Coordinating Multiple Light-Trails in Multicast Elastic Optical Networks With Adaptive Modulation

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***Abstract*—Optical multicasting has been considered resource efficient for multicast services. Light-tree and light-trail are two technologies that support optical multicasting while the former requires many splitters and thus experiences significant power loss. In this paper, we consider using the light-trail technology for the accommodation of multicast requests in elastic optical networks with adaptive modulation. For better spectrum efficiency, we consider accommodating each multicast by multiple light-trails. We formulate the problem by Mixed Integer Linear Programming (MILP) and propose efficient heuristic algorithms. For the impact of accommodation sequence on the algorithm performance, apart from the traditional sequence among different requests, we consider an additional sequence among the destinations of a multicast. For efficient multicast accommodation, we propose several strategies and compare their performances through a range of cases. To avoid a destination occupying excessive resources in certain cases of joining multiple light-trails, we propose an efficient algorithm to delete some duplicated destinations. Numerical results show that the proposed heuristic algorithms significantly outperform a benchmark algorithm and one performs close to the optimal MILP. Also, the algorithm for deleting certain destination replicas largely reduces the spectrum and transmitter usages, up to 41% and 20% for the cases considered, respectively.**

***Index Terms*—Adaptive modulation, elastic optical network, light-trail, multicast, routing and spectrum assignment.**

I. Introduction

INTERNET traffic has been exhibited enormous growth in the past decades. Cisco predicted that global IP traffic would increase threefold from 2017 to 2022 reaching 4.8 ZB [1]. IP video traffic was predicted to be 82% of all IP traffic by 2022, among which live Internet video traffic would increase 15-fold from 2017 to 2022 accounting for 17% of Internet video traffic. Here, live videos usually involve significant multicasting. Other multicast services involving such as ultra-high-definition TV, virtual reality/augmented reality, and synchronization and backup of database among geographically-distributed datacenters are usually bandwidth-hungry.

To support multicast in a more efficient way, the light-tree and light-trail technologies have been proposed that are capable of optical multicasting. A light-tree is an all-optical multicast channel with a tree structure where the optical signal power passing through a network node is split into portions for multiple outgoing ports. A light-trail is also an all-optical multicast channel but with a trail structure. The destinations tap only a very small amount of power for local usage while the remaining power with negligible loss is switched to only one outgoing port. Compared with the method of using lightpath technology, both the light-tree and light-trail are more spectrum-efficient since the nodes could share the same resources [2], [3].

There are some differences between the light-tree and light-trail technologies. The light-tree technology employs power splitters while the light-trail we consider in this paper is based on tap splitters [4]. The power splitters divide the signal power evenly into multiple copies. These attenuated signal copies require the usage of Erbium-Doped Fiber Amplifiers (EDFAs) for amplification before being sent to output ports. However, EDFAs degrade the Optical Signal-to-Noise Ratio (OSNR) greatly, typically four to six dB for every single EDFA traversed. A light-tree going through multiple such nodes could result in very low OSNR which entails the usage of low-level Modulation Schemes (MSs) and therefore requires large spectrum bandwidth. Meanwhile, EDFAs that light-tree networks heavily depend on are one of the main contributors to the network power consumption. Different from the light-tree, the light-trail technology adopts a Tap-and-Continue (TaC) structure [4] employing tap splitters. The optical signal loses negligible amount of power, e.g., 0.5%, after being tapped by the local station. The lossless signal requires no EDFAs for power amplification before being sent to an output port. Many EDFAs can be saved by the light-trail technology and the signal maintains a relatively high OSNR to the light-tree one. The light-trail network node is thus much simpler than the light-tree node [4]. In addition, the light-trail is extended for IP transport [5], [6] and it presents degrees of flexibility, for example, the intermediate nodes could time-share

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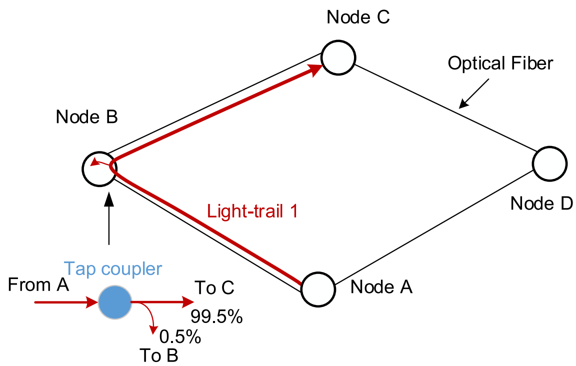

Fig. 1. An example light-trail in a TaC-based optical network.

the bandwidth for sub-wavelength connections. The drawback of the light-trail is that compared to the light-tree, the trail length could be long when it covers many destinations. To mitigate this drawback, multiple light-trails can be used. As demonstrated in [7], the solution of using multiple light-trails based on tap splitters outperforms the one of using multiple light-trees. We present an example of using a light-trail for a multicast request from node A to nodes B and C as shown in Fig. 1. Node B taps a small portion of the signal power and the remaining signal continues to node C. A light-tree solution could have the same routing, but the signal split for node C experiences significant power loss.

To better support the evergrowing traffic, Elastic Optical Networks (EONs) [8], [9] have been considered a potential candidate for next-generation optical transport networks due to its various degrees of flexibility that improve resource utilization. Compared with traditional Wavelength-Division Multiplexing (WDM) optical networks, EONs adopt a finer-granularity grid and allow for a flexible bandwidth allocation to connections which entails a better spectrum utilization. Connections with different data rate requirements are efficiently supported in EONs by adjusting the amount of spectral bandwidth. Also, for a fixed amount of spectrum bandwidth, EONs can still provide varied capacity by an adaptive modulation technique. With adaptive modulation, connections adaptively select modulation for the signals according to their channel conditions. Distance-adaptive spectrum allocation [10] was proposed where channel conditions are mainly affected by transmission distances. For the distance-adaptive spectrum allocation, more spectrally-efficient modulations can be utilized for connections with shorter distances, which saves spectrum resources. Further, these flexibilities of EONs provide ways of establishing supper channels to support future requests with bit rates beyond 1 Tbps. The flexible bandwidth allocation makes problems such as Routing and Spectrum Assignment (RSA) [11] in EONs much more complicated than those in traditional WDM networks. And it becomes even more complicated when the adaptive modulation technique is considered, and the problem is evolved into Routing, Modulation, and Spectrum Assignment (RMSA) problem [12], [13].

A. Related Work

There are generally three technologies, namely, lightpath, light-tree, and light-trail, which support multicast services in optical networks. Different technologies require the network nodes to be capable of different levels of multicasting. When the node architecture only supports unicast capability, the lightpath technology is used to accommodate a multicast demand. A lightpath is an optical channel between two end nodes. For the accommodation using the lightpath technology, a multicast demand is considered as a set of unicast demands, each occupying dedicated resources for a transmission of the data from the source to a destination [14].When the node architecture is *Multicast-Capable (MC)*, the multicast demand can be accommodated by the light-tree technology. In a light-tree, the optical signal is split into multiple copies at a splitting node of the tree and each of the copies feeds an egress link at the node. It is more efficient than the lightpath since the resources can be shared among different destinations [2]. Extensive studies were performed based on the light-tree technology. Lin [15] enhanced a three-stage wavelength-space-wavelength node architecture to be capable of multicasting using the subtree scheme. Integer Linear Programming (ILP) formulations and algorithms were proposed for the light-tree solutions in EONs with adaptive modulation. In [16], a set of candidate paths is considered given for each Source-Destination (SD) while in [17], a set of candidate trees is considered given for a multicast. Choudhury et al. [18] proposed an approximation based Steiner tree approach for RSA of multicast requests and compared with the shortest path tree algorithm. Ruiz and Velasco [19] evaluated the three schemes for multicast demands, namely, path, tree, and subtree. Fan et al. [20] provided an arc-path ILP formulation for the static Multicast-Capable Routing, Modulation and Spectrum Assignment (MC-RMSA) problem and compared the performance of using multiple light-trees to that of using a single light-tree for a multicast. Li et al. [21] considered the problem of on-demand RMSA for light-tree based multicast service aggregation to reduce spectrum and transceiver usage. Moreover, the impact of the number of MC nodes and the multicast degree was investigated on the network design problem of minimizing the spectrum requirement in links [22]. Our earlier work [23] presented MILP and efficient heuristic algorithms for a shared protection scheme for multicast in EONs with adaptive modulation. Habibi and Beyranvand [24] considered physical layer impairments for manycast in EONs and proposed ILP formulations and heuristic algorithms. Deep neural network were utilized to predict the OSNR and the availability of light-trees [25].

When the node adopts a TaC structure, the light-trail technology could be used. The inherent support of optical multicasting and sub-wavelength traffic attracts research interest in multicast and dynamic traffic grooming recently. Ali and Deogun [4] proved the NP-completeness of the problem of finding in WDM networks the optimal minimum-cost trail that traverses a given set of destinations and also proposed a 4-approximation algorithms. Le et al. [26] proposed algorithms to the problem of establishing a minimum number of light-trails for the accommodation of a multicast in WDM optical networks. Majumdar et al. [27] considered elastic light-trails that could increase the spectral bandwidth if needed for dynamic traffic grooming in EONs. Majumdar and De [28] investigated a dynamic traffic grooming problem for unicast requests in EONs and proposed a multi-hop algorithm to reduce spectrum usage and compared with algorithms considering elastic light-trail. Lin and Zhuang [29] considered a problem of dynamic multicast traffic grooming problem in light-trail-based WDM networks and proposed efficient algorithms and compare with existing ones. Sharma et al. [30] introduced the light-trail as a 5 G backhaul solution for the benefit of dynamic provisioning, optical multicasting and sub-wavelength grooming. In our earlier work, we compared the transmitter and spectrum usage of the five methods of using lightpaths, a single light-tree, multiple light-trees, a single light-trail, and multiple light-trails for establishing a single multicast in EONs [3]. In addition, light-hierarchy is a technology for network nodes with different multicast capabilities [31]. A multihop lightpath method is considered in the context of overlay networks [32] where data transmissions are relayed at member destinations for better spectrum efficiency at the cost of longer delay and more transmitters.

