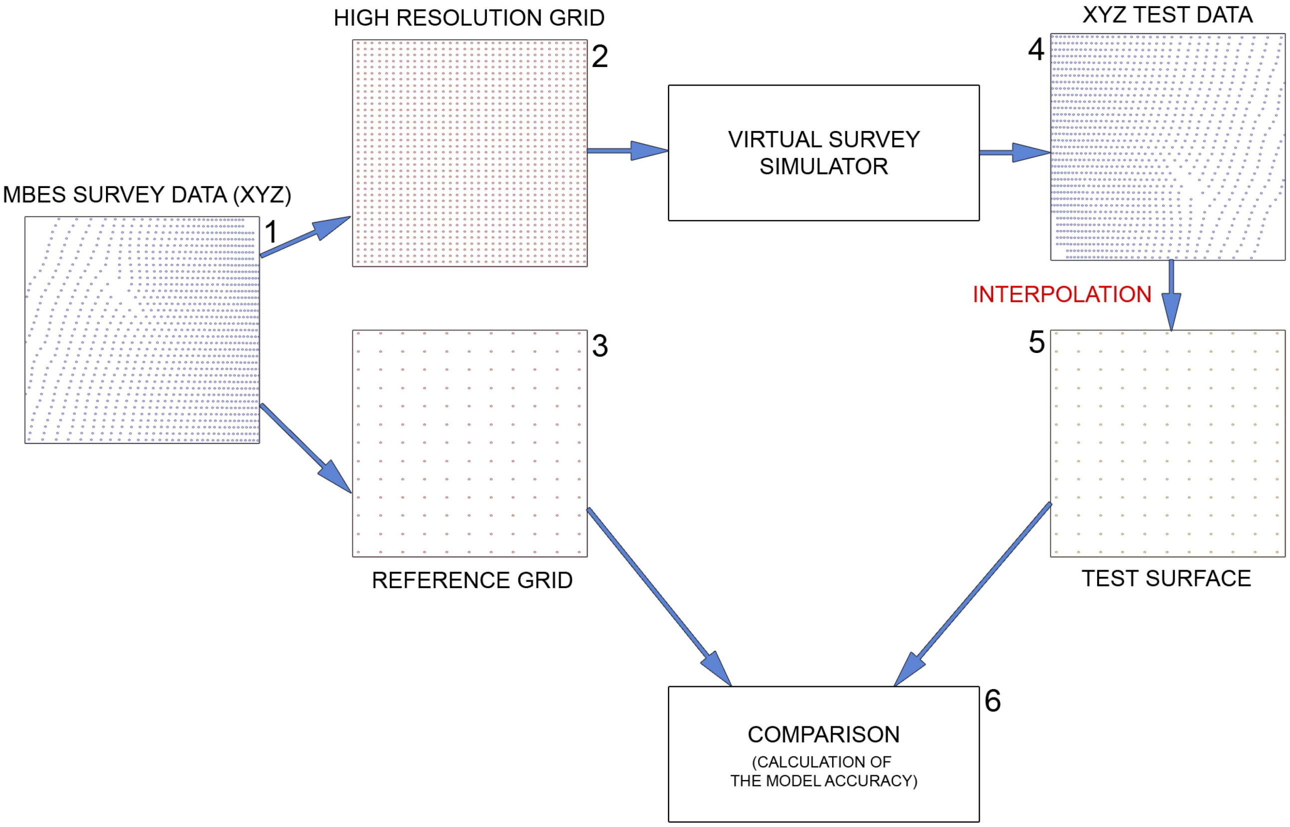


Fig. 1. Virtual sea survey—a scheme for preparing reference, high-resolution grid and XYZ test data.

generated models, and a *high-resolution grid*, which will be used to simulate the process of collecting measurement data in the *virtual sea survey* program, with which a test XYZ dataset will be generated (and used for further research). The general scheme of preparing the reference grid, *high-resolution grid* and XYZ test data are presented in Fig. 1.

The principle of preparing test data using a *virtual sea survey* is as follows (see Fig. 1).

1) Based on actual MBES survey data (1), two surveys covering the same area are generated. The first one is the *reference grid* (3), which has a standard resolution, the same as later generated in the *test surfaces* (5). The second surface is the *high-resolution grid* (2), whose resolution is 10 times higher than the *reference grid*. Such high model accuracy is necessary to calculate accurate XYZ test data (4). When creating both surfaces, three different data interpolation methods were used: MA, IDW, and kriging, and the obtained models were first averaged and then additionally smoothed with a median filter (5 × 5 for *reference grid* and 7 × 7 for *high-resolution grid*). This approach essentially eliminates distortions resulting from inaccurate MBES (device noise) measurements, so the resulting surfaces look smooth and natural [26].
2) We assume that the *high-resolution grid* (2) is a surface describing a certain real area with high accuracy, and we conduct a *virtual sea survey* over its surface using a proprietary program. It simulates the process of movement and data collection by a measuring vessel equipped with an MBES device. Over the indicated digital area (*high-resolution grid*), a virtual measuring vessel moves along the indicated route and, in accordance with the set simulation parameters, reads the depth at the calculated points. It is possible to set many *virtual sea survey* parameters, such as vessel velocity, swath angle, no. of beams in the swath, ping rate and MBES error (regular or depending on the beam angle and depth). As a result of this simulation, we obtain XYZ test data (4), which has very similar properties to actual MBES survey data, i.e., similar data density, spatial distribution, and slightly noisy depth values.
3) The XYZ data files obtained from the *virtual sea survey* are our test data. Using them, we can calculate any DTM using various interpolation methods (also with different parameters). Each of the obtained models can then be compared with the *reference grid*, and the differences between these models can be calculated. Thanks to this, we obtain not only the value of the total error of the model but also their distribution. Since both models have the exact resolution and size, it is possible to compute an error value for each mesh node as well as many statistical data. In the studies presented below, the mean error and model error were calculated at the CL of 95% ($CL = 95\%$).The developed solution makes it possible to generate test data that are very close to real in nature. On their basis, we can create many different test models (using different interpolation methods or different parameters of the same interpolation method), and the ability to compare

the generated test surfaces to the *reference grid* enables accurate estimation of errors (including their distribution). It should be clearly noted that in the described approach, the values of the calculated errors do not result only from the data processing methods used (interpolation), but they are total errors, also taking into account many parameters related to the entire process of data collection and processing (MBES device error, survey parameters, grid resolution, processing data). By changing only, the parameters related to the interpolation method, we can assess their impact on the TVU.

To sum up, without a simulator, we cannot count the errors of the created models or examine the impact of any parameters on the accuracy of these models. The introduction of the simulator made it possible to create test data very similar in nature to the real ones and then calculate the errors of the models created on their basis and perform various tests related to testing the accuracy of MBES.

The author believes that the developed approach significantly eliminates the problems encountered when using other methods of verifying the accuracy of the generated DTMs. More on the operation of the simulator and verification of its correctness can be found in [26] (basis of the sea survey simulator) and [28] (simulator with an extra noise generator).

B. *High-Resolution Grid, Reference Grid and XYZ Test Data*

MBES survey data was collected in 2013 by the hydrographic vessel of the Szczecin Maritime Office, around the West Oder River in Szczecin. A Simrad EM 3000 echosounder was used, and all data was saved with the UTM coordinate system. During that session, 6 082 594 points were measured in an area of 173×180 m^2. The density of this data is ~195 points per 1 m^2, and the average distance between adjacent points is ~7 cm. Thus, it can be considered as high-density data.

Based on the above data, a *high-resolution grid* was then calculated, where X and Y spacing is 2 cm, and thus the grid size is 8650×9000 points. When calculating it, the averaged interpolation of MA, IDW and kriging was used (then averaged), and the obtained area was additionally smoothed with a median filter of size 7×7 points.

The *reference grid* was also calculated in an analogous way. This time the X and Y spacing is 10 cm, and the grid size is 1731×1801 points. All test surfaces obtained during the tests were compared with this surface. When calculating it, the averaged interpolation of MA, IDW, and kriging was used (then averaged), and the obtained area was additionally smoothed with a median filter of 5×5 points.

The test surfaces (as well as *reference grid* and *high-resolution grid*) are characterized by variable shape. There are fragments where it is almost flat, in other places there are slight changes in shape, and in some fragments, there are clear rapid changes in depth (see Fig. 2). It can therefore be concluded that this surface is varied and represents various forms of landforms occurring in reality.

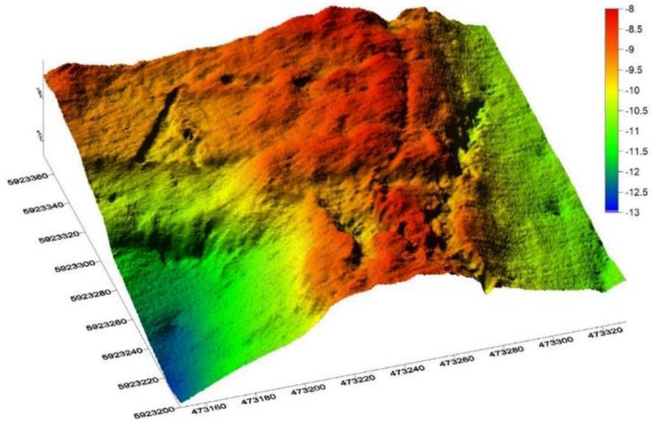


Fig. 2. Shape of the reference grid.

Finally, based on a *high-resolution grid* and using a *virtual sea survey* simulator, six XYZ test data files were generated, differing primarily in the number of measurement points, and thus their density. Efforts were made to reproduce the parameters adopted during the actual sea survey. Table II presents a set of adopted simulation parameters and properties of the generated XYZ test data. The prepared XYZ test data files resemble in their characteristics data obtained during real measurements, using various measuring devices and various values of survey parameters. Distribution (and density) of the prepared XYZ test data are presented in Fig. 3.

C. *Testing Procedure*

All calculations, including the determination of model errors, were performed in *Golden Software SURFER v8.0* [26]. During the research, 47 models were generated based on XYZ data files with different variable densities and different numbers of measurement points involved in interpolation. All the generated surfaces (DTMs) were compared with the *reference surface*, and the mean and 95% CL of error were determined (in cm), as well as the computation time (in s).

The research was calculated using a personal computer equipped with an Intel Core i7 processor (model 870), HDD 2TB, 4 GB RAM and Windows 11. The performance index for this configuration, calculated using *Cinebench R15 software* is equal to 478 points [30].