B. Key Contributions

To the best of our knowledge, there is no published work on the static problem of light-trail routing, modulation and spectrum assignment for a given set of multicast requests in EONs considering adaptive modulation. In this paper we address this important problem. The objective is to minimize the spectrum requirement for the accommodation of all the given multicast requests. A general case is considered that each multicast is accommodated by multiple light-trails. We formulate the problem by Mixed Integer Linear Programming (MILP) for small instances and propose efficient heuristic algorithms that scale to large ones. The proposed algorithms make an adequate tradeoff between light-trail sharing and spectrum requirement in the traversed links to reduce spectrum usage. We investigate the accommodation sequence that affects the algorithm performance and the characteristics of multicast, and propose two-level sequences which is unique to multicast. We also inspect the scenario that a destination joins multiple light-trails leading to the occupation of excessive resources. An algorithm is proposed to delete certain node replicas that reduces transmitters and spectrum usage. Numerical results demonstrate the efficient performance of the proposed algorithm achieving close to the optimal MILP for small instances. This work also provides insights to a similar problem using the light-tree technology.

C. Organization

The remainder of this paper is organized as follows. We state the problem in Section II. A MILP formulation is presented in Section III, and heuristic algorithms are provided in Section IV. Numerical results and analysis are presented in Section V. Section VI concludes the paper.

TABLE I
TRANSPARENT REACH AND CAPACITY PER FREQUENCY SLOT (FS) AT A GRANULARITY OF 12.5 GHZ FOR EACH MS [12]

Modulation Level	MS	Transparent Reach [km]	Capacity per FS [Gb/s]
1	BPSK	4000	12.5
2	QPSK	2000	25
3	8QAM	1000	37.5
4	16QAM	500	50

II. PROBLEM STATEMENT

In this paper, we focus on the RMSA problem of minimizing the spectrum required to serve a set of multicast requests.

The EON model is represented by a directed graph $G = (\mathbf{V}, \mathbf{L})$ where $\mathbf{V}$ is the set of nodes and $\mathbf{L}$ is the set of directed fiber links. A directed fiber link (i, j) is accompanied by one in the opposite direction, i.e., (j, i), and the lengths are assumed the same, $\ell_{ij} = \ell_{ji}$. We consider a set of MSs denoted by $\mathbf{M}$. Given a multicast request r, we denote the transparent reach by τ_m^r, and the capacity per FS by $\mathcal{C}_m$ when the signal is modulated by MS $m, m \in \mathbf{M}$. Let g denote a number of FSs that are used as the guard band placed between two neighboring channels to avoid interference.

We denote a multicast r by $\langle s_r; \mathbf{D}_r; b_r \rangle$ requesting a transmission from the source s_r to all the destinations in set $\mathbf{D}_r$ at the bit rate b_r. We calculate by $\omega_m^r = \lceil b_r / \mathcal{C}_m \rceil$ the number of FSs to be assigned to serve the multicast request r for the required bit rate given that MS m is utilized as presented in [12]. Here, $\lceil x \rceil$ is the smallest integer that is no smaller than x.

In this paper, we focus on using the light-trail technology for planning a network with multicast requests. We assume that for each SD pair the shortest path distance is within the longest transparent reach of the MSs considered, e.g., 4,000 km for Table I. Given the above-mentioned network model and a set of multicast requests, the objective is to minimize the maximum number of FSs among the links so that all the requests can be accommodated in an EON without spectrum conversion capability. We accommodate each multicast by multiple light-trails and adaptive modulation is considered where light-trails with shorter transmission distances may use more spectrum-efficient MSs.

This network planning problem involves light-trail routing, modulation, and spectrum assignment. To set up a light-trail for a given multicast, routing trails should be found that connects the source with the destinations. An MS should be selected so that a signal transmitted via the trail meets the quality-of-transmission requirement. Following that, spectrum should be allocated to carry the signal where three constraints should be satisfied, namely, *spectrum contiguity, spectrum continuity, and spectrum non-overlapping*. The spectrum contiguity constraint ensures that FSs allocated should be contiguous. The spectrum continuity constraint guarantees that the spectrum allocated to a connection should be the same in all the links traversed. The spectrum non-overlapping constraint ensures that no spectrum in a link can be allocated to two or more connections.

A. Finding a Set of Light-Trails

In this paper, multiple light-trails are used for the accommodation of every multicast. Hence, an important problem is to find a set of light-trails with the minimum cost in EONs considering adaptive modulation. This problem is NP-hard since a special case in the context of WDM optical networks is proven NP-hard [26] where the amount of spectrum requested is fixed and no distance limit is considered. New degrees of flexibility provided by EONs make this problem much more complicated. Specifically, there exists a tradeoff between sharing of a light-trail and the spectrum requirement in links. Improving the sharing of a light-trail among destinations does not necessarily reduce resource usage. The reasons are the following. On the one hand, a light-trail covering more destinations traverses more links and has a longer transmission distance. It on the other hand uses more spectrum in each link since a longer transmission requires a less spectrum-efficient MS to be used. Thus, a light-trail that covers many nodes could use excessive spectrum. For example, adding a node that increases the distance beyond the maximum distance of the current light-trail MS results in a change to a less spectrum-efficient one. This increases the spectrum usage in all the links, i.e., existing and newly-added ones. Thus, to accommodate a multicast by an optimal set of light-trails, a proper balance should be found between the sharing of a light-trail and spectrum requirement in links.

B. Avoiding Unnecessary Destination Replicas

Another important problem involving the usage of multiple light-trails for a multicast is that a destination may join multiple light-trails causing excessive spectrum usage. Including unnecessary destinations in a light-trail not only requires an occupation of extra link resources but also prolongs the length causing the usage of a lower modulation level and thus a higher spectrum requirement on every trail link. Efficient algorithms are required to avoid the unnecessary destination replicas in multiple light-trails.

To solve the overall network planning problem, in the following we provide MILP formulations and propose an efficient heuristic algorithm that properly solves the above problems achieving close performance to the optimal MILP.

III. MILP Formulation

We model the problem by a MILP formulation where multicast requests are accommodated by light-trails. In the following, we present the details of the formulation.

A. Notations

$\mathbf{R}$ A set of requests.

$\mathbf{T}^r$ A set of transmitters that are available for multicast request r.

Δ A big number.

B. Variables

$\mathbb{F}_{ij}^{rt}$ Integer; denotes the number of signal flows sourcing from transmitter t for multicast request r transmitted via link ij, $ij \in \mathbf{L}$; $0 \leq \mathbb{F}_{ij}^{rt} \leq |\mathbf{D}_r|$.

$\mathbb{G}_d^{rt}$ Binary; equals one if destination d of multicast request r, $d \in \mathbf{D}_r$, drops a copy of the signal flow sourcing from transmitter t; zero, otherwise.

$\mathbb{D}_t^r$ Real; denotes the light-trail distance via transmitter t for multicast request r; $\mathbb{D}_t^r \geq 0$.

$\mathbb{L}_{ij}^{rt}$ Binary; equals one if link ij, $ij \in \mathbf{L}$, is a part of the light-trail using transmitter t for multicast request r; zero, otherwise.

$\mathbb{K}_m^{rt}$ Binary; equals one if MS m, $m \in \mathbf{M}$, is assigned to the light-trail using transmitter t for multicast request r; zero, otherwise.

$\mathbb{U}_t^r$ Binary; equals one if transmitter t is used for the accommodation of multicast request r.

$\mathbb{N}_t^r$ Integer; denotes the number of FSs utilized by the light-trail using transmitter t for multicast request r, $\mathbb{N}_t^r \geq 0$.

$\mathbb{S}_t^r$ Integer; denotes the start index of the FSs utilized by the light-trail using transmitter t for multicast request r, $\mathbb{S}_t^r \geq 1$.

$\mathbb{E}_t^r$ Integer; denotes the end index of the FSs utilized by the light-trail using transmitter t for multicast request r; $\mathbb{E}_t^r \geq 1$.

$\mathbb{X}_{r_2t_2}^{r_1t_1}$ Binary; equals to one if the light-trail using transmitter t_1 for multicast request r_1 and the light-trail using transmitter t_2 for multicast request r_2 share one or more common links.

$\mathbb{O}_{r_2t_2}^{r_1t_1}$ Binary; equals to zero if the start index of the FSs occupied by the light-trail using transmitter t_1 for multicast request r_1 is greater than the end index of the FSs occupied by the light-trail using transmitter t_2 for multicast request r_2, i.e., $\mathbb{E}_{t_2}^{r_2} < \mathbb{S}_{t_1}^{r_1}, \mathbb{U}_{t_1}^{r_1} = \mathbb{U}_{t_2}^{r_2} = 1$ and if the two light-trails share common link(s), i.e., $\mathbb{X}_{r_2t_2}^{r_1t_1} = 1$.

$\mathbb{B}_{ij}^{rt}$ Integer; denotes a number that is greater than or equal to the number of FSs in link (i,j) utilized by the light-trail using transmitter t for multicast request r; $\mathbb{B}_{ij}^{rt} \geq 0$.

$\mathbb{C}$ Integer; denotes a number that is no smaller than the largest end index of the FSs occupied by the light-trails.

C. Objective

$$\textbf{Minimize } \mathbb{C}$$

The objective is to minimize the number of FSs required in fiber links.