D. *Results*

As mentioned earlier, the modified IDW interpolation method was used in the study. In the traditional approach, the user specifies the size of the local neighborhood (*radius size*), and then all points within it take part in calculating the new value. As shown in [19], slightly better results are obtained when considering the N nearest points. It is obvious that the value of N for which we obtain the most accurate model will depend on the density of the measurement data. On the one hand, it is good to have a lot of data, thanks to which they will be averaged, which in turn reduces the impact of noise on the accuracy of

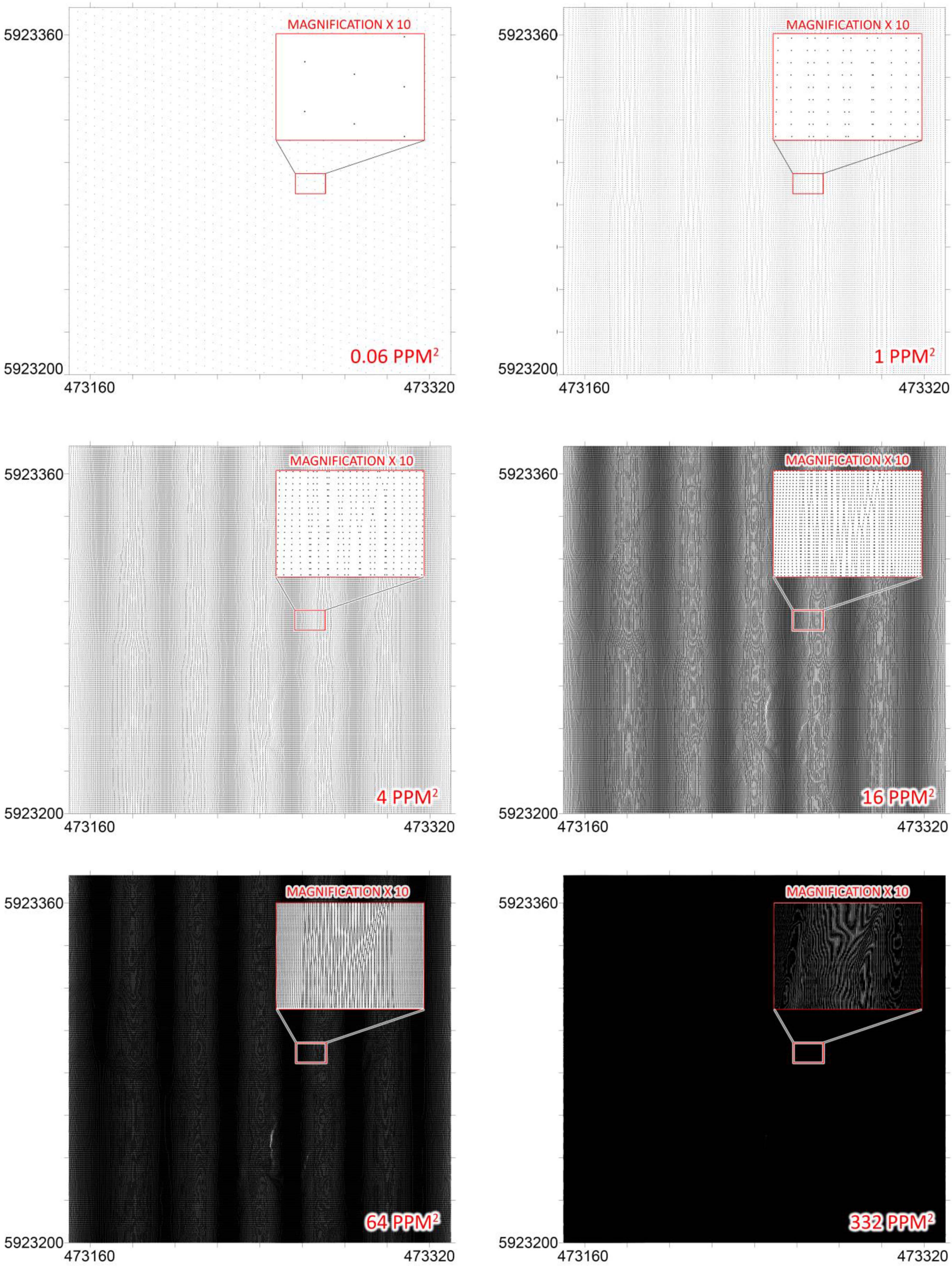
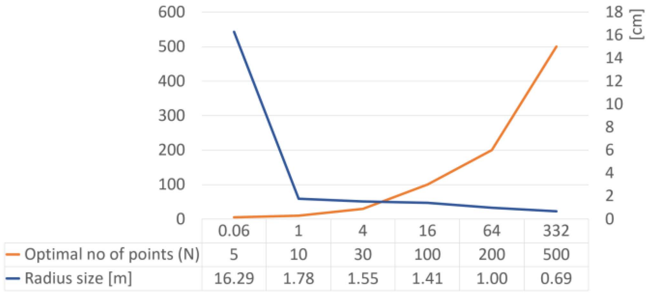


Fig. 3. Distribution and density of XYZ test data.

TABLE II
PROPERTIES OF GENERATED XYZ TEST DATA SETS

measurement density	0.06 PPM^2	1 PPM^2	4 PPM^2	16 PPM^2	64 PPM^2	332 PPM^2
description	very low density *(corresponds to SBES)*	low density	medium density	high density *(corresponds to EM3000)*	very high density	extremely high density *(corresponds to EM 2040C MKII)*
vessel speed [kn]	5.0	5.0	5.0	5.0	5.0	5.0
swath angle	130^{O}	130^{O}	130^{O}	130^{O}	130^{O}	130^{O}
no of beams	1	32	62	127	247	512
ping rate [s]	1.0	0.4	0.2	0.1	0.05	0.02
noise range [cm]	0–5	0–5	0–5	0–5	0–5	0–5
total number of points	1.969	32.355	125 217	516.545	2.002.434	10.340.227
avg. distance between points [cm]	400	98	50	25	12	5



	0.06	1	4	16	64	332
Optimal no of points (N)	5	10	30	100	200	500
Radius size [m]	16.29	1.78	1.55	1.41	1.00	0.69

Fig. 4. Effect of the density of measurement data on the optimal number of points involved in interpolation and the radius size, in which these points are located.

the model, but on the other hand, we do not want points too distant to have a significant impact on the newly calculated value. Therefore, in the first part of the research, the dependence of the optimal number of measurement points (N) on the density of measurement points was checked. The optimal number of points means that using them we generate the most accurate model. For this purpose, 47 models were calculated for the number of points $N = 1, 2, 5, 10, 20, 30, 50, 100, 200$, and 500, using six XYZ test data files with different density of points (density = 0.06, 1, 4, 16, 64, 332 points per 1 m^2). The interpolation did not take into account the *radius size*, but only the N closest measurement points to the node. The power of IDW was set to 2. The obtained results are presented in Fig. 5 (light green background in the tables below the graphs indicates the best results).

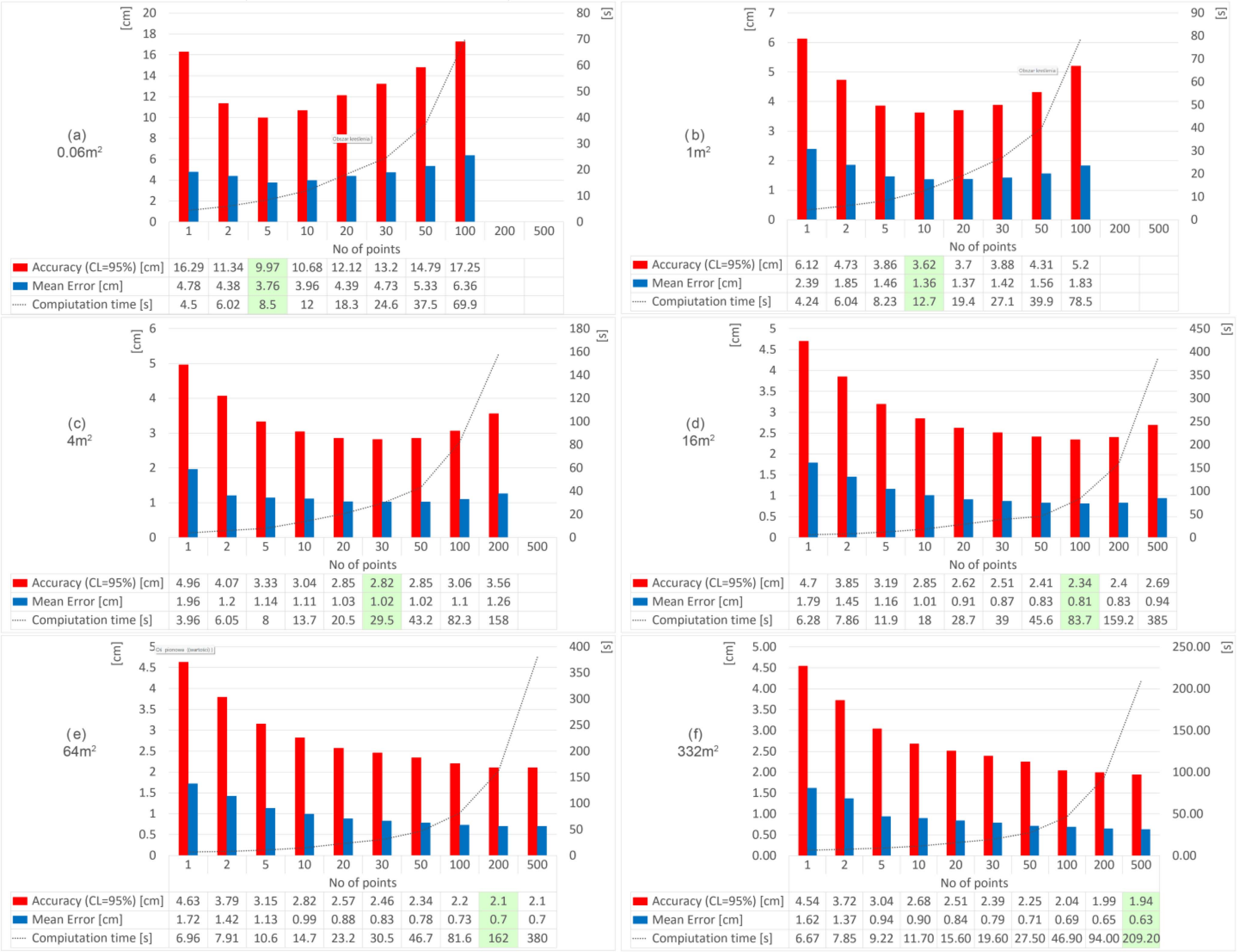
As expected, with the increase in the density of measurement points, the optimal number of points N involved in interpolation increases (for low density $N = 5$, for medium density $N = 30$, and for extremely high-density $N = 500$). Analyzing the detailed results, the question arises whether these points are not located in an environment of comparable size? Will using a specific *radius size* to select the interpolation points give us similar model accuracy? To check this, for each XYZ test data, the size of the area containing the optimal number of N closest points was calculated. The obtained results are presented in Fig. 4.