D. Constraints

We divide the constraints into five groups, namely, *flow conservation*, *trail construction*, *modulation determination*, *spectrum allocation* and *lower bound*. The first group guarantees the flow conservation for a light-trail. The second group ensures that data flows via a routing trail. The third and fourth groups guarantee an MS and the corresponding spectrum are adaptively assigned to each light-trail, respectively. The last group is the lower bound.

1) Flow Conservation:

$$\sum_{t\in\mathbf{T}^r}\left(\sum_{(s_r,j)\in\mathbf{L}}\mathbb{F}_{s_rj}^{rt}-\sum_{(i,s_r)\in\mathbf{L}}\mathbb{F}_{is_r}^{rt}\right)=|\mathbf{D}_r|,\quad\forall r\in\mathbf{R}\quad(1)$$

$$\sum_{(s_r,j)\in\mathbf{L}}\mathbb{F}_{s_rj}^{rt}-\sum_{(i,s_r)\in\mathbf{L}}\mathbb{F}_{is_r}^{rt}=\sum_{d\in\mathbf{D}_r}\mathbb{G}_d^{rt},\quad\forall r\in\mathbf{R},t\in\mathbf{T}^r\quad(2)$$

$$\sum_{(x,j)\in\mathbf{L}}\mathbb{F}_{xj}^{rt}=\sum_{(i,x)\in\mathbf{L}}\mathbb{F}_{ix}^{rt},\quad\forall r\in\mathbf{R},t\in\mathbf{T}^r,x\in\mathbf{V}\setminus\mathbf{D}_r:x\neq s_r\quad(3)$$

$$\mathbb{G}_x^{rt}+\sum_{(x,j)\in\mathbf{L}}\mathbb{F}_{xj}^{rt}=\sum_{(i,x)\in\mathbf{L}}\mathbb{F}_{ix}^{rt},\quad\forall r\in\mathbf{R},t\in\mathbf{T}^r,x\in\mathbf{D}_r\quad(4)$$

$$\sum_{t\in\mathbf{T}^r}\mathbb{G}_d^{rt}=1,\quad\forall r\in\mathbf{R},d\in\mathbf{D}_r.\quad(5)$$

Constraints (1), (2), (3), (4) and (5) guarantee commodity flow conservation of every light-trail. Constraint (1) ensures that the number of net egress flows via all transmitters at the source equals to the number of destinations that drop the signal flow in each light-trail. Constraint (2) ensures that for each light-trail, the number of net egress flows at the source equals to the number of destinations dropping the flows from the light-trail. Constraint (3) guarantees that for each light-trail, the numbers of ingress and egress flows at an intermediate node are the same. Please note that the intermediate nodes of a light-trail are the network nodes that do not drop the signal from it and a destination could be an intermediate node of some light-trail but drops a signal flow from a different light-trail. Constraint (4) ensures that for every light-trail, the number of net ingress flows at a destination is one if the destination drops a signal flow of the light-trail; zero otherwise. Constraint (5) ensures that every destination node of a multicast request drops a signal flow from one of the light-trails sourcing from transmitters.

2) Trail Construction:

$$|\mathbf{D}_r|\cdot\mathbb{L}_{ij}^{rt}\geq\mathbb{F}_{ij}^{rt},\quad\forall r\in\mathbf{R},t\in\mathbf{T}^r,(i,j)\in\mathbf{L}\quad(6)$$

$$\mathbb{L}_{ij}^{rt}\leq\mathbb{F}_{ij}^{rt},\quad\forall r\in\mathbf{R},t\in\mathbf{T}^r,(i,j)\in\mathbf{L}\quad(7)$$

$$\mathbb{U}_t^r\geq\mathbb{G}_d^{rt},\quad\forall r\in\mathbf{R},t\in\mathbf{T}^r,d\in\mathbf{D}_r\quad(8)$$

$$\mathbb{U}_t^r\leq\sum_{d\in\mathbf{D}_r}\mathbb{G}_d^{rt},\quad\forall r\in\mathbf{R},t\in\mathbf{T}^r\quad(9)$$

$$\sum_{(s_r,j)\in\mathbf{L}}\mathbb{L}_{s_rj}^{rt}-\sum_{(i,s_r)\in\mathbf{L}}\mathbb{L}_{is_r}^{rt}=\mathbb{U}_t^r,\quad\forall r\in\mathbf{R},t\in\mathbf{T}^r\quad(10)$$

$$\sum_{(x,j)\in\mathbf{L}}\mathbb{L}_{xj}^{rt}=\sum_{(i,x)\in\mathbf{L}}\mathbb{L}_{ix}^{rt},\quad\forall r\in\mathbf{R},t\in\mathbf{T}^r,x\in\mathbf{V}\setminus\mathbf{D}_r:x\neq s_r\quad(11)$$

$$\sum_{(d,j)\in\mathbf{L}}\mathbb{L}_{dj}^{rt}\leq\sum_{(i,d)\in\mathbf{L}}\mathbb{L}_{id}^{rt},\quad\forall r\in\mathbf{R},t\in\mathbf{T}^r,d\in\mathbf{D}_r\quad(12)$$

$$\mathbb{U}_t^r+\sum_{d\in\mathbf{D}_r}\sum_{(d,j)\in\mathbf{L}}\mathbb{L}_{dj}^{rt}=\sum_{d\in\mathbf{D}_r}\sum_{(i,d)\in\mathbf{L}}\mathbb{L}_{id}^{rt},\quad\forall r\in\mathbf{R},t\in\mathbf{T}^r.\quad(13)$$

Constraints (6), (7), (8), (9), (10), (11), (12), and (13) guarantee that a routing trail is constructed. Constraints (6) and (7) ensure that a link is a part of a light-trail if the signal flows through it. Constraints (8) and (9) ensure that the transmitter is utilized and the light-trail is active if at least one destination drops a signal flow from it. Here, a light-trail is active it is used to transmit data to some destination. Constraint (10) ensures that the number of egress links at the source minus the number of the ingress links is one if a light-trail is active or utilized, zero otherwise. Constraint (11) guarantees that the numbers of ingress and egress links are equal at a node that is not the source or a destination of a multicast. Constraints (12) and (13) ensure that an active light-trail ends at one of the destinations where the number of ingress links equals to the number of egress links plus one.

3) Modulation Determination:

$$\sum_{m\in\mathbf{M}}\mathbb{K}_m^{rt}=\mathbb{U}_t^r,\quad\forall r\in\mathbf{R},t\in\mathbf{T}^r\quad(14)$$

$$\mathbb{D}_t^r=\sum_{(i,j)\in\mathbf{L}}(\ell_{ij}\cdot\mathbb{L}_{ij}^{rt}),\quad\forall r\in\mathbf{R},t\in\mathbf{T}^r\quad(15)$$

$$\tau_m^r-\mathbb{D}_t^r\geq\Delta\cdot(\mathbb{K}_m^{rt}-1),\quad\forall r\in\mathbf{R},m\in\mathbf{M},t\in\mathbf{T}^r.\quad(16)$$

Constraints (14), (15), and (16) guarantee that an MS is assigned to an active light-trail with a transmission distance shorter than the corresponding transparent reach. Constraint (14) guarantees that an MS is assigned to an active light-trail. Constraint (15) guarantees that the distance of a trail is the sum of the lengths of the trail links. Constraint (16) ensures that the trail distance cannot exceed the transparent reach of the assigned MS.

4) Spectrum Allocation:

$$\mathbb{N}_t^r=\sum_{m\in\mathbf{M}}\mathbb{K}_m^{rt}\cdot(\omega_m^r+g),\quad\forall r\in\mathbf{R},t\in\mathbf{T}^r\quad(17)$$

$$\mathbb{E}_t^r=\mathbb{S}_t^r+\mathbb{N}_t^r-1,\quad\forall r\in\mathbf{R},t\in\mathbf{T}^r\quad(18)$$

$$\mathbb{X}_{r_1t_1}^{r_2t_2}+\mathbb{X}_{r_2t_2}^{r_1t_1}\geq2\cdot\left(\mathbb{L}_{ij}^{r_1t_1}+\mathbb{L}_{ij}^{r_2t_2}-1\right),\quad\forall r_1,r_2\in\mathbf{R},\ t_1\in\mathbf{T}^{r_1},t_2\in\mathbf{T}^{r_2},(i,j)\in\mathbf{L}:r_1\neq r_2\text{ or }t_1\neq t_2\quad(19)$$

$$\mathbb{O}_{r_2t_2}^{r_1t_1}+\mathbb{O}_{r_1t_1}^{r_2t_2}=1,\quad\forall r_1,r_2\in\mathbf{R},t_1\in\mathbf{T}^{r_1},t_2\in\mathbf{T}^{r_2}:r_1\neq r_2\text{ or }t_1\neq t_2\quad(20)$$

$$\mathbb{E}_{t_2}^{r_2}-\mathbb{S}_{t_1}^{r_1}\leq\Delta\cdot\left(\mathbb{O}_{r_2t_2}^{r_1t_1}+3-\mathbb{U}_{t_1}^{r_1}-\mathbb{U}_{t_2}^{r_2}-\mathbb{X}_{r_2t_2}^{r_1t_1}\right)-1,\quad\forall r_1,r_2\in\mathbf{R},t_1\in\mathbf{T}^{r_1},t_2\in\mathbf{T}^{r_2}:r_1\neq r_2\text{ or }t_1\neq t_2.\quad(21)$$

Constraints (17), (18), (19), (20), and (21) guarantee that the requirements of spectrum continuity, spectrum continuity, and spectrum non-overlapping are satisfied. Constraints (17) and (18) ensure the spectrum continuity and the spectrum contiguity. Constraint (17) ensures that a number of FSs corresponding to the MS assigned are allocated to a light-trail. Constraint (18) ensures that the end index is equal to the start index plus the number of the allocated FSs minus one. With the use of these two variables $\mathbb{S}_t^r$ and $\mathbb{E}_t^r$ indicating the start and end FS indices of a connection using transmitter t for multicast r, every connection is assigned with a contiguous block of spectrum ranging from the start FS index to the end FS index, thus the spectrum contiguity constraint is satisfied. Again, we take the advantage of the two index variables assigned at connection level, the spectrum blocks of the links traversed by a connection are the same, thereby the spectrum continuity is guaranteed. Constraints (19), (20), and (21) guarantee that the requirement of spectrum non-overlapping between any two light-trails is satisfied. If two light-trails traverse one or more common links, the start index of the FSs used by one connection is greater than the end index of FSs used by the other. Please note that the two light-trails can be for the same multicast request or different ones.

5) Lower Bound:

$$\mathbb{C} \geq \mathbb{E}_t^r, \quad \forall r \in \mathbf{R}, t \in \mathbf{T}^r \tag{22}$$

$$\mathbb{B}_{ij}^{rt} \geq \mathbb{N}_t^r - \Delta \cdot (1 - \mathbb{L}_{ij}^{rt}), \quad \forall r \in \mathbf{R}, t \in \mathbf{T}^r, (i,j) \in \mathbf{L} \tag{23}$$

$$\mathbb{C} \geq \sum_{r \in \mathbf{R}} \sum_{t \in \mathbf{T}^r} \mathbb{B}_{ij}^{rt}, \quad \forall (i,j) \in \mathbf{L}. \tag{24}$$

Constraint (22) ensures that the number of FSs required should be greater than or equal to the maximum of the end indices of FSs assigned. Constraints (23) and (24) are redundancy constraints for faster solutions. They guarantee that the number of FSs required should be greater than or equal to the total number of FSs in each link allocated to connections.

E. MILP Problem Sizes

We calculate the problem size of the MILP formulation by its dominant numbers of variables and constraints. The dominant number of variables is in the order of $O(|\mathbf{R}| \cdot |\mathbf{L}| \cdot |\bar{\mathbf{D}}| + |\mathbf{R}|^2 \cdot |\bar{\mathbf{T}}|^2)$ while the dominant number of constraints is in the order of $O(|\mathbf{R}|^2 \cdot |\bar{\mathbf{T}}|^2 \cdot |\mathbf{L}|)$. The notations used, namely, $|\mathbf{R}|$, $|\mathbf{L}|$, $|\mathbf{M}|$, $|\bar{\mathbf{D}}|$, and $|\bar{\mathbf{T}}|$, are the numbers of requests, network links, considered MSs, and the average numbers of destinations and transmitters available per request, respectively.

IV. Heuristic Algorithms for Multicast Light-Trails

MILP formulations were presented for the design of multicast light-trail networks. Since solving MILP is computationally prohibitive for real-size networks, in this section we provide efficient heuristic algorithms that are scalable to large instances.

The problem is to minimize the number of FSs required to serve a given set of multicast requests by light-trails in EONs. We consider the adaptive modulation for better spectrum efficiency and propose efficient heuristic algorithms. The idea of the heuristic algorithms is to improve resource sharing by adding as many destinations as possible to light-trails and by deleting unnecessary resource usage for cases of destinations joining multiple light-trails. In the following, we first describe the heuristic algorithms in general where a couple of inter-request sequence strategies are considered for the accommodation of multiple requests. Then, to accommodate each multicast we present an efficient routing algorithm that adds to a given light-trail the destinations with the minimum length increase. Following that, for each multicast we provide and compare three inter-destination sequence strategies for sequentially creating light-trails. The inter-request sequence and the inter-destination sequence form a two-level hierarchy sequence that affects the performance of the heuristic algorithms. Finally, we propose an algorithm to deal with certain cases when a destination join multiple light-trails occupying excessive resources.

A. Accommodation of Multiple Multicast Requests

We introduce a heuristic algorithm that deals with multiple multicast requests and discuss the accommodation sequences that affect the algorithm performance. The algorithm accommodates multicast requests in a greedy way that it does not increase spectrum capacity requirement of each fiber unless requests cannot be accommodated by existing resources. The algorithm details are presented as follows.

1) Initialization: For every SD pair we find the shortest path and obtain the *Best-Effort Modulation Scheme (BEMS)*. Here, the BEMS of a destination from a source is the most spectrum-efficient MS that the connection between the two nodes could possibly use in a given network topology while satisfying the Quality of Transmission (QoT) requirement, e.g., the connection distance is shorter than the transparent reach of the used MS.

2) Requests Ordering: We arrange the requests in a sequence and accommodate them one by one. The accommodation sequence is a two-level sequence. The level-1 sequence is an inter-request one among different requests, under which is the level-2 sequence. The level-2 sequence is unique to multicast and is an inter-destination sequence among different destinations of a single multicast. We investigate three inter-request sequence cases, namely, the Highest Bandwidth First (HBF), the Most Destination First (MDF) and a random one. For the HBF, we arrange a given set of requests in the decreasing order of the number of FSs required. Please note that this FS requirement is entailed by the lowest-level one among the BEMSs of all the SD pairs of a multicast. For the MDF, we arrange the requests in the decreasing order of the number of destinations. For the random case, the requests are shuffled for a random sequence. When a request is to be accommodated for a given level-1 sequence, an inter-destination sequence should be provided for destinations accommodation. We investigate three strategies with regards to the BEMSs of the destinations which is discussed in details in Section IV-C.

3) Requests Accommodation: Given an ordering strategy, we accommodate requests sequentially. We start from a network with no spectrum resources and propose an algorithm to create multiple light-trails for every multicast which will be present later. For a given multicast, we add a new FS to each fiber link if no light-trail could be found under current network condition.

After all the multicast requests are accommodated, we obtain the number of FSs required in each link.

4) Multi-Iteration Process: The accommodation sequence of requests affects the performance of the proposed heuristic algorithm. To achieve a better performance, a multi-iteration process is adopted where we consider multiple random sequences of the requests and select the best one of the results generated by the sequences as the final result. The more sequences we consider, the better performance the algorithm can achieve. To reduce computation time caused by multiple iterations, technologies such as parallel computing can be utilized where each sequence is dealt with by a machine. Such a multi-iteration method could achieve a time requirement close to the single-iteration one [33].

Before presenting the algorithm for accommodating multiple light-trails for each multicast, we provide a routing algorithm for a single light-trail as follows.

Algorithm 1: ADMLI: Add Destinations to a trail with Minimum Length Increase.

Input: $\boldsymbol{G(V,E)}$, a trail $\boldsymbol{T(N,P)}$, $\boldsymbol{N}$ is the sequence of traversed destination nodes and $\boldsymbol{P}$ is the corresponding path sequence (e.g., path $p_{n_i}^{n_{i-1}}$ from n_{i-1} to n_i (n_{-1} is the trail source node)), nodes to be added $\boldsymbol{D}$, trail length limit;
Output: A list of destination nodes added, $\boldsymbol{A}$.

if $|\boldsymbol{N}| = 0$; **then**
  Try to find in $\boldsymbol{G}$ the shortest path from the source to every destination in $\boldsymbol{D}$;
  Record the path that covers the largest set of destinations $\boldsymbol{D_1}, \boldsymbol{D_1} \subseteq \boldsymbol{D}$ and the shortest distance while satisfying the length limit;
  $\boldsymbol{A} \leftarrow \boldsymbol{A} + \boldsymbol{D_1}$; $\boldsymbol{D} \leftarrow \boldsymbol{D} - \boldsymbol{D_1}$;
  Add the nodes and routes to $\boldsymbol{N}$ and $\boldsymbol{P}$, respectively;
end if
while there are new nodes added into $\boldsymbol{A}$; **do**
  Remove all the trail links from $\boldsymbol{G}$ and obtain $\boldsymbol{G'}$;
  for $i = 1 \ldots |\boldsymbol{N}|$; **do**
    Remove the links of paths in $\boldsymbol{P} - \{p_{n_i}^{n_{i-1}}\}$ from $\boldsymbol{G}$ and obtain $\boldsymbol{G'}$ (n_0 is the source node);
    Find in $\boldsymbol{G'}$ the shortest paths $\boldsymbol{P'}$ from n_{i-1} to all the nodes in $\boldsymbol{D}$;
    for $p_d^{n_{i-1}} \in \boldsymbol{P'}$, $d \in \boldsymbol{D}$; **do**
      Remove the links of $p_d^{n_{i-1}}$ from $\boldsymbol{G'}$ and obtain $\boldsymbol{G''}$;
      Find the shortest path $p_{n_i}^d$ from node d to n_i in $\boldsymbol{G''}$;
    end for
    Record i^* and node d^* where $L(p_{d^*}^{n_{i^*-1}}) + L(p_{n_{i^*}}^{d^*}) - L(p_{n_{i^*}}^{n_{i^*-1}})$ is minimized; //minimum length increase
  end for
  Find the solution of adding a node in $\boldsymbol{D}$ after the trail end node with minimum length increase while maintaining the length limit;
  Choose the one of the two solutions (one adding after the trail end node and the other inserting between the trail segment) with the minimum trail length increase within the trail length limit, update the trail information by adding the corresponding node d'' to $\boldsymbol{A}$ and path(s) to $\boldsymbol{P}$ according to the trail directional order;
  $\boldsymbol{D} \leftarrow \boldsymbol{D} - \{d''\}$; $\boldsymbol{A} \leftarrow \boldsymbol{A} + \{d''\}$;
end while
return $\boldsymbol{A}$;

B. Routing Algorithm for a Single Light-Trail

In this subsection, we present a routing algorithm called ADMLI that Adds Destinations with Minimum Length Increase to enable better resource sharing. The pseudocode is presented by Algorithm 1. As we know that different transmission distances entail different MSs that can be used for a connection and thus different spectrum requirements. We assume that the MS and the maximum transmission distance (or the optical reach) are given. To achieve the goal of better sharing, we add destinations to the trail one by one. For every destination, we attempt to insert it between one of the trail segments (from the source to the first downstream destination or between two sequential destinations) and to add it after the trail end node. We add the destination that introduces the minimum increase to the trail length while ensuring that the length of the trail after the addition is within the maximum transmission distance entailed by the used MS. The algorithm terminates when all nodes are added in the trail or when no destinations can be added since the requirement of the trail length limit is not satisfied.