Based on these results, we can clearly state that the *radius size* of the N nearest measurement points clearly decreases with the increase in the density of measurement points. Consequently, the use of the IDW *fixed radius size* method in the standard approach will not give us optimal models, because we can increase their accuracy by using the N parameter, and for low-density measurement data this number should be low, for example in the range of 5 to 10, for medium density files in the range of 30 to 100, and for very high density files even more than 200 points. On the other hand, we can make an additional conclusion that using the standard IDW interpolation method using the *radius size* parameter, we should set it for low-density files to over 2 m, for medium-density files to about 1–2 m, and for files with very high density less than 1 m.

Another aspect that was checked in the research is the effect of the density of measurement points on the accuracy of the generated model and the calculation time, assuming that we use the optimal number of points N in interpolation. For this purpose, the accuracy of the models (CL = 95%) was calculated depending on the density of measurement points and the time of these calculations was measured. The obtained results are presented in Fig. 6.

The obtained results clearly show that only data with a low density of measurement points generate entirely inaccurate models. In the presented studies, models made using measurements from SBES (density 0.06) have an accuracy of TUV = 10 cm. On the one hand, this value is within the IHO standards; however, the distance between adjacent profiles is 4 m, which does not meet the requirement of 100% bathymetric coverage, and significant changes in shape and depth between profiles or existing obstacles can be omitted or distorted.

Based on files with an average density of measurement points, we get sufficiently accurate models, in which the TUV is about 2–3 cm. This is only 10% of the total allowable error according to IHO standards. Further increasing the density of measurement data only slightly affects the accuracy of the model, which increases by less than 1 cm. So, it can be considered that any measurements made using MBES, even if we use slightly older devices that collect less measurement data, they are dense



(a) 0.06m²

	1	2	5	10	20	30	50	100	200	500
🟥 Accuracy (CL=95%) [cm]	16.29	11.34	9.97	10.68	12.12	13.2	14.79	17.25		
🟦 Mean Error [cm]	4.78	4.38	3.76	3.96	4.39	4.73	5.33	6.36		
┈ Compiutation time [s]	4.5	6.02	8.5	12	18.3	24.6	37.5	69.9		

(b) 1m²

	1	2	5	10	20	30	50	100	200	500
🟥 Accuracy (CL=95%) [cm]	6.12	4.73	3.86	3.62	3.7	3.88	4.31	5.2		
🟦 Mean Error [cm]	2.39	1.85	1.46	1.36	1.37	1.42	1.56	1.83		
┈ Compiutation time [s]	4.24	6.04	8.23	12.7	19.4	27.1	39.9	78.5		

(c) 4m²

	1	2	5	10	20	30	50	100	200	500
🟥 Accuracy (CL=95%) [cm]	4.96	4.07	3.33	3.04	2.85	2.82	2.85	3.06	3.56	
🟦 Mean Error [cm]	1.96	1.2	1.14	1.11	1.03	1.02	1.02	1.1	1.26	
┈ Compiutation time [s]	3.96	6.05	8	13.7	20.5	29.5	43.2	82.3	158	

(d) 16m²

	1	2	5	10	20	30	50	100	200	500
🟥 Accuracy (CL=95%) [cm]	4.7	3.85	3.19	2.85	2.62	2.51	2.41	2.34	2.4	2.69
🟦 Mean Error [cm]	1.79	1.45	1.16	1.01	0.91	0.87	0.83	0.81	0.83	0.94
┈ Compiutation time [s]	6.28	7.86	11.9	18	28.7	39	45.6	83.7	159.2	385

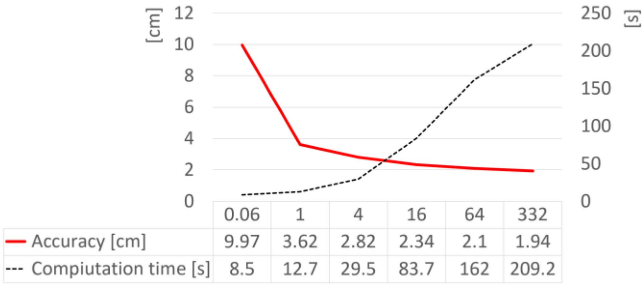
(e) 64m²

	1	2	5	10	20	30	50	100	200	500
🟥 Accuracy (CL=95%) [cm]	4.63	3.79	3.15	2.82	2.57	2.46	2.34	2.2	2.1	2.1
🟦 Mean Error [cm]	1.72	1.42	1.13	0.99	0.88	0.83	0.78	0.73	0.7	0.7
┈ Compiutation time [s]	6.96	7.91	10.6	14.7	23.2	30.5	46.7	81.6	162	380

(f) 332m²

	1	2	5	10	20	30	50	100	200	500
🟥 Accuracy (CL=95%) [cm]	4.54	3.72	3.04	2.68	2.51	2.39	2.25	2.04	1.99	1.94
🟦 Mean Error [cm]	1.62	1.37	0.94	0.90	0.84	0.79	0.71	0.69	0.65	0.63
┈ Compiutation time [s]	6.67	7.85	9.22	11.70	15.60	19.60	27.50	46.90	94.00	209.20

Fig. 5. Accuracy of models and calculation time for test data with different densities and different numbers of nearest points involved in interpolation.