1) Adding Nodes of Different Requirements to a Light-Trail: To efficiently set up a light-trail that covers multiple destinations of a multicast, ways are to minimize resource usage and to maximize resource sharing. We minimize the resource usage by setting the MS of a light-trail as the BEMS of the destination group that we aim to add. We then add the destinations whose BEMSs are the same as the MS of the light-trail as many as possible. The resource usage is minimized since the light-trail provides an amount of spectrum that is the minimum requirement by the destinations. We also improve the resource sharing by firstly adding as many as possible the destinations we aim to add and then allowing the light-trail to be shared by other destinations. To efficiently utilize a given optical light-trail, we group the destinations by their BEMSs and add them as many as possible into the light-trail group by group with Algorithm 1. Destinations of a lower-level modulation group are prioritized over those of a higher-level one. More specifically, destinations whose BEMSs are the same as the light-trail MS are added first. It is very efficient that the destinations join a light-trail with their BEMSs. This is because they use the most-spectrum efficient MS that they can achieve and thus consume spectrum at the minimum requirement. Meanwhile, the efficient MS utilized put a hard limit on the trail length which in return limits the length of the paths to the destinations and thus reduces the spectrum usage. Please note that the destination with a lower BEMS cannot be added to the light-trail since no path can be found from the source that satisfies the QoT requirement as it extends the trail length beyond the limit entailed by the light-trail MS. The BEMS for each destination of a multicast is determined by the highest-level one of the MSs that the signal can use to transmit via the shortest path while meeting the QoT requirement.

A single light-trail does not usually cover all the destinations since the trail length would exceed the limit entailed by the used MS. To support all the destinations of a multicast, we present strategies that produce multiple light-trails in certain sequences as follows.

C. Accommodating a Multicast by Multiple Light-Trails

Based on the trail routing algorithm, a heuristic algorithm is proposed that serves a single multicast request with multiple light-trails. It not only can be used in a static traffic environment when a set of requests is given as we consider in this paper, but also applies to dynamic cases where connections undergo a birth-and-death process.

A multicast may require multiple light-trails to support the communication, especially when it has many destinations. To evaluate the impact of the accommodation sequence of the destinations of a multicast on the algorithm performance, we propose three inter-destination (level-2) sequence strategies with regards to their BEMSs, namely, the Highest-level Modulation scheme First (HMF) and the Lowest-level Modulation scheme First (LMF), and Random Modulation scheme First (RMF). The HMF prioritizes the establishment of light-trails with higher-level MSs (shorter transmission distances) and thus the accommodation of destinations with higher-level BEMSs. Similarly, the LMF prioritizes the setup of light-trails with lower-level MSs and thus the accommodation of destinations with lower-level BEMSs. For the RMF, light-trails are established with random MSs. For the LMF strategy, light-trails are created in a sequence to use the MSs from lower levels to higher levels of the BEMSs of the destinations uncovered where the farther destinations are prioritized. The idea of the LMF is to reduce resource usage by increasing the number of destinations in light-trails as a lower-level MS allows for a longer transmission distance. However, adding the destinations of higher-level BEMSs to a light-trail with a lower-level MS could result in excessive spectrum usage in some links as they use spectrum more than the minimum requirement. To reduce the excessive usage, in the LMF we prioritize adding the farther destinations with a lower-level BEMS over the closer ones with a higher-level BEMS as the former destinations use the spectrum provided by the light-trail more efficiently than the latter. In this way, the LMF improves the resource sharing of a light-trail and also reduce the additional resource consumption. For the HMF, light-trails are created in a sequence to use MSs from higher levels to lower levels. The idea of the HMF is to allocate spectrum at their minimum requirements for the destinations and to share a light-trail only among the destinations whose BEMSs are the same as the MS of the light-trail to avoid the excessive spectrum usage in the LMF. In the HMF, the destinations of a lower-level BEMS can only be accommodated after those of a higher-level one. Thus, a light-trail cannot be shared to destinations with a different BEMS. This could result in excessive usage of transmitters. Meanwhile, for the spectrum consumption, there exists a contradiction in the HMF strategy. It is spectrum efficient that the destinations consume a small amount of spectrum on each link traversed. However, limiting the resource sharing only among the destination of the same BEMS could result in excessive spectrum usage. The third one is the RMF which lies inbetween the HMF and the LMF.

For a given MS, we calculate for a multicast the number of FSs required by Table I. We consider the *first-fit* spectrum allocation by checking a spectrum window from the first FS. Here, a spectrum window is a block of spectrum containing a number of contiguous FSs. We obtain a spectrum window plane [23] that is a network topology derived by removing from the original topology the links the spectrum window of which is not available. A spectrum window is considered available only when all its FSs are available, otherwise it is unavailable. The benefit of using the spectrum window plane is that the three spectrum assignment constraints, namely, spectrum continuity, contiguity and non-overlapping are satisfied. As long as a routing trail is successfully found, it can be accommodated with the FSs of the spectrum window. We input the network topology and the destinations whose BEMSs are the same as the given MS to Algorithm 1 for setting up a light-trail. The destinations of a higher-level BEMS are attempted for being adding to the light-trail after destinations of a lower-level BEMS cannot be added any more. When no destinations can be added to the light-trail, we establish a new light-trail. We try every spectrum window to find a light-trail for the destinations. If no light-trail can be found under current network condition, we try a lower-level MS which relaxes the limit of the trail length. If there are destinations that still cannot be added to light-trails, we add FSs to each of the fiber links as we mentioned in Section IV-A.

The complexity of Algorithm 1 is $O((|\mathbf{A}| + |\mathbf{N}|)|\mathbf{A}||\mathbf{D}||\mathbf{S}|)$, where $|\mathbf{A}|$ is the number of destination nodes that are added in the light-trail, $|\mathbf{N}|$ is the number of destinations in the trail before running the algorithm, $|\mathbf{D}|$ is the number of destinations to be added, and $|\mathbf{S}|$ is the complexity of the shortest path algorithm used. Then the complexity of the algorithm that accommodates a single multicast request is $O(F|\mathbf{M}|^2|\mathbf{D}|^3|\mathbf{S}|)$ and thus $O(F|\mathbf{M}|^2|\mathbf{D}|^3(|\mathbf{V}| + |\mathbf{L}|)\log|\mathbf{V}|)$ when Dijkstra's algorithm is used. Here, $|\mathbf{V}|$, $|\mathbf{L}|$, $|\mathbf{M}|$, and F are the numbers of nodes, links, MSs considered and FSs in the network links, respectively.

D. Algorithm for Deleting Certain Duplicated Nodes

With the help of the above-mentioned three strategies, a multicast communications can be supported by establishing multiple light-trails. To accommodate a multicast request, a spectrum-efficient solution usually entails multiple light-trails. For heuristic algorithms, these light-trails are created one by one. Each time a new light-trail is created, it may cover destinations that have been covered by previous light-trails. The occurrence of a node joining multiple light-trails could result in resource waste. In this case, destinations involved in multiple light-trails could be deleted in certain light-trails to reduce resource usage. If a destination is covered in multiple light-trails, the replicas included as the light-trail end nodes can be deleted to save resources as long as we guarantee that the destination is covered by at least one light-trail. For the heuristic algorithms proposed

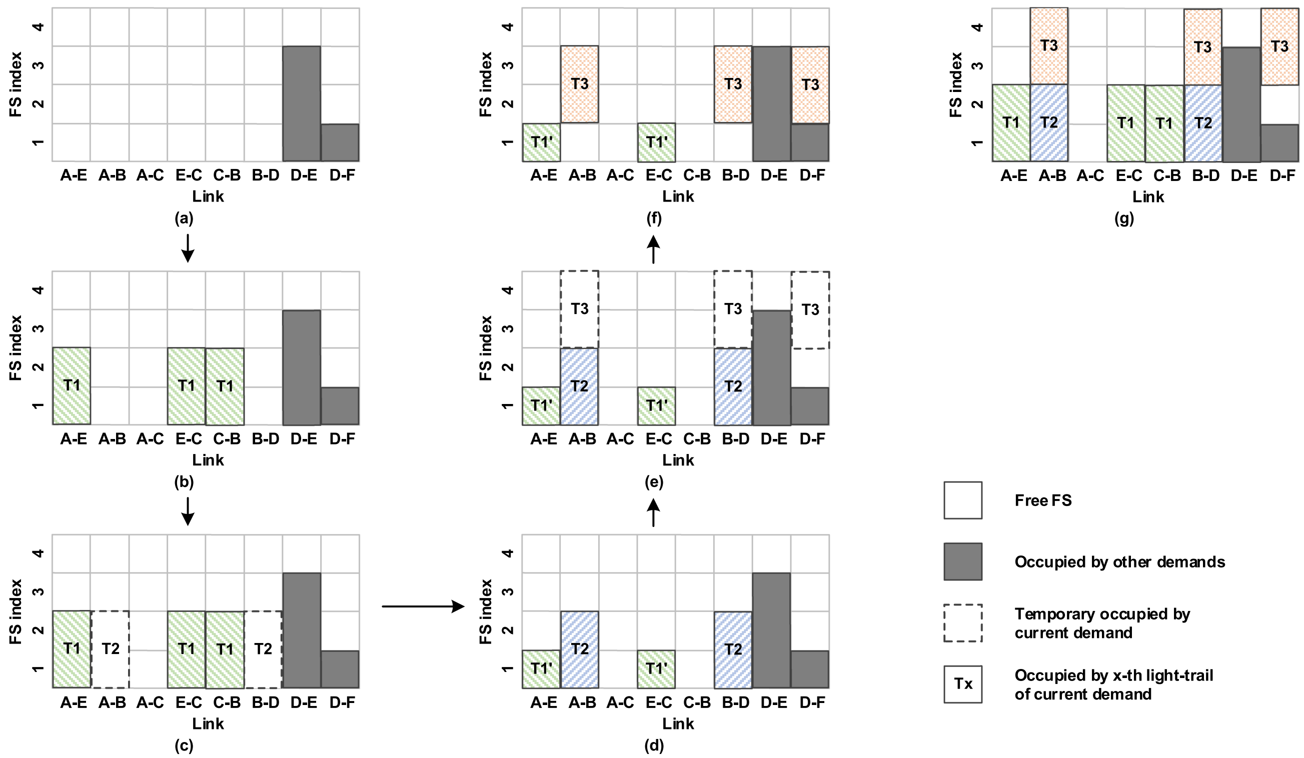

Fig. 2. An example of the process of deleting destination replicas in multiple light-trails and filling spectrum voids: (a) network utilization, (b-f) the process, (f) the process result, (g) no deletion of the replicas.