	0.06	1	4	16	64	332
— Accuracy [cm]	9.97	3.62	2.82	2.34	2.1	1.94
--- Compiutation time [s]	8.5	12.7	29.5	83.7	162	209.2

Fig. 6. Effect of measurement data density on the accuracy of generated models and calculation time.

enough to create a very accurate model of the bottom. The use of much newer and more expensive devices, a speed reduction of the measuring vessel, additional profiles, or the use of two heads will enable the collection of many times more data, but only slightly affect the accuracy of the DTM generated. Therefore, it can be considered whether the costs of creating or changing the sea survey system in relation to the slightly more accurate models obtained are justified.

It is also worth noting that the average error for most models is ∼1 cm. This can be considered a very good result, especially considering that the measurements are burdened with a random error of 0–5 cm (average 2.5 cm), additionally increased with increasing depth and for extreme beam angles. This means that the use of a large number of measurement points (N) averages the data sufficiently effectively and generates a fairly smooth surface, as shown in Fig. 7.

For the defined grid size and the density of measurement data, the model generation time depends primarily on the number of nearest N points involved in the interpolation. On the other hand, for the exact value of N, the higher the measurement data density, the longer the DTM generation time. In both cases, this relationship is close to linear. Considering how the IDW method works, we can conclude that the total calculation time depends primarily on the number of input points (density), the number of points involved in interpolation (N), and the grid size. In the extreme case, the DTM calculation time was as much as 209 s for a small area occupied by the *reference surface*. Taking the above aspects into account, we can conclude that the interpolation duration should also be an important element in the