above for finding light-trails, including a destination again in a new light-trail is unintentional if it has already been covered by a previous one. Here we propose an algorithm that deals with the cases when a destination is the end node of a light-trail and it is also covered by other light-trails. We revise the light-trails to ensure the end node of a light-trail is a destination but is not covered in other light-trails. We guarantee that a destination exists in at least one light-trail.

The algorithm for node deletion is provided as follows.

- *Step 1:* Given a new light-trail covering one or more destinations, for each of the previous light-trails, we perform a Maximum Tail Deletion (MTD). In the MTD, the node is repeatedly deleted from the trail end and the related links until the end node (or tail) is a destination but is not covered by other light-trails (or the new light-trail in this case).
- *Step 2:* We consider the deletion again for all the light-trails except the new one. This is because after the deletion in Step 1, the end nodes of the previous light-trails change and may again be covered in multiple light-trails. For each of the existing light-trails, we check if the trail end destination node exists in the other light-trails and calculate the spectrum savings assume that the MTD is performed. We perform the deletion for the light-trail that gives the maximum spectrum savings. Repeat this step until deletion is performed for all the light-trails or until the spectrum savings is none.

If deletion is performed, the trail length should be shortened. If the shortened trail length allows for a more spectrum-efficient MS, we use the highest-level MS that satisfies the transmission distance requirement and change the spectrum usage accordingly by freeing some high-index FSs for the light-trail for resource savings. Please note that for some cases, an entire light-trail is deleted, and the transmitter and spectrum resources should be released or freed.

1) Filling Spectrum Voids: Following the operation of freeing FSs mentioned above, spectrum voids are created making the spectrum more fragmented. To provide a large number of contiguously free FSs for better accommodation of future requests, we iteratively attempt to drag light-trails to fill the newly freed spectrum if they use the FSs right after the spectrum.

Here we provide an example of the algorithm showing the procedure of deleting certain destination replicas and filling spectrum voids as presented in Fig. 2. Given a part of network resource utilization as shown in Fig. 2(a), assume that a multicast request $< A; \{B, C, D, E, F\}; 100$ Gbps$>$ is to be accommodated. We first find a light-trail T1: A-E-C-B as shown in Fig. 2(b). Then we find the second light-trail T2: A-B-D as shown in Fig. 2(c). This new light-trail covers a destination, i.e., node B, which is the end node of the previous trail. Thus, we perform the MTD for the previous light-trail. We delete node B and the associated link C-B from light-trail T1, and failed to delete the new trail end node C since it is not covered by any other light-trail. Here, the revised trail T1' is A-E-C. As the trail is shorter, the light-trail may use a more spectrum-efficient MS and thus less spectrum as shown in Fig. 2(d). After that, another new light-trail aiming for adding destination F is found as T3: A-B-D-F as shown in Fig. 2(e). We then perform MTD for light-trails T1' and T2. There is no destination of T1' covered

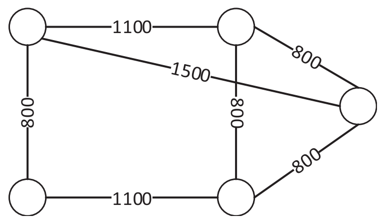

Fig. 3. A five-node seven-span (N5S7) network.

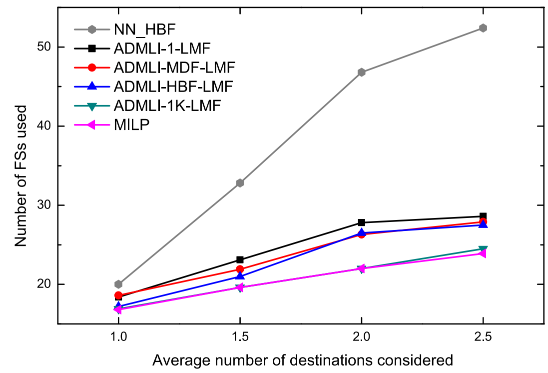

Fig. 4. Spectrum usage in the N5S7 network.

by T3, thus no nodes can be deleted for T1'. Light-trail T2 is entirely deleted as T2' since all its destinations, namely, B and D, are covered by T3. We then check T1' and T2' again for possible deletion. For T1', the trail end node C does not exist in any other light-trails, i.e., it is neither in T2' nor in T3, thus no deletion is performed. And no deletion can be performed for T2' since it has no destination covered. After the links were deleted from the light-trails and the associated FSs are freed, the connections that use the FS right after them can be dragged to fill the void. We drag the T3 to a lower FS index if the spectrum allocation constraints are satisfied. We repeatedly perform the drag for the new spectrum voids created by the previous drag until no drag can be performed. The final result of the algorithm is shown in Fig. 2(f). It requires two light-trails with a total of eight FSs and two transmitters. We also provide the result if no such deletion of certain destination replicas is performed as shown in Fig. 2(g). It requires three light-trails with a total of 16 FSs and three transmitters. In this example, deleting certain destination replicas in multiple light-trails and filling the spectrum voids help save one transmitter and eight FSs.

V. Numerical Results

In this section, we present the test conditions and the performance comparison of the proposed methods.

A. Test Conditions

To evaluate the performance of the proposed methods, we consider the following conditions. Since MILP is computationally prohibitive for large instances, we consider a five-node seven-span (N5S7) network as shown in Fig. 3. The numbers on the link are the lengths in kilometers. In addition to the N5S7 network, the 11-node COST239 [34] and the 24-node USNET [35] networks are considered additionally for the proposed algorithms. Each FS occupies a frequency bandwidth of 12.5 GHz. We consider four MSs, namely, BPSK, QPSK, 8QAM, and 16QAM as shown in Table I. Without loss of generality, multicast requests are generated randomly. In order for MILP algorithms, we consider small instances. For the N5S7 network instance, we consider a set of six multicast requests. For the COST239 and USNET networks, we consider a set of 50 multicast demands. The bit-rate requirements follow a uniform distribution in the range between 100 Gbps and 400 Gbps. For each set of demands, we run the algorithms to obtain a result. The number of destinations of a multicast request is generated randomly between one and a upper bound. For the case of multi-iteration process, we consider up to 10,000 iterations, and select the best result value as the final result. We name the algorithm by the algorithm name, inter-request sequence, light-trail MS sequence, and duplicated node deletion strategies. For example, algorithm "ADMLI-1-LMF" stands for an algorithm based on ADMLI considering the strategies of a single random sequence and the lowest-level modulation scheme first among multiple destinations of each request and deletion of duplicated nodes (default). In particular, "MILP" stands for the optimal algorithm by a Gurobi optimization software [36]. For a heuristic performance comparison, a Nearest Neighbor (NN) algorithm with the ordering strategy of the Highest Bandwidth First called "NN_HBF" is considered as a benchmark. The NN-HBF searches the original network topology for a routing trail that iteratively visits a nearest destination until all the destinations are visited or until it reaches the length limit of the lowest MS. The first fit strategy is utilized for the spectrum allocation. It repeats the above process for the usage of multiple light-trails if necessary.

B. Performance Comparison for the N5S7 Network

We compare the performances of the heuristics NN_HBF, ADMLI-1-LMF, ADMLI-MDF-LMF, ADMLI-HBF-LMF, and ADMLI-1K-LMF with the optimal MILP algorithm in the N5S7 network. Fig. 4 presents the performance of spectrum usage for different average numbers of destinations considered for multicast requests. For all the six algorithms, the required spectrum increases with more destinations. Compared with the MILP optimum, NN_HBF, ADMLI-1-LMF, ADMLI-MDF-LMF, ADMLI-HBF-LMF, and ADMLI-1K-LMF consume on average 79.6%, 18.3%, 14.7%, 11.2%, and 0.8% more spectrum consumption, respectively. Specifically, the percentage of additional spectrum required by the NN_HBF compared with the optimum raises with the increase of average number of destinations, reaching 119% for the case of averagely two and a half destinations. The reason is that the algorithms proposed in this paper achieve a better balance between the light-trail sharing and the MS usage to reduce the spectrum requirement while the NN_HBF tends to maximize sharing that causes many destinations close to the source to use much less efficient MSs and thus excessive spectrum due to the extended trail length.

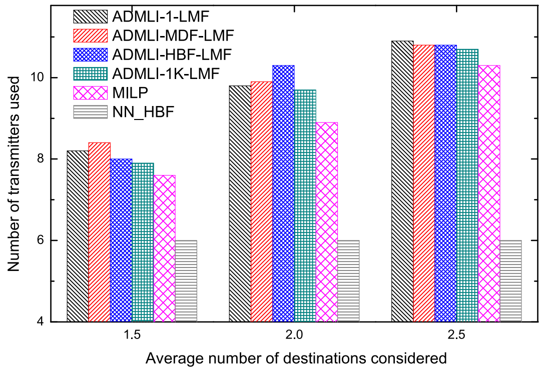

Fig. 5. Transmitter usage in the N5S7 network.

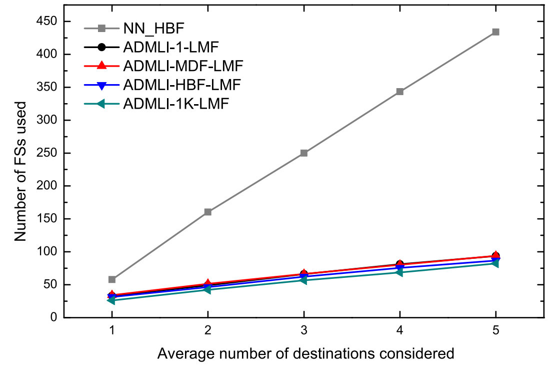

Fig. 6. Spectrum usage in the COST239 network.

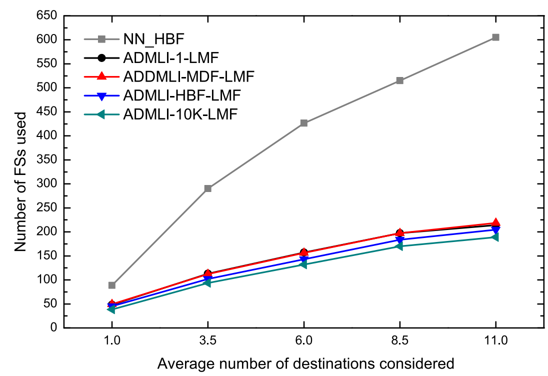

Fig. 7. Spectrum usage in the USNET network.

The algorithms proposed in this paper set up more light-trails but with higher spectrum-efficiency MSs to reduce spectrum usage while the NN_HBF uses fewer modulation-inefficient light-trails causing very high spectrum usage because the latter light-trails not only traverse more links and but also occupy more spectrum in each of the links. Even for the unicast case when the destination count is one, the NN_HBF require 18% more spectrum when compared with the ADMLI-1K-LMF. This is because the NN_HBF based on fixed routing does not utilize the knowledge of spectrum resources in the network in searching routing trails, while the algorithms proposed in this paper do.

Among the heuristic algorithms, the one with 1,000 random shuffled request sequences, i.e., ADMLI-1K-LMF, performs the best and closely to the optimum. This also indicates that the LMF strategy is very effective to save spectrum resources. Compared with a single shuffled sequence, a thousand shuffled ones provide significant performance improvement, i.e., 14.6%. Meanwhile, for the single demand sequence case, the ADMLI-HBF-LMF performs the best.

We also compare the transmitter usage as shown in Fig. 5. The average transmitter usages of the heuristic algorithms proposed in this paper are up to 9% more than that of the MILP, while the NN_HBF uses only one transmitter for every multicast request. Thus, the NN_HBF maximizes the light-trail sharing among the destinations but requires more spectrum. Please note that for MILP algorithm, the transmitter usage is minimized as the secondary objective where the primary one is to minimize the spectrum requirement. The unicast case is excluded from the figure as all the algorithms require the same number of transmitters.

C. Performance Comparisons for the COST239 and USNET Networks

We also consider two larger networks. Due to the computation prohibitiveness, the optimal MILP algorithm is not compared here. In the following, we first evaluate the ordering strategies among multiple multicast requests, namely, HBF, MDF, Random, multi-iteration process. Then, we assess the benefit of deletion of duplicated nodes in multiple light-trails. We also investigate the ordering strategies of multiple destinations of a single multicast, namely, LMF, HMF and RMF. We also examine the performance of the multi-iteration process.

We compare different ordering strategies in terms of spectrum consumption among the heuristic algorithms as shown in Figs. 6 and 7 for the COST239 and USNET networks, respectively. For the both networks, algorithm ADMLI-10K-LMF performs the best, the ADMLI-HBF-LMF is the second, and the two algorithms, namely, ADMLI-1-LMF, and ADMLI-MDF-LMF, follow. The observation for the COST239 and USNET cases is similar to that for the six-node network. The NN_HBF performs the worst, and it is much worse than all the algorithms proposed in this paper. For the COST239 network, compared with the ADMLI-10K-LMF, the algorithms NN_HBF, ADMLI-HBF-LMF, ADMLI-1-LMF, and ADMLI-MDF-LMF require on average 326%, 15%, 22%, and 24% more spectrum, respectively. For the USNET, the numbers are 197%, 10%, 19%, and 20%. In particular for the unicast case, the NN_HBF require around 130% more spectrum than the ADMLI-10K-LMF for both networks. We observe that the performance gain of the algorithms proposed in this paper over the NN_HBF is larger for the COST239 and the USNET than for the N5S7. There are three main reasons as follows. Multiple light-trails are used for the two larger networks while a single light-trail is used in the N5S7 network and the deletion of the certain destination replicas that saves spectrum resources is not performed for the NN_HBF. Also, the NN_HBF is basically a fixed routing algorithm for a

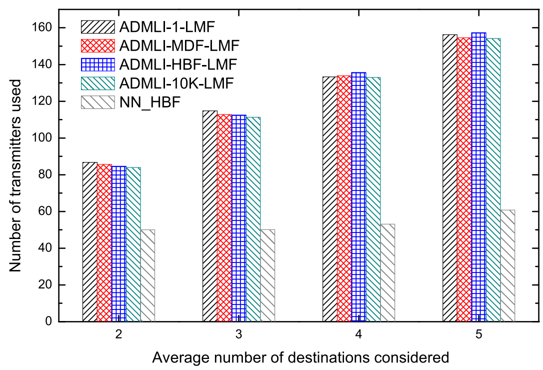

Fig. 8. Transmitter usage in the COST239 network.

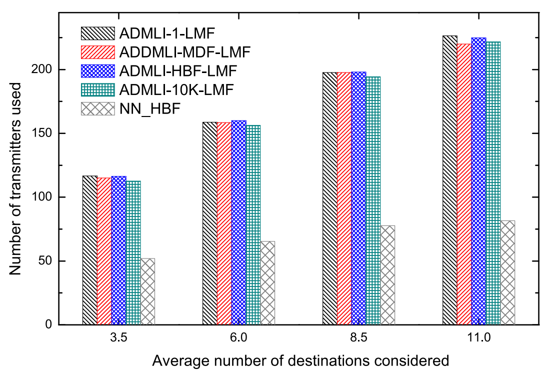

Fig. 9. Transmitter usage in the USNET network.

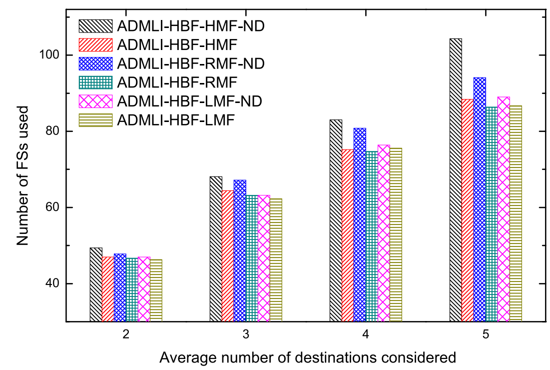

Fig. 10. Spectrum usage in the COST239 network.

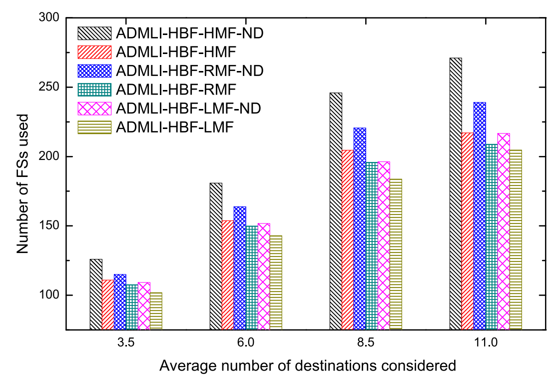

Fig. 11. Spectrum usage in the USNET network.

given multicast while the proposed algorithms search routing trails with the knowledge of network resource usage and have many more choices of potential trails in a larger network. In addition, almost all the destinations in the COST239 network can be reached within QPSK, but the NN_HBF uses less spectrum-efficient MSs as it adds as many destinations as possible to a light-trail. For the both networks, ten thousand (10 K) shuffles provide a significant performance improvement over a single shuffle (random). Another observation is that the HBF strategy helps reduce spectrum consumption when compared with the random and the MDF strategies. Although the MDF strategy does not present desired improvement over the random one, the strategy of prioritizing the demand with more destinations does increase sharing since the transmitter usage is lower than ADMLI-1-LMF as shown in Figs. 8 and 9. The reason is that the MDF strategy has a higher probability of sharing a low-modulation-level light-trail with more destinations that could use some high-level MSs where these destinations are served with spectrum resources more than the requirements causing a higher spectrum consumption. Please note that as we mentioned earlier, improving sharing does not necessarily reduce resource consumption when adaptive modulation is considered. This is because when many destinations with higher-level BEMSs join a lower-modulation-level light-trail, the destinations consume excessive spectral bandwidth.