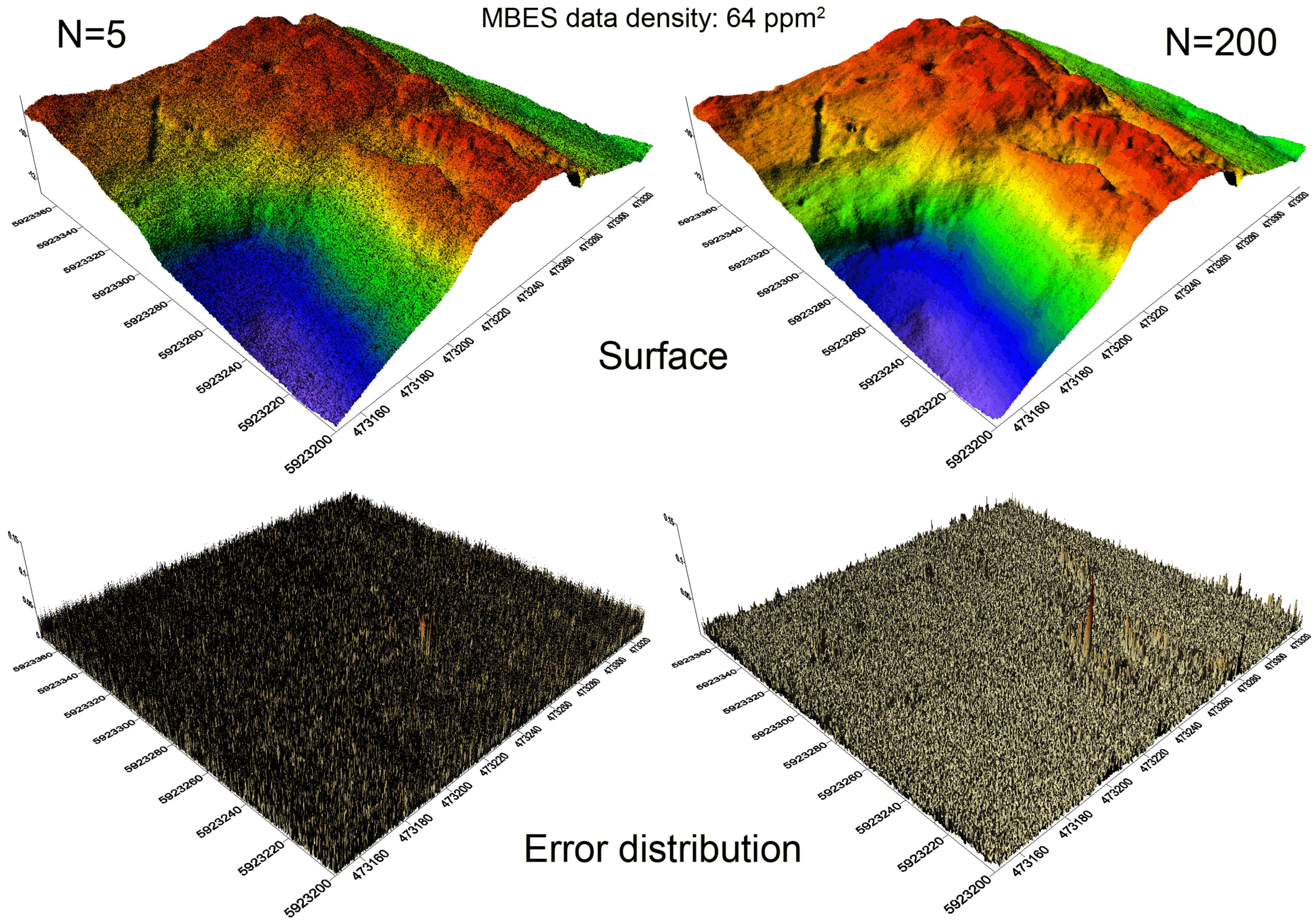


Fig. 7. Visualization of the surface and the distribution of model errors obtained on the basis of measurement data with a density of 64 ppm^2 and $N = 5$ points (left) and $N = 200$ points (right).

selection of interpolation parameters. The author believes that in particular for very high-density data, we can consider using a smaller number of the nearest N points in the interpolation, which will cause a slight deterioration in the accuracy of the model, but calculations will be many times faster. For example, for a 64 ppm^2 file and $N = 200$ points (optimal value), TUV = 2.1 cm, and the calculation time is 162 s. If we use $N = 20$ points, TUV = 2.57 cm, and calculation time 23 s. Therefore, the accuracy of the model decreased by only 0.47 cm (which is 2% of the allowed total error of the model), while the calculation time decreased 7 times.

Comparing the results obtained at this research (modified IDW interpolation and optimal N points involving in interpolation) with regular IDW (search radius = 1, and max $N = 10$) could be noticed that it gives us 5%–15% more accurate models, with similar or slightly longer calculation times.

Based on the results obtained and the author's experience (also from other similar studies), it can be said that the following.

1) The use of low-density datasets (corresponding to SBES) does not meet the standards defined by IHO (for special class), so such data are insufficient.
2) The use of sets with a medium density of measurement points (corresponds to EM3000 echosounder) enables the creation of high-accuracy models. In the presented example, using the optimal number of points involved in the interpolation N = 100, a model with an accuracy of 2.34 cm (at CL = 95%) was obtained. This accuracy is only 9% of the model error allowed by IHO (26 cm). The use of fewer points in interpolation (e.g., N = 10) only slightly reduces the accuracy of the model (2.85 cm at CL = 95%), shortening the calculation time by almost 5 times (from 83.7 to 18 s).
3) The use of sets with an extremely high density of measurement points (corresponds to the EM 2040C MKII echo sunder) enables the creation of models with the highest accuracy. In the presented example, using the optimal number of points involved in the interpolation, N = 500, a model with an accuracy of 1.94 cm (at CL = 95%) was obtained, which is 17% more accurate than for medium-density sets. However, this is paid for by 2.5 times longer calculation time of 209 s. Using a much smaller number of points in interpolation (e.g., N = 50) only slightly decreases the accuracy of the model (2.25 cm at CL = 95%), shortening the calculation time by almost 8 times (from 209 to 27 s).

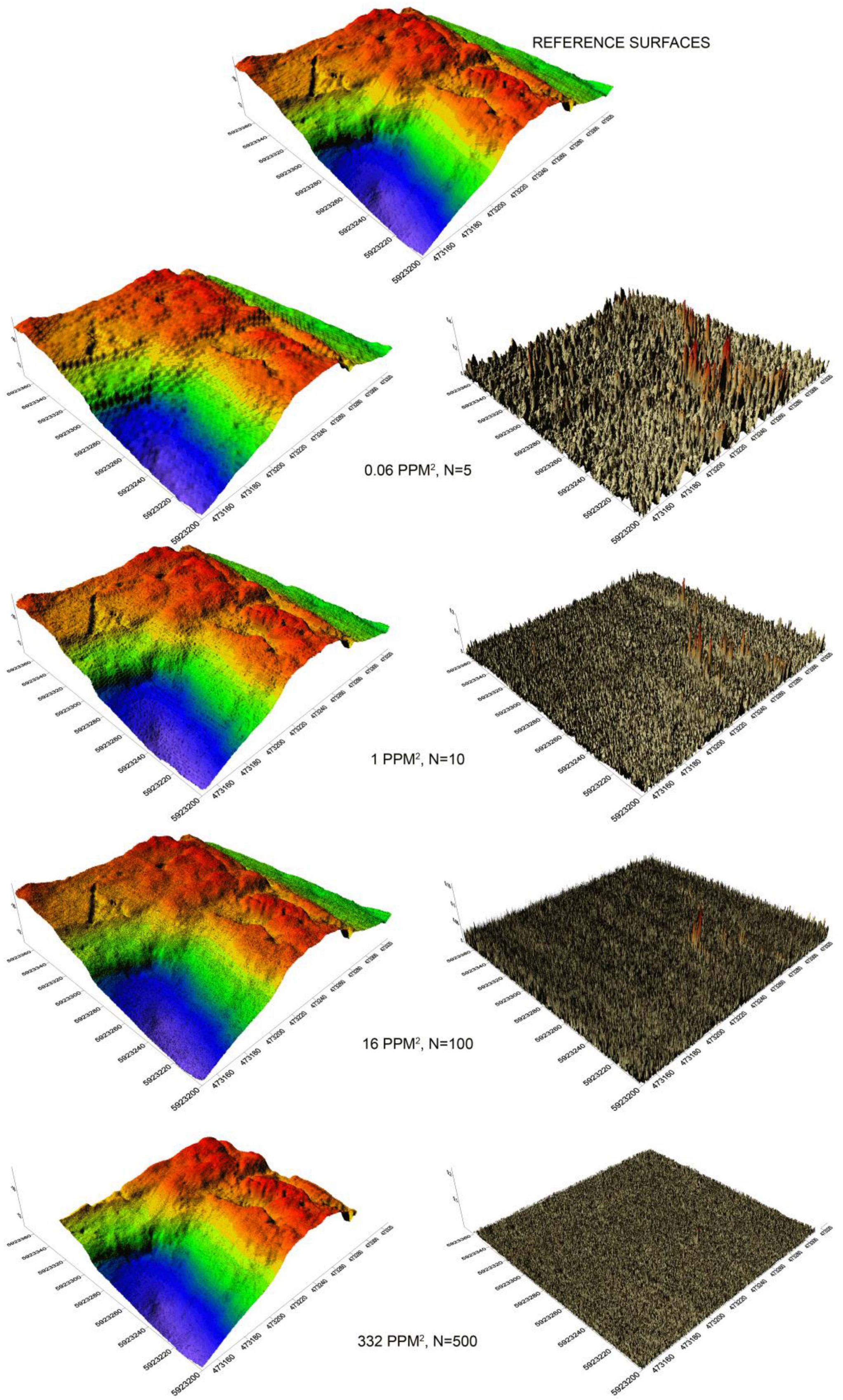


Fig. 8. Sample surfaces and error distribution generated using modified IDW interpolation using the *N* closest points for different measurement data densities.

4) Reducing the number of N points involved in the interpolation slightly decreases the model accuracy, additionally surfaces look rougher. This undesirable effect can be easily eliminated by using one of known surface smoothing methods, which should not only improve the visual look of the surface but also slightly increase the accuracy of the model. However, these theses are only the author's predictions; the use of smoothing methods in the process of creating DTMs was not the subject of this research.

It can therefore be concluded that denser sets of measurement points as well as a greater number of N points involved in interpolation, have a positive effect on the accuracy of the models. However, the increase in their accuracy is small, which is related to a clearly noticeable increase in the calculation time. It seems that when creating such accurate models (at the level of 10% of the error allowed by IHO standards), the calculation time may be more important in practice than increasing the accuracy of the model by another few millimetres. It should be up to the operator of the hydrographic system to decide whether it is reasonable to use optimal interpolation parameters that allow a slight increase in model accuracy, at the expense of much longer calculations.

The presented research also showed that the size of the area (*radius size*), in which the optimal number of measurement points involved in interpolation (N) lies depends on the density of measurement points. The denser they are, the smaller the *radius size*. The *radius size* $= 1$ m, often used in calculations, seems to be a good compromise; however, the author suggests that for very high density MBES files, it may be smaller, e.g., 0.5 m, and for low density MBES files, about 2 m.

Examples of surfaces (DTMs) and the distribution of errors that were generated using modified IDW interpolation using the N nearest measurement points for different measurement data densities are presented in Fig. 8.

V. CONCLUSION

The article presents research related to the creation of bathymetric models based on measurement data with different densities from MBES with the use of IDW interpolation. In particular, the effect of the density of measurement data on the accuracy of the generated DTM and the selection of the optimal number of measurement points used for interpolation of subsequent grid nodes depending on the density of measurement points were examined.

To be able to estimate the accuracy of the generated DTMs accurately, the author proposed an innovative method of preparing test data using actual MBES data and proprietary *virtual survey simulator* software.

The work showed that the increase in the density of measurement points enables the creation of more accurate bathymetric models. This is an obvious relationship, but the detailed results allow to precisely assess the value of this improvement, and thus answer the question of whether it is worth using very dense datasets, which usually involve a higher cost of obtaining them as well as a much longer calculation time to obtain only slightly more accurate models.

Following the IHO standards, the TVU of the generated test models should not be larger than 26 cm. This error includes all components of inaccuracies arising in the sea survey and postprocessing, including the MBES data interpolation. For all models generated during the research, the TVU was much lower and amounted to approx. 1–6 cm (except very low density MBES file, where TVU was 12–17 cm). It shows that the optimized IDW method can be used in the process of DTM creation based on data collected from an MBES.

In general, the density of measuring points obtained from MBES should be considered when selecting the optimal interpolation method and its parameters. Knowing the density of measurement points, we can more accurately select the number of measurement points involved in interpolation (or similar *radius size*), while expecting shorter/longer calculation time. Also, when planning survey work, we can select such equipment and survey parameters to obtain files with a specific density, and thus, models with a specific accuracy. Perhaps it will turn out that cheaper / simpler equipment (devices) and faster measurements give sufficiently dense sets of measurement points and sufficiently accurate bathymetric models. On the other hand, expecting the highest accuracy, the latest technology should be used, and the sea survey should be properly planned. However, we should be aware that the obtained models will be only slightly more accurate, and the calculation time will be much longer.

The IDW method presented in this article uses a pinpoint set of optimal parameters for different density input data. As experiments proved, its application slightly increases the accuracy of the resulting grid and fits the actual seabed surface better. Users in GIS software can use the proposed method and presented dependencies for DTM creation based on large data sets.

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