1) Benefit of Deleting Certain Destination Replicas: To investigate the benefit of deleting certain duplicated destinations, we compare two cases, one with such deletion and the other with none. Algorithms that adopt the case of No Deletion have a postfix of "ND," while the others have no such postfix. We also assess the impact of modulation strategies by considering the three modulation strategies, namely, HMF, LMF, and RMF. The results are presented in Figs. 10 and 11. The percentage of spectrum savings by deleting the duplicated destinations are the highest for the HMF, and the lowest for the LMF. For the COST239 network, deleting duplicated destinations reduces the spectrum consumption of the algorithms with the HMF, the RMF, and the LMF strategies by on average 8.7%, 6.0%, and 1.6%, respectively. The numbers are around 16%, 10%, and 6% for the USNET network. For both of the networks, we also observe that the saving percentages grow as the number of multicast destinations increases except for the LMF strategy whose saving percentage line fluctuates. This can be explained by that the LMF achieves good performance leaving little space for improvements. Specifically, for ADMLI-HBF-HMF when the average number of destinations is 11, the saving percentage is about 20% for the USNET. Apart from the spectrum consumption aspect, we also compare the transmitter usage as shown in Figs. 12 and 13. For the COST239 network, the deletion of duplicated destinations saves on average 18, 12, and 3.2 percent of the transmitters for the algorithms with the HMF, the RMF, and the LMF strategies, respectively. For the USNET networks, the percentages of the saved transmitters

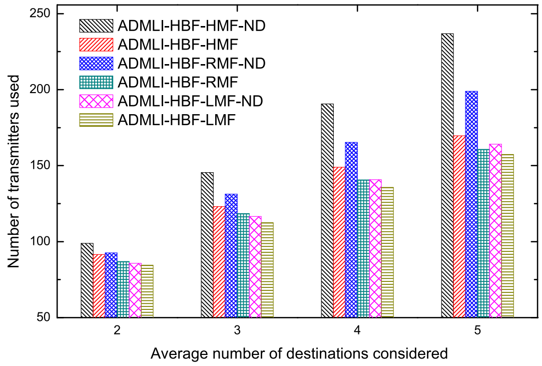

Fig. 12. Transmitter usage in the COST239 network.

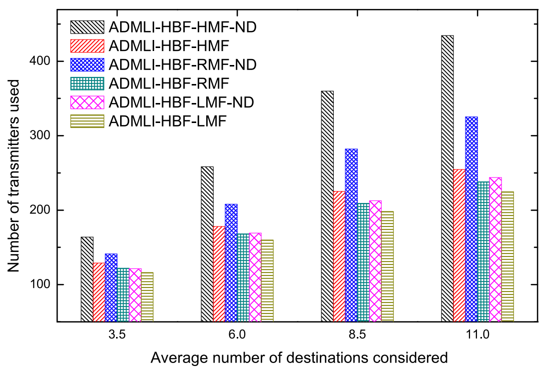

Fig. 13. Transmitter usage in the USNET network.

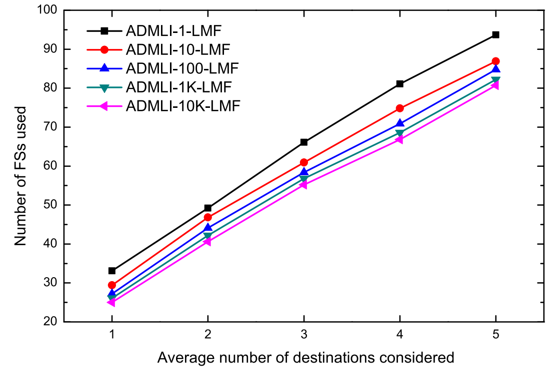

Fig. 14. Spectrum usage in the COST239 network.

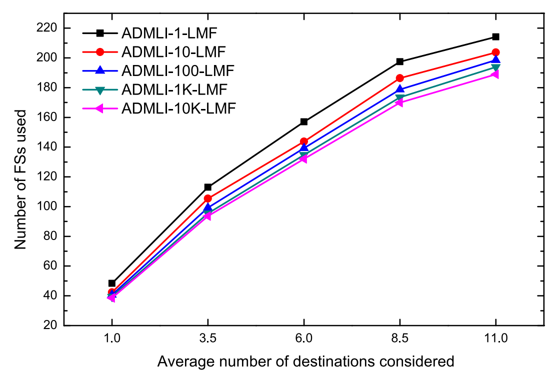

Fig. 15. Spectrum usage in the USNET network.

are around 33%, 21%, and 6%. And similar observations are made to the spectrum consumption. With the increase of the destination count, the percentage of transmitter savings grows for all the three modulation strategies. In particular, the highest transmitter saving percentage of all the considered cases is over 41% for the USNET network case considering the HMF strategy when the average number of destinations is 11. The strategy of deleting duplicated destinations in multiple connections reduce the spectrum and the transmitter usages.

2) Impact of Destinations Ordering: For the assessment of ordering among different destinations of each multicast, we compare the performance of spectrum consumption as shown in Figs. 10 and 11. For the COST239 network, compared with the LMF strategy the HMF and the RMF consume on average 9.7% and 4.9% more spectrum, respectively, for the case with no deletion of duplicated destinations, while for the other case, the percentages are small. This also verifies that deletion of the duplicated destinations helps reduce resource usage. For the transmitter usage, the HMF and the RMF require over 44% and 21% more transmitters than the LMF, respectively, for the case of no deletion of duplicated destinations while for the other case, the values are 8.8% and 3.5%. Similar observations are made for the USNET network except that the percentages are higher. Compared with the LMF strategy, the HMF and RMF require on average 21% and 9% more spectrum, respectively, for the case with no deletion of duplicated destinations, while for the other case, the percentages are 8.5% and 4.8%. For the transmitter usage, the HMF and the RMF require about 59% and 26% more transmitters than the LMF, respectively, for the case of no deletion of duplicated destinations while for the other case, the values are 12% and 5.5%. In particular, for the US-NET network considering no deletion of duplicated destinations, ADMLI-HBF-HMF-ND requires over 25% more spectrum and uses 78% more transmitters than ADMLI-HBF-LMF-ND for the case of 11 destinations on average per multicast. Overall, the LMF strategy outperforms the other two in reducing spectrum requirement and transmitter usage.

3) Impact of Iteration Times: We also investigate the impact of iteration times on the performance of the algorithm ADMLI-x-LMF with multi-iteration process as shown in Figs. 14 and 15, where x stands for the number of iterations considered. We consider five cases, namely, one, ten, a hundred, a thousand, and ten thousand iterations. With the increase of number of iterations, the spectrum usage drops. For the COST239 network case as shown in Fig. 14, compared with the case of a single iteration, the cases of 10, 100, 1 K, and 10 K saves about 7.8, 12.4, 15.4, and 18.0 percent of spectrum, respectively. A small number of iterations could provide a sufficiently good performance improvement. Specifically, compared to a single random iteration, a hundred iterations help save 12.4% spectrum resources. Further, more spectrum savings can be achieved by increasing the number of iterations. For the USNET as shown in Fig. 15, the percentages are 8.8%, 12.0%, 14.4%, and 16.5%.

TABLE II
RUNNING TIMES OF THE HEURISTIC ALGORITHMS FOR THE USNET NETWORK (IN MILLISECONDS)

Algorithms \ Average Number of Destinations	1	3.5	6	8.5	11
NN_HBF	10.2	16.6	19.6	26.7	32.8
ADMLI-HBF-LMF	36.2	118.7	184.7	292.3	383.4
ADMLI-MDF-LMF	31.1	98.5	164.9	260.9	335.3
ADMLI-1-HBF	21.7	89.3	163	258.3	330.6
ADMLI-10-HBF	139.8	777.3	1,437.9	2,394.5	3,073.3
ADMLI-100-HBF	1,099.3	6,940.4	13,028.4	21,470.6	28,355.7
ADMLI-1K-HBF	9,807.1	63,301.5	124,072.8	204,245.8	272,286.8
ADMLI-10K-HBF	92,175.9	591,434.5	1,181,901.8	1,952,425	2,606,518.9

For transmitter usage, the savings brought by more iterations are small, i.e., less than 3%, for the both networks.

4) Running Times: We also provide the running times of the heuristic algorithms for the USNET network as shown in Table II. The platform is Eclipse running on a PC with an Intel i9-9900 CPU @3.1 GHz and 16-GB RAM. The NN_HBF presents a shorter running time than the others. This is due to the fact that the NN_HBF calculates the routing trails in the original network topology. It does not encounter cases that no routing trail is found while the heuristic algorithms proposed in this paper do and to find a routing trail the algorithms were attempted for multiple times. We also observe that with the increase of the destination count, the running time are longer for all the algorithms. And for the proposed algorithms considering a single demand sequence, the running times are roughly the same. For those considering multiple sequences, the running times are linear to the number of considered sequences.

VI. CONCLUSION

In this paper, we solved a multicast routing and spectrum allocation problem in elastic optical networks by the light-trail technology. We considered multiple light-trails for every single multicast. MILP formulations were presented for small problem instances. To scale for large ones, heuristic algorithms were proposed. We proposed a routing algorithm called ADMLI that adds destinations to a light-trail with minimum length increase to improve resource sharing. We evaluated the heuristics by comparing them with the optimal MILP and with a benchmark heuristic algorithm based on the nearest neighbor. Test results show that the heuristics proposed can achieve performances close to the optimum and significantly outperforms the benchmark. We also discussed the strategies used in the heuristic algorithms. We investigated the impact of the order of requests accommodation on the algorithm performance. We considered two levels of ordering, one among different requests, the other among different destinations of a single multicast. Multiple strategies were considered and comparisons were made for a range of cases. Results demonstrate that the highest bandwidth first (HBF) and the lowest-level modulation scheme first (LMF) achieve the best performances among the strategies considered for the ordering of multiple requests and for the ordering of multiple destinations of a multicast, respectively. We also considered deleting certain duplicated destinations involving multiple light-trails. Such deletion saves significant spectrum and transmitter resources, up to 20% spectrum and 41% transmitters among the considered cases. Moreover, the multi-iteration process was considered that the best one of the results produced by multiple orders of the requests was selected as the final result. Generally, a better performance can be achieved by increasing the number of iterations. And the first few hundred iterations provide a satisfactory performance for the considered cases.

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